Dynamical Λ models of structure formation

Kimberly Coble
Department of Astronomy and Astrophysics, The University of Chicago, Chicago, Illinois 60637
and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500

Scott Dodelson
NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500

Joshua A. Frieman
Department of Astronomy and Astrophysics, The University of Chicago, Chicago, Illinois 60637
and NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500
(Received 23 August 1996)

Models of structure formation with a cosmological constant Λ provide a good fit to the observed power spectrum of galaxy clustering. However, they suffer from several problems. Theoretically, it is difficult to understand why the cosmological constant is so small in Planck units. Observationally, while the power spectra of cold dark matter plus Λ models have approximately the right shape, the COBE-normalized amplitude for a scale-invariant spectrum is too high, requiring galaxies to be antibiased relative to the mass distribution. Attempts to address the first problem have led to models in which a dynamical field supplies the vacuum energy, which is thereby determined by fundamental physics scales. We explore the implications of such dynamical Λ models for the formation of large-scale structure. We find that there are dynamical models for which the amplitude of the COBE-normalized spectrum matches the observations. We also calculate the cosmic microwave background anisotropies in these models and show that the angular power spectra are distinguishable from those of standard cosmological constant models. [S0556-2821(97)03704-1]

PACS number(s): 98.80.Bp, 98.80.Cq

I. INTRODUCTION

The cosmological constant has had a long and tortured history since Einstein first introduced it in 1917 in order to obtain static cosmological solutions [1]. Under observational duress, it has been periodically invoked by cosmologists and then quickly forgotten when the particular crisis passed. Historical examples include the first ''age crisis'' arising from Hubble's large value for the expansion rate, the apparent clustering of quasistellar objects (QSO's) at a specific redshift, and early cosmological tests which indicated a negative deceleration parameter.

Recently, a cosmological model with substantial vacuum energy, a relic cosmological constant Λ, has again come into vogue for several reasons [2]. First, dynamical estimates of the mass density on the scales of galaxy clusters, the largest gravitationally bound systems, suggest that $\Omega_m=0.2\pm0.1$ for the matter (m) which clusters gravitationally (where Ω is the present ratio of the mean mass density of the Universe to the critical Einstein–de Sitter density, $\Omega=8\pi G\rho/3H^2$) [3]. However, if a sufficiently long epoch of inflation took place during the early Universe, the present spatial curvature should be negligibly small, $\Omega_{\rm tot}=1$. A cosmological constant, with effective density parameter $\Omega_\Lambda\equiv\Lambda/3H_0^2=1-\Omega_m$, is one way to resolve the discrepancy between Ω_m and $\Omega_{\rm tot}$.

The second motivation for the revival of the cosmological constant is the ''age crisis'' for spatially flat $\Omega_m=1$ models. Current estimates of the Hubble expansion parameter from a variety of methods appear to be converging to $H_0\simeq70\pm10$ km/sec Mpc^{-1}, while estimates of the age of the Universe from globular clusters are holding at $t_{gc}\simeq13$–15 Gyr or more. Thus, observations imply a value for the ''expansion age'' $H_0t_0=(H_0/70\text{ km/sec Mpc}^{-1})(t_0/14\text{ Gyr})\simeq1.0\pm0.2$. This is higher than that for the standard Einstein–de Sitter model with $\Omega_m=1$, for which $H_0t_0=2/3$. On the other hand, for models with a cosmological constant, H_0t_0 can be larger. For example, for $\Omega_\Lambda=0.6=1-\Omega_m$, $H_0t_0=0.89$.

Third, cold dark matter (CDM) models for large-scale structure formation, which include a cosmological constant (hereafter, ΛCDM), provide a better fit to the shape of the observed power spectrum of galaxy clustering than does the ''standard'' $\Omega_m=1$ CDM model [4]. Figure 1 shows the inferred galaxy power spectrum today (based on a recent compilation [5]), compared with the matter power spectra predicted by standard CDM and a ΛCDM model with $\Omega_\Lambda=0.6$. In both cases, the Hubble parameter has been fixed to $h\equiv H_0/(100\text{ km/sec Mpc}^{-1})=0.7$ and the baryon density to $\Omega_B=0.0255$, in the center of the range allowed by primordial nucleosynthesis. Linear perturbation theory has been used to calculate the model power spectra $P(k)$, defined by $\langle\delta(\mathbf{k})\delta^*(\mathbf{k}')\rangle=(2\pi)^3P(k)\delta_D(\mathbf{k}-\mathbf{k}')$, where $\delta(\mathbf{k})$ is the Fourier transform of the spatial matter density fluctuation field and δ_D is the Dirac δ function. Here and throughout, we have taken the primordial power spectrum to be exactly scale invariant, $P_{\rm primordial}(k)\propto k^n$ with $n=1$. Standard CDM clearly gives a poor fit to the shape of the observed spectrum [6], while the ΛCDM model gives a good fit to the shape of the observed spectrum. The amplitudes of the model spectra in Fig. 1 have been fixed at large scales by observations of cosmic microwave background (CMB) anisotropies by the Cosmic Background Explorer (COBE) satellite [7,8].

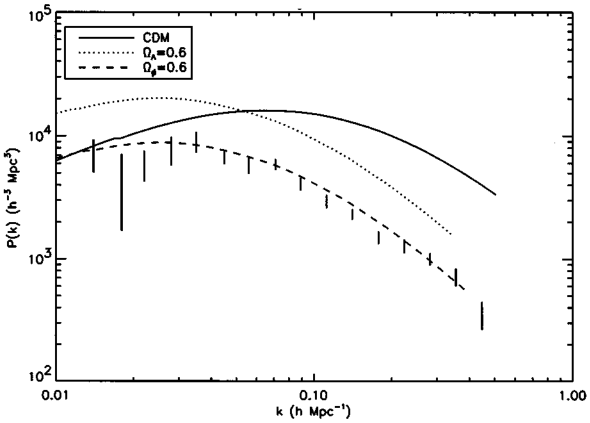

FIG. 1. COBE-normalized power spectra for standard CDM, ΛCDM with $\Omega_\Lambda = 0.6$, and scalar field $\Omega_\phi = 0.6$ models. In all models $h = 0.7$, $\Omega_B = 0.0255$, and $n = 1$. The data points are based on a recent compilation of galaxy clustering data by Peacock and Dodds [5].

Despite these successes, cosmological constant models face several difficulties of their own. On aesthetic grounds, it is difficult to understand why the vacuum energy density of the Universe, $\rho_\Lambda \equiv \Lambda/8\pi G$, should be of order $(10^{-3}\ \mathrm{eV})^4$, as it must be to have a cosmological impact $(\Omega_\Lambda \sim 1)$. On dimensional grounds, one would expect it to be many orders of magnitude larger, of order m_{Pl}^4 or perhaps m_{SUSY}^4. Since this is not the case, we might plausibly assume that some physical mechanism sets the ultimate vacuum energy to zero. Why then is it not zero today?

The cosmological constant is also increasingly observationally challenged. Preliminary results from ongoing searches [9] for distant Type Ia supernovae indicate that $\Omega_\Lambda < 0.47$ (at 95% C.L.) for spatially flat Λ models. Furthermore, in Λ models a larger fraction of distant QSO's would be gravitationally lensed than that in a $\Lambda = 0$ universe; surveys for lensed QSO's have been used to infer the bound $\Omega_\Lambda \lesssim 0.7$ [10].

In this paper, we focus on a third problem of cosmological constant models, the amplitude of the power spectrum of galaxy clustering. The shape of the ΛCDM power spectrum in Fig. 1 matches the galaxy power spectrum; however, the COBE-normalized amplitude is too high. Indeed, a number of analyses have found that this problem persists on all scales.

On the largest scales ($k < 0.1\ h\ \mathrm{Mpc}^{-1}$), linear theory should be adequate, and Fig. 1 suggests that the amplitude is too high by at least a factor of 2.

On intermediate scales, we can quantify the amplitude through the dispersion of the density field smoothed over top-hat spheres of radius $R = 8\ h^{-1}\ \mathrm{Mpc}^{-1}$, denoted σ_8, where $\sigma^2(R) = 4\pi \int_0^\infty k^2 P(k) W^2(kR) dk$, and $W(kR)$ is the Fourier transform of the spatial top-hat window function of radius R. In the ΛCDM model of Fig. 1, COBE normalization yields $\sigma_8 \simeq 1.3$ [8], while galaxy surveys generally indicate $\sigma_{8,\mathrm{gal}} \simeq 1$ for optically selected galaxies and ~ 0.8 for galaxies selected by infrared flux. This high COBE normalization also marginally conflicts with the abundance of rich galaxy clusters [11]. Using the observed cluster x-ray temperature distribution function and modeling cluster formation using Press-Schechter theory, for this ΛCDM model the cluster abundance implies $\sigma_8 \simeq 1.0^{+0.35}_{-0.26}$ [12], where the errors are approximate 95% C.L.

N-body simulations indicate that the power spectrum amplitude is higher by a factor of 2 to 3 than that found in galaxy surveys at small scales, $k \gtrsim 0.4$ h Mpc^{-1} [13]. Thus, the cosmological constant model would require galaxies to be substantially *antibiased* with respect to the mass distribution, $\sigma_{\mathrm{gal}} < \sigma_\rho$. Models of galaxy formation, however, suggest that the bias parameter, $b \equiv \sigma_{\mathrm{gal}}/\sigma_\rho$, is greater than unity [14,15].

Motivated by these difficulties, we consider models in which the energy density resides in a dynamical scalar field rather than in a pure vacuum state. These *dynamical* Λ models [16,17] were proposed in response to the aesthetic difficulties of cosmological constant models. They were also found [16] to partially alleviate their observational problems as well; for example, the statistics of gravitationally lensed QSO's yields a less restrictive upper bound on $H_0 t_0$ in these models [18]. We emphasize here that they may also solve the galaxy clustering amplitude problem.

To get a preview of this conclusion, Fig. 1 also shows the COBE-normalized power spectrum for a dynamical Λ model with present scalar field density parameter $\Omega_\phi = 0.6$ (see Sec. III for a discussion of these models). While the shape of the spectrum is identical to that of the ΛCDM model with $\Omega_\Lambda = 0.6$, the scalar field model has a lower amplitude, and thus provides a better fit to the galaxy clustering data. In Sec. II, we explain these features of the power spectrum for the standard ΛCDM model and for generic dynamical Λ models. The remaining sections investigate in detail a specific class of models as a worked example. Section III reviews the scalar field model, based on ultralight, pseudo-Nambu-Goldstone bosons (PNGB's) [16]. To explore the parameter space of this model, we have adapted a code which solves the linearized Einstein-Boltzmann equations for perturbations to a Friedman-Robertson-Walker (FRW) background. The appendices contain details of these modifications. Section IV discusses the qualitative features of cosmic evolution in the PNGB models and presents results of our calculation for the amplitude of the power spectrum in this model. In Sec. V we present the cosmic microwave background (CMB) power spectrum for a particular set of model parameters, followed by the conclusion.

II. THE POWER SPECTRUM

A. The shape of $P(k)$

Figure 1 suggests that standard CDM could be improved by simply shifting the turnover in the power spectrum to larger scales (smaller wave number k). This is a plausible fix, for the location of the turnover corresponds to the scale that entered the Hubble radius when the Universe became matter dominated. On scales smaller than this, the fluctuation amplitude is suppressed compared to that on larger scales, because matter perturbations inside the Hubble radius cannot grow in a radiation-dominated universe. This scale is determined by the ratio of matter to radiation energy density at early times. To ''fix'' CDM, one must decrease the ratio $\overline{\rho_m}/\overline{\rho_r}$ in the Universe today below that predicted by the standard Einstein–de Sitter model. The matter and radiation

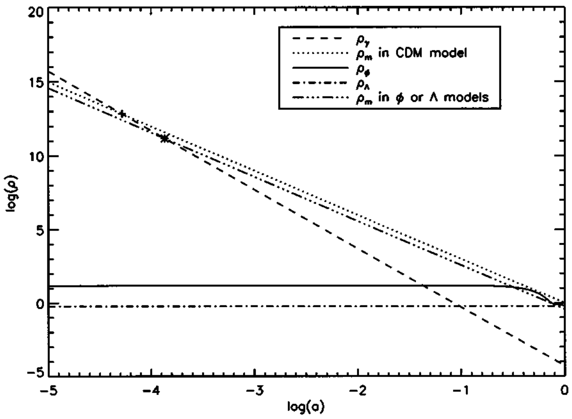

FIG. 2. Density $\overline{\rho}$ vs cosmic scale factor a. Fixing Ω_Λ or Ω_ϕ to 0.6 lowers Ω_m from the standard CDM value of 1.0, pushing the epoch of matter-radiation equality $a_{\rm eq}$ closer to today. The cross denotes $a_{\rm eq}$ for the standard CDM model and the asterisk denotes $a_{\rm eq}$ for the Λ and ϕ models. The logarithm is to base e.

densities scale as $\overline{\rho_m}=\overline{\rho_{m,0}}a^{-3}$ and $\overline{\rho_r}=\overline{\rho_{r,0}}a^{-4}$, where the cosmic scale factor a is normalized to unity today ($a_0=1$) and the subscript 0 denotes the present. Thus, the epoch of matter-radiation equality is determined by the present energy densities of matter and radiation:

$$a_{\rm eq}=\frac{\overline{\rho_{r,0}}}{\overline{\rho_{m,0}}}=\frac{4.3\times10^{-5}}{\Omega_m h^2}. \tag{1}$$

Decreasing the matter to radiation density ratio shifts the epoch of matter-radiation equality closer to the present, thereby moving the turnover in the power spectrum to larger scales.

Indeed, this shift is precisely what is done in several currently popular models of structure formation. Examples include (i) models with a lower Hubble constant than that indicated by observations [19], (ii) models with extra relativistic degrees of freedom [20], and (iii) models with a cosmological constant [4]. Since $\overline{\rho_m}\propto\Omega_m h^2$, a lower Hubble constant decreases the ratio of matter to radiation density today. Adding more relativistic degrees of freedom adds to the radiation content, decreasing the ratio of matter to radiation. Finally, in spatially flat Λ models, $\Omega_m\equiv1-\Omega_\Lambda$ is reduced from its standard CDM value ($\Omega_m=1$), achieving a similar effect.

Thus, the main benefit of Λ models for the shape of the power spectrum is that Ω_m is smaller than that in the standard CDM model. For the purpose of the power spectrum shape, the value of the vacuum energy density at early times is irrelevant, as long as it is negligible compared to the matter and radiation densities at matter-radiation equality. While the time dependence of the vacuum energy density is different for various dynamical Λ models, all such models yield the same power spectrum shape for a fixed value of the present vacuum energy density. We emphasize this point in Fig. 2, which shows the energy densities of matter, radiation, Λ, and a specific dynamical Λ model (scalar field ϕ), as a function of scale factor a. With Ω_Λ and $\Omega_\phi=0.6$ today, the standard and dynamical Λ models have the same shape for $P(k)$ (shown in Fig. 1), since they have identical values of $a_{\rm eq}$. As we will see below, however, the amplitudes of the power spectra in these models differ substantially.

B. The amplitude of $P(k)$

Compared to standard CDM, three new physical effects [21] conspire to change the amplitude of the matter power spectrum in COBE-normalized Λ models: (i) the suppression of growth of perturbations when the Universe becomes Λ dominated, (ii) the reduced gravitational potential, and (iii) the integrated Sachs-Wolfe (ISW) effect. We review these effects in turn.

The equations governing large scale perturbations in a flat universe with matter and vacuum energy are

$$\ddot{\delta}+Ha\dot{\delta}-\frac{3}{2a^2}H^2\Omega_m\delta=0, \tag{2}$$

$$H^2=\frac{H_0^2}{a^3}\left[\Omega_m+\Omega_\Lambda\frac{\rho_\Lambda}{\rho_{\Lambda,0}}a^3\right]. \tag{3}$$

Here, overdots denote derivatives with respect to conformal time τ, where $\tau\equiv\int dt/a(t)$, ρ_Λ is the vacuum energy density, not necessarily equal to its present value $\rho_{\Lambda,0}$, the density fluctuation amplitude $\delta(\mathbf{x},\tau)\equiv[\rho_m(\mathbf{x},\tau)-\overline{\rho_m}(\tau)]/\overline{\rho_m}(\tau)$, and H is the Hubble expansion rate [we use units in which $\hbar=c=1$].

Equation (2) essentially describes the behavior of a damped harmonic oscillator. When the energy density of the Universe becomes dominated by a Λ or a dynamical Λ, i.e., the second term on the the right-hand side (RHS) in Eq. (3) becomes important, the damping becoming more severe. When this happens, the growth of perturbations is suppressed. As a function of Ω_m, this suppression can be described by the scaling

$$\delta_0/\delta_{(z=100)}\propto\Omega_m^p, \tag{4}$$

where δ_0 is the perturbation amplitude today, and $\delta_{(z=100)}$ is the amplitude at the epoch $z\equiv(1/a)-1=100$, chosen as an arbitrary early epoch before the vacuum energy becomes dynamically important. In ΛCDM models, $p\simeq0.2$. For dynamical Λ models, the suppression exponent depends on the details of the specific model, but it is generally greater than that in ΛCDM models, because the dynamical Λ dominates earlier in the history of the Universe for fixed $\rho_{\Lambda,0}$. For the model shown in Fig. 1, $p\simeq0.56$. For open CDM models (with $\Lambda=0$), the scaling is also $p\simeq0.56$.

As a result of this suppression, one might expect the amplitude of the power spectrum in ΛCDM and dynamical Λ models to be smaller than that in standard CDM. However, from the Poisson equation,

$$\nabla^2\Phi=\frac{3}{2a^2}H^2\Omega_m\delta, \tag{5}$$

we have $\Phi\propto\Omega_m\delta$, where Φ is the gravitational potential associated with large-scale density fluctuations. Since the CMB anisotropy at large angle is a well-defined function of the potential [22], COBE normalization corresponds to fixing the potential, i.e., to fixing $\Omega_m\delta$. For COBE-normalized models, the growth suppression and Poisson's equation combine to yield the scale-independent relation $\delta \propto \Omega_m^{p-1}$. Thus, the power spectrum $P(k) \propto \delta^2 \propto \Omega_m^{-1.6}$ in ΛCDM models. A larger cosmological constant implies a smaller Ω_m, which in turn implies a larger amplitude for the power spectrum. In dynamical Λ models, p is not fixed at 0.2, so the amplitude of the power spectrum can be smaller than that in standard Λ models. For the model of Fig. 1, with $p=0.56$, $P(k) \propto \Omega_m^{-0.9}$.

The integrated Sachs-Wolfe (ISW) effect, which is due to time evolution of the potential, also affects the amplitude of the power spectrum. The changing potential at late times in Λ models increases the anisotropy on the large angular scales probed by COBE. Thus, for fixed COBE normalization, the amplitude of the power spectrum decreases, changing the dependence of the power spectrum on Ω_m to $P \propto \Omega_m^{-1.4}$ in the ΛCDM model. In dynamical Λ models, where the potential typically changes more than it does in standard Λ models, the ISW effect tends to be larger and is not a power-law function of Ω_m. Hence, dynamical Λ models have less power than ΛCDM and can even have less power than standard CDM.

III. ULTRALIGHT SCALAR FIELDS

A number of models with a dynamical Λ have been discussed in the literature [17]. We will focus on a particular class of models motivated by the physics of pseudo-Nambu-Goldstone bosons (PNGB's) [16,23].

It is conventional to assume that the fundamental vacuum energy of the Universe is zero, owing to some as yet not understood mechanism, and that this mechanism ''commutes'' with other dynamical effects that lead to sources of energy density. This is required so that, e.g., at earlier epochs there can temporarily exist nonzero vacuum energy which allows inflation to take place. With these assumptions, the effective vacuum energy at any epoch will be dominated by the heaviest fields which have not yet relaxed to their vacuum state. At late times, these fields must be very light.

Vacuum energy is most simply stored in the potential energy $V(\phi) \sim M^4$ of a scalar field, where M sets the characteristic height of the potential, and we set $V(\phi_m)=0$ at the minimum of the potential by the assumptions above. In order to generate a nonzero Λ at the present epoch, ϕ must initially be displaced from the minimum ($\phi_i \neq \phi_m$ as an initial condition), and its kinetic energy must be small compared to its potential energy. This implies that the motion of the field is still overdamped, $m_\phi \equiv \sqrt{|V''(\phi_i)|} \lesssim 3H_0 = 5\times 10^{-33} h$ eV. In addition, for $\Omega_\Lambda \sim 1$, the potential energy density should be of order the critical density, $M^4 \sim 3H_0^2 m_{\rm Pl}^2/8\pi$, or $M \simeq 3\times 10^{-3} h^{1/2}$ eV. Thus, the characteristic height and curvature of the potential are strongly constrained for a classical model of the cosmological constant.

This argument raises an apparent difficulty for such a model: why is the mass scale m_ϕ 30 orders of magnitude smaller than M? In quantum field theory, ultralow-mass scalars are not *generically* natural: radiative corrections generate large mass renormalizations at each order of perturbation theory. To incorporate ultralight scalars into particle physics, their small masses should be at least ''technically'' natural, that is, protected by symmetries, such that when the small masses are set to zero, they cannot be generated in any order of perturbation theory, owing to the restrictive symmetry.

From the viewpoint of quantum field theory, PNGB's are the simplest way to have naturally ultralow-mass, spin-0 particles. PNGB models are characterized by two mass scales, a spontaneous symmetry-breaking scale f (at which the effective Lagrangian still retains the symmetry) and an explicit breaking scale μ (at which the effective Lagrangian contains the explicit symmetry-breaking term). In terms of the mass scales introduced above, generally $M \sim \mu$ and the PNGB mass $m_\phi \sim \mu^2/f$. Thus, the two dynamical conditions on m_ϕ and M above essentially fix these two mass scales to be $\mu \sim M \sim 10^{-3}$ eV, interestingly close to the neutrino mass scale for the Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem, and $f \sim M_{\rm Pl} \simeq 10^{19}$ GeV, the Planck scale. Since these scales have a plausible origin in particle physics models, we may have an explanation for the ''coincidence'' that the vacuum energy is dynamically important at the present epoch. Moreover, the small mass m_ϕ is technically natural.

An example of this phenomenon is the ''schizon'' model [23], based on a Z_N-invariant low-energy effective chiral Lagrangian for N fermions, e.g., neutrinos, with mass of order M, in which the small PNGB mass, $m_\phi \simeq M^2/f$, is protected by fermionic chiral symmetries. The potential for the light scalar field ϕ is of the form

$$V(\phi) = M^4[\cos(\phi/f)+1]. \tag{6}$$

Since ϕ is extremely light, we assume that it is the only classical field which has not yet reached its vacuum expectation value. The constant term in the PNGB potential has been chosen to ensure that the vacuum energy vanishes at the minimum of the ϕ potential, in accord with our assumption that the fundamental vacuum energy is zero.

IV. COSMIC EVOLUTION AND LARGE-SCALE POWER SPECTRUM IN PNGB MODELS

To study the cosmic evolution of these models, we focus on the spatially homogeneous, zero-momentum mode of the field, $\phi^{(0)}(\tau) = \langle \phi(\mathbf{x},\tau) \rangle$, where the angular brackets denote spatial averaging. We are assuming that the spatial fluctuation amplitude $\delta\phi(\mathbf{x},\tau)$ is small compared to $\phi^{(0)}$, as would be expected after inflation if the post-inflation reheat temperature $T_{\rm RH} < f \sim M_{\rm Pl}$. The scalar equation of motion is given in Appendix A.

The cosmic evolution of ϕ is determined by the ratio of its mass, $m_\phi \sim M^2/f$, to the instantaneous expansion rate, $H(\tau)$. For $m_\phi \lesssim 3H$, the field evolution is overdamped by the expansion, and the field is effectively frozen to its initial value ϕ_i. Since ϕ is initially laid down in the early Universe (at a temperature $T \sim f \gg M$), when its potential was dynamically irrelevant, its initial value in a given Hubble volume will generally be displaced from its vacuum expectation value $\phi_m = \pi f$ (vacuum misalignment). Thus, at early times, the field acts as an effective cosmological constant, with vacuum energy density and pressure $\rho_\phi \simeq -p_\phi \sim M^4$. At late times, $m_\phi \gg 3H(\tau)$, the field undergoes damped oscillations

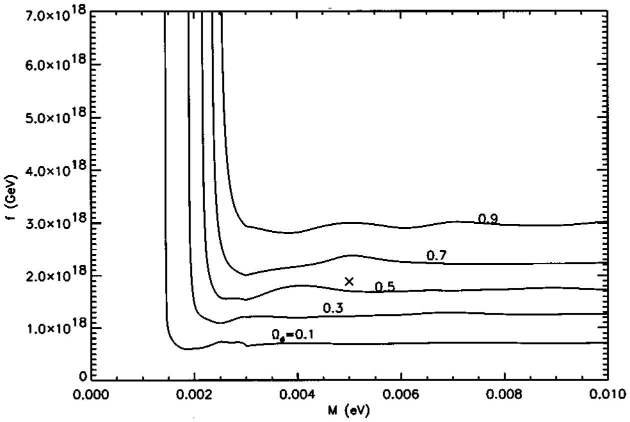

FIG. 3. Contours of Ω_ϕ in the PNGB parameter space, assuming an initial field value $\phi_i/f=1.6$. The cross marks the choice $M=0.005$ eV, $f=1.885\times10^{18}$ GeV, yielding $\Omega_\phi=0.6$, which is the model shown in Figs. 1, 2, and 6–9.

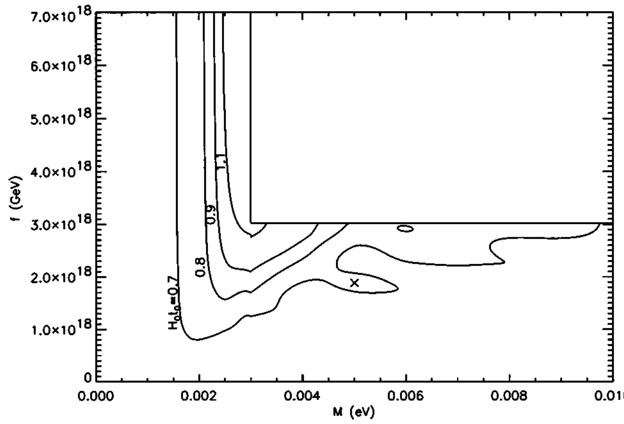

FIG. 4. Contours of H_0t_0 in the PNGB parameter space for $\phi_i/f=1.6$. The cross indicates the same model as in Fig. 3.

about the potential minimum; at sufficiently late times, these oscillations are approximately harmonic, and the stress-energy tensor of ϕ averaged over an oscillation period is that of nonrelativistic matter, with energy density $\rho_\phi\sim a^{-3}$ and pressure $p_\phi\simeq0$.

Let τ_x denote the epoch when the field becomes dynamical, $m_\phi=3H(\tau_x)$, with corresponding redshift $1+z_x=1/a(\tau_x)=(M^2/3H_0f)^{2/3}$. For comparison, the Universe makes the transition from radiation to matter domination at $z_{\rm eq}\simeq2.3\times10^4\Omega_mh^2$, much earlier than when the field becomes dynamical. The $f-M$ parameter space is shown in Fig. 3. In the far right portion of the figure, the field becomes dynamical before the present epoch and currently redshifts as nonrelativistic matter; on the far left, ϕ is still frozen and acts as an ordinary cosmological constant. In the dynamical region, the present density parameter for the scalar field is approximately $\Omega_\phi\sim24\pi(f/M_{\rm Pl})^2$, independent of M [24]. The quasihorizontal lines show contours of constant Ω_ϕ, assuming a typical initial field value $\phi_i/f=1.6$ (we will use this value of ϕ_i/f for all the plots below; the quoted limits and results depend slightly on it). The limit $\Omega_\phi<1$ corresponds approximately to $f<3.5\times10^{18}$ GeV. In the frozen region, on the other hand, Ω_ϕ is determined by M^4, independent of f, and the contours of constant Ω_ϕ are nearly vertical. In this region, the bound $\Omega_\phi<1$ corresponds roughly to $M<0.003$ eV.

Figure 4 shows contours of constant H_0t_0 in the same parameter space. As expected, models with large H_0t_0 are concentrated toward the middle to left-hand portion of the figure; as one moves to the right in the lower portion of the figure, H_0t_0 asymptotically approaches the Einstein–de Sitter value 2/3, since the scalar field currently redshifts as nonrelativistic matter and we have assumed a spatially flat universe. Consequently, the ''interesting'' region of parameter space is the area near the ''corner'' in Figs. 3 and 4, in which the field becomes dynamical at recent epochs, $z_x\sim0-3$. This has new consequences, compared to Λ models, for the classical cosmological tests, the expansion age H_0t_0, and large-scale structure. In this region, the mass of the PNGB field is miniscule, $m_\phi\sim3H_0\sim4\times10^{-33}$ eV, and (by construction) its Compton wavelength is of order the current Hubble radius, $\lambda_\phi=m_\phi^{-1}=H_0^{-1}/3\sim1000\ h^{-1}$ Mpc. Note that, as Fig. 3 indicates, the empty rectangular box in Figs. 4 and 5 approximately bounds the region of parameter space for which $\Omega_\phi>1$; this region is, therefore, excluded.

Figure 5 shows contours of the amplitude of galaxy clustering in the $f-M$ parameter space. The amplitude shown is the quantity

$$\lim_{k\to0}\left[\frac{(P(k)/k)_\phi}{(P(k)/k)_\Lambda}\right],\tag{7}$$

i.e., the amplitude on large scales relative to that for a ΛCDM model with the same effective density as the PNGB model, $\Omega_\Lambda=\Omega_\phi$. This amplitude ratio goes to unity in the left-hand portion of the figure since that region corresponds to a ΛCDM model. However, the amplitude ratio can be substantially below one in the dynamical region on the right. The cross marks the specific choice $M=0.005$ eV, $f=1.885\times10^{18}$ GeV, with initial field value $\phi_i/f=1.6$, yielding $\Omega_\phi=0.6$, which corresponds to the parameters used

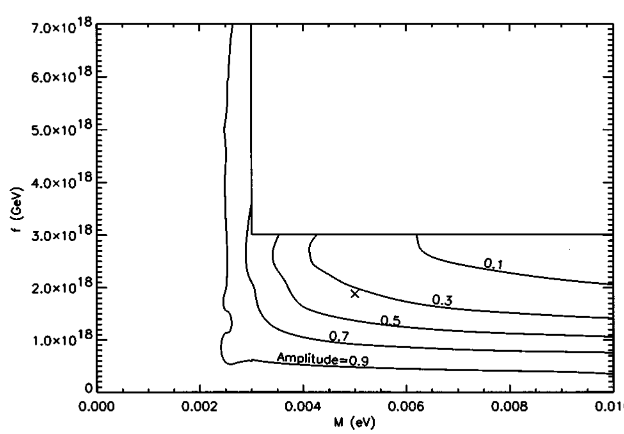

FIG. 5. Amplitude contours in the PNGB parameter space for $\phi_i/f=1.6$. The amplitude shown is defined as $\lim_{k\to0}\{[P(k)/k]_\phi/[P(k)/k]_\Lambda\}$, the amplitude on large scales relative to that of a ΛCDM model with the same effective density as the PNGB model, $\Omega_\Lambda=\Omega_\phi$. Again, the cross marks the sample model for which both the power spectrum shape and amplitude provide a good fit to the galaxy clustering data.

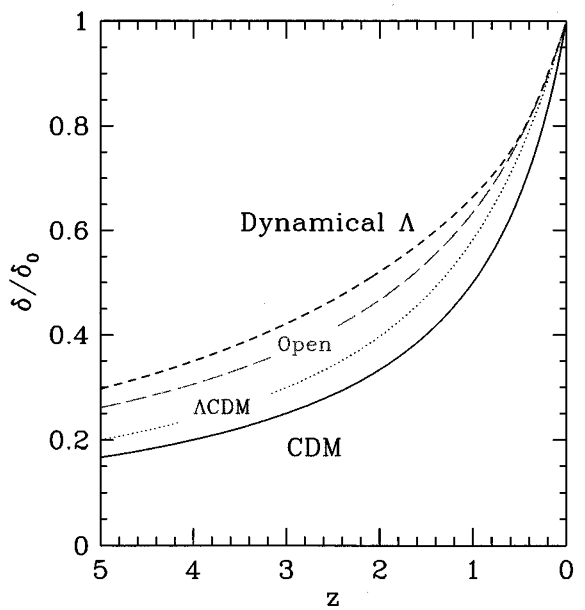

FIG. 6. Evolution of density perturbations. Shown is the density fluctuation amplitude at redshift z normalized to its present amplitude, $\delta(z)/\delta_0$, vs z. The models shown are standard $\Omega_m=1$ CDM (solid), ΛCDM with $\Omega_\Lambda=0.6=1-\Omega_m$ (dotted), an open CDM model with $\Omega_m=0.4$ (short dashed), and the dynamical Λ model with $\Omega_\phi=0.6$ (long dashed).

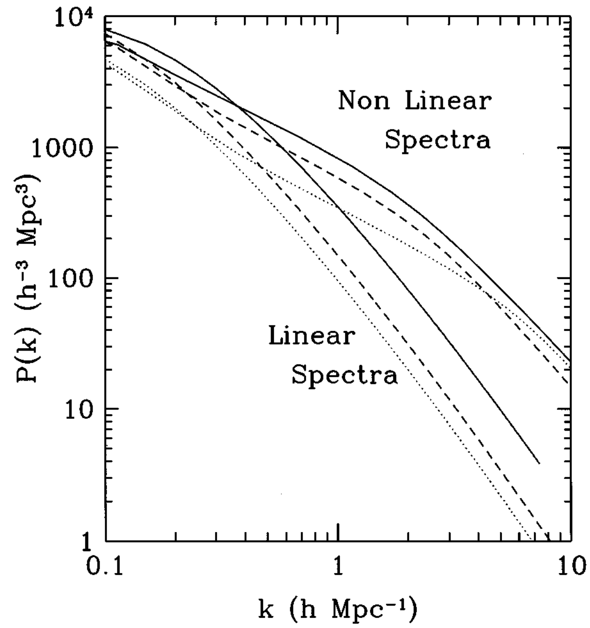

FIG. 7. Power spectra for COBE-normalized standard CDM (solid) with $\sigma_8=1.2$; ΛCDM with $\Omega_\Lambda=0.6$ (dashed), for which $\sigma_8=1.0$; and the dynamical Λ model with $\Omega_\phi=0.6$ (dotted), for which $\sigma_8=0.8$. The latter two models are normalized to the cluster abundance and have $h=0.7$. Lower curves show the linear theory power spectra, upper curves the nonlinear spectra obtained from scaling relations extracted from N-body simulations.

for the dynamical Λ curves in Figs. 1 and 2. For this case, the x-ray cluster abundance yields $\sigma_8^{\rm cl}\simeq 0.9^{+0.3}_{-0.2}$ in good agreement with the COBE normalization $\sigma_8^{\rm COBE}\simeq 0.8$ for this model. Figure 6 shows how density perturbations grow in different models. From Eq. (4) and the text following, the dynamical Λ model has a higher amplitude at early times than does a ΛCDM model with the same amplitude today. As a consequence, there should be no problem accounting for high-redshift objects such as QSO's and Lyman-α clouds in this model.

Note that the factor $\delta(z)/\delta_0$, relative to its value in the standard CDM model, approaches Ω_m^{-p} at $z\gg 1$, where p is the scaling exponent discussed in Sec. II. As a result, the nonlinear behavior of the dynamical Λ model follows that of an open model with the same value of Ω_m. We estimate the nonlinear behavior by using the fitting formula of Ref. [25], following the original treatment [26] of Hamilton *et al.* Figure 7 shows these nonlinear spectra. On scales $k\leqslant 1$ h Mpc^{-1}, the amplitude of the power spectrum is indeed a factor of 2 smaller in the dynamical Λ model than in the corresponding ΛCDM model.

We note by comparing Figs. 4 and 5 that the region of parameter space in which the amplitude (antibias) problem is solved, i.e., in which the amplitude ratio is approximately in the range 0.3–0.5, is the one in which the age of the Universe is only slightly greater than in the Einstein–de Sitter $\Omega_m=1$ case. For our specific model above, $H_0t_0=0.73$. For the corresponding ΛCDM model with the same value of Ω_m, $H_0t_0=0.89$, more comfortably within the observational limits. This is a general feature of the dynamical models considered here: for fixed Ω_m, the standard Λ model gives an upper bound on H_0t_0. Thus, the amplitude problem in this model is resolved partially at the expense of the age problem.

On the other hand, the q_0 constraints from supernovae and gravitational lensing translate into weaker upper bounds on H_0t_0 for the dynamical as opposed to the standard Λ models. Although we have not thoroughly examined all models, it is clear that one could explore the PNGB model parameter space to obtain a more balanced compromise between the age problem and the antibias problem. For example, for $f\simeq 2.5\times 10^{18}$ GeV and $M\simeq 0.0035$ eV, the amplitude ratio is about 0.5, and one has $\Omega_\phi\simeq 0.75$ and $H_0t_0\simeq 0.9$. In this case, with $h=0.7$, the power spectrum shape is reasonable ($\Omega_m h\simeq 0.15$) and the age of the universe is $t\simeq 12.6$ G yr.

Comparing Figs. 3 and 5, and focusing on the dynamical region near the ''corner'' of the parameter space, we see that the power spectrum shape and amplitude constraints fix the free parameters of the model. That is, as noted in Sec. II, the shape of the spectrum is fixed by requiring $\Omega_\phi\simeq 0.6$, which determines the scale f. Near the corner, fixing the amplitude then determines the other mass scale M. While these figures correspond to a specific choice of the initial field value ϕ_i/f, the scalar field evolution is universal in the sense that a shift in the mass scale f, accompanied by an appropriate rescaling of ϕ_i, leads to essentially identical evolution. Consequently, compared to ΛCDM models, these dynamical models have only one additional free parameter, the mass M, to solve the amplitude (antibias) problem.

V. CMB ANISOTROPY

The angular power spectra of the cosmic microwave background (CMB) anisotropy for dynamical Λ models are

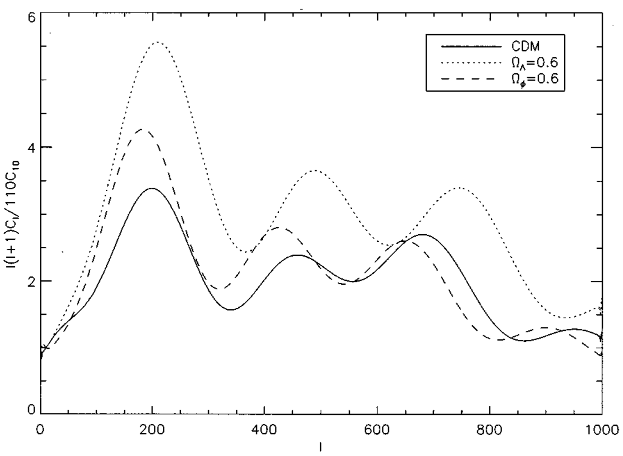

FIG. 8. CMBR angular power spectra for standard CDM, ΛCDM with $\Omega_\Lambda = 0.6$, and scalar field $\Omega_\phi = 0.6$ models. In all models $h = 0.7$, $\Omega_B = 0.0255$, and $n = 1$. Plotted is $l(l+1)C_l$ vs l, normalized at $l = 10$.

distinguishable from those of standard CDM and ΛCDM models. CMB angular power is usually expressed in terms of the angular multipoles C_l. If the sky temperature is expanded in terms of spherical harmonics as $T(\theta,\phi) = \Sigma_{lm} a_{lm} Y_{lm}(\theta,\phi)$, then $C_l = \langle |a_{lm}|^2 \rangle$, where large l corresponds to small angular scales. The angular power spectra for standard CDM ($\Omega_m = 1$), ΛCDM, and dynamical Λ models (the latter two with $\Omega_m = 0.4$) are shown in Figs. 8 and 9 for $h = 0.7$, $\Omega_B = 0.0255$, and primordial spectral index $n = 1$. Following standard practice, we plot the product $l(l+1)C_l$, normalized to its value at $l = 10$, vs l.

The appendices contain the details of the alterations required in the standard Boltzmann code to calculate the CMB anisotropy in scalar field dynamical Λ models. We can, however, identify two physical effects primarily responsible for the differences in the CMB signature between the ΛCDM and dynamical Λ models shown in Figs. 8 and 9. First, the present ages in conformal time coordinates τ_0 are different in the two models. Even though the acoustic oscillations responsible for the peaks in the CMB angular spectrum occur at the same physical scales (or same Fourier wave numbers k), the correspondence between k and angular multipole l differs. Typically, in a flat universe, a given multipole l corresponds to a fixed value of $k\tau_0$. Thus, the dynamical Λ angular spectra are shifted in l by the ratio of the present conformal times in the two models. Second, since the scalar field evolves at late times, the gravitational potential changes more rapidly in the dynamical Λ model. This leads to an enhanced ISW effect and, therefore, a relatively larger C_l at large scales (low l), as shown in Fig. 9. Thus, for models normalized by COBE, which approximately fixes the spectrum at $l \simeq 10$, the angular amplitude $l(l+1)C_l$ at small scales (large l) is smaller in the dynamical Λ model.

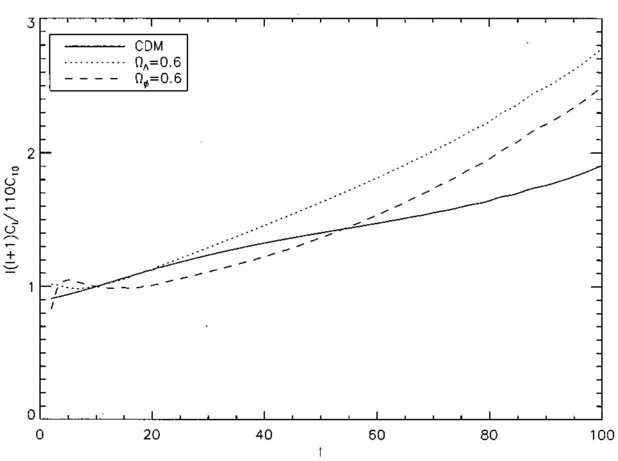

FIG. 9. Same as Fig. 8, but showing only the low l multipoles to emphasize the enhanced ISW effect in the PNGB model.

VI. CONCLUSIONS

The observational arguments in favor of the resurrection of the cosmological constant apply to dynamical Λ models as well. In addition, the dynamical Λ models offer a potential physical explanation for the curious coincidence that Ω_Λ is close to 1, by relating the present vacuum energy density to mass scales in particle physics. In the ultralight pseudo-Nambu-Goldstone boson models, this is achieved through spontaneous symmetry breaking near the Planck scale, $f \sim M_{\rm Pl}$, and explicit breaking at a scale reminiscent of MSW neutrino masses, $M \sim 10^{-3}$ eV. In combination with the assumption that the true vacuum energy vanishes (due to an as yet unknown physical mechanism), such a model provides an example of a dynamical Λ.

We have shown that such dynamical models can lead to a lower amplitude for density fluctuations compared to standard Λ models, thereby alleviating the antibias problem. The advantages of the cosmological constant for the shape of the power spectrum are retained in the dynamical models as well. Such dynamical models are, moreover, distinguishable from constant-Λ models by virtue of their CMB angular spectra.

ACKNOWLEDGMENTS

We thank Andrew Liddle and Martin White for useful conversations. This work was supported in part by the U.S. DOE (at Fermilab) and the NASA (at Fermilab through Grant No. NAG 5-2788).

APPENDIX A: CHANGES IN STANDARD BOLTZMANN CODE

This appendix and the following briefly outline the new physics incorporated into the Boltzmann code in ΛCDM and dynamical Λ models. Since the Hubble parameter is determined by the sum over densities of all species, $H^2 = (8\pi G)\Sigma_i \rho_i$, inclusion of a cosmological constant Λ or scalar field ϕ changes the relationship between the cosmic scale factor a and conformal time τ, since $(da/d\tau)/a^2 = H$. In addition to the species included in the standard Boltzmann code, namely, baryons, cold dark matter, photons, and three massless neutrinos, the density in a cosmological constant or scalar field ϕ is now included. In ΛCDM models, the vacuum energy density $\rho_\Lambda = \Lambda/8\pi G$ is constant. In the dynamical models, the scalar field energy density ρ_ϕ can be solved for with the scalar equation of motion for the homogeneous part $\phi^{(0)}(\tau)$ of the field,

$$\ddot{\phi}^{(0)}+2Ha\dot{\phi}^{(0)}+a^2dV(\phi^{(0)})/d\phi^{(0)}=0, \qquad \text{(A1)}$$

where the scalar field potential is

$$V(\phi)=M^4[\cos(\phi/f)+1], \qquad \text{(A2)}$$

and the scalar energy density

$$\rho_\phi=\frac{1}{2a^2}\dot{\phi}^{(0)2}+V(\phi^{(0)}). \qquad \text{(A3)}$$

Here, overdots denote derivatives with respect to conformal time τ.

APPENDIX B: PERTURBATION EQUATIONS FOR DYNAMICAL Λ MODELS

The general equation of motion for the scalar field $\phi(\mathbf{x},\tau)$ is derived by minimizing the action

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi-V(\phi)\right] \qquad \text{(B1)}$$

with respect to variations in ϕ. The metric is that of a perturbed Friedmann-Robertson-Walker universe:

$$g_{\mu\nu}(\mathbf{x},\tau)=g^{(0)}_{\mu\nu}(\tau)+\delta g_{\mu\nu}(\mathbf{x},\tau), \qquad \text{(B2)}$$

where $g^{(0)}_{\mu\nu}$ is the homogeneous part which describes the Hubble expansion, and $\delta g_{\mu\nu}$ is the metric perturbation. In synchronous gauge, the latter can be parametrized by the variables h,h_{33} as in [27]. The scalar field can be similarly decomposed into a homogeneous part and a spatial perturbation:

$$\phi(\mathbf{x},\tau)=\phi^{(0)}(\tau)+\delta\phi(\mathbf{x},\tau), \qquad \text{(B3)}$$

where $\phi^{(0)}$ is the solution to the spatially homogeneous equation of Appendix A. Keeping only terms linear in $h,\ h_{33}$, and $\delta\phi$, and taking the Fourier transform yields the equation of motion for the Fourier amplitude $\delta\phi_k$:

$$(\delta\ddot{\phi}_k)+2Ha(\dot{\bar{\phi}}_k)+(k^2+a^2[d^2V/d\phi^2]_{\phi=\phi^{(0)}(\tau)})(\delta\phi_k)$$
$$=\frac{\dot{h}\dot{\phi}^{(0)}}{2}. \qquad \text{(B4)}$$

There will also be an additional source term in the Einstein equation for the metric perturbation. Again following the notation of [27], the Einstein equation becomes

$$\ddot{h}+Ha\dot{h}=8\pi G(S_\phi+S_u), \qquad \text{(B5)}$$

where the source term due to ϕ is given by

$$S_\phi=4(\dot{\delta}\phi)\dot{\phi}-2a^2(\delta\phi)[dV/d\phi]_{\phi=\phi^{(0)}(\tau)}, \qquad \text{(B6)}$$

and S_u contains the usual source terms for matter and radiation [27].

[1] A. Einstein, Sitzungsber. Preuss. Akad. Wiss. **1917**, 142 (1917). This article is translated into English in J. Bernstein and G. Feinberg, *Cosmological Constants* (Columbia University, New York, 1986).

[2] J. P. Ostriker and P. J. Steinhardt, Nature (London) **377**, 600 (1995); L. Krauss and M. S. Turner, Gen. Relativ. Gravit. **27**, 1137 (1995). For earlier discussions, see M. S. Turner, G. Steigman, and L. Krauss, Phys. Rev. D **52**, 2090 (1984); P. J. E. Peebles, Astrophys. J. **284**, 439 (1984); L. Kofman and A. A. Starobinskii, Sov. Astron. Lett. **11**, 271 (1985); G. Efstathiou, Nature (London) **348**, 705 (1990); M. S. Turner, Phys. Scr. **T36**, 167 (1991).

[3] R. Carlberg *et al.*, Astrophys. J. **462**, 32 (1996).

[4] G. Efstathiou, S. Maddox, and W. Sutherland, Nature (London) **348**, 705 (1990); L. Kofman, N. Gnedin, and N. Bahcall, Astrophys. J. **413**, 1 (1993).

[5] J. A. Peacock and S. J. Dodds, Mon. Not. R. Astron. Soc. **267**, 1020 (1994).

[6] Strictly speaking, ''standard'' CDM usually refers to the parameter choice $\Omega_m=1$ and $h=0.5$; while such a model yields a better shape for $P(k)$ than does an $\Omega_m=1$ model with $h=0.7$, the fit to the data is still poor.

[7] See K. M. Gorski *et al.*, Astrophys. J. **464**, L11 (1996); E. L. Wright *et al.*, *ibid*. **464**, L21 (1996); G. Hinshaw *et al.*, *ibid.* **464**, L17 (1996) for analyses of the four-year COBE data.

[8] R. Stompor, K. M. Gorski, and A. J. Banday, Mon. Not. R. Astron. Soc. **277**, 1225 (1995).

[9] S. Perlmutter *et al.*, Astrophys. J. **440**, L1 (1995).

[10] C. S. Kochanek, Astrophys. J. **466**, 638 (1996); **419**, 12 (1993); D. Maoz and H.-W. Rix, *ibid.* **416**, 425 (1993).

[11] S. D. M. White, G. Efstathiou, and C. Frenk, Mon. Not. R. Astron. Soc. **262**, 1023 (1993).

[12] P. Viana and A. Liddle, Mon. Not. R. Astron. Soc. **281**, 323 (1996).

[13] A. Klypin, J. Primack, and J. Holtzman, Astrophys. J. **466**, 13 (1996).

[14] H. J. Mo and S. D. M. White, Mon. Not. R. Astron. Soc. (1996); G. Kauffmann, A. Nusser, and M. Steinmetz, Report No. astro-ph/9512009 (unpublished).

[15] Another fix for the amplitude problem is to introduce a small tilt to the primordial power spectrum, $P_i(k)\sim k^n$, with $n<1$. In the models of Fig. 1, we have assumed a scale-invariant primordial spectrum with $n=1$, but inflationary models typically predict small deviations from this, with n less than unity in models with a single scalar field; see, e.g., F. Adams *et al.*, Phys. Rev. D **47**, 426 (1993). Tilted spectra in Λ models have been examined in A. R. Liddle *et al.*, Mon. Not. R. Astron. Soc. **282**, 281 (1996); S. Dodelson, E. Gates, and M. S. Turner, Science **374**, 69 (1996).

[16] J. Frieman *et al.*, Phys. Rev. Lett. **75**, 2077 (1995).

[17] For discussions of other scalar field and phenomenological models of a time-varying vacuum energy, see, e.g., K. Freese *et al.*, Nucl. Phys. **B287**, 797 (1987); M. Ozer and M. Taha, *ibid.* **B287**, 776 (1987); B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); W. Chen and Y. Wu, *ibid.* **41**, 695 (1990); J. Carvalho, J. Lima, and I. Waga, *ibid.* **46**, 2404 (1992); V. Silveira and I. Waga, *ibid.* **50**, 4890 (1994); J. Lopez and D. Nanopoulos, Mod. Phys. Lett. A **11**, 1 (1996); M. Fukugita and T. Yanagida, Report No. YITP/K-1098, 1995 (unpublished).

[18] The constraints from Type Ia supernova observations are also different in these models from conventional Λ models; J. Frieman and I. Waga (in preparation).

[19] J. Bartlett *et al.*, Science **267**, 980 (1995). A less extreme proposal was put forth by M. White *et al.*, Mon. Not. R. Astron. Soc. **276**, L69 (1995).

[20] J. R. Bond and G. Efstathiou, Phys. Lett. B **265**, 245 (1991); S. Dodelson, G. Gyuk, and M. S. Turner, Phys. Rev. Lett. **72**, 3578 (1994).

[21] M. White and E. F. Bunn, Astrophys. J. **450**, 477 (1995); G. Efstathiou, J. R. Bond, and S. D. M. White, Mon. Not. R. Astron. Soc. **258**, 1P (1992).

[22] W. Hu and N. Sugiyama, Phys. Rev. D **51**, 2599 (1995).

[23] C. Hill and G. Ross, Nucl. Phys. **B311**, 253 (1988); Phys. Lett. B **203**, 125 (1988).

[24] J. Frieman, C. Hill, and R. Watkins, Phys. Rev. D **46**, 1226 (1992).

[25] J. A. Peacock and S. J. Dodds, Report No. astro-ph/9603031, 1996 (unpublished). See also B. Jain, H. J. Mo, and S. D. M. White, Mon. Not. R. Astron. Soc. **276**, L25 (1995).

[26] A. J. S. Hamilton, P. Kumar, E. Lu, and A. Matthews, Astrophys. J. **374**, L1 (1991).

[27] G. Efstathiou in *Physics of the Early Universe*, edited by J. A. Peacock, A. F. Heavens, and A. T. Davies (Edinburgh University Press, Edinburgh, England, 1990).