Primordial fluctuations from nonlinear couplings

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(Received 26 August 1996)

We study the spectrum of primordial fluctuations in theories where the inflaton field is nonlinearly coupled to massless fields and/or to itself. Conformally invariant theories generically predict a scale-invariant spectrum. Scales entering the theory through infrared divergences cause logarithmic corrections to the spectrum, tilting it towards the blue. We discuss in some detail whether these fluctuations are quantum or classical in nature. [S0556-2821(97)08404-X]

PACS number(s): 98.80.Cq, 03.65.Sq, 05.40.+j, 04.62.+v

I. INTRODUCTION

The inflationary scenario [1] allows us to consider how the primordial seeds of macroscopic structures were generated in the Universe, since, because of its quantum nature, the field that drives inflation can be decomposed into a mean field and fluctuations around it. The former gives a homogeneous background of matter and the latter induces the production of local inhomogeneities. These fluctuations evolve and are amplified during the inflationary era. At the end of this epoch, the inflaton field decays into relativistic ordinary matter. Heuristic arguments show that, as a first approximation, a scale-invariant spectrum of density fluctuations results, in rough agreement with observations [2].

Despite this success, the conventional method of identifying the structure creating fluctuations with the quantum fluctuations of the inflaton field during slow roll is conceptually unsatisfactory, and eventually leads to an overestimation of the produced density contrast ([3,4]). The basic point is that when we talk of the ''metric,'' or the density profile of the Universe (as in ''a Friedmann-Robertson-Walker metric,'' or even today, when we say ''space-time is flat'' when talking about local physics) we are referring to a macroscopic construct whereby microscopic (quantum) fluctuations of geometry and matter fields are skipped over or coarse grained away [5]. The difference between microscopic and macroscopic fluctuations is not merely one of wavelength: the real difference is that macroscopic fluctuations, when left to unfold over the relevant space and time scales, effectively decohere from each other and thus acquire individual reality. Indeed, this is the process by which a quantum homogeneous state (such as the de Sitter invariant vacuum during slow roll) may evolve into an inhomogeneous universe: decoherence gives a formal device, such as the harmonic analysis of quantum fluctuations, its physical content. Now, given that macroscopic and microscopic fluctuations are to be distinguished (and structure formation definitively belongs to the physics of the former, as cosmic structures are ''classical,'' individually existing objects), the relationship between them is not obvious and requires elucidation. In the same way that the usual Brill-Hartle waves of general relativity [6], being a first-order effect, only react on the background metric at second order, we should not expect microscopic fluctuations by themselves to be lifted into the macroscopic level, but rather that they will act on the macroscopic level as some higher-order effect. The goal of this paper is to present a detailed analysis of the action of microscopic fluctuations on the macro level, obtaining from it an improved estimate of the produced density contrast.

This issue can not even be posed correctly unless an open system view of the inflaton dynamics is adopted. In this approach, the ''decoherence,'' that is, the conversion into c-number, of the q-number fluctuations is due to the interaction of the inflaton field with a partially unknown and uncontrolled environment. There are several proposals as to how the exact separation of system and environment should be carried out ([3,7,8]).

In this paper we shall present an improved discussion of to what extent primordial fluctuations are ''quantum'' or ''classical,'' from the viewpoint of the ''consistent histories'' approach to quantum mechanics [9]. As it turns out, a detailed analysis of the conceptual difficulties of inflation points the way to the solution of the quantitative problems as well ([3,4,10]).

The consistent histories approach views quantum evolution as the coherent unfolding of individual histories for a given system, the main physical input being the specification of the particular histories relevant to the description of a concrete observer's experiences. For example, we could choose our histories as containing an exhaustive description of the values of all the fields in the theory at every space-time location. A description in terms of these ''fine grained'' histories is equivalent to a full quantum field theoretic account of the dynamics. We shall rather assume that the relevant histories for cosmological modeling are ''coarse grained.''

Concretely, we shall assume that close enough fine grained histories are physically indistinguishable and should be bundled together as a single coarse grained history. Each coarse grained history is thus labeled by the value of a typical or representative history within the bundle. The actual histories in a given bundle will differ from this representative

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by amounts of the order of the quantum fluctuations of the corresponding fields (we could consider also tighter or looser coarse graining, but as a matter of fact these histories dominate the actual evolution of the system [11]).

Given a pair of coarse-grained histories, we can compute the so-called decoherence functional (df) between them. The df measures the quantum overlap between these two histories. If the df between any two histories of a given set is strongly suppressed, then quantum interference effects will be unobservable, and it will be possible to treat each history classically, that is, to assign individual probabilities to each of them. Moreover, the most likely histories will be those for which the phase of the df is stationary, which yields the ''equations of motion'' for the representative history [12].

Going back to the problem of generation of fluctuations in inflation, our starting point is to assume that the evolution of the model is described in terms of coarse grained histories as said, and to compute the df between two generic coarse grained histories. We shall show that, for a variety of models involving coupling the inflaton to massless fields of different spin, coarse grained histories are indeed mutually consistent, and that the equations of motion, as derived from the decoherence functional, are stochastic. Thus, the representative fields naturally evolve fluctuations, and these are responsible for the creation of primordial density inhomogeneities at reheating.

It should be stressed that we are not assuming that the representative fields are ''classical''; on the contrary, its classical nature is a consequence of the theory itself, and follows from the suppression of the df between generic coarse grained histories. Physically, the representative field is decohered by its progressive entanglement with the microscopic quantum fluctuations which surround it. This entanglement is a necessary consequence of the nonlinear interaction between the two (for generic initial conditions), and at the level of the equations of motion for the representative field it appears as damping, and noise. Thus, decoherence, damping, and noise are just different manifestations of the same process, a point further elaborated elsewhere ([4,11]).

In what follows, we shall consider inflationary models where the inflaton field is nonlinearly coupled to itself, and to spin 1/2 and 1 massless fields, respectively (the spin 2 case has been dealt with in Ref. [4]).

The paper is organized as follows. In the next section, we consider in some detail a simple model of inflation, where the inflaton interacts with itself through a cubic coupling. Treating the fluctuations around the representative or physical value of the inflaton as a massless, minimally coupled field, we shall derive the density contrast generated and discuss both the amplitude of the scale invariant spectrum and the corrections to it. In the following two sections, we briefly present the necessary adaptations when the inflaton is coupled to massless, conformally invariant spin 1/2 and 1 fields, respectively, and discuss the corresponding changes in the predictions of the theory. We summarize our results in Sec. V.

II. FLUCTUATION GENERATION FROM INFLATON SELF-COUPLING

A. The model

The production of the primordial seeds for structure generation began soon after the setup of inflation and ended in the radiation dominated era. Although realistic description of the phenomena that took place during this epoch requires a detailed knowledge of the inflationary potential, it is common to consider toy models that simplify the mathematical aspects of the problem, but are still accurate enough to give a qualitative description of the related physics. We first consider a cubic field theory as a model for the inflationary universe:

$$V(\phi)=V(0)-\tfrac{1}{6}g\phi^3, \tag{1}$$

where ϕ is a c-number, homogeneous field, whose precise meaning shall be discussed below. The dynamics of geometry is governed by the Friedmann equation

$$H^2=\frac{V(\phi)}{m_P^2}, \tag{2}$$

where H is the Hubble constant (we assume a spatially flat Friedmann-Robertson-Walker (FRW) universe and work, in this subsection, in the cosmological time frame) and m_P is Planck's mass. This equation assumes vacuum dominance: namely,

$$V(\phi)\gg\dot{\phi}^2. \tag{3}$$

We shall also assume potential flatness, that is

$$V(\phi)\sim V(0)\gg g\phi^3. \tag{4}$$

The field begins inflation at some small positive value and then ''rolls down'' the slope of the potential (at some point the potential must bend upwards again, but that concerns the physics of reheating and shall not be discussed here [13,14]). The dynamics of the homogenous field is described by the Klein-Gordon equation

$$\ddot{\phi}+3H\dot{\phi}+(1/2)g\phi^2=0 \tag{5}$$

(quantum corrections to this equation shall be discussed below). Under slow rollover conditions ($\ddot{\phi}\ll 3H\dot{\phi}$), we find the solution

$$\phi(t)=\phi_0\left\{1-\frac{g\phi_0 t}{6H}\right\}^{-1}. \tag{6}$$

Slow roll over breaks down when

$$1-\frac{g\phi_0 t}{6H}\sim\frac{g\phi_0}{9H^2}. \tag{7}$$

Vacuum dominance applies to the whole slow rolling period under the mild bound $\phi_0\leq m_P$. Potential flatness requires $H^4/g^2m_P^2\leq 10^{-3}$. Since H is essentially constant during slow roll, the condition for enough inflation $Ht\geq 60$ implies $H^2\geq 10g\phi_0$. Current bounds on Ω suggest that this bound is probably saturated; in this regime the flatness condition is already satisfied given the other ones. The final requirement on the model is enough reheating, namely $m_P^2H^2\leq(T_{\mathrm{GUT}})^4$.

The density contrast in the Universe is given in terms of the fluctuations in ϕ by the formula [15]

$$\left(\frac{\delta\rho}{\rho}\right)_k\Bigg|_{\rm in}=H\frac{\delta\phi_k}{\dot\phi}\Bigg|_{\rm out}, \tag{8}$$

which relates the density contrast at horizon entry to the amplitude of fluctuations at horizon exit. Conventional accounts of the fluctuation generation process estimate $\delta\phi_k$ from the value of the free quantum fluctuations of a scalar field in a de Sitter universe ($Hk^{-3/2}$ at horizon crossing) and thus find a Harrison-Zel'dovich (HZ) scale invariant spectrum with amplitude

$$\frac{H^2}{\dot\phi}\sim\frac{g}{H}\sim\sqrt{\frac{g}{\phi_0}}. \tag{9}$$

Thus, the observational bound of 10^{-6} on the density contrast implies $g\leqslant 10^{-12}\phi_0$.

One of the main aims of this paper is to present a different estimate. In the approach to be presented below, the actual fluctuations in ϕ are much less than expected (of order $gk^{-3/2}$), which leads to a revised estimate $\delta\rho/\rho\sim(g/\phi_0)$ (no square root), and thus relaxing the bounds on the self-coupling by six orders of magnitude. This is consistent with recent findings by Matacz and by Calzetta and Hu [3,4].

We proceed now to show how the revised estimate is found.

B. Consistent histories account of fluctuation generation

Let us now upgrade the inflaton field ϕ to a full fledged quantum field Φ with a potential

$$V(\Phi)=V(0)+c\Phi-\tfrac{1}{6}g\Phi^3 \tag{10}$$

(we have added the linear term for renormalization purposes). The massless quantum field Φ obeys the Heisenberg equation of motion

$$-\Box\Phi-\frac{dV}{d\Phi}=0. \tag{11}$$

As described in the Introduction, we shall assume that the fine details of the evolution of the inflaton are inaccessible to cosmological observations. Thus, we shall split the field as in

$$\Phi=\phi+\varphi,$$

where ϕ represents a typical field history within a bundle of indistinguishable configurations, and φ describes the unobserved microscopic fluctuations. We identify ϕ with the classical inflaton field of the previous subsection. φ obeys linearized equations

$$-\Box\varphi-g\phi\varphi=0. \tag{12}$$

The equation for ϕ is obtained by subtracting Eq. (12) from Eq. (11):

$$-\Box\phi+c-\tfrac{1}{2}g\phi^2-\tfrac{1}{2}g\langle\varphi^2\rangle_\phi=\tfrac{1}{2}g(\varphi^2-\langle\varphi^2\rangle_\phi),$$

where $\langle\dots\rangle_\phi$ means the expectation value of the quantity between brackets, evaluated around a particular configuration ϕ of the physical field. If the constant c takes the value $c=\frac{1}{2}g\langle\varphi^2\rangle_0$ (which corresponds to evaluate $\langle\varphi^2\rangle$ for the false-vacuum configuration $\phi=0$), and the right-hand side is neglected, this equation admits the false vacuum solution $\phi=0$ in a de Sitter geometry $g_{\mu\nu}=[1/(H\tau)^2]\,\eta_{\mu\nu}$.

We can also linearize this last expression to get the wave equation for small fluctuations in ϕ. The additional hypothesis that the phases of the microscopic field φ are aleatory assures that the right-hand side of this equation is always small. Indeed, if we were to identify ϕ with the expectation value of Φ, we would drop this term altogether. Since we are not doing such an identification, we shall retain it a little longer, simply observing that we can evaluate this term at the false vacuum $\phi=0$ configuration:

$$-\Box\phi+\tfrac{1}{2}g(\langle\varphi^2\rangle_\phi-\langle\varphi^2\rangle_0)=gj(x), \tag{13}$$

where

$$j(x)\equiv\tfrac{1}{2}[\varphi^2(x)-\langle\varphi^2\rangle_\phi(x)] \tag{14}$$

is seen as a noise source. The self-correlation of this source is given by the so-called noise kernel [4,16]:

$$N(x_1,x_2)\equiv\tfrac{1}{2}\langle\{j(x_1),j(x_2)\}\rangle_\phi\approx\tfrac{1}{2}\langle\{j(x_1),j(x_2)\}\rangle_0. \tag{15}$$

The last term is a valid approximation provided the physical field ϕ remains close to its false vacuum configuration.

It is common to write Eq. (13) as

$$-\Box_x\phi(x)+g^2\int d^4x'\sqrt{{}^{(4)}_{-}g}\,D(x,x')\phi(x')=gj(x), \tag{16}$$

where

$$D(x,x')\equiv-\frac{1}{2g}\frac{\delta\langle\varphi^2\rangle(x)}{\delta\phi(x')}\Bigg|_{\phi=0}$$

is the dissipation kernel ([4,16]). The physical meaning of the noise and dissipation kernels is borne out by the df between two histories described by different typical fields

$$\mathcal{D}[\phi,\phi']=\int D\varphi D\varphi' e^{i[S(\phi+\varphi)-S(\phi'+\varphi')]},$$

where the integral is over fluctuation fields matched on a constant time surface in the far future. Actual evaluation yields ([4,16])

$$\mathcal{D}(\phi,\phi')\sim e^{(iI-R)},$$

$$I=S(\phi)-S(\phi')+(g^2/2)\int d^4x\sqrt{{}^{(4)}_{-}g}\,d^4x'\times\sqrt{{}^{(4)}_{-}g'}(\phi-\phi')(x)D(x,x')(\phi+\phi')(x'),$$

$$R=(g^2/2)\int d^4x\sqrt{{}^{(4)}_{-}g}\,d^4x'\sqrt{{}^{(4)}_{-}g'}(\phi-\phi')(x)N(x,x')\times(\phi-\phi')(x').$$

We see that the dissipation kernel contributes to the phase of the df close to the diagonal, and thus to the equations of motion for the most likely histories, while the noise kernel directly determines whether interference effects are suppressed or not, and thus the consistency of the chosen coarse grained histories.

C. Actual estimates of fluctuation generation

The above treatment of fluctuation generation implies that there are essentially two sources of fluctuations in ϕ, namely, uncertainties in the initial value data of ϕ at the beginning of inflation, and fluctuations induced by stochastic sources during the slow roll period (as we shall see below, noise generation cuts off naturally after horizon crossing).

Let us assume that decoherence is efficient (see below), and thus that we can deal with each history individually. Then we must conclude that only those histories where the initial value of ϕ is exceptionally smooth may lead to inflation (see the Appendix). This limitation on initial data for inflation has been discussed by several authors, most notably from numerical simulations by Goldwirth and Piran [17], and from general arguments by Calzetta and Sakellariadou, Deruelle and Goldwirth [18], and others. Discarding the fluctuations in the initial conditions, we find the solution

$$\phi(x)=g\int d^4x_1\sqrt{\underset{-}{{}^{(4)}}g}\,G_{\rm ret}(x,x_1)j(x_1),$$

where $G_{\rm ret}$ is the scalar field retarded propagator, and the two-point correlation function

$$\frac{1}{2}\langle\{\phi(\vec{x},\tau),\phi(0,\tau)\}\rangle$$
$$\approx g^2\int d^4x_1\sqrt{\underset{-}{{}^{(4)}}g}\int d^4x_2\sqrt{\underset{-}{{}^{(4)}}g}\,G_{\rm ret}((\vec{x},\tau),x_1)$$
$$\times G_{\rm ret}((0,\tau),x_2)N(x_1,x_2).$$

The noise and dissipation kernels can be written as

$$N(x_1,x_2)\approx{\rm Re}[\langle j(x_1)j(x_2)\rangle_0]={\rm Re}(\langle\varphi_1\varphi_2\rangle_0^2),\qquad(17)$$

$$D(x_1,x_2)\approx{\rm Im}(\langle\varphi_1\varphi_2\rangle_0^2)\,\theta(\tau_1-\tau_2).\qquad(18)$$

Returning to the fluctuation field associated to the $g\Phi^3$ coupling, we can write

$$\langle\varphi(x_1)\varphi(x_2)\rangle_0=H^2\Lambda(r,\tau_1,\tau_2),\qquad(19)$$

where Λ is the dimensionless function

$$\Lambda(r,\tau_1,\tau_2)=\frac{1}{2\pi^2}\int_0^\infty\frac{dk}{k^2r}\sin(kr)f_k(\tau_1)f_k^*(\tau_2).\qquad(20)$$

The f_k are the positive frequency modes for the free field in a de Sitter geometry and are solutions for Eq. (12) valid to first order. These f_k are functions of one single variable $k\tau_i$:

$$f_k(\tau_i)=e^{ik\tau_i}(1-ik\tau_i).$$

In the same ''first-order'' approach, we can consider that $G_{\rm ret}$ is well described by the free-field retarded propagator:

$$G_{\rm ret}(x,x_1)=-i\frac{H^2}{(2\pi)^3}\theta(\tau-\tau_1)\int\frac{d^3k}{2k^3}e^{i\vec{k}\cdot(\vec{x}-\vec{x}_1)}$$
$$\times[f_k(\tau)f_k^*(\tau_1)-f_k^*(\tau)f_k(\tau_1)].$$

It is easy to see that the spatial Fourier transform of this quantity can be written as $H^2k^{-3}\mathcal{G}(k\tau,\beta_i)$, where we have defined the dimensionless variable $\beta_i=k\tau_i$ and the wave-number dependence has been factorized out of $\mathcal{G}$. Moreover, if we look at Eqs. (15), (19), and (20), we conclude that the spatial Fourier transform of the noise kernel can be written in principle as $H^4k^{-3}\mathcal{N}(\beta_1,\beta_2)$; i.e., it depends on k only through the k^{-3} factor. The Fourier transform of $\langle\phi(x_1)\phi(x_2)\rangle$ becomes

$$\Delta_k(k\tau)=\frac{1}{4}\frac{g^2}{k^3}\frac{1}{(2\pi)^6}\int_{-\infty}^{k\tau}\frac{d\beta_1}{\beta_1^4}\int_{-\infty}^{k\tau}\frac{d\beta_2}{\beta_2^4}\mathcal{G}(k\tau,\beta_1)\mathcal{G}(k\tau,\beta_2)$$
$$\times\mathcal{N}(k,\beta_1,\beta_2).\qquad(21)$$

The double integral in Eq. (21) represents a function of the comoving wave number k and the conformal time τ which appears in the one-variable combination $k\tau$.

As we shall show below, fluctuation generation is effective only until horizon crossing. As the k mode of the field becomes greater than the horizon when $k\tau=-1$, the last consideration suggests that the integrals in Eq. (21) can be truncated at this value, and will therefore take the form

$$\Delta_k(k\tau=-1)=\frac{1}{4}\frac{g^2}{k^3}\frac{1}{(2\pi)^6}\int_{-\infty}^{-1}\frac{d\beta_1}{\beta_1^4}\int_{-\infty}^{-1}\frac{d\beta_2}{\beta_2^4}\mathcal{G}(-1,\beta_1)$$
$$\times\mathcal{G}(-1,\beta_2)\mathcal{N}(k,\beta_1,\beta_2).\qquad(22)$$

If we take the above equations at face value, we find no explicit k dependence within the integrand, and therefore the spectrum of field fluctuations can be written as

$$\Delta_k^{\rm sca}(\tau)\propto g^2\frac{1}{k^3},\qquad(23)$$

where the superscript indicates that this prediction corresponds to a scalar field theory. This is, of course, the well-established prediction of a scale-invariant spectrum of density fluctuations.

However, in a de Sitter geometry a minimally coupled massless scalar field is not well defined at the infrared limit, and the propagators associated to it are divergent [19]. We can handle this problem by introducing an infrared cutoff and studying the way in which this new parameter modifies the Harrison-Zel'dovich spectrum (a small inflaton mass would have the same physical effects). Our new propagator is

$$\Lambda_{\rm cut}(r,\tau_1,\tau_2)=\frac{1}{2\pi^2}\int_{k_{\rm infra}}^\infty\frac{dk}{k^2r}\sin(kr)f_k(\tau_1)f_k^*(\tau_2).\qquad(24)$$

We want to find the k dependence of Δ_k for the noise kernel associated to the cubic coupling between two scalar fields, ϕ and φ. N is obtained immediately from Eq. (17) as $N(x_1,x_2)=H^4$ Re$[\Lambda^2_{\rm cut}(r,\tau_1,\tau_2)]$. As it was already noted, if we consider the retarded propagators for the free field, then only $\mathcal{N}$ will have a nontrivial k dependence. Of course, k will always appear as an adimensional quantity $k_{\rm infra}/k$.

In order to analyze the emergence of corrective terms to a HZ spectrum, it is convenient to note that $\mathcal{N}$ can be written as

$$\mathcal{N}=\mathcal{N}_{\rm HZ}+\mathcal{N}_{\rm infra},$$

where $\mathcal{N}_{\rm HZ}$ is independent of $k_{\rm infra}$, and $\mathcal{N}_{\rm infra}$ contains $k_{\rm infra}$ only as $\ln(k/k_{\rm infra})$. It is now evident that, after performing the double integration for the $\mathcal{N}_{\rm HZ}$ term in Eq. (22), one will arrive to the usual $\Delta_k\propto k^{-3}$ Harrison-Zel'dovich spectrum. Furthermore, the logarithmic terms can be factored out of the integral, i.e., the corrective terms to the spectrum will have the form $\ln(k/k_{\rm infra})$, and Eq. (22) will just give the amplitude for this corrections. This means that, provided the phenomena that induce the generation of fluctuations are effective only until horizon crossing and that it is a good approximation to consider free-retarded propagators for the field, the spectrum of fluctuations takes the form

$$\Delta_k^{\rm sca}(k\tau=-1)=\frac{C}{k^3}\left[1+B\ln\left(\frac{k}{k_{\rm infra}}\right)\right], \tag{25}$$

where C is the Harrison-Zel'dovich amplitude and B is the amplitude of the corrections. An actual evaluation shows that B is positive and that its numerical value is about 5×10^{-3}. This result tells us that the logarithmic corrections increase the spectral power, specially for small scales (large k), and the spectrum moves slightly to the blue.

As for the amplitude of the scale invariant part of the spectrum, we may adopt the simple estimate Eq. (23). This leads to the revised bound $g\leqslant10^{-6}\phi_0$ discussed at the beginning of this section.

D. Loose ends

The method developed in this section to describe the generation of primordial fluctuations can be applied with only trivial modifications to other nonlinear theories involving the inflaton, as shall be demonstrated below by considering couplings to spin 1/2 and 1 fields. However, before we proceed, it is convenient to discuss in full two essential elements of our argument, namely, that coarse grained histories described by generic values of ϕ are truly consistent, and that superhorizon fluctuations are dynamically decoupled from the noise sources [more concretely, we must show that on superhorizon scales $\delta\phi_k\sim\delta\tau_k\dot{\phi}(t)$, since this formula enters the derivation of Eq. (8)].

Let us first consider the issue of consistency. We wonder if the history we have considered, starting from vanishing initial conditions at the beginning of inflation, is truly decohered from any other history differing from it by amounts of the order of the quantum fluctuations of a scalar field in de Sitter space. If this is the case, then we are justified to treat this history classically.

The answer to this question lies on whether the df between any such two histories is strongly suppressed or not. In other terms, we must compute

$$-2\ln[|\mathcal{D}(\phi,\phi')|]\equiv g^2\int d^4x\sqrt{-g(x)}d^4x'\sqrt{-g(x')}\times(\phi-\phi')(x)N(x,x')(\phi-\phi')(x'). \tag{26}$$

Or, Fourier transforming on the space variables

$$g^2\int\frac{d^3k}{(2\pi)^3}\frac{d\tau}{(H\tau)^4}\frac{d\tau'}{(H\tau')^4}(\phi_k-\phi_k')(t)\times N_k(\tau,\tau')(\phi_k-\phi_k')(\tau'). \tag{27}$$

For each mode, the integral extends from the beginning of inflation up to horizon crossing. Due to the τ^4 suppression factor, the integral is actually dominated by the upper limit. In this regime

$$N_k(\tau,\tau')\sim H^4/k^3. \tag{28}$$

By choice, the value of the product of the fields is close to the expectation value of quantum fluctuations, namely

$$(\phi_k-\phi_k')(\tau)(\phi_k-\phi_k')(\tau')\sim(H^2/k^3)\,\delta(0)\sim(H^2/k^3)k_{\rm infra}^{-3} \tag{29}$$

(as follows from conventional quantization in the de Sitter background)

$$\int^{k^{-1}}\frac{d\tau}{(H\tau)^4}\sim k^3/H^4. \tag{30}$$

Finally,

$$-\ln[|\mathcal{D}(\phi,\phi')|]\equiv\int\frac{d^3k}{(2\pi k)^3}\left(\frac{g}{H}\right)^2\left(\frac{k}{k_{\rm infra}}\right)^3. \tag{31}$$

As we have seen, $g^2/H^2\sim g/\phi_0\sim10^{-6}$, and so decoherence obtains for all modes $k\gg10^2k_{\rm infra}$. For example, if we take $k_{\rm infra}$ as corresponding to the horizon length at the beginning of inflation, and fine tune the model so that this will also correspond to the horizon today, all modes entering the horizon prior to recombination would be classical in this sense. Of course, in a realistic model $k_{\rm infra}$ would be much larger than today's horizon, and all physically meaningful modes will be decohered. In this case, moreover, we would obtain decoherence even between histories much closer to each other than the quantum limit.

Let us consider now the issue of noise on super horizon scales. In order to arrive to the previous results, we have considered the integration of our expression for the power spectrum of the fluctuations of the field [Eq. (21)] from the beginning of inflation up to the moment in which each mode k crossed the horizon. The full expression can be rewritten as

$$\Delta_k(\tau)=-\frac{g^2}{H^4}\frac{1}{(2\pi)^6}\left\{\int_{-\infty}^{-1}\frac{d\beta_1}{\beta_1^4}\int_{-\infty}^{-1}\frac{d\beta_2}{\beta_2^4}\mathcal{G}_1\mathcal{G}_2\mathcal{N}\right.$$
$$+\int_{-1}^{k\tau}\frac{d\beta_1}{\beta_1^4}\int_{-1}^{k\tau}\frac{d\beta_2}{\beta_2^4}\mathcal{G}_1\mathcal{G}_2\mathcal{N}$$
$$\left.+2\int_{-1}^{k\tau}\frac{d\beta_1}{\beta_1^4}\int_{-\infty}^{-1}\frac{d\beta_2}{\beta_2^4}\mathcal{G}_1\mathcal{G}_2\mathcal{N}\right\}, \tag{32}$$

where the second and third terms represent the contribution of a given mode when it is outside the horizon. In the previous sections, we have ignored these terms. If we consider the behavior of the noise kernel $\mathcal{N}$ far away from the horizon, it is easy to verify that the last term may be effectively ignored. The second term requires some additional considerations. First we observe that the noise kernel is not oscillatory outside the horizon, so the sources at different times are strongly correlated. We can write $j_k(\tau)\sim j_k\sqrt{\mathcal{N}(k\tau)}$, where the j_k are time-independent Gaussian variables. The wave equation that governs the evolution of each mode may be written as

$$-\ddot{\phi}_k+(H\tau)^2k^2\phi_k(\tau)+g^2\int_{-\infty}^{\tau}\frac{d\tau'}{(H\tau')^4}\mathcal{D}(\tau-\tau')\phi_k(\tau')$$
$$=gj_k(\tau)\simeq gj_k\sqrt{\mathcal{N}(k\tau)}, \tag{33}$$

where $\mathcal{D}$, j_k, and $\mathcal{N}$ indicate the spatial Fourier transforms of the dissipation kernel, the source and the noise kernel, respectively. When the mode is outside the horizon ($|k\tau|\ll 1$), we can write the last equation as

$$-\ddot{\phi}_k+g^2\int_{-\infty}^{\tau}\frac{d\tau'}{(H\tau')^4}\mathcal{D}(\tau-\tau')\phi_k(\tau')\simeq gj_k\sqrt{\mathcal{N}(k\tau)}. \tag{34}$$

The dissipative term is dominated by the contribution close to the upper limit, and it can be written as

$$\frac{g^2}{H^4}\delta\phi_k(\tau)\int_{-\infty}^{\tau}\frac{d\tau'}{\tau'^4}\mathcal{D}(\tau-\tau'). \tag{35}$$

The dissipation kernel can be obtained from Eq. (18). The asymptotic expressions near and far away from the coincidence limit $\tau\simeq\tau'$ are, respectively,

$$\sqrt{-{}^{(4)}g}\mathcal{D}_{\rm near}(\tau-\tau')\approx\frac{\pi}{\tau'^4}\left(\frac{5}{3}\ln[k(\tau-\tau')]-1.50\right)\times(\tau-\tau')^3,$$

$$\sqrt{-{}^{(4)}g}\mathcal{D}_{\rm away}(\tau-\tau')\approx-\frac{\pi\tau}{2k}\frac{1}{\tau'^3}\sin[k(\tau-\tau')]\times\ln[k(\tau-\tau')].$$

The upper formula holds when $\tau-\tau'\leqslant k^{-1}$. $\mathcal{D}_{\rm near}$ goes to zero rapidly as $\tau-\tau'\rightarrow 0$ and its contribution to the integral in Eq. (35) will be completely negligible. Moreover, the oscillatory part of $\mathcal{D}_{\rm away}$ cancels the contribution of the dissipative term far away from the coincidence limit. From these observations, we conclude that dissipation is not effective for modes that are outside the horizon, i.e., those modes behave as a free field.

Since $\mathcal{N}(k\tau)$ grows at most logarithmically, we find that the particular solution to Eq. (33) vanishes faster than $O(\tau)$, while the homogeneous (growing) solution is ‘‘frozen’’ into a constant value. Thus the value of ϕ_k obeys the usual (classical) Klein-Gordon equation while beyond the horizon, and the conventional derivation of Eq. (8) holds [15].

III. YUKAWA COUPLING

Now we consider the interaction between the inflaton field and a massless Dirac field. The Lagrangian density for a theory in which two Dirac fields are coupled to a scalar massless field is

$$\mathcal{L}=\partial_\mu\Phi\partial^\mu\Phi+\frac{i}{2}(\overline{\Psi}\gamma^\mu\partial_\mu\Psi-\partial_\mu\overline{\Psi}\gamma^\mu\Psi)+f\overline{\Psi}\Psi\Phi,$$

where f is an arbitrary coupling constant. The equation of motion for the inflaton Φ is

$$-\Box\Phi+m^2\Phi-f\overline{\Psi}\Psi=0.$$

If we consider the separation of Φ in a mean field and fluctuations $\Phi=\phi+\varphi$, the linearized equation of motion for the physical field is

$$-\Box\phi(x)+m^2\phi(x)=fj_{\rm Yuk}(x),$$

where

$$j_{\rm Yuk}(x)=\overline{\Psi}(x)\Psi(x).$$

The noise kernel, defined as the mean value of the anticommutator of the sources [see Eq. (15)], takes the form

$$N_{\rm Yuk}(x_1,x_2)\approx\tfrac{1}{2}\langle\{j_{\rm Yuk}(x_1),j_{\rm Yuk}(x_2)\}\rangle_0$$
$$\approx\tfrac{1}{2}[\langle\overline{\Psi}(x_1)\Psi(x_1)\overline{\Psi}(x_2)\Psi(x_2)\rangle_0+(1\leftrightarrow 2)]. \tag{36}$$

The four-point function can be reduced to a product of two-point functions which correspond to the fermionic propagators:

$$\langle\overline{\Psi}(x_1)\Psi(x_1)\overline{\Psi}(x_2)\Psi(x_2)\rangle=\langle\overline{\Psi}(x_1)\Psi(x_2)\rangle\times\langle\Psi(x_1)\overline{\Psi}(x_2)\rangle,$$

where

$$\langle\overline{\Psi}(x_1)\Psi(x_2)\rangle\equiv-iS^+(x_2-x_1),$$

$$\langle\Psi(x_1)\overline{\Psi}(x_2)\rangle\equiv-iS^-(x_1-x_2).$$

These expressions allow us to write the noise kernel as

$$N_{\rm Yuk}(x_1,x_2)\approx-\tfrac{1}{2}f^2S^-(x_1-x_2)S^+(x_2-x_1). \tag{37}$$

This expression is valid provided the scalar field remains near its false vacuum configuration. As the spinor field is conformally invariant, the propagators corresponding to a curved space-time can be written in terms of those associated to a Minkowskian geometry [20]. For a de Sitter background geometry, we have

$$S^{\pm}_{\rm dS}(x_1,x_2)=H^3(\tau_1\tau_2)^{3/2}S^{\pm}_{\rm Mink}(x_1,x_2).$$

The Minkowskian propagators for the spinor field can be written as derivatives of the scalar field propagators

$$S^{\pm}=-i\gamma^{\mu}\partial_{\mu}D^{\pm}=\pm\frac{1}{(2\pi)^3}\gamma^{\mu}\partial_{\mu}\int\frac{d^3k}{2k}e^{i(\pm kx_0-\vec{k}\cdot\vec{x})}.$$

The noise kernel takes the form

$$N_{\rm Yuk}=-f^2H^4(\tau_1\tau_2)^3\partial_{\mu}D^{-}_{\rm Mink}(x_1-x_2)\partial^{\mu}D^{-}_{\rm Mink}(x_1-x_2).$$

To arrive to a specific integral for the power spectrum generated by the Yukawa coupling, we can proceed in close analogy to the scalar field case [Eq. (22)]:

$$\Delta_k^{\rm Yuk}(k\tau=-1)=-\frac{1}{4}\frac{f^2}{k^3}\frac{H^4}{(2\pi)^6}\int_{-\infty}^{-1}\frac{d\beta_1}{\beta_1}\int_{-\infty}^{-1}\frac{d\beta_2}{\beta_2}\times\mathcal{G}(-1,\beta_1)\mathcal{G}(-1,\beta_2)\mathcal{N}^{\rm Yuk}$$

$$\mathcal{N}^{\rm Yuk}=\frac{1}{2}\mathcal{F}(\partial_{\mu}D^{-}_{\rm Mink}\partial^{\mu}D^{-}_{\rm Mink}+\text{c.c.}),$$

where $\mathcal{F}(\cdots)$ represents the three-dimensional Fourier transform of $(\cdots)$.

As we are now considering conformal fields, the propagators are perfectly defined and the last expression will produce a "pure" HZ spectrum. As in the scalar field, there are no relevant corrections coming from the ultraviolet limit. The spectrum produced by this coupling will be of the scale-invariant form

$$\Delta_k^{\rm Yuk}(k\tau=-1)=\frac{C'}{k^3}.$$

As a rough approximation, we may take $C'\approx f^2H^4$, leading to

$$\frac{\delta\rho}{\rho}\sim\frac{H\delta\phi}{\dot{\phi}}\sim\frac{H^3f}{g\phi^2}.$$

As we can write $H\sim\sqrt{g\phi}$, we obtain

$$\frac{\delta\rho}{\rho}\sim\sqrt{\frac{g}{\phi}}f.$$

Given our previous estimate for the self-coupling, agreement between this expression and the observational data requires that the coupling constant $f\sim10^{-3}$.

IV. ELECTROMAGNETIC COUPLING

As a last example, let us consider the coupling between the inflaton field and a massless vectorial field. The Lagrangian density for a theory with massless scalar and electromagnetic fields is

$$\mathcal{L}=(\partial_{\mu}+ieA_{\mu})\Phi(\partial_{\mu}-ieA_{\mu})\Phi^*-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

from which we can deduce the equation of motion

$$-(\Box-e^2A^{\mu}A_{\mu}-ie\partial_{\mu}A^{\mu})\Phi-2ieA^{\mu}\partial_{\mu}\Phi=0.$$

If we decompose the inflaton field in its physical and virtual components $\Phi=\phi+\varphi$ and write linearized equations, we obtain

$$-\Box\varphi=0,$$

where we are assuming that A^{μ} is always small and can be thought as a fluctuation in the electromagnetic potential $V^{\mu}=0$. The more general case $A^{\mu}=V^{\mu}+\delta A^{\mu}$ would give essentially the same results for the small deviations δA^{μ}. The equation for the physical field is

$$-(\Box-e^2A^{\mu}A_{\mu})\phi=2ieA^{\mu}\partial_{\mu}\varphi+ie(\partial_{\mu}A^{\mu})\varphi.$$

The right-hand side of this equation defines the source

$$j(x)=(A^{\mu}\partial_{\mu}\varphi+\tfrac{1}{2}(\partial_{\mu}A^{\mu})\varphi).$$

As usual, the noise kernel associated to this source is $N(x_1,x_2)\simeq\langle[j(x_1),j(x_2)]\rangle_0$ where

$$\begin{aligned}\langle j(x_1)j(x_2)\rangle_0 &=\langle A^{\mu}(x_1)A^{\nu}(x_2)\rangle_0\partial_{\mu,1}\partial_{\nu,2}\langle\varphi(x_1)\varphi(x_2)\rangle_0\\ &\quad+\tfrac{1}{4}\partial_{\mu,1}\partial_{\nu,2}\langle A^{\mu}(x_1)A^{\nu}(x_2)\rangle_0\langle\varphi(x_1)\varphi(x_2)\rangle_0\\ &\quad+\tfrac{1}{2}\partial_{\nu,2}\langle A^{\mu}(x_1)A^{\nu}(x_2)\rangle_0\partial_{\mu,1}\langle\varphi(x_1)\varphi(x_2)\rangle_0\\ &\quad+\tfrac{1}{2}\partial_{\mu,1}\langle A^{\mu}(x_1)A^{\nu}(x_2)\rangle_0\partial_{\nu,2}\langle\varphi(x_1)\varphi(x_2)\rangle_0.\end{aligned}\tag{38}$$

Before we proceed, it will be convenient to write this expression in terms of the propagators of the interacting fields. The scalar propagator has been considered in a previous section. It can be shown that a massless vectorial field couples to the space-time curvature conformally. This result implies that the covariant electromagnetic propagators for a de Sitter geometry are identical to the Minkowskian ones [20]:

$$\langle A_{\alpha}(x_1)A_{\beta}(x_2)\rangle_{\rm dS}=\langle A_{\alpha}(x_1)A_{\beta}(x_2)\rangle_{\rm Mink}\equiv\langle A_{\alpha}(x_1)A_{\beta}(x_2)\rangle.$$

Rising indexes with $g^{\mu\nu}(x_i)=(H\tau_i)^2\eta^{\mu\nu}$ and adopting the Feynman gauge, where the Minkowskian electromagnetic and scalar propagators are related by

$$\langle A_{\alpha}(x_1)A_{\beta}(x_2)\rangle=-i\eta_{\alpha\beta}D^{+}(x_1-x_2),$$

we obtain

$$\langle A_{\alpha}(x_1)A_{\beta}(x_2)\rangle_{\rm dS}=iH^4(\tau_1\tau_2)^2\eta^{\mu\nu}D^{+}_{\rm Mink}(x_1-x_2).$$

As this is a propagator for a conformal field, it is well defined and will not produce any correction to the power spectrum. Only the factors which correspond to the inflaton in Eq. (38) will produce corrections. In order to get these corrections, we must consider the truncated scalar propagators defined in a previous section [see Eqs. (19) and (24)]. As we have already seen, the cutoff dependence can be isolated as $\log(k/k_{\rm infra})$, where k is a parameter that will be associated to the Fourier transform of the noise kernel. Thus we can say that the sought for corrections will be logarithmic:

$$\Delta_k^{\rm Em}(k\tau=-1)=\frac{C''}{k^3}\left[1+B''\ln\left(\frac{k}{k_{\rm infra}}\right)\right].$$

As in the previous examples we considered, C'' is undetermined because it includes the square of the coupling constant. Roughly, $C''\sim H^4e^2$, leading to $e\lesssim 10^{-3}$ to match density production bounds. The ultraviolet contribution is always irrelevant. B'' measures the relative importance of the logarithmic corrections compared with the HZ background.

V. CONCLUSIONS

We considered fluctuation generation in the context of three elementary regularizable field theories that represent the interaction of the inflaton with itself and other massless fields of different spin. In each case, we obtained the power spectrum for the field fluctuations Δ_k, which can be easily related to the primordial density inhomogeneities that constituted the seeds for structure generation. These fluctuations are produced by a random noise source. We found that the predicted spectrum is scale invariant when only conformal fields contribute to the noise term; in a more general situation, such as when the source includes the virtual scalar field, there appear logarithmic corrections.

Two features of our results stand out, namely, that we satisfy current observational bounds on the amplitude of the primordial spectrum for values of the inflaton self-coupling much larger than previously reported, and that the corrections to the HZ spectrum depend not only on the shape of the inflaton potential, but also on what exactly the inflaton is coupled to.

Concerning the first issue, it should be clear that the drastic relaxation on the bounds for the inflaton self-coupling we have obtained is related to much tighter bounds on the initial conditions for the inflaton field than previously used. Of course, this is not the only factor that determines this relaxation, for which we would also have to consider, at least, the *rôle* of the dissipation terms, which have been ignored so far. In this sense, it might seem that we have just traded one fine tuning for another. However, it should be remembered that the fine tuning of initial conditions is not added ad hoc to match the COBE observations, but it is independently necessary to obtain inflation at all. As a matter of fact, this fine tuning is necessary even if we accept the usual estimate of $g/\phi_0\sim 10^{-12}$. So, even if not yet totally satisfactory, it may be said that the model has improved in regard to fine tuning. As we mentioned previously, a similar result concerning fine tuning has been obtained by Matacz [3] and by Calzetta and Hu [4]. Matacz considered a phenomenological model of inflation consisting of a system surrounded by an environment of time-dependent harmonic oscillators that back-react on the former acting as a stochastic source of white noise. The approach by Calzetta and Hu consisted on coarse graining the graviton degrees of freedom associated to the geometry of space-time. The latter methodology is followed closely in the present work. We complement its results in some aspects such as making the explicit calculation of the most relevant physical quantities, generalizing the possible interactions of the scalar field, and computing the main corrections to the scale invariant spectrum.

In the long run, it may well be that the second aspect of our conclusions, namely, the much wider scope to seek corrections to the fundamental Harrison-Zel'dovich spectrum, will prove to be more relevant. Indeed, it is well known that for any observed spectrum it is possible to ''taylor'' an inflationary potential that will reproduce it [21]. But these *ad hoc* potentials have no other motivation than matching this result, and more often than not are unmotivated or even pathological from the standpoint of current high-energy physics. The extra freedom afforded by the possibility that the primordial spectrum of fluctuations could depend on the coupling of the inflaton to other fields (which must exist if we are to have reheating) could be the key to building simpler and yet more realistic theories of the generation of primordial fluctuations.

Of course, the massless theories considered in this paper are too simplistic to live up to this promise. Couplings to massive fields, and even the possibility that the inflaton could be part of a larger, maybe grand unified, theory, ought to be considered before actual predictions may be extracted. We continue our research on this key issue in early universe cosmology.

ACKNOWLEDGMENTS

It is a pleasure to thank Antonio Campos, Jaume Garriga, Salman Habib, Bei-lok Hu, Alejandra Kandus, Andrew Matacz, Diego Mazzitelli, Emil Mottola, Juan Pablo Paz, and Enric Verdaguer for multiple exchanges concerning this project. We also wish to thank the hospitality of the Universidad de Barcelona and the Workshop on Non Equilibrium Phase Transitions (Saint John's College, Santa Fe, New Mexico), where parts of it were completed. This work has been partially supported by Universidad de Buenos Aires, CONICET and Fundación Antorchas, and by the Commission of the European Communities under Contract No. CI1*-CJ94-0004.

APPENDIX: HOMOGENEOUS INITIAL CONDITIONS

In this appendix we try to clarify the reasons why we have considered homogeneous initial conditions at the beginning of inflation. We show that the requirement of vacuum dominance at the beginning of inflation excludes classical fluctuations larger than 10^{-6} of the conventional vacuum fluctuations on any interesting scale.

To start with, we consider the energy density associated to these classical fluctuations

$$\rho\sim\dot{\phi}^2+\nabla\phi^2+V(\phi)\tag{A1}$$

as measured by a comoving observer. The fluctuations in the energy pick up first- and second-order terms, which are denoted as $\delta\rho_1$ and $\delta\rho_2$, respectively:

$$\delta\rho_1 \sim \dot{\phi}_0 \delta\dot{\phi} + V'(\phi)\delta\phi,$$

$$\delta\rho_2 \sim \delta\dot{\phi}^2 + \nabla\delta\phi^2 + V''(\phi_0)\delta\phi^2. \tag{A2}$$

We will only consider the second-order terms. These terms dominate over $\delta\rho_1$ for small $\dot{\phi}_0$. For modes far inside the horizon the last term can be neglected. If the fluctuations behave as a massless field, it follows that

$$\delta\rho_k \sim (\delta\dot{\phi}_k)^2 = k_{\rm phys}^2 (\delta\phi_k)^2. \tag{A3}$$

As usual, the scales k refer to comoving quantities, while the $k_{\rm phys} = k/a = k|H\eta|$ stand for quantities measured in terms of physical lengths. The spatial average for the field fluctuations can be written as

$$\langle \delta\phi_k \delta\phi_{k'} \rangle \approx \frac{(H\eta)^2}{k} \sigma_k \delta(k-k'), \tag{A4}$$

where σ_k measures the ratio between the classical fluctuations in question and the quantum vacuum fluctuations in the de Sitter invariant vacuum (which we include here only to have something familiar to compare against). This expression allows us to write

$$\left.\frac{\delta\rho}{\rho}\right|_{\rm in} \sim \left(\frac{1}{m_P H}\right)^2 \int d^3k \left(\frac{k}{a}\right)^2 \frac{(H\eta)^2}{k} \sigma_k \Big|_{\rm out}, \tag{A5}$$

where we have used the Einstein equation to substitute $\rho \sim (m_P H)^{-2}$, and we have assumed that the δ function in $\langle \delta\phi_k^2 \rangle$ cancels the divergence associated to the infinite volume over which this average is taken. If we also assume that σ_k obeys a power law, the last expression can be written as

$$\frac{\delta\rho}{\rho} \sim \left(\frac{k^4}{a^4 m_P^2 H^2}\right) \sigma_k, \tag{A6}$$

where k corresponds to the lowest (in wavelength) fluctuation scale. The scale factor a and the quantity σ_k are evaluated at the beginning of inflation. In terms of the physical wavelength, we have

$$\frac{\delta\rho}{\rho} \sim \left(\frac{l_P}{l_{\rm hor}}\right)^2 \left(\frac{l_{\rm hor}}{\lambda_{\rm phys}}\right)^4 \sigma_k. \tag{A7}$$

Obviously, the model is consistent only if this expectation value is lower than one, i.e.:

$$\sigma_k \leqslant \left(\frac{l_P}{l_{\rm hor}}\right)^{-2} \left(\frac{l_{\rm hor}}{\lambda_{\rm phys}}\right)^{-4}. \tag{A8}$$

This means that we have an upper limit for the amount of classical fluctuations which are present (at a given scale) at the beginning of inflation, if there is to be inflation at all.

The question we must face now is whether this bound still allows for fluctuations the size of those which will build up subsequently through matching to an effective stochastic source, as we have demonstrated in the main part of this paper. These latter fluctuations amount to around 10^{-6} of the quantum zero point fluctuations in the models we have considered.

The wavelengths where it is possible to have $\sigma_k \geqslant 10^{-6}$ (so that initial classical fluctuations can dominate the fluctuations generated from the stochastic source) must obey

$$\lambda_{\rm phys} \geqslant 10^{-3/2} \sqrt{l_p \times l_{\rm hor}}. \tag{A9}$$

We now recall the known result

$$l_P \times l_{\rm hor} = T_r^{-2}, \tag{A10}$$

where T_r is the reheating temperature. Thus, we have the condition

$$\lambda_{\rm phys} \geqslant 10^{-3/2} \frac{1}{T_r}. \tag{A11}$$

It is convenient to phrase this condition in terms of the present wavelength of the same fluctuation

$$\lambda_{\rm phys}\big|_{\rm today} \geqslant 10^{-3/2} \frac{1}{T_r} \times e^N \times \frac{T_r}{T_0}, \tag{A12}$$

where N is the number of e foldings during inflation, T_r the temperature at reheating, and T_0 the temperature today.

In natural units, $T_0^{-1} \sim 10^{-28} d_0$, where d_0 is the present size of the horizon. Thus, we must have

$$\lambda_{\rm phys}\big|_{\rm today} \geqslant 10^{-29.5} \times e^N \times d_0. \tag{A13}$$

Most inflationary models predict values of N over 60 and even larger. Thus, classical fluctuations that have $\sigma_k \geqslant 10^{-6}$ at the beginning of inflation are excluded on any cosmologically relevant scale.

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