Cold dark matter in SUSY theories: The role of nuclear form factors and the folding with the LSP velocity

T. S. Kosmas*
Theoretical Physics Section, University of Ioannina, GR 451 10, Ioannina, Greece

J. D. Vergados
*Theoretical Physics Section, University of Ioannina, GR 451 10, Ioannina, Greece
and Department of Natural Sciences, University of Cyprus, 1678 Nicosia, Cyprus*

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The momentum transfer dependence of the total cross section for elastic scattering of cold dark matter candidates, i.e., lightest supersymmetric particles (LSP's), with nuclei is examined. The presented calculations of the event rates refer to a number of representative nuclear targets throughout the periodic table and have been obtained in a relatively wide phenomenologically allowed SUSY parameter space. For the coherent cross sections it is shown that, since the momentum transfer can be quite big for a large mass of the LSP and heavy nuclei even though the energy transfer is small ($\leqslant 100$ keV), the total cross section can in such instances be reduced by a factor of about 5. For the spin-induced cross section of odd-A nuclear targets, as is the case of ^{207}Pb studied in this work, we found that the reduction is less pronounced, since the high multipoles tend to enhance the cross section as the momentum transfer increases (for a LSP mass <200 GeV) and partially cancel the momentum retardation. The effect of the Earth's revolution around the Sun on these event rates is also studied by folding with a Maxwellian LSP-velocity distribution which is consistent with its density in the halos. We thus find that the convoluted event rates do not appreciably change compared to those obtained with an average velocity. The event rates increase with A and, in the SUSY parameter space considered, they can reach values up to 140 $\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$ for Pb. The modulation effect, however, was found to be small (less than $\pm 5\%$). [S0556-2821(97)00904-1]

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I. INTRODUCTION

There is ample evidence that about 90% of the matter of the Universe is dark [1–3]. There are also numerous arguments indicating that our galaxy is immersed in a dark halo which outweighs the luminous component by a factor of about 10. Furthermore, the large scale structure of the Universe may be accommodated supposing two kinds of dark matter [3]. One kind is composed of particles which were relativistic at the time of the structure formation. This is called hot dark matter (HDM). The other kind is composed of particles which were nonrelativistic at the time of structure formation. These constitute the cold dark matter (CDM) component of the Universe. In any case, the CDM component of the Universe is at least 60%. Obviously, the galactic halo is composed of dark matter, since HDM particles will be moving too fast to be trapped in the galaxy. There are two candidates for CDM. The first is massive compact halo objects (MACHO's), i.e., white dwarfs, Jupiter-like objects etc. and the other is more exotic, i.e., weak interacting massive particles (WIMP's). Recent preliminary experiments [4] suggest that about one-half of the mass of the halo is made of WIMP's. The most appealing possibility linked with supersymmetry (SUSY) of a WIMP candidate is the lightest supersymmetric particle (LSP) [5–7] (see Ref. [7] for a recent review).

In recent years, the phenomenological implications of supersymmetry are being taken very seriously [5,6]. More or less, accurate predictions at low energies are now feasible in terms of few input parameters in the context of SUSY models [5–8]. Such predictions do not appear to depend on arbitrary choices of the relevant parameters or untested assumptions. In any case the SUSY parameter space is somewhat restricted [5–7].

In such theories derived from supergravity the LSP is expected to be a neutral Majorana fermion with mass in the 10–500 GeV/c^2 region traveling with nonrelativistic velocities ($\langle\beta\rangle \approx 10^{-3}$), i.e., with energies in the keV region. In practice, however, one expects a velocity distribution which is supposed to be Maxwellian (see Sec. IV). In the absence of R-parity-violating interactions this particle is absolutely stable. But even in the presence of R-parity violation, it may live long enough to be a CDM candidate.

The detection of the LSP, which is going to be denoted by χ_1, is extremely difficult, since this particle interacts with matter extremely weakly. One possibility is the detection of secondary high energy neutrinos which are produced by pair annihilation in the Sun where this particle is trapped. Such high energy neutrinos can be detected via neutrino telescopes.

The other possibility, to be examined in the present work, is the detection of the energy of the recoiling nucleus (A,Z) in the reaction

$$\chi_1 + (A,Z) \rightarrow \chi_1 + (A,Z)^*. \tag{1}$$

*Present address: Institute of Theoretical Physics, University of Tübingen, D-72076, Germany.

This energy can be converted into phonon energy and detected by a temperature rise in a cryostatic detector with sufficiently high Debye temperature [3,9,10]. The detector should be large enough to allow a sufficient number of counts but not too large to permit anticoincidence shielding to reduce background. A compromise of about 1 kg is achieved.

Another possibility is the use of superconducting granules suspended in a magnetic field. The heat produced will destroy the superconductor and one can detect the resulting magnetic flux. Again, a target of about 1 kg is favored.

There are many targets which can be employed. The most popular ones contain the nuclei $^{3}_{2}$He, $^{19}_{9}$F, $^{23}_{11}$Na, $^{29}_{14}$Si, $^{40}_{20}$Ca, $^{73,74}_{32}$Ge, $^{75}_{33}$As, $^{127}_{53}$I, $^{134}_{54}$Xe, and $^{207}_{82}$Pb.

In order to be able to calculate the event rate for the process (1) the following ingredients are necessary.

(1) One must be able to construct the effective Lagrangian at the elementary particle level in the framework of supersymmetry [11–17]. We will follow the procedure adopted in Ref. [17]. For the readers' convenience we will provide the important elements in Sec. II.

(2) One must make the transition from the quark to the nucleon level [18–24]. This is not straightforward for the scalar couplings, which dominate the coherent part of the cross section, and the isoscalar axial current which is important for the incoherent cross section for odd targets.

(3) One must properly treat the nucleus. Admittedly, the uncertainties here are smaller than those of even the most restricted SUSY parameter space. One, however, would like to put as accurate nuclear physics input as possible in order to constrain the SUSY parameters as much as possible when the data become available.

For the coherent production, if one ignores the momentum transfer dependence, the procedure is straightforward. The spin matrix element, however, is another story. For its evaluation, practically every known nuclear model has been employed.

At first, the independent single particle shell model (ISPSM) had been employed [11,13,25,26]. Subsequent calculations using the odd group method (OGM) and the extended odd group method (EOGM), utilizing magnetic moments and mirror β decays, by Engel and Vogel [27], showed that the ISPSM was inadequate (see also Ref. [13]). Eventually, however, by performing shell model calculations [28], this model was also found lacking (see Ref. [29]). Iachello, Krauss, and Maino [30] employed the interacting boson fermion model (IBFM) and Nikolaev and Klapdor-Kleingrothaus [14,31] the finite fermion theory in order to reliably evaluate the spin matrix elements.

One additional complication arises from the fact that the LSP appears to be quite massive, perhaps heavier than 100 GeV. For such heavy LSP and sufficiently heavy nuclei, the dependence of the nuclear matrix elements on the momentum transfer cannot be ignored even if the LSP has energies as low as 100 keV [12,25,26]. This affects both the coherent and the spin matrix elements. For the coherent mode the essential new features can be absorbed in the nuclear form factor. The evaluation of the spin matrix elements is quite a bit more complicated. Quite a number of high multipoles can now contribute, some of them getting contributions from components of the wave function which do not contribute in the static limit (i.e., at $q=0$, see Sec. III). Thus, in general, sophisticated shell model calculations are needed to account for both the observed retardation of the static spin matrix element and its correct q dependence. For the experimentally interesting nuclear systems $^{29}_{14}$Si and $^{73}_{32}$Ge, very elaborate calculations have been performed by Ressell *et al.* [32]. In the case of $^{73}_{32}$Ge a further improved calculation by Dimitrov, Engel, and Pittel has recently been performed [33] by suitably mixing variationally determined triaxial Slatter determinants. Indeed, for this complex nucleus many multipoles contribute and the needed calculations involve techniques which are extremely sophisticated.

From the above discussion the necessity for more detailed calculations, especially for the spin component of the cross sections for heavy nuclei, is motivated. The aim of the present paper is to calculate LSP-nucleus-scattering cross section using some representative input in the restricted SUSY parameter space as outlined above. The coherent matrix elements are computed throughout the periodic table (Sec. III). The necessary form factors were obtained using the method of Ref. [34], which are in good agreement with experiment. For the spin matrix elements we have chosen $^{207}_{82}$Pb as target. This target, in addition to its experimental qualifications, has the advantage of a rather simple nuclear structure [35,36] (see Sec. III). Thus, only two multipoles can contribute.

Finally, the counting rates are folded with a reasonable LSP-velocity distribution [7] (see Sec. IV) in order to estimate the modulation due to the Earth's velocity. Convoluted rates are obtained (Sec. IV) in the framework of models discussed in the Introduction and Sec. II.

II. BRIEF DESCRIPTION OF THE OPERATORS

It has recently been shown that process (1) can be described by a four-fermion interaction [11–16] of the type [17]

$$L_{\mathrm{eff}}=-\frac{G_F}{\sqrt{2}}[J_\lambda\overline{\chi_1}\gamma^\lambda\gamma^5\chi_1+J\overline{\chi_1}\chi_1], \tag{2}$$

where

$$J_\lambda=\overline{N}\gamma_\lambda[f_V^0+f_V^1\tau_3+(f_A^0+f_A^1\tau_3)\gamma_5]N \tag{3}$$

and

$$J=\overline{N}(f_S^0+f_S^1\tau_3)N. \tag{4}$$

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, because of the Majorana nature of the LSP, $\overline{\chi_1}\gamma^\lambda\chi_1=0$ (identically). The vector and axial vector form factors can arise out of Z exchange and s-quark exchange [11–17] (s quarks are the SUSY partners of quarks with spin zero). They have uncertainties in them (for three choices in the allowed parameter space of Ref. [5] see Ref. [17]). In our choice of the parameters the LSP is mostly a gaugino. Thus, the Z contribution is small. It may become dominant in models in which the LSP happens to be primarily a Higgsino. Such a possibility will be examined elsewhere. The transition from the quark to the nucleon level is

TABLE I. The parameters βf_V^0, f_S^0, f_A^0, f_A^1 and f_V^1/f_V^0, f_S^1/f_S^0 for three SUSY solutions (see Ref. [17]). The value of $\beta=10^{-3}$ was used. For the definition of f_S^0 and f_S^1 (models A, B, and C) see Ref. [17] and for the values of $\tan\beta$, m_h, m_H, and m_A employed see Ref. [5].

Quantity	Solution No. 1	Solution No. 2	Solution No. 3
$\beta f_V^0(Z)$	0.475×10^{-5}	1.916×10^{-5}	0.966×10^{-5}
$f_V^1(Z)/f_V^0(Z)$	-1.153	-1.153	-1.153
$\beta f_V^0(\tilde{q})$	1.271×10^{-5}	0.798×10^{-5}	1.898×10^{-5}
$f_V^1(\tilde{q})/f_V^0(\tilde{q})$	0.222	2.727	0.217
$f_s^0(\tilde{q})/\beta f_V^0(\tilde{q})$ (model A)	6.3×10^{-3}	3.6×10^{-3}	2.4×10^{-3}
$f_s^0(\tilde{q})/\beta f_V^0(\tilde{q})$ (model B)	0.140	3.5×10^{-2}	5.8×10^{-2}
βf_V^0	1.746×10^{-5}	2.617×10^{-5}	2.864×10^{-5}
f_V^1/f_V^0	-0.153	-0.113	-0.251
LSP mass (GeV)	126.0	27.0	102.0
$\tan\beta$	10.0	1.5	5.0
$\tan 2a$	0.245	6.265	0.528
m_h, m_H, m_A	116, 346, 345	110, 327, 305	113, 326, 324
$f_S^0(H)$ (model A)	1.31×10^{-5}	1.30×10^{-4}	1.38×10^{-5}
f_S^1/f_S^0 (model A)	-0.275	-0.107	-0.246
$f_S^0(H)$ (model B)	5.29×10^{-4}	7.84×10^{-3}	6.28×10^{-4}
$f_S^0(H)$ (model C)	7.57×10^{-4}	7.44×10^{-3}	7.94×10^{-4}
$f_A^0(Z)$	-	-	-
$f_A^1(Z)$	1.27×10^{-2}	5.17×10^{-2}	2.58×10^{-2}
$f_A^0(\tilde{q})$ (NQM)	0.510×10^{-2}	3.55×10^{-2}	0.704×10^{-2}
$f_A^0(\tilde{q})$ (EMC)	0.612×10^{-3}	0.426×10^{-2}	0.844×10^{-3}
$f_A^1(\tilde{q})$	0.277×10^{-2}	0.144×10^{-2}	0.423×10^{-2}
f_A^0 (NQM)	0.510×10^{-2}	3.55×10^{-2}	0.704×10^{-2}
f_A^0 (EMC)	0.612×10^{-3}	0.426×10^{-2}	0.844×10^{-3}
f_A^1	1.55×10^{-2}	5.31×10^{-2}	3.00×10^{-2}

quite straightforward in this case. This is, in general, the case of vector current contribution. We will see later that, because of the Majorana nature of the LSP, the contribution of the vector current, which can lead to a coherent effect of all nucleons, is suppressed [10–16]. The vector current is effectively multiplied by a factor of $\beta=v/c$, v is the velocity of LSP (see Table I). Thus, the axial current, especially in the case of light and medium mass nuclei, cannot be ignored.

For the isovector axial current one is confident about how to go from the quark to the nucleon level. We know from ordinary weak decays that the coupling merely gets renormalized from $g_A=1$ to $g_A=1.24$. For the isoscalar axial current the situation is not completely clear. The naive quark model (NQM) would give a renormalization parameter of unity (the same as the isovector vector current). This point of view has, however, changed in recent years due to the so-called spin crisis, i.e., the fact that in the EMC data [18] it appears that only a small fraction of the proton spin arises from the quarks. Thus, one may have to renormalize f_A^0 by $g_A^0=0.28$, for u and d quarks, and $g_A^0=-0.16$ for the strange quarks [19], i.e., a total factor of 0.12. These two possibilities, labeled as NQM and EMC, are listed in Table I. One cannot completely rule out the possibility that the actual value may be anywhere in the above-mentioned region [20].

The scalar form factors arise out of the Higgs exchange or via s-quark exchange when there is a mixing between s quarks $\tilde{q}_L$ and $\tilde{q}_R$ [11–13] (the partners of the left-handed and right-handed quarks). They have two types of uncertainties in them. One, which is the most important, is at the quark level due to the uncertainties in the Higgs sector. The other is in going from the quark to the nucleon level [15,16]. Such couplings are proportional to the quark masses, and hence sensitive to the small admixtures of $q\bar{q}$ (q other than u and d) present in the nucleon. Again, values of f_S^0 and f_S^1 in the allowed SUSY parameter space are considered [17].

The actual values of the parameters f_S^0 and f_S^1 used here, arising mainly from Higgs exchange, were obtained by considering one-loop corrections in the Higgs sector. As a result, the lightest Higgs boson mass is now a bit higher, i.e., more massive than the value of the Z boson [21,22]. Thus the obtained values of the parameters f_S^0 and f_S^1 are smaller than those of Ref. [17] (see Table I). The next source of ambiguities involves the step of going from the quark to the nucleon level for the scalar and isoscalar couplings. Here, we adopt the procedure described in Ref. [17] as a result of the analysis of Refs. [15,23,24].

III. TOTAL CROSS SECTION

The invariant amplitude in the case of nonrelativistic LSP's takes the form [17]

$$|m|^2=\frac{E_f E_i - m_1^2+\mathbf{p}_i\cdot\mathbf{p}_f}{m_1^2}|J_0|^2+|\mathbf{J}|^2+|J|^2$$
$$\simeq\beta^2|J_0|^2+|\mathbf{J}|^2+|J|^2, \quad (5)$$

TABLE II. The quantity q_0 (forward momentum transfer) in units of fm^{-1} for three values of m_1 and three typical nuclei. In determining q_0 the value $\sqrt{\langle\beta^2\rangle}=10^{-3}$ was employed [see Eq. (12)].

	q_0 (fm^{-1})		
Nucleus	$m_1=30.0$ GeV	$m_1=100.0$ GeV	$m_1=150.0$ GeV
Ca	0.174	0.290	0.321
Ge	0.215	0.425	0.494
Pb	0.267	0.685	0.885

where m_1 is the LSP mass, $|J_0|$ and $|\mathbf{J}|$ indicate the matrix elements of the time and space components of the current J_λ of Eq. (2), respectively, and J represents the matrix element of the scalar current J of Eq. (3). Notice that $|J_0|^2$ is multiplied by β^2 (the suppression due to the Majorana nature of LSP's mentioned above). It is straightforward to show that

$$|J_0|^2=A^2|F(\mathbf{q}^2)|^2\left(f_V^0-f_V^1\frac{A-2Z}{A}\right)^2, \qquad (6)$$

$$J^2=A^2|F(\mathbf{q}^2)|^2\left(f_S^0-f_S^1\frac{A-2Z}{A}\right)^2, \qquad (7)$$

$$|\mathbf{J}|^2=\frac{1}{2J_i+1}|\langle J_i||[f_A^0\mathbf{\Omega}_0(\mathbf{q})+f_A^1\mathbf{\Omega}_1(\mathbf{q})]||J_i\rangle|^2, \qquad (8)$$

with $F(\mathbf{q}^2)$ the nuclear form factor, and

$$\mathbf{\Omega}_0(\mathbf{q})=\sum_{j=1}^{A}\sigma(j)e^{-i\mathbf{q}\cdot\mathbf{x}_j}, \quad \mathbf{\Omega}_1(\mathbf{q})=\sum_{j=1}^{A}\sigma(j)\tau_3(j)e^{-i\mathbf{q}\cdot\mathbf{x}_j}, \qquad (9)$$

where $\sigma(j)$, $\tau_3(j)$, $\mathbf{x}_j$ are the spin, third component of isospin ($\tau_3|p\rangle=|p\rangle$), and coordinate of the jth nucleon and $\mathbf{q}$ is the momentum transferred to the nucleus.

The differential cross section in the laboratory frame takes the form [17]

$$\frac{d\sigma}{d\Omega}=\frac{\sigma_0}{\pi}\left(\frac{m_1}{m_p}\right)^2\frac{1}{(1+\eta)^2}\xi \times\left\{\beta^2|J_0|^2\left[1-\frac{2\eta+1}{(1+\eta)^2}\xi^2\right]+|\mathbf{J}|^2+|J|^2\right\}, \qquad (10)$$

where m_p is the proton mass, $\eta=m_1/m_pA$, $\xi=\hat{\mathbf{p}}_i\cdot\hat{\mathbf{q}}\geqslant 0$ (forward scattering), and

$$\sigma_0=\frac{1}{2\pi}(G_Fm_p)^2\simeq 0.77\times 10^{-38}\ \mathrm{cm}^2. \qquad (11)$$

The momentum transfer $\mathbf{q}$ is given by

$$|\mathbf{q}|=q_0\xi, \quad q_0=\beta\frac{2m_1c}{1+\eta}. \qquad (12)$$

Some values of q_0 (forward momentum transfer) for some characteristic values of m_1 and representative nuclear systems (light, medium, and heavy) are given in Table II. It is clear from Eq. (12) that the momentum transfer can be sizable for large m_1 and heavy nuclei (η small).

The total cross section can be cast in the form

$$\sigma=\sigma_0\left(\frac{m_1}{m_p}\right)^2\frac{1}{(1+\eta)^2}\Bigg(A^2\Bigg\{\Bigg[\beta^2\left(f_V^0-f_V^1\frac{A-2Z}{A}\right)^2 +\left(f_S^0-f_S^1\frac{A-2Z}{A}\right)^2\Bigg]I_0(q_0^2)-\frac{\beta^2}{2}\frac{2\eta+1}{(1+\eta)^2} \times\left(f_V^0-f_V^1\frac{A-2Z}{A}\right)^2I_1(q_0^2)\Bigg\}+[f_A^0\Omega_0(0)]^2I_{00}(q_0^2) +2f_A^0f_A^1\Omega_0(0)\Omega_1(0)I_{01}(q_0^2)+[f_A^1\Omega_1(0)]^2I_{11}(q_0^2)\Bigg). \qquad (13)$$

The quantities entering Eq. (13) are explained in Secs. III A and III B.

A. The coherent matrix element

The terms in the square brackets of Eq. (13) describe the coherent cross section, the momentum dependence of which is involved in the integrals $I_\rho(q_0^2)$, where $\rho=0$ for the isoscalar and $\rho=1$ for the isovector component. These integrals are given by

$$I_\rho(q_0^2)=2(\rho+1)\int_0^1\xi^{1+2\rho}|F(q_0^2\xi^2)|^2d\xi, \quad \rho=0,1. \qquad (14)$$

The integrals I_ρ are normalized so that $I_\rho(0)=1$ [following the normalization $F(0)=1$] and they can be calculated by using Ref. [34] where we have shown that the nuclear form factor $F(q^2)$ can be adequately described within the harmonic oscillator model as

$$F(q^2)=\left[\frac{Z}{A}\Phi(qb,Z)+\frac{N}{A}\Phi(qb,N)\right]e^{-q^2b^2/4} \qquad (15)$$

($N=A-Z$) where $b\approx 1.0A^{1/6}$ fm and Φ is a polynomial of the form

$$\Phi(qb,\alpha)=\sum_{\lambda=0}^{N_{\max}(\alpha)}\theta_\lambda^{(\alpha)}(qb)^{2\lambda}, \quad \alpha=Z,N. \qquad (16)$$

$N_{\max}(Z)$ and $N_{\max}(N)$ depend on the major harmonic oscillator shell occupied by protons and neutrons [34], respectively. The integrals $I_\rho(q_0^2)$ can be written as

$$I_\rho(q_0^2)=I_\rho(u)=(1+\rho)u^{-(1+\rho)}\int_0^u x^{1+\rho}|F(2x/b^2)|^2dx, \qquad (17)$$

where

$$u=q_0^2b^2/2. \qquad (18)$$

With the use of Eqs. (15) and (16) we obtain

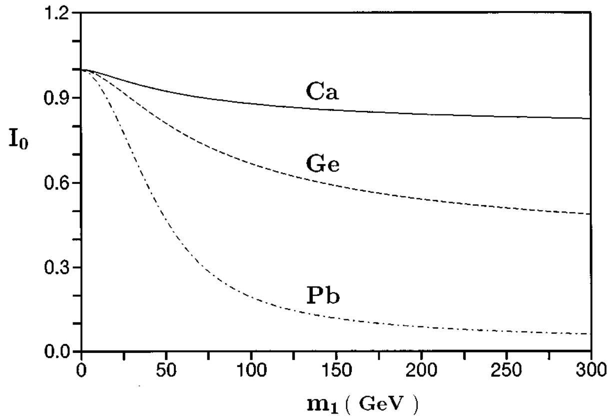

FIG. 1. The integrals I_0 [see Eq. (17)], which describe the dominant scalar contribution (coherent part) of the total cross section as a function of the LSP mass (m_1), for three typical nuclei: Ca, Ge, and Pb. The value $\sqrt{\langle\beta^2\rangle}=10^{-3}$ was used.

$$I_\rho(u)=\frac{1}{A^2}\{Z^2 I_{ZZ}^{(\rho)}(u)+2NZI_{NZ}^{(\rho)}(u)+N^2 I_{NN}^{(\rho)}(u)\}, \quad (19)$$

where

$$I_{\alpha\beta}^{(\rho)}(u)=(1+\rho)\times\sum_{\lambda=0}^{N_{\max}(\alpha)}\sum_{\nu=0}^{N_{\max}(\beta)}\frac{\theta_\lambda^{(\alpha)}}{\alpha}\frac{\theta_\nu^{(\beta)}}{\beta}\frac{2^{\lambda+\nu+\rho}(\lambda+\nu+\rho)!}{u^{1+\rho}}\times\left[1-e^{-u}\sum_{\kappa=0}^{\lambda+\nu+\rho}\frac{u^\kappa}{\kappa!}\right] \quad (20)$$

$(\alpha,\beta=N,Z)$. The coefficients $\theta_\lambda^{(\alpha)}$ are given in Ref. [34] for light and medium nuclei, and in Ref. [35] for heavy nuclei.

The integrals I_0 for three typical nuclei ($^{40}_{20}$Ca, $^{72}_{32}$Ge, and $^{208}_{82}$Pb) are presented as a function of m_1 in Fig. 1. The values of the harmonic oscillator parameter b used are: $b=1.849$ fm for Ca, $b=2.039$ fm for Ge, and $b=2.434$ fm for Pb. We see that for light nuclei the modification of the cross section by the inclusion of the form factor is small. For heavy nuclei and massive m_1 the form factor has a dramatic effect on the cross section and may decrease it by a factor of about 5. The integral I_1 is even more suppressed but it is less important (see Ref. [35]).

B. The spin matrix element

The other terms in Eq. (13) describe the spin dependence of the cross section in terms of the $I_{\rho\rho'}$, with $\rho,\rho'=0,1$. The latter integrals result by following the standard procedure of the multipole expansion of the $e^{-i\mathbf{q}\cdot\mathbf{r}}$ in Eq. (9) and are defined by

$$I_{\rho\rho'}(q_0^2)=2\int_0^1\xi d\xi\sum_{\lambda,\kappa}\frac{\Omega_\rho^{(\lambda,\kappa)}(q_0^2\xi^2)}{\Omega_\rho(0)}\frac{\Omega_{\rho'}^{(\lambda,\kappa)}(q_0^2\xi^2)}{\Omega_{\rho'}(0)}, \quad \rho,\rho'=0,1, \quad (21)$$

where we have made the identification

$$\Omega_\rho^{(0,1)}=\Omega_\rho(q_0^2\xi^2)=(2J_i+1)^{-1/2}\left\langle J_f\left|\left|\sum_{j=1}^{A}j_0(q_0\xi r_j)\omega_\rho(j)\sigma(j)\right|\right|J_i\right\rangle, \quad \rho=0,1, \quad (22)$$

with $\omega_0(j)=1$ and $\omega_1(j)=\tau_3(j)$. In general,

$$\Omega_\rho^{(\lambda,\kappa)}=(2J_i+1)^{-1/2}\sqrt{4\pi}\left\langle J_f\left|\left|\sum_{j=1}^{A}[\mathbf{Y}^\lambda(\hat{\mathbf{r}}_j)\otimes\sigma(j)]^\kappa j_\lambda\times(q_0\xi r_j)\omega_\rho(j)\right|\right|J_i\right\rangle, \quad \rho=0,1. \quad (23)$$

With the above expressions we have managed to separate the elementary parameters f_A^0 and f_A^1 from the nuclear parameters.

We warn the reader that the integrals of Eq. (21) are normalized to unity as $q\rightarrow0$, i.e., $I_{\rho\rho'}(q_0=0)=1$. This normalization is different from that found in the previous literature. In the above limit ($q_0=0$) the spin matrix element takes the simple expression

$$|\mathbf{J}|^2=|f_A^0\Omega_0(0)+f_A^1\Omega_1(0)|^2.$$

The spin matrix elements, unfortunately, depend in general rather sensitively on the details of the nuclear structure, which is included in the integrals $I_{\rho\rho'}(q_0^2)$ [see Eq. (21)]. As we mentioned in the Introduction, the first attempt for quantitative description was based on the odd group method [27]. Subsequent shell model calculations demonstrated that the OGM was not adequate and showed that more elaborate calculations were needed [32,33]. Furthermore, since the matrix elements at $q=0$ are often quenched, the momentum dependence of the matrix elements was more important than it was naively expected. As a matter of fact, one has to include many configurations to accommodate all multipoles, which in complex nuclei such as ^{29}Si and ^{73}Ge, result in very large Hilbert spaces. It will be, therefore, a very difficult task to substantially improve the calculations of Refs. [32,33].

Among the targets which have been considered for LSP detection, ^{207}Pb stands out as an important candidate. The spin matrix element of this nucleus has not been evaluated, since one expects the relative importance of the spin versus the coherent mode to be more important on light nuclei. But, as we have mentioned, the spin matrix element in the light systems is quenched. On the other hand, the spin matrix element of ^{207}Pb, especially the isoscalar one, does not suffer from unusually large quenching, as is known from the study of the magnetic moment. Thus, we view it as a good theoretical laboratory since (i) it is believed to have simple structure, one $2p_{1/2}$ neutron hole outside the doubly magic nucleus ^{208}Pb and (ii) because of its low angular momentum, only two multipoles $\lambda=0$ and $\lambda=2$ with a J rank of $\kappa=1$ can contribute even at large momentum transfers. One can thus view the information obtained from this simple nucleus as complementary to that of ^{73}Ge, which has a very complex nuclear structure.

To a good approximation [36] the ground state of the $^{207}_{82}$Pb nucleus can be described as a $2p_{1/2}$ neutron hole in the $^{208}_{82}$Pb closed shell. Then, for $\lambda=0$ one finds

$$\Omega_0(\mathbf{q})=-(1/\sqrt{3})F_{2p}(\mathbf{q}^2), \quad \Omega_1(\mathbf{q})=(1/\sqrt{3})F_{2p}(\mathbf{q}^2), \tag{24}$$

and

$$I_{00}=I_{01}=I_{11}=2\int_0^1 \xi[F_{2p}(q^2)]^2 d\xi. \tag{25}$$

Even though the probability of finding a pure $2p_{1/2}$ neutron hole in the $(\frac{1}{2})^-$ ground state of $^{207}_{82}$Pb is greater than 95%, the ground-state magnetic moment is quenched due to the 1^+ p-h excitation involving the spin orbit partners. Hence, we expect a similar suppression of the isovector spin matrix elements. Thus, we write

$$\begin{aligned}|(\tfrac{1}{2})^-\rangle_{gs}=C_0&|(2p_{1/2})^{-1}\rangle\\ &\times[1+C_1|[0i_{11/2}(n)(0i_{13/2})^{-1}(n)]1^+\rangle\\ &+C_2|[0h_{9/2}(p)(0h_{11/2})^{-1}(p)]1^+\rangle+\cdots].\end{aligned} \tag{26}$$

Because of angular momentum and parity selection rules, we have $\kappa=1$ and $\lambda=0,2$. Retaining terms which are at most linear in the coefficients C_1, C_2 we obtain the following:

(i) $\lambda=0$.

$$\begin{aligned}\Omega_0(\mathbf{q})=C_0^2\{&F_{2p}(q^2)/\sqrt{3}-8[(7/13)^{1/2}C_1F_{0i}(q^2)\\ &+(5/11)^{1/2}C_2F_{0h}(q^2)]\},\end{aligned} \tag{27}$$

$$\begin{aligned}\Omega_1(\mathbf{q})=-C_0^2\{&F_{2p}(q^2)/\sqrt{3}-8[(7/13)^{1/2}C_1F_{0i}(q^2)\\ &-(5/11)^{1/2}C_2F_{0h}(q^2)]\},\end{aligned} \tag{28}$$

where

$$F_{nl}(q^2)=e^{-q^2b^2/4}\sum_{\mu=0}^{N_{\max}}\gamma_\mu^{(nl)}(qb)^{2\mu}. \tag{29}$$

The coefficients $\gamma_\mu^{(nl)}$ are given in Ref. [35] and the coefficients C_0, C_1, and C_2 were obtained by diagonalizing the Kuo-Brown G matrix [37,38] in a model space of $2h$-$1p$ configurations. They are given by

$$C_0=0.973350, \quad C_1=0.005295, \quad C_2=-0.006984.$$

We also find

$$\Omega_0(0)=-(1/\sqrt{3})(0.95659) \quad \text{(small quenching)}, \tag{30}$$

$$\Omega_1(0)=(1/\sqrt{3})(0.83296) \quad \text{(sizable quenching)}. \tag{31}$$

The amount of retardation of the total matrix element depends on the values of f_A^0 and f_A^1.

(ii) $\lambda=2$.

In this case, in addition to the leading $(2p_{1/2})^{-1}$ configuration, the first leading correction to the nuclear matrix element is linear in the mixing coefficients $C_{j_1j_2}$ appearing in the expression

$$\begin{aligned}|(\tfrac{1}{2})^{-1}\rangle=C_0\Big\{&|(2p_{1/2})^{-1}(n)\rangle\\ &+\sum_{j_1j_2}C_{j_1j_2}|(2p_{1/2})^{-1}(n);(j_1^{-1}j_2)J_{12}=1;\tfrac{1}{2}\rangle\Big\},\end{aligned} \tag{32}$$

i.e.,

$$\begin{aligned}\Omega_\rho^{(2,1)}=\frac{C_0^2}{\sqrt{2J_i+1}}\Big\{&2\sum_{j_1j_2}C_{j_1j_2}G(j_1,j_2,\rho)\\ &\times\langle n_1l_1|j_2(q_0\xi r)|n_2l_2\rangle\\ &+(-1)^\rho\langle(2p_{1/2})^{-1}||T^k||(2p_{1/2})^{-1}\rangle\Big\},\end{aligned} \tag{33}$$

where $G(j_1,j_2,\rho)$ are isospin-dependent geometrical factors which can be evaluated by standard techniques. The radial integrals $\langle n_1l_1|j_2(q_0\xi r)|n_2l_2\rangle$ can be cast in the form of Eq. (29), but they depend on two single-particle quantum numbers (see Appendix). The relevant coefficients used in the present work are given in Ref. [35].

Notice, however, that unlike the $\lambda=0$ case, many amplitudes can contribute if the quadrupole modes coupled to the single-hole wave function are admixed in the ground state of the system.

In the simple model of Ref. [36], in addition to the C_1, C_2 encountered above, one needs the amplitudes of the two additional configurations, $0j_{13/2}0j_{11/2}(n)$ and $0i_{11/2}0g_{9/2}(p)$, which are $C_3=0.000239$ and $C_4=-0.000642$. Obviously, this is a simplification, since one should consider the spin giant quadrupole resonance (GQR), which may have a small admixture in the ground state of the nucleus but a very large transition matrix element. Such a detailed calculation including all $2\hbar\omega$ excitations is in progress and it will be reported elsewhere.

Using Eqs. (21), (27), (28), and (33), we can evaluate the integrals I_{00}, I_{01}, and I_{00} for ^{207}Pb. The results for I_{11} are presented in Fig. 2 (the other two are practically indistinguishable, see Ref. [35]). In Fig. 2(a) I_{11} is plotted as a function of the LSP mass while in Fig. 2(b) it is plotted (together with I_0) as a function of the parameter u given in Eq. (18). We see that for a heavy nucleus and high LSP mass the momentum transfer dependence of the spin monopole $(\lambda=0)$ matrix elements is quite large. It is, however, to a large extent, neutralized by the spin quadrupole $(\lambda=2)$. So, the overall effect is not dramatic for LSP mass less than 100 GeV. As a matter of fact, from Fig. 2(b) we see that the retardation of the spin matrix element is quite a bit less than that of the coherent mode for almost all values of u.

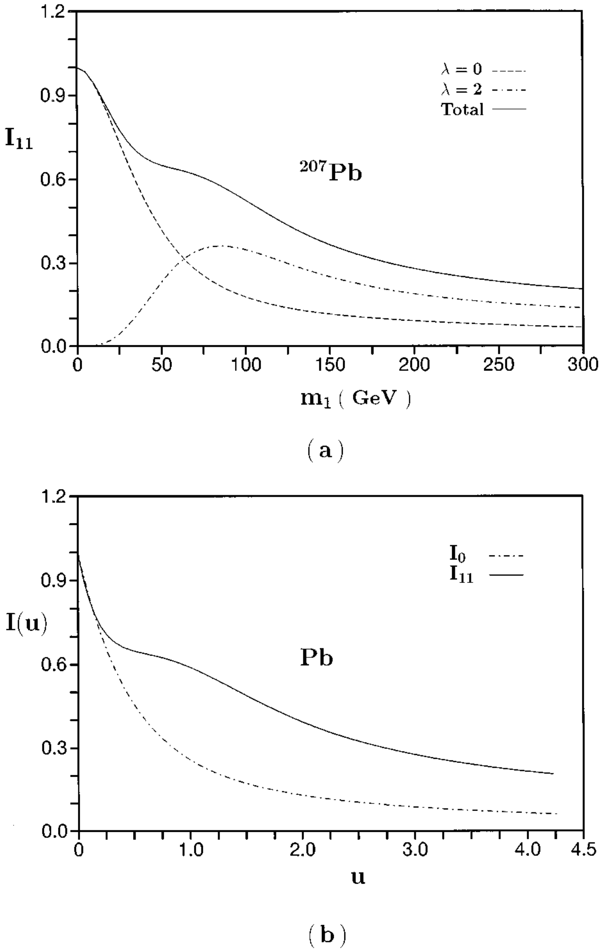

FIG. 2. (a) Plot of the integrals I_{11} as a function of the LSP mass m_1. This integral gives the spin contribution to the LSP-nucleus total cross section for $^{207}_{82}$Pb [for the definition see Eq. (21)] in the model described in Sec. III. The integrals I_{00} and I_{01} are similar. (b) Plot of the integrals $I_{11}(u)$ and $I_0(u)$ for Pb, where u is given by Eq. (18). Note that I_{11} is quite a bit less retarded compared to I_0.

IV. CONVOLUTION OF THE CROSS SECTION WITH THE VELOCITY DISTRIBUTION

The cross sections which would be given from an LSP detector participating in the revolution of the Earth around the Sun would appear retarded. In this section we are going to study this effect by using the method of folding. To this aim let us assume that the LSP is moving with velocity v_z with respect to the detecting apparatus. Then, the detection rate for a target with mass m is given by

$$\frac{dN}{dt}=\frac{\rho(0)}{m_1}\frac{m}{Am_p}|v_z|\sigma(v), \tag{34}$$

where $\rho(0)=0.3$ GeV/cm^3 is the LSP density in our vicinity. This density has to be consistent with the LSP velocity distribution. Such a consistent choice can be a Maxwell distribution

$$f(v')=(\sqrt{\pi}v_0)^{-3}e^{-(v'/v_0)^2}, \tag{35}$$

provided that [30]

$$v_0=\sqrt{(2/3)\langle v^2\rangle}=220 \text{ km /s}. \tag{36}$$

For our purposes it is convenient to express the above distribution in the laboratory frame; i.e.,

$$f(\mathbf{v},\mathbf{v}_E)=(\sqrt{\pi}v_0)^{-3}e^{-(\mathbf{v}+\mathbf{v}_E)^2/v_0^2}, \tag{37}$$

where $\mathbf{v}_E$ is the velocity of the Earth with respect to the center of the distribution. Choosing a coordinate system in which $\hat{\mathbf{x}}_2$ is the axis of the galaxy, $\hat{\mathbf{x}}_3$ is along the Sun's direction of motion ($\mathbf{v}_0$), and $\hat{\mathbf{x}}_1=\hat{\mathbf{x}}_2\times\hat{\mathbf{x}}_3$, we find that the position of the axis of the ecliptic is determined by the angle $\gamma\approx 29.80$ (galactic latitude) and the azimuthal angle $\omega=186.3°$ measured on the galactic plane from the $\hat{\mathbf{x}}_3$ axis [39].

Thus, the axis of the ecliptic lies very close to the x_2-x_3 plane and the velocity of the Earth is

$$\mathbf{v}_E=\mathbf{v}_0+\mathbf{v}_1=\mathbf{v}_0+v_1(\sin\alpha\hat{\mathbf{x}}_1-\cos\alpha\cos\gamma\hat{\mathbf{x}}_2+\cos\alpha\sin\gamma\hat{\mathbf{x}}_3) \tag{38}$$

and

$$\mathbf{v}_0\cdot\mathbf{v}_1=v_0v_1\frac{\cos\alpha}{\sqrt{1+\cot^2\gamma\cos^2\omega}}\approx v_0v_1\sin\gamma\cos\alpha, \tag{39}$$

where v_0 is the velocity of the Sun around the center of the galaxy, v_1 is the speed of the Earth's revolution around the Sun, α is the phase of the Earth's orbital motion, $\alpha=2\pi(t-t_1)/T_E$, where t_1 is around the second of June and $T_E=1$ yr.

The mean value of the event rate of Eq. (34) is defined by

$$\left\langle\frac{dN}{dt}\right\rangle=\frac{\rho(0)}{m_1}\frac{m}{Am_p}\int f(\mathbf{v},\mathbf{v}_E)|v_z|\sigma(|\mathbf{v}|)d^3\mathbf{v}. \tag{40}$$

Then, we can write the counting rate as

$$\left\langle\frac{dN}{dt}\right\rangle=\frac{\rho(0)}{m_1}\frac{m}{Am_p}\sqrt{\langle v^2\rangle}\langle\Sigma\rangle, \tag{41}$$

where

$$\langle\Sigma\rangle=\int\frac{|v_z|}{\sqrt{\langle v^2\rangle}}f(\mathbf{v},\mathbf{v}_E)\sigma(|\mathbf{v}|)d^3\mathbf{v}. \tag{42}$$

Thus, taking the polar axis in the direction $\mathbf{v}_E$, we get

$$\langle\Sigma\rangle=\frac{4}{\sqrt{6\pi}v_0^4}\int_0^\infty v^3dv\int_{-1}^1|\xi|d\xi e^{-(v^2+v_E^2+2vv_E\xi)/v_0^2}\sigma(v) \tag{43}$$

or

$$\langle\Sigma\rangle=\frac{2}{\sqrt{6\pi}v_E^2}\int_0^\infty vdvF_0\left(\frac{2vv_E}{v_0^2}\right)e^{-(v^2+v_E^2)/v_0^2}\sigma(v), \tag{44}$$

with

$$F_0(\chi)=\chi\sinh\chi-\cosh\chi+1. \tag{45}$$

One can also write Eq. (44) as

$$\langle\Sigma\rangle=\left(\frac{2}{3}\right)^{1/2}\int_0^\infty \frac{v}{v_0} f_1(v)\sigma(v)dv, \tag{46}$$

with

$$f_1(v)=\frac{1}{\sqrt{\pi}}\frac{v_0}{v_E^2}F_0\left(\frac{2vv_E}{v_0^2}\right)e^{-(v^2+v_E^2)/v_0^2}. \tag{47}$$

In the case in which the first term in Eq. (45) becomes dominant, we get

$$f_1(v)=\frac{1}{\sqrt{\pi}}\frac{v}{v_0 v_E}\left\{\exp\left[-\frac{(v-v_E)^2}{v_0^2}\right]-\exp\left[-\frac{(v+v_E)^2}{v_0^2}\right]\right\} \tag{48}$$

in agreement with Eq. (8.15) of Ref. [7]. In Eq. (44) the nuclear parameters are implicit in the cross section $\sigma(v)$ given from Eq. (13). The nuclear physics dependence of $\langle\Sigma\rangle$ could be disentangled by taking note of the extra velocity dependence of the coherent vector contribution in $\sigma(v)$ and introducing the parameters

$$\delta=\frac{2v_E}{v_0}=0.27, \quad \psi=\frac{v}{v_0}, \quad u=u_0\psi^2, \tag{49}$$

where the quantity u_0 is the one entering the nuclear form factors of Eq. (17) for $v=v_0$, which in this case is given by

$$u_0=\frac{1}{2}\left(\frac{2\beta_0 m_1 c^2}{(1+\eta)}\frac{b}{\hbar c}\right)^2, \quad \beta_0=\frac{v_0}{c}. \tag{50}$$

Afterwards, we can write Eq. (44) as

$$\begin{aligned}\langle\Sigma\rangle=&\left(\frac{m_1}{m_p}\right)^2\frac{\sigma_0}{(1+\eta)^2}\Bigg\{A^2\Bigg[\langle\beta^2\rangle\left(f_V^0-f_V^1\frac{A-2Z}{A}\right)^2\\&\times\left(J_0-\frac{2\eta+1}{2(1+\eta)^2}J_1\right)+\left(f_S^0-f_S^1\frac{A-2Z}{A}\right)^2\tilde{J}_0\Bigg]\\&+[f_A^0\Omega_0(0)]^2J_{00}+2f_A^0f_A^1\Omega_0(0)\Omega_1(0)J_{01}\\&+[f_A^1\Omega_1(0)]^2J_{11}\Bigg\}.\end{aligned} \tag{51}$$

If we assume that $J_{00}=J_{01}=J_{11}$, as seems to be the case for ^{207}Pb, the spin-dependent part of Eq. (47) is reduced to the familiar expression $[f_A^0\Omega_0(0)+f_A^1\Omega_1(0)]^2J_{11}$, where the quantity in the square brackets represents the spin matrix element at $q=0$.

The parameters $\tilde{J}_0$, J_ρ, $J_{\rho\sigma}$ describe the scalar, vector, and spin part of the counting rate, respectively, and they are given by

$$\tilde{J}_0(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}\frac{e^{-\lambda^2}}{\lambda^2}\int_0^\infty \psi e^{-\psi^2}F_0(2\lambda\psi)I_0(u_0\psi^2)d\psi, \tag{52}$$

$$J_\rho(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}\frac{e^{-\lambda^2}}{\lambda^2}\int_0^\infty \psi^3 e^{-\psi^2}F_0(2\lambda\psi)I_\rho(u_0\psi^2)d\psi, \tag{53}$$

$$J_{\rho\sigma}(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}\frac{e^{-\lambda^2}}{\lambda^2}\int_0^\infty \psi e^{-\psi^2}F_0(2\lambda\psi)I_{\rho\sigma}(u_0\psi^2)d\psi, \tag{54}$$

$$\lambda=\frac{v_E}{v_0}=[1+\delta\cos\alpha\sin\gamma+(\delta/2)^2]^{1/2}. \tag{55}$$

The parameters I_ρ, $I_{\rho\sigma}$ have been discussed in the previous section. The above integrals are functions of λ and u_0. The latter depends on v_0, the nuclear parameters, and the LSP mass. These integrals can only be done numerically. Since, however, λ is close to unity, we can expand in powers of δ and make explicit the dependence of these integrals on the Earth's motion. Thus,

$$\tilde{J}_0(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}B_1[\tilde{K}_0^{(0)}(u_0)+\delta\sin\gamma\cos\alpha\tilde{K}_0^{(1)}(u_0)], \tag{56}$$

$$J_\rho(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}B_2[K_\rho^{(0)}(u_0)+\delta\sin\gamma\cos\alpha K_\rho^{(1)}(u_0)], \tag{57}$$

$$J_{\rho\sigma}(\lambda,u_0)=\frac{2}{\sqrt{6\pi}}B_1[K_{\rho\sigma}^{(0)}(u_0)+\delta\sin\gamma\cos\alpha K_{\rho\sigma}^{(1)}(u_0)]. \tag{58}$$

The integrals $\tilde{K}_0^0$, K_ρ^0, and $K_{\rho\sigma}^0$ are normalized so that they become unity at $u_0=0$ (negligible momentum transfer). We find

$$B_1=\frac{1}{e}\int_0^\infty \psi e^{-\psi^2}F_0(2\psi)d\psi=\frac{1}{e}+2\nu\approx 1.860, \tag{59}$$

$$B_2=\frac{2}{3e}\int_0^\infty \psi^3 e^{-\psi^2}F_0(2\psi)d\psi=\frac{2}{3}\left(\frac{3}{e}+7\nu\right)\approx 4.220, \tag{60}$$

with

$$\nu=\int_0^1 e^{-t^2}dt\approx 0.747. \tag{61}$$

Furthermore,

$$\tilde{K}_0^l=\frac{1}{eB_1}\int_0^\infty \psi e^{-\psi^2}F_l(2\psi)I_0(u_0\psi^2)d\psi, \quad l=0,1, \tag{62}$$

$$K_\rho^l=\frac{2}{3eB_2}\int_0^\infty \psi^3 e^{-\psi^2}F_l(2\psi)I_\rho(u_0\psi^2)d\psi, \quad l=0,1, \tag{63}$$

$$K^l_{\rho\sigma}=\frac{1}{eB_1}\int_0^\infty \psi e^{-\psi^2}F_l(2\psi)I_{\rho\sigma}(u_0\psi^2)d\psi, \quad l=0,1, \tag{64}$$

with $F_0(\chi)$ given in Eq. (45) and

$$F_1(\chi)=2\left[\left(\frac{\chi^2}{4}+1\right)\cosh\chi-\chi\sinh\chi-1\right]. \tag{65}$$

The counting rate can thus be cast in the form

$$\left\langle\frac{dN}{dt}\right\rangle=\left\langle\frac{dN}{dt}\right\rangle_0(1+h\cos\alpha), \tag{66}$$

where $\langle dN/dt\rangle_0$ is the rate obtained from the $l=0$ multipole and h the amplitude of the oscillation, i.e., the ratio of the component of the multipole $l=1$ to that of the multipole $l=0$. Below (see also Tables III and IV) we compute separately the amplitude of oscillation for the scalar, vector, and spin parts of the event rate, i.e., the quantity $h=\delta\sin\gamma K^1(u_0)/K^0(u_0)$. Note the presence of the geometric factor $\sin\gamma=1/2$, which reduces the modulation effect.

In order to get some idea of the dependence of the counting rate on the Earth's motion, we will evaluate the above expressions at $u_0=0$. We get

$$\tilde{K}^0_0=K^0_{\rho\sigma}\approx K^0_\rho=1, \tag{67}$$

$$\tilde{K}^1_0=K^1_{\rho\sigma}=\frac{\nu}{1/e+2\nu}\approx 0.402, \tag{68}$$

$$K^1_\rho=\frac{3/(2e)+(11/2)\nu/2}{3/e+7\nu}\approx 0.736. \tag{69}$$

Thus, for $\sin\gamma\approx 0.5$

$$\tilde{J}_0\approx J_{\rho\sigma}=\frac{2}{\sqrt{6\pi}}1.860(1+0.054\cos\alpha)$$
$$=0.857(1+0.054\cos\alpha), \tag{70}$$

$$J_\rho=\frac{2}{\sqrt{6\pi}}4.220(1+0.099\cos\alpha)=1.944(1+0.099\cos\alpha). \tag{71}$$

We see that the modulation of the detection rate due to the Earth's motion is quite small ($h\approx 0.05$). The corresponding amplitude of oscillation in the coherent vector contribution, Eq. (70), is a bit bigger ($h\approx 0.10$). However, this contribution is suppressed due to the Majorana nature of LSP (through the factor β^2). The modulation due to the Earth's rotation is expected to be even smaller.

The exact K^l integrals, for the $l=0$ and $l=1$, are shown in Figs. 3(a)–3(c). The most important of these integrals, those of Eq. (62) associated with the scalar interaction, are shown in Fig. 3(a). In Fig. 3(b) we present the integrals of Eq. (63) for $\rho=0$ associated with the vector interaction (the integral for $\rho=1$ is analogous but it is less important). Finally, in Fig. 3(c) the integrals of Eq. (64) for $\rho=1$ and $\sigma=1$ are shown. The others are practically indistinquishable from these and are not shown (see Ref. [35]).

Before closing this section we should mention that the folding procedure can also be applied in the differential rate in order to obtain the corresponding convoluted expression for $d\sigma/d\Omega$, i.e., before doing the angular integration in Eq. (10) and obtaining the total cross section equation (13). The resulting expressions are, however, a bit more complicated and they will not be given here.

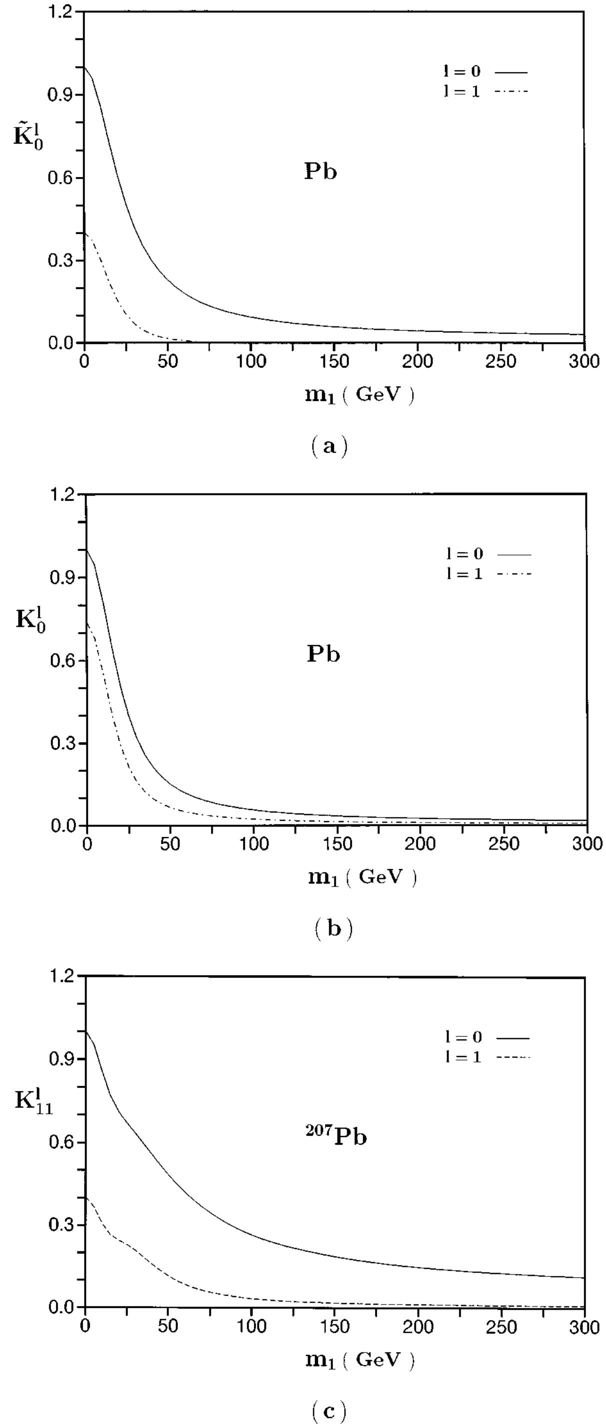

FIG. 3. Contributions of K integrals (for $l=0$ and $l=1$) defined in Eqs. (62)–(64) and entering the event rate due to Earth's revolution around the Sun: $\tilde{K}^l_0$ in (a), K^l_0 in (b), and K^l_{11} in (c). The other integrals K^l_{00} and K^l_{01} are similar to K^l_{11}.

V. RESULTS AND DISCUSSION

The main goal of the present work was the calculation of the cross sections for the scattering of LSP, a cold dark matter candidate, with nuclei. The coherent scattering was evaluated for three typical nuclei, Ca, Ge, and Pb, and the spin matrix element for ^{207}Pb. The momentum dependence of the matrix elements was taken into account. Special attention has been paid to evaluating the modulation of the event rates due to the Earth's motion. Our results are summarized as follows.

A. Coherent scattering

As we have seen in Sec. III, the coherent scattering depends on the isoscalar scalar f_S^0 and vector f_V^0 parameters. The latter is effectively multiplied by the average velocity $\langle\beta^2\rangle^{1/2}$ of the LSP due to its Majorana nature. These parameters depend on the SUSY model considered. They were evaluated in the allowed SUSY parameter space of Kane *et al.* [5]. The construction of the dominant scalar parameters suffers from additional uncertainties, which involve the step of going from the quark to the nucleon level. In other words, the results are very sensitive to the presence of quarks other than u and d in the nucleon. Three such choices indicated by A, B, C are presented in Table I.

From a nuclear structure input point of view, the coherent scattering does not depend on the details of the nuclear wave function. It does, however, depend on the nuclear density, i.e., the assumed form factor, for fairly massive LSP's and correspondingly heavy nuclei. This is because the momentum transfer in such cases can be quite high (by nuclear standards) even though the energy transfer is small. The form factors used were as realistic as possible [34]. The inclusion of the form factor results in sizable retardation of the cross section which, for case No. 1, can be a factor of 3 for Ge, and a factor of 14 for Pb. This conclusion is in agreement with previous estimates although they do not include detailed calculations for heavy nuclear targets [32].

B. Spin contribution

As we have mentioned in the Introduction, the relative importance of the spin matrix element is expected to decrease as one goes to the heavier systems vis-à-vis the coherent scattering. We have seen, however, that the increase due to the mass number in the coherent scattering is offset by the decline due to form factor. Thus, the spin contribution may not be completely negligible.

For the spin matrix contribution of ^{207}Pb, we find that its ground state (a $\frac{1}{2}^-$ state), in a $1h$ and $2h$-$1p$ model space, is more than 95% a $2p_{1/2}$ neutron configuration. We have evaluated the spin matrix elements up to terms linear in the small amplitudes. Out of the 250 components, only two of them contribute to the $\lambda=0$ multipole while two more contribute to the $\lambda=2$ multipole. Thus, we find that the isoscalar static matrix element suffers from very little retardation, while the isovector matrix element is reduced by 17%. Since, in the SUSY parameter space we have used, the isovector coupling f_A^1 is larger, this results in a similar retardation of the total matrix element. The isoscalar matrix element has other uncertainties. In passing from the quark to the nucleon level the amplitude must be multiplied by a factor g_A which ranges from 1 in the naive quark model (NQM) to 0.12 if extracted from the EMC data. In our calculations we considered both of these extremes. Also, since the dominant configuration is of the neutron variety, the isoscalar tends to subtract from the isovector, but this is not so dramatic in our case since the isoscalar is smaller in absolute value, especially in the EMC case.

Our spin matrix elements at $q=0$ are

$$[\Omega_1(0)]^2=0.231 \quad \text{and} \quad [\Omega_0(0)]^2=0.301.$$

These are a bit larger than the values

$$[\Omega_1(0)]^2\approx[\Omega_0(0)]^2=0.2 \quad (\tfrac{1}{2}^{+}\ ^{29}\text{Si}),$$

extracted from Fig. 3 of Ref. [32]. They are, however, smaller than the values

$$[\Omega_1(0)]^2=1.00 \quad \text{and} \quad [\Omega_0(0)]^2=1.03 \quad (\tfrac{9}{2}^{+}\ ^{73}\text{Ge}),$$

extracted from Fig. 3 of Ref. [32].

As it has been found in Eq. (13), the momentum dependence of the spin part can be described in terms of three integrals I_{00}, I_{01}, I_{11}, where the subscripts indicate the isospin channels. In our case, these integrals receive contributions from two multipoles, $\lambda=0$ (spin monopole) and $\lambda=2$ (spin quadrupole). They were judiciously normalized to unity at $q=0$. When so normalized these integrals are approximately equal (see Fig. 3 and also Ref. [35]). We notice that the monopole contribution falls with momentum transfer, as expected, quite fast, in fact, for a large LSP mass. On the contrary, the quadrupole contribution starts out at zero and keeps increasing up to about 90 GeV. As a result, its contribution in this mass regime is crucial, since it tends to partly compensate for the suppression of the monopole term. We expect these trends to persist even in the most elaborate calculations which include the giant quadrupole resonance (GQR) and are currently under way.

As a consequence, the momentum transfer suppression is small (a factor of about 4 for $m_1=100$ GeV). Thus, the above two matrix elements take the values 0.120 and 0.153 for the purely isovector and isoscalar channels. From Table IV of Ref. [32] one extracts at $y=0.174$ the values 0.335 and 0.402. Thus, the spin matrix element Ω^2 in ^{207}Pb is less than three times smaller compared to that of Ref. [32] for ^{73}Ge. On the other hand, the corresponding matrix elements for the $A=29$ and $A=207$ are similar. When it comes to the cross section the spin contribution in the case of the $A=207$ and $A=73$ may be similar due to the presence of η in Eq. (13), which favors the $A=207$ system by a factor of about 2.6 for $m_1=100$ GeV.

C. Convoluted rates

For the coherent cross section in Table III we present results for the parameters $\langle dN/dt\rangle_0$ and h for three typical and experimentally interesting nuclei, Ca, Ge, and Pb. From this table we see that the event rate is highest when the intermediate particles and the LSP's are lightest ($m_1=27$ GeV, case No. 2). We notice that even within the allowed parameter space, the event rate may vary by two orders of magnitude. We also notice that, with the possible exception

TABLE III. The quantity $\langle dN/dt \rangle_0$ in $\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$ and the parameter h (oscillation due to the Earth's motion around the Sun) for the coherent vector and scalar contributions. For the definition of A, B, C, see text. NFF and WNFF stand for nuclear form factor and without nuclear form factor, respectively.

		Vector contribution		Scalar contribution			
		$\left\langle \frac{dN}{dt} \right\rangle_0$ ($\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$)	h	$\left\langle \frac{dN}{dt} \right\rangle_0$ ($\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$)			h
	Solution			Model A	Model B	Model C	
Pb	No. 1 (NFF)	0.264×10^{-3}	0.029	0.151×10^{-3}	0.220	0.450	-0.002
	No. 1 (WNFF)	0.378×10^{-2}	0.102	0.212×10^{-2}	3.087	6.322	0.054
	No. 2 (NFF)	0.162×10^{-3}	0.039	0.410×10^{-1}	142.860	128.660	0.026
	No. 2 (WNFF)	0.332×10^{-3}	0.102	0.872×10^{-1}	303.390	273.220	0.054
	No. 3 (NFF)	0.895×10^{-3}	0.038	0.200×10^{-3}	0.377	0.602	-0.001
	No. 3 (WNFF)	0.970×10^{-2}	0.102	0.220×10^{-2}	4.114	6.576	0.054
Ge	No. 1 (NFF)	0.151×10^{-3}	0.043	0.779×10^{-4}	0.120	0.245	0.017
	No. 1 (WNFF)	0.518×10^{-3}	0.102	0.230×10^{-3}	0.353	0.723	0.054
	No. 2 (NFF)	0.527×10^{-4}	0.057	0.146×10^{-1}	51.724	46.580	0.041
	No. 2 (WNFF)	0.708×10^{-4}	0.102	0.196×10^{-1}	69.506	62.595	0.054
	No. 3 (NFF)	0.481×10^{-3}	0.045	0.101×10^{-3}	0.198	0.316	0.020
	No. 3 (WNFF)	0.144×10^{-2}	0.102	0.267×10^{-3}	0.525	0.839	0.054
Ca	No. 1 (NFF)	0.790×10^{-4}	0.053	0.340×10^{-4}	0.055	0.114	0.037
	No. 1 (WNFF)	0.135×10^{-3}	0.102	0.520×10^{-4}	0.085	0.174	0.054
	No. 2 (NFF)	0.264×10^{-3}	0.060	0.612×10^{-2}	22.271	20.056	0.048
	No. 2 (WNFF)	0.307×10^{-3}	0.102	0.704×10^{-2}	25.601	23.055	0.054
	No. 3 (NFF)	0.241×10^{-3}	0.053	0.435×10^{-4}	0.090	0.144	0.038
	No. 3 (WNFF)	0.388×10^{-3}	0.102	0.642×10^{-4}	0.133	0.212	0.054

of the not so realistic model A, the Higgs contribution becomes dominant. This is even more so in models where quarks other than u and d are present in the nucleus with appreciable probabilities, due to their large masses. In the most favorable cases, one may have more than 140 events per year per kilogram of Pb target. We notice that the amplitude due to the Earth's annual motion depends on m_1 and A. In Pb it is less than $\pm5\%$.

Finally, we should mention that, for cases No. 1 and No. 3 (massive LSP's), the rate due to the nuclear form factor can be reduced by a factor of approximately 6. Because the form factor dependence is more pronounced in heavy nuclei, if the LSP turns out to be massive, the benefit of going to heavier targets is somewhat diminished.

TABLE IV. The spin contribution in the LSP-nucleus scattering in ^{207}Pb for two cases: EMC data and NQM model (see Sec. II). For other notations see caption of Table III.

	EMC Data		NQM Model	
Solution	$\left\langle \frac{dN}{dt} \right\rangle_0$ ($\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$)	h	$\left\langle \frac{dN}{dt} \right\rangle_0$ ($\mathrm{yr}^{-1}\,\mathrm{kg}^{-1}$)	h
No. 1 (NFF)	0.285×10^{-2}	0.014	0.137×10^{-2}	0.015
No. 1 (WNFF)	0.130×10^{-1}	0.054	0.551×10^{-2}	0.054
No. 2 (NFF)	0.041	0.046	0.384×10^{-2}	0.056
No. 2 (WNFF)	0.062	0.054	0.405×10^{-2}	0.054
No. 3 (NFF)	0.012	0.016	0.764×10^{-2}	0.017
No. 3 (WNFF)	0.047	0.054	0.269×10^{-1}	0.054

The detection rates obtained, after folding the spin-induced cross section, for ^{207}Pb are shown in Table IV. The folding makes the spin dependence even less sensitive on the momentum transfer. The event rate, however, remains much smaller than the coherent scattering for models B and C and even for model A in the case No. 2. These results may suggest that, in the SUSY parameter space given in Ref. [5], the spin-induced LSP-nucleus scattering may not be detectable. The spin contribution, however, may be somewhat enhanced in models in which the LSP is primarily a Higgsino [40].

In order to estimate the total effect of the nuclear density on the event rates, in Tables III and IV the results ignoring the nuclear form factor (WNFF) are shown.

VI. SUMMARY AND CONCLUSIONS

In the present work we have performed calculations of the event rate for LSP-nucleus scattering for three typical experimentally interesting nuclei, i.e., Ca, Ge, and Pb. The three basic ingredients of our calculation were the input SUSY parameters, a quark model for the nucleon, and the structure of the nuclei involved.

The input SUSY parameters were calculated in a phenomenologically allowed parameter space (cases No. 1, No. 2, No. 3 of Table I) as explained in the text. In going from the quark to the nucleon level, the quark structure of the nucleon was essential, in particular its content in quarks other than

u and d. For the scalar interaction we considered three models (labeled A, B, C in Table I) as described in the text. For the isovector axial coupling one encounters the so-called nucleon spin crisis. Again, we considered two possibilities depending on the assumed portion of the nucleon spin which is due to the quarks (indicated by EMC and NQM in Table I) as described in the text.

As regards nuclear structure, we employed as detailed as feasible nuclear wave functions. For the coherent part (scalar and vector), we used realistic nuclear form factors. For the ^{207}Pb system we also computed the spin matrix element. The ground-state wave function was obtained by diagonalizing the nuclear Hamiltonian in a $2h$-$1p$ space which is standard for this doubly magic nucleus. The momentum dependence of the matrix elements was taken into account and all relevant multipoles were retained (in this system one encounters only two, $\lambda=0$ and $\lambda=2$, due to selection rules).

From our discussion in the previous section we conclude that, even though the spin matrix elements Ω^2 are a factor of 3 smaller than those for ^{73}Ge obtained in Ref. [32], their contribution to the cross section is almost the same (at least for LSP masses around 100 GeV).

Finally, the obtained results were convoluted with a suitable Maxwell-Boltzmann velocity distribution of the LSP's. This convolution was necessary to partially neutralize the form factor retardation, but mainly to compute the modulation of the event rate due to the Earth's motion. We find that in almost all cases the event rate due to the Earth's revolution around the Sun is very small, less than $\pm 5\%$ around its average value. The event rates thus obtained (see Tables III and IV) are, unfortunately, sensitive to the input parameters.

The inclusion of the nuclear form factor significantly retards the event rates for heavy nuclei ($A>100$) and fairly massive LSP's ($m_1>100$ GeV). However, this retardation does not outweigh completely the advantages of using a heavy target. For the spin matrix elements the form factor retardation of the usual $\lambda=0$ multipole is partially neutralized by the enhancement of the $\lambda=2$ multipole.

From the data of Tables III and IV we see that it is possible to have detectable rates (>20 yr^{-1} kg^{-1}) only for case No. 2 and the realistic nucleon models B and C, associated with the scalar Higgs exchange term. In all other cases, including the spin contribution, the calculated event rates are too small.

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APPENDIX

For the spin contribution of ^{207}Pb, we have to calculate the matrix elements of the operator

$$T^\kappa=\sqrt{4\pi}\, j_\lambda(qr)[Y^\lambda(\hat{\mathbf{q}})\otimes\sigma]^\kappa, \tag{A1}$$

where $\kappa=1$ and $\lambda=0,2$ for the states [see Eqs. (26) and (32)]

$$|\Psi_j\rangle=C_0|j^{-1}(n)\rangle\left\{1+\sum_{j_1j_2}C_{j_1j_2}|j_1(n)j_2^{-1}(n)\kappa\rangle+\sum_{j_3j_4}C_{j_3j_4}|j_3(p)j_4^{-1}(p)\kappa\rangle\right\} \tag{A2}$$

p for the proton component and n for the neutron component, or (to the first order) the matrix elements

$$|\Psi_j||T^\kappa||\Psi_j\rangle=C_0^2\left\{\pm(-)^{\kappa+1}\langle j||T^\kappa||j\rangle+2\left(\frac{2j+1}{2\kappa+1}\right)^{1/2}\times\left[\pm\sum_{j_1j_2}C_{j_1j_2}\langle j_1||T^\kappa||j_2\rangle+\sum_{j_3j_4}C_{j_3j_4}\langle j_3||T^\kappa||j_4\rangle\right]\right\}. \tag{A3}$$

The $+$ sign is for isoscalar and the $-$ for isovector matrix elements. The reduced matrix elements $\langle j_1||T^\kappa||j_2\rangle$ are given in Ref. [41]. The relevant radial matrix elements $\langle n_1l_1|j_l(qr)|n_2l_2\rangle$ for harmonic oscillator basis can be witten in a compact way:

$$\langle n_1l_1|j_l(qr)|n_2l_2\rangle=e^{-\chi}\sum_{\kappa=0}^{\kappa_{\max}}\varepsilon_\kappa\chi^{\kappa+l/2},\quad \chi=(qb)^2/4, \tag{A4}$$

where

$$\kappa_{\max}=n_1+n_2+m,\quad m=(l_1+l_2-l)/2,$$

$$|l_1-l_2|\leqslant l\leqslant l_1+l_2.$$

The coefficients $\varepsilon_\kappa(n_1l_1,n_2l_2,l)$ are given by

$$\varepsilon_\kappa=\left[\frac{\pi n_1!n_2!}{4\Gamma(n_1+l_1+\frac{3}{2})\Gamma(n_2+l_2+\frac{3}{2})}\right]^{1/2}\times\sum_{\kappa_1=\phi}^{n_1}\sum_{\kappa_2=\sigma}^{n_2}n!\Lambda_{\kappa_1}(n_1l_1)\Lambda_{\kappa_2}(n_2l_2)\Lambda_\kappa(nl), \tag{A5}$$

where

$$n=\kappa_1+\kappa_2+m,\quad \Lambda_\kappa(nl)=\frac{(-)^\kappa}{\kappa!}\binom{n+l+1/2}{n-k}$$

(see also Ref. [41]) and

$$\phi=\begin{cases}0, & \kappa-m-n_2\leqslant 0,\\ \kappa-m-n_2, & \kappa-m-n_2>0,\end{cases}$$

$$\sigma=\begin{cases}0, & \kappa-m-\kappa_1\leqslant 0,\\ \kappa-m-\kappa_1, & \kappa-m-\kappa_1>0.\end{cases}$$

For the coefficients ε_κ used in the present work see Ref. [35].

[1] P. F. Smith and J. D. Lewin, Phys. Rep. **187**, 203 (1990).
[2] M. Roman-Robinson, in *Cosmological Dark Matter*, Proceedings of the International School, Valencia, Spain, 1993, edited by J. W. F. Valle and A. Perez (World Scientific, Singapore, 1994), p. 7.
[3] C. S. Frenk, in *Cosmological Dark Matter* [2], p. 65; J. R. Primack, *ibid.* p. 81; J. R. Primack, D. Seckel, and B. Sadoulet, Annu. Rev. Nucl. Part. Sci. **38**, 751 (1988).
[4] MACHO Collaboration, D. P. Bennett *et al.*, Proceedings of the 5th Annual Maryland Conference, edited by S. Holt, 1995, Report No. astro-ph/9606012 (unpublished); G. F. Smoot *et al.*, (COBE data) Astrophys. J. **396**, L1 (1992); MACHO Collaboration, C. Alcock *et al.*, Phys. Rev. Lett. **74**, 2867 (1995); Report No. astro-ph/9606165.
[5] G. L. Kane, C. Kolda, L. Roszkowski, and J. D. Wells, Phys. Rev. D **49**, 6173 (1994); E. Diehl, G. L. Kane, C. Kolda, and J. D. Wells, ''Theory, Phenomenology and prospects of Detection of Supersymmetric Dark Matter,'' Report No. UM-TH-9438, hep-ph 9502339.
[6] V. Barger, M. S. Berger, and P. Ohmann, Phys. Rev. D **49**, 4908 (1994); D. J. Castaño, E. J. Piard, and P. Ramond, *ibid.* **49**, 4882 (1994).
[7] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).
[8] G. K. Leontaris and N. D. Tracas, Phys. Lett. B **342**, 163 (1995); E. G. Floratos, G. K. Leontaris, and S. Lola, *ibid.* **365**, 149 (1996).
[9] D. B. Cline [2], p. 93.
[10] F. von Feilitzch, *Detectors for Dark Matter Interactions Operated at Low Temperatures*, International Workshop on Neutrino Telescopes, Vanezia, 1990, edited by Milla Baldo Ceolin (Padova University Press, Padova, Italy, 1990), p. 257.
[11] M. W. Goodman and E. Witten, Phys. Rev. D **31**, 3059 (1985).
[12] K. Griest, Phys. Rev. Lett. **61**, 666 (1988); Phys. Rev. D **38**, 2357 (1988); **39**, 3802 (1989).
[13] J. Ellis and R. A. Flores, Phys. Lett. B **263**, 259 (1991); **300**, 175 (1993); Nucl. Phys. **B400**, 25 (1993); J. Ellis and L. Roszkowski, Phys. Lett. B **283**, 252 (1992).
[14] V. A. Bednyakov, H. V. Klapdor-Kleingrothaus, and S. G. Kovalenko, Phys. Lett. B **329**, 5 (1994).
[15] M. Drees and M. M. Nojiri, Phys. Rev. D **48**, 3483 (1993); **47**, 4226 (1993).
[16] J. Engel, Phys. Lett. B **264**, 114 (1991).
[17] J. D. Vergados, J. Phys. G **22**, 253 (1996).
[18] EMC Collaboration, J. Ashman *et al.*, Nucl. Phys. **B328**, 1 (1989).
[19] R. L. Jaffe and A. Manohar, Nucl. Phys. **B337**, 509 (1990).
[20] P. M. Gensini, ''In Search of the Quark Spins in the Nucleon: A Next-to-Next-to Leading Order in QCD Analysis of the Ellis-Jaffe Sum Rule,'' Report No. DFUPG-95-GEN-01, hep-ph/9512440 (unpublished).
[21] J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. B **257**, 83 (1991).
[22] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991).
[23] T. P. Cheng, Phys. Rev. D **38**, 2869 (1988).
[24] H-Y. Cheng, Phys. Lett. B **219**, 347 (1989).
[25] A. K. Drukier, K. Freese, and D. N. Spergel, Phys. Rev. D **33**, 3495 (1986).
[26] K. Freese, J. Frieman, and A. Gould, Phys. Rev. D **37**, 3388 (1988).
[27] J. Engel and P. Vogel, Phys. Rev. D **40**, 3132 (1989).
[28] A. F. Pacheco and D. Strottman, Phys. Rev. D **40**, 2131 (1989).
[29] J. Engel, S. Pittel, and P. Vogel, Int. J. Mod. Phys. E **1**, 1 (1992).
[30] F. Iachello, L. M. Krauss, and G. Maino, Phys. Lett. B **254**, 220 (1991).
[31] M. A. Nikolaev and H. V. Klapdor-Kleingrothaus, Z. Phys. A **345**, 373 (1993).
[32] M. T. Ressell *et al.*, Phys. Rev. D **48**, 5519 (1993).
[33] V. I. Dimitrov, J. Engel, and S. Pittel, Phys. Rev. D **51**, R291 (1995).
[34] T. S. Kosmas and J. D. Vergados, Nucl. Phys. **A536**, 72 (1992).
[35] T. S. Kosmas and J. D. Vergados, in Proceedings of the 6th Hellenic Symposium on Nuclear Physics, Athens, 1995 (in press).
[36] J. D. Vergados, Phys. Lett. **36B**, 12 (1971); **34B**, 121 (1971).
[37] T. T. S. Kuo and G. E. Brown, Nucl. Phys. **85**, 40 (1966).
[38] G. H. Herling and T. T. S. Kuo, Nucl. Phys. **A181**, 113 (1972).
[39] C. Alissandrakis (private communication).
[40] G. L. Kane and J. D. Wells, Phys. Rev. Lett. **76**, 4458 (1996).
[41] T. S. Kosmas, J. D. Vergados, O. Civitarese, and A. Faessler, Nucl. Phys. **A570**, 637 (1994).