# Jet algorithms and top quark mass measurement

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Mass measurements of objects that decay into hadronic jets, such as the top quark, are shown to be improved by using a variant of the  $k_i$  jet algorithm in place of standard cone algorithms. The possibility and importance of better estimating the neutrino component in tagged *b* jets is demonstrated. These techniques will also be useful in the search for Higgs boson  $\rightarrow b\overline{b}$ . [S0556-2821(97)03401-2]

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## I. INTRODUCTION

It is often necessary to measure the mass of an object that decays into hadronic jets. An example of current importance is the decay of the top quark  $t \rightarrow bW$ , where the b quark materializes as a jet and the W boson decays either leptonically or into two light-quark jets. The accuracy with which the jets can be measured governs the error in the top quark mass measurement, which is crucial to the study of electroweak physics—e.g., knowing  $m_t$  allows a logarithmic estimate of the Higgs boson mass in the minimal model. Accurate measurement of the jet decays of the W is also valuable here because good W mass resolution can reduce the combinatoric and other backgrounds in the analysis. Futhermore, the hadronically decaying W can provide an alternative measure of  $m_t$  based on the jet angles in the top rest frame [1]: these angles determine  $m_t/m_W$  in each event with errors that are largely independent from the errors of the traditional measure, so the two methods can be averaged to improve resolution. At the same time,  $t\bar{t}$  events offer a sample of hadronic W decays that can be compared against the known W mass to test the theoretical and experimental assumptions underlying all jet spectroscopy. This opportunity is unique because hadronic W decays are otherwise obscured by large QCD backgrounds and triggering problems [2,3].

A second important application of jet spectroscopy occurs in the search for Higgs boson  $\rightarrow b\overline{b}$ . A moderate improvement in dijet mass resolution has been shown to extend the range of possible discovery to  $m_{\text{Higgs}} \approx 80-100 \text{ GeV}/c^2$  in Fermilab Tevatron run II [4].

The important sources of error in jet spectroscopy are (1) QCD radiation and hadronization effects, (2) jet definitions, and (3) detector effects. We will compare these sources of error quantitatively, using Monte Carlo simulation events for which the true partonic momenta are known, and we will study the degree to which the jet finding algorithm can be improved. There is an interplay between the first two sources of error because acceptable jet algorithms differ from one another at next-to-leading order in  $\alpha_s$  and in the nonperturbative hadronization corrections they require. Previous top quark analyses [5] have used cone algorithms for jet defini-

tion [6,7]. But I will show in this paper that a particular version of the  $k_{\perp}$  successive recombination algorithm [8,9] instead promises superior results.

The detector effects studied here are generic ones that arise from the basic segmented calorimeter design of all contemporary detectors. Particular attention is paid to the unseen neutrino component of b jets, which is found to be significant and partially correctable. Dealing with the additional foibles of each specific apparatus must be left to the experimentalists.

### **II. SIMULATION**

Throughout this paper we investigate the experimentally favorable single-lepton ( $\ell = e$  or  $\mu$ ) top quark channel  $p\overline{p} \rightarrow t\overline{t}X$  with  $t \rightarrow W^+b \rightarrow jjj$  and  $\overline{t} \rightarrow W^-\overline{b} \rightarrow \ell^-\overline{\nu}_\ell j$  or their charge conjugates at the present Tevatron energy  $\sqrt{s} = 1.8$  TeV. The results also apply rather directly to the six-jet channel where both *t* and  $\overline{t}$  decay hadronically.

Because of color confinement, the quarks from top decay show themselves as jets of hadrons [10]. One must infer the momenta of the quarks from measurements of the observed jets. Because of the collinear and soft singularities of QCD, a quark naturally shares its momentum with accompanying gluons and/or  $q\bar{q}$  pairs. It is necessary to include these as much as possible in order to capture the momentum of the original quark. Sometimes the QCD radiation is so hard as to produce an extra separate isolated jet. In such events, reconstructing the mass of the original state is generally hopeless because the number of combinatoric possibilities resulting from the many possible sources of extra radiation is so large. In many events, however, the effect of the QCD radiation is simply to broaden the jets in the  $(\eta, \phi)$  plane.

Some of this territory has been explored previously [11]. However, we use here a significantly improved simulation program with an up-to-date estimate of the top-quark mass, and make a fuller study of the effect of different options and parameters in the jet definitions. Also, we include the step of making "jet energy corrections" which has become standard experimental practice.

#### A. Event generation and cuts

Events were simulated using the HERWIG 5.8 [12] Monte Carlo event generator, which models both hard and soft QCD effects. HERWIG is known to agree well with jet data

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FIG. 1.  $p_{\perp}$  distributions in  $t \rightarrow bW \rightarrow bq\overline{q}$  for quarks from W decay (*solid*) and b quarks (*dotted*). The dashed line shows the minimum  $p_{\perp}$  cut used in this study.

from  $e^+e^-$  interactions at values of  $Q^2$  comparable to those that arise in top quark decay [13]. It also agrees well with next-to-leading order perturbative calculations of the distributions in  $p_{\perp}^t$ ,  $\eta_t$ , and  $m_{t\bar{t}}^-$  for  $t\bar{t}$  production [14]. The default HERWIG parameters were used, but I have checked that substituting parameters that have been tuned to fit jet data from  $e^+e^-$  interactions [13] causes negligible change. HER-WIG does not include decay correlations between the t and  $\bar{t}$  [15], or the finite width of the top; but these effects are probably not important for our purposes.

Using HERWIG for top-quark production is not without risk in view of discrepancies with perturbative calculations that appear specifically for top quark production [16]. I have incorporated a "bug fix" recently circulated by the authors of HERWIG [17], which substantially increases the amount of hard gluon radiation in top decays and removes the strong discrepancy shown in Fig. 2 of Ref. [16].

I assume  $m_t = 175$  in the simulation. To approximate standard experimental cuts, I restrict the discussion to events in which the lepton from W decay has transverse momentum  $p_{\perp}^{\vee} > 20$  and pseudorapidity  $|\eta| < 2$ , and its neutrino has  $p_{\perp}^{\nu} > 20$ . These cuts keep 73% of the single-lepton  $t\bar{t}$  events. (Units with GeV=c=1 are used throughout this paper.)

Figure 1 shows the  $p_{\perp}$  distribution for the two *b* quarks and the two quarks from *W* decay. Typical values are comparable to those for which HERWIG has been tested and tuned using data from the CERN  $e^+e^-$  collider LEP [13]. I impose a cut requiring all four of these partons to have  $p_{\perp} > 20$ . This cut keeps 67% of the events that pass the lepton cuts. It is intended to simulate the effect of a cut on the minimum  $p_{\perp}$ of the four highest  $p_{\perp}$  jets observed in each event. The cut is made at the parton level in this simulation so that the different jet algorithms are compared fairly, by applying them to the same set of events. The partonic cut should be very similar to experimentally possible cuts on observed jet  $p_{\perp}$  — at least for the events that contribute to the signal, for which the four highest  $p_{\perp}$  jets in fact correspond to the primary partons.

The reduction in signal due to a fairly strong cut on the minimum  $p_{\perp}$  of the observed four primary jets is a price worth paying, particularly as the total number of observed events rises, for several reasons: (1) It avoids the need to measure jets of low  $p_{\perp}$ , which have intrinsically large fractional uncertainties as is quantified below; (2) it increases the fraction of events for which the observed jets will be correctly matched to their original partons, especially since only the four jets with highest  $p_{\perp}$  observed in each event will be analyzed to reduce the combinatoric background in assigning the jets; and (3)  $p_{\perp}$  cuts have been shown effective in suppressing the major background from W+ jets processes without  $t\bar{t}$  [18].

### **B.** Detector models

The detector is modelled as an array of  $0.1 \times 0.1$  cells in pseudorapidity  $\eta = -\ln \tan \theta/2$  and azimuthal angle  $\phi$ . This granularity in the  $(\eta, \phi)$  plane is similar to that of the current DØ detector, while Collider Detector at Fermilab (CDF) detector cells have width 0.26 in  $\phi$ . The detector is assumed to have no ability to identify particles, so the energy deposited in each cell according to the simulation is analyzed as if it came from a massless particle whose momentum direction pointed toward the center of the cell. (In real life, corrections must be made for the spreading of energy into neighboring cells due to the finite size of the shower generated by a single particle. This spreading also creates a possibility in principle to locate the direction of momentum more accurately than the cell size would predict.)

We consider three different models for the energy resolution of the detector cells. In model A (ideal), the total energy deposited in each cell is measured exactly, even including the contribution from neutrinos. In models B and C, the total energy in each cell is smeared by realistic Gaussian errors of standard deviation  $\Delta E$  given by

$$\frac{\Delta E}{E} = \sqrt{\frac{c_1^2}{E} + c_2^2} \tag{1}$$

with  $c_1=0.55$ ,  $c_2=0.03$  for charged hadrons (mostly  $\pi^{\pm}$ ), and  $c_1=0.15$ ,  $c_2=0.003$  for  $\gamma$ , *e* or  $\mu$  (mostly  $\gamma$  from  $\pi^0$ ). These parameters are approximately those of the DØ detector [19].

Models B and C differ only in that neutrinos are treated like electrons in B, while in C the detector is blind to neutrinos like a real detector. The purpose for this distinction is that we will find a sizable difference between these two models because of the frequent presence of neutrinos in b jets, and it may be possible to compensate for some of the neutrino component on an event-by-event basis using leptonic information that is acquired as a part of some b tags.

Cells that receive  $p_{\perp} < 0.75$  are ignored in the analysis. This mimics a limitation of the DØ detector due to noise levels from its uranium calorimeter. But it may be a good idea anyway to drop contributions from very low  $p_{\perp}$  particles, which are at best poorly associated with any jet direction in part because of hadron resonance decay effects and the difference between rapidity and pseudorapidity; and be-



cause extraneous low  $p_{\perp}$  particles are present from soft hadronic interactions that are additional to the hard scattering that produced  $t\overline{t}$  ("background event") and from independent  $p\overline{p}$  interactions at high luminosity ("pileup"). The dependence on this  $p_{\perp}$  threshold will be discussed in Sec. II E.

Additional limitations that depend on experimental details of real detectors, such as differences in the response to charged and neutral particles in a shower, nonlinearity of that response, small regions where there is no response, etc., are not included here. The mass resolutions we find therefore represent an optimistic limit for what can be expected. However, the neglected effects are generally small compared to those included, and they should in particular not affect our conclusions on the relative merits of different methods of analysis.

### C. Jet definition

For jet spectroscopy, I advocate a particular version of the  $k_{\perp}$  jet finding algorithm [8,9] that is defined by the following explicit steps.

(1) Begin with a list of "jets" that consists simply of the four-momentum from each cell above the  $p_{\perp} > 0.75$  threshold, treated as a zero-mass particle. (There are typically  $\sim 40-60$  such cells, but more in a real detector where the energy of a single particle is spread over several cells.)

(2) Compute  $d_i$  for each jet and  $d_{ij}$  for each pair of jets, where  $d_i$  is the jet transverse momentum and

$$d_{ij} = \min(d_i, d_j) \Delta R / R_0, \qquad (2)$$

where

$$\Delta R = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2} \tag{3}$$

is the angular separation in the  $(\eta, \phi)$  "lego" plane. The parameter  $R_0$  was introduced in Ref. [9] to generalize the  $k_{\perp}$  algorithm. It sets the scale for the size of the jets in the  $(\eta, \phi)$  plane. Although it does not create a sharp cutoff, cells that are farther than  $R_0$  from their final jet axis seldom contribute. In this analysis, I mainly use  $R_0=1$ , which corresponds to the original algorithm. The dependence on  $R_0$  will be discussed in Sec. II E.

(3) Find the minimum of all  $\{d_i, d_{ij}\}$ . If the minimum value is less than  $P_{\perp}^0$  the procedure is finished and the current list contains the final jet momenta. This termination rule is different from some other versions of the  $k_{\perp}$  algorithm. The parameter  $P_{\perp}^0$  defines a hardness scale at which the algorithm terminates. In particular, the final jet list will contain no jets with  $p_{\perp}$  below  $P_{\perp}^0$  I find that  $P_{\perp}^0 = 10$  GeV/*c* works well for the top quark analysis.

(4) Otherwise, if the minimum is a  $d_i$ , that jet is deemed to be a fragment of one of the original beam particles (initial state radiation) and it is dropped from the list.

(5) Otherwise, the minimum is a  $d_{ij}$ . That pair of jets is combined into a single jet by adding their four-momenta.

[The simple choice of adding the four-momenta to combine protojets has an obvious good feature that the invariant mass of a multijet object will be stable with respect to changing the assignment of a cell or group of cells from one jet to another within the object. A customary alternative to this choice is to combine protojets according to the "Snowmass Accord" [7] formulas

$$p_{\perp} = p_{\perp}^i + p_{\perp}^j , \qquad (4)$$

$$\eta = (\eta_i p_\perp^i + \eta_j p_\perp^j) / (p_\perp^i + p_\perp^j), \qquad (5)$$

$$\phi = (\phi_i p_\perp^i + \phi_j p_\perp^j) / (p_\perp^i + p_\perp^j), \qquad (6)$$

where  $\phi_j$  must be shifted by  $\pm 2\pi$  here and in Eq. (3) if possible to minimize  $|\phi_i - \phi_j|$ . I find this rule to give slightly poorer mass resolution than simply adding the fourmomenta.]

(6) Go to step (2).

Only the four highest  $p_{\perp}$  jets found by the  $k_{\perp}$  algorithm are used in the analysis. This causes a very small fraction (~2%) of events to be dropped immediately because fewer than four jets are found. This can happen even though we are looking for jets down to  $p_{\perp} = 10$  from partons with  $p_{\perp} > 20$ , because one jet can split into two or more by hard radiation, or because two jets can lie so close together in ( $\eta$ , $\phi$ ) that they appear as one. (It will eventually be desirable to keep more than the four highest  $p_{\perp}$  jets, to allow for initial state radiation at higher  $p_{\perp}$  than one of the four primary decay partons or hard radiation from the t,  $\overline{t}$ , b, or  $\overline{b}$ [20], in order to test our understanding of QCD radiation; but because of its combinatoric richness, this will not be helpful for the mass measurement.)

The four hardest jets are matched to the four original parton momenta, which are of course known in the simulation, by trying all 4!=24 assignments and keeping the one with the smallest root mean square error in fitting the four parton directions in the  $(\eta, \phi)$  plane. The jet energies are not considered in this matching process, so as not to bias our study of the accuracy of jet energy measurement.

The distribution in the rms error of the best fitting assignment shows a strong peak at small values, above a background that extends to large ones. We impose a cut  $\leq 0.8$  on the total rms error, which is equivalent to a cutoff at  $\leq 0.4$  for the average deviation in  $(\eta, \phi)$  from each of the four parton directions. This cut keeps 67% of the events. The events it removes are mainly those in which the four highest  $p_{\perp}$  jets are not the right ones because of initial state radiation of a gluon with higher  $p_{\perp}$  than one of the top decay quarks. Thus our procedure of keeping only the four jets with highest  $p_{\perp}$  captures the desired two *b* jets and two *W* decay jets about 2/3 of the time.

The events that survive the rms fit cut are used to study the  $p_{\perp}$  resolution for jets, and the resulting mass resolution for  $t \rightarrow bW \rightarrow jjj$ , in the next two sections. To compare the effects of different jet algorithm parameters or detector parameters fairly, the location of the cut is adjusted slightly to keep the fraction of events that pass the cuts constant.

#### **D.** Jet energy resolution

Figures 2–4 show the ratio  $p_{\perp}^{\text{jet}}/p_{\perp}^{\text{parton}}$  at  $p_{\perp}^{\text{parton}} \approx 50$ . The solid curves are for jets from W decay (light quarks), while the dotted curves are for b jets. The three figures correspond to the three models for calorimeter energy resolution: Figure



FIG. 2. Distribution of the ratio of observed jet transverse momentum  $(p_{\perp}^{\text{jet}})$  to original parton transverse momentum  $(p_{\perp}^{\text{parton}})$  in  $t \rightarrow b W \rightarrow b q \bar{q}$  for quarks from W decay (solid) and b quarks (dotted) at  $p_{\perp}^{\text{parton}} \approx 50 \text{ GeV}/c$ , for the ideal calorimeter model.

2 assumes perfect resolution, while Figs. 3 and 4 both include the realistic energy resolution given in Eq. (1). The detector is assumed capable of detecting neutrinos in Figs. 2 and 3, while it is blind to them in Fig. 4.

All of the curves peak at  $p_{\perp}^{\text{jet}}/p_{\perp}^{\text{parton}}$  below 1 because of the assumed  $p_{\perp}$  threshold of the cells and because QCD radiation can cause a significant fraction of the jet energy to appear at large angles where it is omitted by the jet algorithm. The peaks in Fig. 3 are more than twice as wide as the peaks in Fig. 2. This indicates that the energy resolution of



FIG. 3. Like Fig. 2 except that the calorimeter model includes realistic energy resolution.



FIG. 4. Like Fig. 3 except that the calorimeter is blind to neutrinos, which is realistic unless the neutrino component can be estimated from leptonic information.

the calorimeter cells is the major source of error in the jet energy measurement: e.g., if the QCD and calorimeter cell size errors included in Fig. 2 and the resolution errors were equal, the peak width would increase only by a factor  $\sqrt{2}$  in going from Fig. 2 to 3.

Figure 2 shows only a small difference between b jets (dotted) and the light quark jets from W decay (solid). The difference remains small when energy resolution is included in Fig. 3. In going from Fig. 3 to Fig. 4, there is almost no change in the W decay jets (solid), as expected because there is not much neutrino component in light quark jets. But a dramatic difference appears between Figs. 3 and 4 for the b jets (dotted). The loss in b-jet resolution due to varying amounts of missing neutrino energy is very significant. It will therefore be useful to investigate the possibility of correcting for the neutrinos on a jet-by-jet basis, using information that is acquired as a part of b-jet identification.

To study the dependence on partonic  $p_{\perp}$ , we can characterize peaks like those shown in Figs. 2–4 by the value of  $p_{\perp}^{\text{jet}}/p_{\perp}^{\text{parton}}$  corresponding to the 50th percentile (median) of the distribution, and the values corresponding to the 16th and 84th percentiles which define the middle 68% of the probability distribution. These would be the  $\pm 1\sigma$  points if the distributions were Gaussian. The result is shown in Figs. 5–7, expressed in terms of the difference  $p_{\perp}^{\text{jet}}-p_{\perp}^{\text{parton}}$  instead of the ratio for convenience.

One sees that the 50th percentile curves in Figs. 5–7 can be reasonably well approximated by straight lines. Those straight line fits can be used to make average "jet energy corrections" of a linear form

$$p_{\perp}^{\text{parton}} \simeq A + B p_{\perp}^{\text{jet}} \tag{7}$$

to better estimate the partonic energy from an observed jet energy. The appropriate parameters A and B are somewhat different for b jets and W-decay jets, and vary with the parameters of the jet algorithm.



FIG. 5. Three solid curves for *W* decay jets and three dotted curves for *b* jets show the 16th, 50th, 84th percentile points (i.e., the middle 68%) for the distributions of  $p_{\perp}^{\text{jet}} - p_{\perp}^{\text{parton}}$  as a function of  $p_{\perp}^{\text{parton}}$ . The calorimeter model is the ideal one as in Fig. 2.

After average jet energy corrections have been made, fluctuations from jet to jet remain due to different amounts of QCD radiation falling outside the identified jet. These fluctuations contribute to the energy resolution errors, and hence to the width of peaks in multijet mass distributions. The " $\pm 1\sigma$ " spread in  $p_{\perp}^{\text{jet}} - p_{\perp}^{\text{parton}}$  is seen in Figs. 5–7 to grow only slowly with  $p_{\perp}^{\text{parton}}$  so the *fractional* accuracy of the  $p_{\perp}$  measurement improves significantly with increasing  $p_{\perp}$ . The spread in  $p_{\perp}^{\text{jet}} - p_{\perp}^{\text{parton}}$  is larger for *b* jets. This is dramatically so in the case of the most realistic detector



FIG. 6. Like Fig. 5 but the calorimeter model includes energy resolution as in Fig. 3.



FIG. 7. Like Fig. 6 but the calorimeter is blind to neutrinos as in Fig. 4.

model C, which admits the possibility of large energy escape in the form of neutrinos.

## E. Top mass resolution

We concentrate on the mass measurement of the hadronically decaying top, since it is a good example of "jet spectroscopy" in general, and since the treatment of the leptonically decaying top is complicated by errors in the measurement of the neutrino momentum. (The transverse momentum of the neutrino is inferred from missing  $p_{\perp}$ , which can be strongly affected by detector imperfections and by the presence of neutrinos in the *b* or *c* jets. The longitudinal momentum of the neutrino is subsequently obtained by assuming  $m_{\ell\nu} = m_W$ , which acquires serious uncertainties from the error in  $p_{\perp}^{\nu}$  and the finite *W* width in addition to the two-fold ambiguity in the sign of  $\eta_{\nu} - \eta_{\ell}$ .)

Three-jet mass distributions from  $t \rightarrow bW \rightarrow jjj$  are shown in Fig. 8 for the three models of calorimeter energy resolution. In generating these histograms, the best match to the four parton directions was again used to infer the jet assignments. But this time the best-fitting assignment is plotted for every event, without a cut on the quality of the fit. This makes the simulation more realistic, since it includes backgrounds of a type that will be present in actual data analysis. The jet assignments are needed to know which three of the four jets come from the hadronic top decay, and also because linear jet energy corrections are made using Eq. (7) with parameters A and B that are slightly different for b jets and light-quark jets according to Figs. 5–7.

Thanks to the jet energy corrections, the peaks are centered very close to the input value  $m_t = 175$ . Their shapes are not symmetrical, but are instead skewed toward low masses since QCD radiation and loss due to neutrinos can substantially reduce the observed energy of a jet, but cannot increase it. The widths of these peaks can be measured by fitting the histograms to a Gaussian plus a linear background over the



FIG. 8. Invariant mass distribution for  $t \rightarrow jjj$  for the three models of calorimeter energy resolution.

fairly narrow mass range  $160 < M_{jjj} < 190$ : this is useful for purposes of comparison, even though the resulting fits are not statistically adequate at the high statistics at which the histograms have been computed. The resulting Gaussian peaks correspond to standard deviations of  $\Delta M = 4.0$ , 7.3, and 9.1 for the three models of resolution. Fitting over a different mass range results in somewhat different numbers, but leads to the same qualitative conclusions.

The mass resolution for  $m_t$  can be improved by replacing the usual invariant mass estimate, which is based on the sum of the four-momenta of the three jets, by the average of that value and a mass estimate based on the jet angles in the top rest frame [1]. Three-jet mass distributions obtained using this average variable are shown in Fig. 9. These peaks are more symmetrical than those of Fig. 8 because fluctuations in the jet angle part of the mass measurement have no definite sign. The peaks are narrower in each case, with widths  $\Delta M = 3.9$ , 5.7, and 7.3 for the three models of resolution. This demonstrates the value of the jet angle method.

The dependence on the assumed calorimeter cell threshold is not large. For example, raising the threshold from  $p_{\perp} > 0.75$  to  $p_{\perp} > 1.00$  increases the width of the mass peak by only  $\approx 5\%$  in the case of model *B* for the energy resolution. Similarly, lowering the threshold to  $p_{\perp} > 0.50$  narrows the peak by  $\approx 5\%$ . The actual effect would be even less than that because the "background event," which contributes random noise at low  $p_{\perp}$ , has not been included in the simulation.

The dependence on the jet radius parameter  $R_0$  of the  $k_{\perp}$  algorithm is also not large. The original choice  $R_0 = 1$  is found to be close to optimal. Going to  $R_0 = 0.8$  or  $R_0 = 1.2$  results in mass peaks that are a few percent broader.

One might wonder if the  $k_{\perp}$  algorithm could be improved in some cases by revising its assignment of cells to jets according to their proximity to the jet axes it finds. To test this, the following plausible modification was tried: After completing the work of the  $k_{\perp}$  algorithm on each event, any cell



FIG. 9. Similar to Fig. 8, but  $M_{jjj}$  is obtained by averaging the conventional invariant mass and the "jet angle" mass measure of Ref. [1].

above the  $p_{\perp} > 0.75$  threshold was reassigned to the nearest of the four highest  $p_{\perp}$  jet axes if the cell was within 0.7 of that axis and (1) it was previously assigned to a different jet whose axis is farther away than this new one by a factor >1.2, or (2) it was previously not assigned to any jet. This modification affected only 17% of the events, almost entirely through option (1). It produced a small improvement in energy resolution for that subset of events, but the improvement was not large enough to make it worthwhile to "second-guess" the  $k_{\perp}$  algorithm in this way.

# **III. COMPARISON WITH CONE ALGORITHMS**

The analysis of jet data at hadron colliders has traditionally been done using cone algorithms, in which a jet is defined as the final particles within a circle of fixed radius R in the  $(\eta, \phi)$  plane. A typical cone size is R = 0.7; but smaller values like 0.4 have been used for processes like  $t\bar{t}$  production, to improve the sensitivity to configurations where partons lie close together in the  $(\eta, \phi)$  plane at the expense of increased errors in the partonic momentum measurement due to fluctuations in the QCD radiation lying outside the cone.

Cone algorithms are not at all straightforward to design, nor even to describe, because of ambiguities in how to treat situations in which jets overlap. Overlap occurs to some degree whenever two jet axes lie within 2R of each other in  $(\eta, \phi)$ , which happens in the majority of events of the type we are considering.

I have repeated the analysis of Sec. II with the  $k_{\perp}$  algorithm replaced by a cone algorithm [21] that begins with clustering based on equivalence classes [22]. I have also repeated the analysis using a version of the cone algorithm by Seymour [11], which is patterned after current practice. A cone radius R=0.7 was used in both cases. The results achieved by these two cone algorithms, which are alike in

intent but very different in implementation, are strikingly similar to each other.

Cone algorithms generally do not allow the final jet momenta to lie within *R* of each other. This leads to a significant loss of events in the top analysis, where the nearest pair of the four primary partons lie within 0.7 of each other in 20% of the events. It shows up quickly on repeating the analysis of Sec. II, in that 27% of the events for the algorithm of Ref. [21], or 32% for the algorithm of Ref. [11], are rejected because fewer than the required four jets with  $p_{\perp} > 10$  are found, as compared to only <2% for the  $k_{\perp}$  algorithm. Furthermore, the distribution of errors in the best fit to the partonic angles is broader for the cone algorithms than for  $k_{\perp}$ .

For events in which the necessary four jets are found, both cone algorithms perform almost as well as the  $k_{\perp}$  one. In particular, the final  $M_{jjj}$  distributions are quite similar to those shown in Figs. 8 and 9, especially for the cases in which realistic calorimeter energy resolution is included, which masks the differences. The average energy corrections needed for the cone algorithms are also similar to those for the  $k_{\perp}$  algorithm, although slightly larger.

One could therefore say that the  $k_{\perp}$  algorithm provides only slightly better mass resolution than the cone algorithms, but allows approximately 30% more events to be kept. Another way to compare the algorithms would be to impose a cut on the minimum separation between observed jets in  $(\eta, \phi)$  for the  $k_{\perp}$  algorithm, or to raise the  $p_{\perp}$  threshold for defining jets in it, or to make a combination of such cuts that would make the fraction of events kept by the various algorithms the same. The benefits of the  $k_{\perp}$  algorithm would then appear entirely in the form of improved mass resolution.

The solid curve in Fig. 10 shows the fraction of events for which a good match is found between the four highest  $p_{\perp}$ jets found by the  $k_{\perp}$  algorithm and the four primary partons [using a criterion based on the quality of fit to the  $(\eta, \phi)$ direction and  $p_{\perp}$  of all four as a function of the minimum separation between jets as observed by the algorithm. The algorithm is seen to have significant success even at minimum separations below 0.5. Meanwhile, the two versions of cone algorithm with R = 0.7 (dotted and dashed curves in Fig. 10) are somewhat less effective overall, and are completely unable to see separations smaller than the assumed cone size. A smaller cone size could be used to extend the effectiveness of the cone algorithms to smaller minimum separation, as CDF and D0 have both done; but that would reduce the accuracy of the  $p_{\perp}$  measurements, and hence reduce the overall fraction of good matches. (As an aside, the curves shown in Fig. 10 are seen to turn over at large minimum separation. This may at first sight be puzzling, but it only reflects the fact that large separation between all six pairs of partons is very unlikely, so if the jet finder sees such a configuration, it is likely to be mistaken.)

In setting up the definitive top quark data analysis, the best choice of cuts on minimum jet-jet angular separation and minimum jet  $p_{\perp}$  will have to be determined using a full simulation of both the detector and the complete analysis procedure. Optimal choices for the cuts for the purpose of mass measurement will also depend on the number of events available for analysis, since one can afford statistically to cut harder when there are more events to begin with.



FIG. 10. Fraction of events for which a good match is found between the four highest  $p_{\perp}$  observed jets and the four primary partons according to a criterion based on agreement in both angle and energy, as a function of the minimum separation in  $(\eta, \phi)$ between pairs of observed jets. Solid curve is for the  $k_{\perp}$  algorithm. Dashed and dotted curves are for the two versions of cone algorithm (dashed [21], dotted [11]).

Another way to compare the  $k_{\perp}$  and cone algorithms was carried out to study the ability to analyze objects that decay into two jets in the presence of additional jets, which will be necessary in the Higgs search. For this study,  $t\bar{t}$  events were generated as before except for an additional cut requiring the partons from W decay to be separated from each other by >1.0 in the ( $\eta, \phi$ ) plane. This cut is minor because these jets tend to be opposite each other in azimuthal angle and hence well separated. The ideal calorimeter model was used. The events were analyzed as before except that all jets found by the jet finder were kept and there was no requirement that four or more jets be found. The pair of jets (at least two jets were always found) making the best fit in  $(\eta, \phi)$  to the two partons from W decay were identified. Linear jet energy corrections were applied as before to these jets. The invariant mass of the pair was computed and corrected for the deviation of the partonic W mass from its nominal value, to remove the effect of finite W width that is included in the simulation. The resulting distribution in dijet mass is shown in Fig. 11 for the  $k_{\perp}$  algorithm and the two versions of cone algorithm. The distributions are normalized to the same number of events, so the superiority of the  $k_{\perp}$  method is demonstrated by the fact that its peak is significantly higher. This is true even though the width of the peak — measured by full width at half maximum above background or "by eye" - is not obviously better. The point is that many events are so clean that all three jet finders give almost identical results for them. This can be seen in Fig. 12, which shows the distribution of the total root mean square deviation between the two jets identified as coming from the W and their true parton directions, i.e., the quantity that was minimized to identify the "correct" jet pair. Compared to the  $k_{\perp}$  algorithm, the



FIG. 11. Dijet mass distribution from W decays identified by  $k_{\perp}$  algorithm (solid) or cone algorithms (dashed [21], dotted [11]).

two cone algorithms both have relatively strong tails into a region of large deviation where the W decay axes have not been located very well. These tails result mainly from events in which the jet finder includes contributions to a W decay jet from particles actually coming from a b jet that happens to lie nearby in the  $(\eta, \phi)$  plane. This explains the tails extending toward higher  $M_{jj}$  in Fig. 11. The  $k_{\perp}$  algorithm is less easily confused by such particles.

#### **IV. NEUTRINO MOMENTUM DISTRIBUTIONS**

Figsures 8 and 9 show that there is a substantial loss in mass resolution caused by fluctuations in the neutrino com-



FIG. 12. Distribution of total rms deviation in  $(\eta, \phi)$  of best fitting dijet pair to W decay partons using  $k_{\perp}$  algorithm (solid) or cone algorithms (dashed [21], dotted [11]) as in Fig. 11.



FIG. 13. The observable (i.e., non-neutrino) fraction of the jet momentum for *b* jets that contain at least one neutrino: solid = all, dotted = jets containing  $e^{\pm}$  or  $\mu^{\pm}$  of  $p_{\perp} > 2$  GeV/*c*.

ponent of b jets. To study this in more detail, Fig. 13 shows the distribution of the observable (i.e., non-neutrino) fraction of jet momentum

$$z = 1 - p_{\perp}^{\text{neutrinos}} / p_{\perp}^{\text{parton}}$$
(8)

for *b* jets that contain at least one neutrino. The log-log plot reveals that the distribution can be rather well approximated by a power law:  $dP/dz \propto z^A$  with A = 4.4 for z < 0.98. The dotted curve in Fig. 13 shows the distribution for the subset of jets that contain an  $e^{\pm}$  or  $\mu^{\pm}$  with  $p_{\perp} > 2$ , which may be detected experimentally — especially in the case of  $\mu^{\pm}$ . The two distributions are nearly identical. Distributions with stronger or weaker cuts on the  $p_{\perp}$  of  $e^{\pm}$  or  $\mu^{\pm}$ , or with cuts on  $p_{\perp}^{\text{parton}}$  are also about the same.

We can use this power law over the entire range 0 < z < 1 because the neutrino contribution to  $p_{\perp}$  is small compared to other errors in jet energy measurement in the tiny region 0.98 < z < 1 where the power law doesn't fit well. Including the contribution from jets without neutrinos then gives a normalized parametrization of the distribution in observable momentum fraction

$$\frac{dP}{dz} = f \,\delta(z-1) + (1-f) 5.4 z^{4.4},\tag{9}$$

where *f* is the fraction of jets with negligible or zero neutrino contribution. For all *b* jets, f=0.59 which implies that 23% of them hide >10% of their momentum in neutrinos and 12% of them hide >20%. For the 33% of *b* jets that contain an electron or muon with  $p_{\perp}>2$ , *f* is only 0.10 which implies that 51% of them hide >10% of their momentum in neutrinos and 27% of them hide >20%. It is thus clearly advantageous to use different estimates to correct for the missing neutrino energy in a *b* jet, depending on whether or not a lepton is observed in the jet. This has already been

done in the analysis of the top quark signal [23]. A topic worthy of future study would be to see if any further details of the observed jet, in addition to the mere presence or absence of a lepton, can be used to further improve the neutrino momentum estimate.

It is interesting that the distribution in missing neutrino energy fraction when a lepton is observed is nearly independent of the energy of that lepton, except for the difference in probability that the missing energy is negligible or zero. Additionally, the probability distribution for the error in jet momentum measurement is very asymmetric and very far from Gaussian. This should be taken into account in the  $t\bar{t}$  final state reconstruction analysis.

# V. DIRECT COMPARISON OF MASS DISTRIBUTIONS

So far, we have compared jet algorithms by making explicit use of the original parton momenta to infer the correspondence between jets and partons. This facilitates a detailed comparison of the methods, but it is somewhat artificial, since it can never be carried out using real data for which underlying partonic information is unknown. In this section we compare the jet algorithms directly, using no information that exists only in the world of Monte Carlo.

An appealing way to make the comparison would be to simulate a full data analysis recommended for  $t\bar{t}$  events, and see how the choice of jet algorithm affects the uncertainty in measuring  $m_t$ . The treatment of measurement errors in that analysis, however, is very complicated; and further complications arise from the role played by missing  $p_{\perp}$  in identifying the leptonically decaying W, and from the existence of a variety of classes of events with regard to *b*-tagging information (zero, one, or two tags with varying degrees of certainty). The complete comparison can therefore only be done properly by the experimentalists who are in a position to use full simulations of the detector, and who can make the comparison with data as well as with Monte Carlo events.

In order to test our methods directly, but without carrying out the full  $t\bar{t}$  analysis, HEWIG events were generated as before except that all parton-level cuts were removed. The intermediate model of the calorimeter was used, i.e., energy resolution was included, but neutrinos were assumed to be observable. Instead of using partonic information to infer the jet assignments, a trijet mass distribution was found by simply plotting a histogram of  $M_{jjj}$  formed from each subset of three of the four highest  $p_{\perp}$  jets. Events with fewer than four jets were ignored. The minimum jet  $p_{\perp}$  was chosen slightly differently for the different jet algorithms to make the fraction of events kept the same for each algorithm.

The histograms of  $M_{jjj}$  are shown in Fig. 14. The  $k_{\perp}$  algorithm (solid curve) produces a clear peak above the combinatoric background. That background is very large because even events that are analyzed correctly contribute three incorrect combinations to the histogram in addition to the correct one. The two cone algorithms (dashed and dotted curves) produce nearly identical results. They show a peak that is significantly smaller and broader than the result of the  $k_{\perp}$  algorithm.

In a full analysis, b tagging and the constraint from the hadronic W decay mass would greatly reduce the combinatoric background is Fig. 14, and accentuate the difference



FIG. 14. Trijet mass distributions formed from each three of the four highest  $p_{\perp}$  jets observed in each event (four combinations per event), using the  $k_{\perp}$  algorithm (solid) or cone algorithms (dashed [21], dotted [11]).

between the methods. The signal peaks would also be slightly narrower because different jet energy corrections could be made for the b quark and light quark jets, in place of the cruder method of just making an average correction for all jets, which was used in generating Fig. 14.

#### **VI. CONCLUSIONS**

We have seen that a form of the  $k_{\perp}$  successive recombination jet algorithm offers a significant improvement in the fraction of  $t\bar{t}$  events that can be reconstructed and/or offers significantly improved t mass resolution at the same efficiency, compared with cone algorithms like those that have been used up to now for  $t\bar{t}$  data analysis. The basis of this is the flexibility of the  $k_{\perp}$  algorithm with respect to jet radius: it can include final particles in a cone as large as R = 1 or even greater when possible, while maintaining some useable efficiency for resolving jets down to as close as R = 0.2. The improved mass resolution that can be obtained using jet angle variables in the top rest frame [1] has also been confirmed. The size of these improvements and the importance of an accurate top mass measurement are such that the procedure should be carried out in spite of the considerable work that will be necessary to reevaluate the instrumental corrections using the new methods.

The particular form of the  $k_{\perp}$  algorithm advocated here is characterized by a simple rule for when to terminate the process of combining protojets into jets, as described explicitly in Sec. II C. The dependence on parameters appearing in the algorithm is discussed in Sec. II E. With this algorithm, the mass resolution is close to optimal in the sense that the majority of the width of the final mass peak is generated by the nominal energy resolution of a typical detector, so not much further improvement is theoretically possible. We have seen that fluctuations in the momentum carried by neutrinos contributes significantly to the error in measuring the momentum of a b jet. This error is reduced in current practice [23] by using different distributions according to whether or not a lepton is identified in the jet. A matter for future study is to see if any other features of the observed jet can be used to further improve the estimate.

Finally, both the improved jet algorithm and the improved estimate of neutrino contributions can help also in the search

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for other heavy objects that decay into jets, such as Higgs boson  $\rightarrow b\overline{b}$  [4].

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