Is the Zee model the model of neutrino masses?

Alexei Yu. Smirnov*
International Center for Theoretical Physics, I-34100 Trieste, Italy

Morimitsu Tanimoto†
Science Education Laboratory, Ehime University, Matsuyama 790, Japan

(Received 26 April 1996)

The Zee model predicts naturally two heavy, strongly degenerate and almost maximally mixed neutrinos and one light neutrino with small mixing. This pattern coincides with the one needed for a solution of the atmospheric neutrino problem by $\nu_\mu \to \nu_\tau$ oscillations and for existence of the two component hot dark matter in the Universe. Furthermore, the oscillations $\overline{\nu_\mu} \to \overline{\nu_e}$ can be in the range of sensitivity of KARMEN, LSND experiments. The phenomenology of this scenario is considered and the possibility to check it in the forthcoming experiments is discussed. The scenario implies large values and inverse flavor hierarchy of couplings of the Zee boson with fermions: $f_{e\tau} \ll f_{\mu\tau} \leq f_{e\mu} \sim 0.1$. The main signatures of the scenario are a strongly suppressed signal of $\nu_\mu \to \nu_\tau$ oscillation in CHORUS and NOMAD experiments, so that a positive result from these experiments will rule out the scenario, the possibility of the observation of $\nu_e \to \nu_\tau$ oscillations by CHORUS and NOMAD, the corrections to the muon decay and neutrino-electron scattering at the level of experimental errors, and a branching ratio $B(\mu \to e\gamma)$ bigger than 10^{-13}. The solar neutrino problem can be solved by the introduction of an additional very light singlet fermion without appreciable modification of the active neutrino pattern. [S0556-2821(97)04903-5]

PACS number(s): 14.60.Pq, 14.60.St, 14.80.Cp

I. INTRODUCTION

The Zee model [1] is the simplest model which explains the smallness of the neutrino masses by physics at the electroweak scale. It can be considered as an alternative of the seesaw mechanism [2].

The Zee model includes the existence of the charged scalar field h, being a singlet of SU(2), and two doublets of the Higgs bosons Φ_1, Φ_2. The singlet h couples to lepton doublets $\Psi_{lL} = (\nu_l, l^-)$, $(l = e, \mu, \tau)$ as well as to Higgs doublets; leptons are assumed to couple to the doublet Φ_1 only. The appropriate terms in the Lagrangian are

$$\mathcal{L}_{\rm Zee} = f_{\ell\ell'} \Psi^T_{\ell L} i\tau_2 \Psi_{\ell' L} h + c_{12} \Phi_1^T i\tau_2 \Phi_2 h^\dagger + \frac{m_l}{\langle \Phi_1 \rangle} \overline{\Psi_l} \Phi_1 l_R + \text{H.c.}, \tag{1}$$

where $c_{12} = -c_{21}$ are real mass parameters, the couplings $f_{\ell\ell'}$ are antisymmetric in ℓ and ℓ'. The interactions (1) generate neutrino mass terms in one loop.

The Zee model gives a very distinctive pattern of neutrino masses and mixing [3,4]. For a not too strong hierarchy of couplings $f_{\ell\ell'}$ the two heavy neutrinos ν_2, ν_3 are degenerate and mix ν_μ and ν_τ almost maximally. The first neutrino ν_1 practically coincides with ν_e and has a much smaller mass:

$$m_1 \ll m_2 \approx m_3. \tag{2}$$

It was noted [5] that the neutrino mass pattern of the Zee model coincides with the one needed to solve simultaneously the atmospheric neutrino problem [6] and problem of the hot dark matter in the Universe [7]. Indeed, a deficit of the atmospheric muon neutrinos can be explained by the oscillations ν_μ-ν_τ with practically maximal mixing. Two heavy neutrinos with masses $m_2 \approx m_3 \approx (1-5)$ eV compose two component hot dark matter (which may give even better fit of the cosmological data than one component) [7]. Furthermore, oscillations $\nu_\mu \to \nu_e$ and $\overline{\nu_\mu} \to \overline{\nu_e}$ can be at the level of sensitivity of existing experiments: BNL [8] and KARMEN [9] (see [5]). Later it was noted [10,11] that the model can immediately accommodate positive LSND results [12].

A similar pattern of mass and mixing has also been discussed in the context of other mechanisms of neutrino mass generation [13].

In this paper we consider phenomenology of the outlined scenario. In particular, a possibility to check it by forthcoming experiments is studied. (Some previous studies can be found in [13]). In Sec. II we describe the scenario in details. Section III is devoted to oscillations. In Sec. IV we find the bounds on the Zee coupling constants. In Sec. V implications of data on the muon decay, neutrino electron scattering, e-μ-τ universality to the scenario are considered. Predictions for $\mu \to e\gamma$ and $\nu_{3(2)} \to \nu_1 \gamma$ are given. In Sec. VI, we describe a modification of the Zee model which is able to solve the solar neutrino problem. Section VII contains our conclusions.

II. SCENARIO

In flavor basis, $\nu_f = (\nu_e, \nu_\mu, \nu_\tau)$, the neutrino mass matrix of the Zee model can be written as

$$M = m_0 \begin{pmatrix} 0 & \epsilon & \sin\theta \\ \epsilon & 0 & \cos\theta \\ \sin\theta & \cos\theta & 0 \end{pmatrix}. \tag{3}$$

*Electronic address: smirnov@ictp.triese.it
†Electronic address: tanimoto@edserv.ed.ehime-u.ac.jp

where m_0 is the basic mass scale. The mixing angle θ and parameter ϵ can naturally be much smaller than 1. [We will discuss a relation of these parameters with the parameters of the Lagrangian (1) in Sec. IV.]

In the case $\cos\theta \gg \sin\theta, \epsilon$ the eigenvalues of matrix (3) equal to

$$m_1 = -m_0 \epsilon \sin 2\theta, \quad m_{2,3} = m_0(\pm 1 - \tfrac{1}{2}\epsilon \sin 2\theta), \tag{4}$$

and the mixing matrix S which diagonalizes (3) can be written as

$$S \simeq \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\cos\theta & \sin\theta + \epsilon\cos\theta & \sin\theta - \epsilon\cos\theta \\ -\sqrt{2}\sin\theta & \cos\theta & \cos\theta \\ -\sqrt{2}\epsilon & 1 & -1 \end{pmatrix}, \tag{5}$$

[$\nu_f = S\nu$, where $\nu \equiv (\nu_1, \nu_2, \nu_3)$ are the mass eigenstates]. According to Eq. (4) the states ν_2 and ν_3 are approximately degenerate, and moreover their masses ($\sim m_0$) are much larger than mass of ν_1. The mass squared difference is

$$\Delta m_{32}^2 = 2\epsilon \sin 2\theta m_0^2 \ll m_0^2, \tag{6}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. For ν_1 component the model gives $\Delta m_{21}^2 \simeq \Delta m_{31}^2 \simeq m_0^2$ and the ratio of mass differences equals

$$\frac{\Delta m_{32}^2}{\Delta m_{21}^2} = 2\epsilon \sin 2\theta. \tag{7}$$

Thus the Zee mass matrix leads to two different scales of the mass squared differences and to almost maximal mixing between two heaviest neutrinos.

As it was outlined in the Introduction we consider the following scenario.

ν_2 and ν_3 compose two component hot dark matter of the Universe, so that

$$m_0 = m_{\rm HDM} = (1-5) \text{ eV}. \tag{8}$$

The ν_μ-ν_τ oscillations with practically maximal depth explain the atmospheric neutrino deficit and, therefore,

$$\Delta m_{32}^2 = \Delta m_{\rm atm}^2 \sim (0.3-3) \times 10^{-2}. \tag{9}$$

The oscillations ν_μ-ν_e and $\overline{\nu_\mu}$-$\overline{\nu_e}$ with $\Delta m_{21}^2 \approx m_{\rm HDM}^2$ can be in the region of sensitivity of the KARMEN and LSND experiments. For m_0 in the cosmologically interesting domain (8) this implies

$$\sin^2 2\theta \leq \sin^2 2\theta_{e\mu} \sim (1-3) \times 10^{-3}, \tag{10}$$

where $\theta_{e\mu}$ is the experimental bound (or preferable value in the case of positive result) for the ν_e-ν_μ mixing angle.

Substituting m_0 and $\sin^2 2\theta$ from Eq. (8) and Eq. (10) in Eq. (6) we get

$$\epsilon \geq \frac{\Delta m_{\rm atm}^2}{2 m_{\rm HDM}^2 \sin 2\theta_{e\mu}}. \tag{11}$$

Numerically this leads [see Eqs. (9) and (10)] to $\epsilon = 10^{-3} - 0.5$, with typical value 3×10^{-2}. Thus all oscillation parameters of the model (mass squared differences and mixing angles) are fixed by the experimental data.

III. NEUTRINO OSCILLATIONS

In terms of elements of the mixing matrix (5), $S_{\alpha i}$, the oscillation probability can be written as

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} S_{\alpha i} S_{\beta i} S_{\alpha j} S_{\beta j} \sin^2\left(\frac{\Delta m_{ij}^2 L}{4E_\nu}\right), \tag{12}$$

where E_ν is the neutrino energy and L is the distance. We neglect the CP violation, taking the elements $S_{\alpha i}$ to be real. Let us consider the probabilities (12) for short and long distances separately.

(1) In the short distance limit, the phase difference due to Δm_{32}^2 is small: $\Delta m_{32}^2 L/4E_\nu \ll 1$. Taking into account that $\Delta m_{31}^2 \simeq \Delta m_{21}^2$ and using matrix (5) we find

$$P(\nu_\mu \to \nu_e) = P(\overline{\nu_\mu} \to \overline{\nu_e}) \approx \sin^2 2\theta \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right). \tag{13}$$

This result is applied to E776 [8], KARMEN [9], and LSND [12].

For $\nu_\mu - \nu_\tau$ we get

$$P(\nu_\mu \to \nu_\tau) = 4(\epsilon \sin\theta)^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right) + \cos^2\theta \sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right). \tag{14}$$

The ν_μ-ν_τ oscillations due to large mass splitting[1] are doubly suppressed because of $\sin\theta \ll 1$ and $\epsilon \ll 1$. This feature originates from the fact that an admixture of ν_1 in ν_τ is suppressed by ϵ [see Eq. (5)]. The mode of oscillations with the smallest mass splitting [second term in Eq. (14)] may give a comparable contribution. For values of $\sin^2 2\theta$ and ϵ from Eq. (10) and Eq. (11) correspondingly, we obtain

$$P(\nu_\mu \to \nu_\tau) \sim 10^{-7} - 10^{-5}. \tag{15}$$

If both ϵ and Δm^2 are near the upper bounds, the probability can be as big as 10^{-4}. These values are still below the sensitivity of CHORUS and NOMAD [14], but they can be reached by future experiments COSMOS at Fermilab and E889 at BNL [15].

For ν_e-ν_τ channel we get

$$P(\nu_e \to \nu_\tau) = 4(\epsilon \cos\theta)^2 \sin^2\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right). \tag{16}$$

If $\epsilon \geq 10^{-1}$ and $m_0 > 4$ eV, then $P(\nu_e \to \nu_\tau) \simeq 10^{-2} - 10^{-1}$ and the $\nu_e \to \nu_\tau$ oscillations can be detected by CHORUS and NOMAD. Thus the observation of signals of the $\nu_e \to \nu_\tau$ oscillation and absence of signal from $\nu_\mu \to \nu_\tau$ mode in CHORUS and NOMAD are the signatures of the Zee model. The scenario under consideration will be ruled out if

[1] These so-called second order oscillations have been discussed previously in [16].

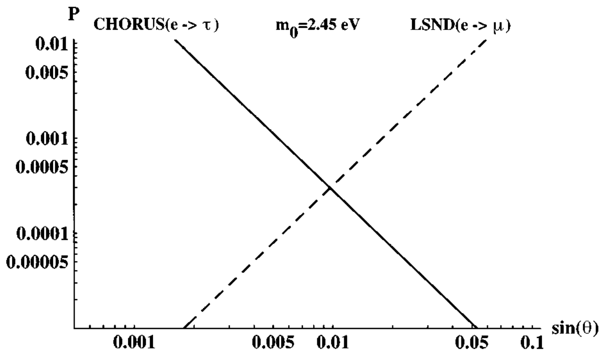

FIG. 1. The dependence of the oscillation probabilities $P(\nu_e\to\nu_\tau)$ at CHORUS and NOMAD (solid line), and $P(\nu_\mu\to\nu_e)$ at LSND (dashed line) on $\sin\theta$ for $\Delta m_{32}^2=10^{-2}$ eV^2 and $m_0=2.45$ eV.

CHORUS and NOMAD find signals of the $\nu_\mu\to\nu_\tau$ oscillations. Similar predictions from other models have been discussed in [16].

According to Eq. (6) for fixed Δm_{32}^2 and m_0, the parameter ϵ is inversely proportional to $\sin\theta$. Therefore $P(\nu_e\to\nu_\tau)$ increases when $P(\nu_\mu\to\nu_e)$ decreases (see Fig. 1).

In particular, if $P(\overline{\nu_\mu}\to\overline{\nu_e})\sim 3\times 10^{-3}$ (the level of the LSND result) and $m_0^2>6$ eV^2, then $P(\nu_e\to\nu_\tau)<3\times 10^{-5}$ which is beyond a sensitivity of CHORUS and NOMAD. On the contrary, for $P(\nu_e\to\nu_\tau)>2\times 10^{-2}$ which can be observed by these experiments one has $P(\nu_\mu\to\nu_e)<10^{-5}$. Thus a comparison of results from searches for $P(\nu_e\to\nu_\tau)$ and $P(\nu_e\to\nu_\mu)$ oscillations can give crucial check of the model.

For the parameters under consideration there are strong resonance transitions $\nu_e\to\nu_\mu,\nu_\tau$ and $\nu_\mu\to\nu_e$ in the inner parts of collapsing stars. As a consequence one predicts (i) the disappearance of the neutronization peak, (ii) hard ν_e spectrum at the cooling stage, and (iii) additional energy release in the inner parts of star which will stimulate shock wave revival desired for a star explosion. (iv) At the same time the $\nu_\mu\to\nu_e$ conversion leads to suppression of the r processes responsible for nucleosynthesis of heavy elements unless $m_0\leqslant 2$ eV [17].

(2) In the long distance limit, experiments are sensitive to oscillations stipulated by small mass difference Δm_{32}^2 and the oscillations due to large mass difference are averaged out. We get the probabilities

$$P(\nu_\mu\to\nu_e)=\tfrac{1}{2}\sin^2 2\theta-\cos^2\theta \times(\sin^2\theta-\epsilon^2\cos^2\theta)\sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right), \quad (17)$$

$$P(\nu_\mu\to\nu_\tau)=2(\epsilon\sin\theta)^2+\cos^2\theta\sin^2\left(\frac{\Delta m_{32}^2 L}{4E_\nu}\right), \quad (18)$$

which are applied to the atmospheric neutrinos. Notice that $P(\nu_\mu\to\nu_e)$ is suppressed due to both $\sin\theta\ll 1$ and $\epsilon\ll 1$, and the dominant effect comes from $\nu_\mu\to\nu_\tau$ oscillations as we suggested in the Introduction.

IV. PARAMETERS OF THE ZEE MODEL

In terms of parameters of the Lagrangian equation (1) the elements of the mass matrix (3) equal

$$\tan\theta\equiv\frac{f_{e\tau}}{f_{\mu\tau}}, \quad \epsilon\equiv\frac{f_{e\mu}}{\sqrt{f_{e\tau}^2+f_{\mu\tau}^2}}\left(\frac{m_\mu}{m_\tau}\right)^2, \quad (19)$$

and [4]

$$m_0\approx m_\tau^2\sqrt{f_{e\tau}^2+f_{\mu\tau}^2}\,\frac{g\sin 2\phi\cot\beta}{64\sqrt{2}M_W\pi^2}\ln\frac{M_2^2}{M_1^2}. \quad (20)$$

Here m_τ is the tau lepton mass, g is the weak coupling constant, m_W is the W-boson mass, $\tan\beta\equiv\langle\Phi_1\rangle_0/\langle\Phi_2\rangle_0$ is the ratio of the vacuum expectation value (VEV) of two Higgs doublets. The angle β determines physical (orthogonal to the would-be Goldstone) charged Higgs boson: $\Phi^+=\Phi_1^+\cos\beta-\Phi_2^+\sin\beta$, where Φ_1^+,Φ_2^+ are two charged Higgs fields from the doublets. The angle ϕ is the mixing angle of the Zee singlet and the doublet state Φ^+:

$$h=\cos\phi H_1+\sin\phi H_2,$$

$$\Phi^+=-\sin\phi H_1+\cos\phi H_2, \quad (21)$$

where H_1 and H_2 are the eigenstates of the mass matrix with masses M_1 and M_2, and the mixing angle is determined by

$$\tan 2\phi=\frac{4\sqrt{2}g^{-1}c_{12}M_W}{\sqrt{(M_1^2-M_2^2)^2-(4\sqrt{2}g^{-1}c_{12}M_W)^2}}. \quad (22)$$

As we have seen in Sec. II, the parameters of the mass matrix (3) m_0, ϵ, θ can be fixed by the data. This in turn allows one to determine ratios of the constants f_{ij} using Eq. (19):

$$\frac{f_{e\tau}}{f_{\mu\tau}}=\tan\theta_{e\mu}\leqslant 1 \quad (23)$$

and

$$\frac{f_{e\mu}}{f_{\mu\tau}}\approx\frac{\Delta m_{\rm atm}^2}{2m_{\rm HDM}^2}\left(\frac{m_\tau}{m_\mu}\right)^2\frac{1}{\sin 2\theta_{e\mu}}. \quad (24)$$

For $\sin^2 2\theta_{e\mu}=2\times 10^{-3}$, $\Delta m_{\rm HDM}^2=6$ eV^2 and $\Delta m_{\rm atm}^2=10^{-2}$ eV^2, Eq. (24) gives $f_{e\mu}/f_{\mu\tau}=5.3$, which means an inverse hierarchy of the Zee boson coupling constants with $f_{e\mu}$ being the largest one [11]. For fixed value $P(\nu_\mu\to\nu_e)$ the mixing angle θ is a function of $\Delta m_{31}^2=m_0^2$ Using this dependence we get from Eq. (23) and Eq. (24), the ratios $f_{e\mu}/f_{e\tau}$ and $f_{\mu\tau}/f_{e\tau}$ as the functions of m_0 for fixed value of $P(\nu_\mu\to\nu_e)$ (see Fig. 2). For $P(\nu_\mu\to\nu_e)=1.5\times 10^{-3}$ (which is in the range of sensitivity of KARMEN and LSND), we find $f_{e\mu}\simeq f_{\mu\tau}\gg f_{e\tau}$ at $m_0=5$ eV. This relation may testify for certain horizontal symmetry. Values of mass m_0 below 5 eV imply an inverse flavour hierarchy of the couplings: $f_{e\mu}\gg f_{\mu\tau}\gg f_{e\tau}$. For $P(\nu_\mu\to\nu_e)\leqslant 10^{-4}$ one gets the inverse flavor hierarchy already below $m_0=10$ eV. Absolute values of the coupling constants can be fixed by Eq. (20). Taking values of parameters: $\sin\phi\simeq 10^{-1}$, $\tan\beta\simeq 10$, $M_1\simeq M_2$

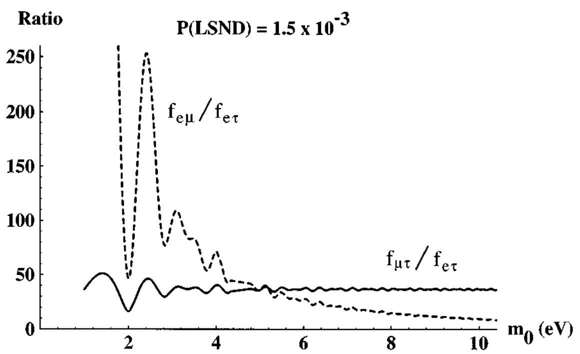

FIG. 2. The ratios $f_{\mu\tau}/f_{e\tau}$ (solid line) and $f_{e\mu}/f_{e\tau}$ (dashed line) as the functions of m_0 for $P(\nu_\mu \to \nu_e) = 1.5 \times 10^{-3}$.

$\simeq$500 GeV we find $f_{e\mu} = 10^{-2} - 1$. Therefore, the scenario implies rather big coupling constants of the Zee boson.

V. CONSTRAINTS FROM THE ELECTROWEAK DATA

Since the constants $f_{e\mu}$, $f_{\mu\tau}$ are rather big the Zee singlet can give observable contributions to different weak processes. The effective four-fermion Lagrangian induced by the Zee boson exchange can be written (after appropriate Fiertz transformation) as

$$\frac{G_F}{\sqrt{2}} \xi [\overline{\nu_\mu} \gamma^\mu (1-\gamma_5) e \overline{\nu_\mu} \gamma_\mu (1-\gamma_5) e - \overline{\nu_\mu} \gamma^\mu (1-\gamma_5) \mu \overline{\nu_e} \gamma^\mu (1-\gamma_5) e + \cdots], \qquad (25)$$

where

$$\xi \equiv \frac{1}{\sqrt{2} G_F} \frac{f_{e\mu}^2}{\overline{M}^2}, \qquad (26)$$

and

$$\frac{1}{\overline{M}_H^2} \equiv \frac{\cos^2\phi}{M_1^2} + \frac{\sin^2\phi}{M_2^2}. \qquad (27)$$

Notice that only usual left-handed components of leptons participate in the interactions with the Zee boson, and therefore the Lagrangian (25) has usual V-A form.

Let us consider the neutrino electron scattering, $\overline{\nu_\mu} e^- \to \overline{\nu_\mu} e^-$ and $\nu_\mu e^- \to \nu_\mu e^-$. The contribution from the Lagrangian (25) changes the g_L^e coupling: $g_L^e \to g_L^e + \xi$. On the other hand the CHARM II data on g_L and g_R [18] agree well with predictions of the standard model. Therefore ξ should be smaller than the experimental error Δg_L^e $\xi < \Delta g_L^e$. Using 1σ error $\Delta g_L^e = 0.025$, we find, explicitly,

$$\frac{f_{e\mu}^2}{\overline{M}^2} < 0.036 G_F. \qquad (28)$$

The Zee singlet exchange leads also to lepton number violating process $\nu_\mu e^- \to \nu_\tau e^-$ which contributes to $\nu_\mu e$ scattering incoherently. Its amplitude is proportional to $f_{e\mu} f_{e\tau}$.

The second term in the Lagrangian (25) gives a renormalization of the four fermion coupling constant G of the muon decay $\mu \to \nu_\mu e \overline{\nu_e}$. Assuming that the effect of the Zee boson on the decay rate is smaller than 0.1%, (so that it does not destroy the agreement in the electroweak precision tests) we find

$$\frac{f_{e\mu}^2}{\overline{M}^2} < 7 \times 10^{-4} G_F. \qquad (29)$$

Also modes of the muon decay with lepton number violation appear: $\mu \to \nu_\tau e \overline{\nu_e}$, $\mu \to \nu_\mu e \overline{\nu_\tau}$, $\mu \to \nu_\tau e \overline{\nu_\mu}$. They contribute to the total decay rate incoherently, and restrictions on constants are much weaker than Eq. (29).

Bound on the model follow also from the $e - \mu - \tau$ universality. A deviation from the universality due to the Zee boson contribution is $|1 - g_\tau / g_\mu| \sim f_{e\mu}^2 / (G_F \overline{M}^2)$, where g_μ and g_τ are the coupling constants of the μ- and τ-weak charged currents. Recent measurement of the branching ratio of the decay $\tau \to e \overline{\nu_e} \nu_\tau$ at OPAL [19] gives the ratio of the couplings $g_\tau / g_\mu = 1.0025 \pm 0.0060$, and the corresponding bound on the parameters of model is Eq. (29) $f_{e\mu}^2 / \overline{M}^2 < 0.006 G_F$.

These bounds corresponding input data are collected in Table I, where we used experimental errors with 1σ(68% C.L.) in CHARM II and OPAL data in order to get bounds for the coupling.

The result (29) allows one to get bounds on masses and mixing of scalar bosons. Indeed, using expression (20) for the mass we can find $f_{e\mu}$ as the function of ϕ, β, and M_i: $f_{e\mu} = f_{e\mu}(\phi, \beta, M_i)$. Inserting this function into Eq. (29), we find the lower bound on $\sin\phi$ as the function of M_1 for different values of $P(\nu_\mu \to \nu_e)$, m_0, $\tan\beta$, and fixed $M_2 = 300$ GeV [see Figs. 3(a)–3(c)]. Notice that the strongest bound is for $M_1 = M_2$. The forbidden region becomes larger with an increase of $\tan\beta$ as well as with a decrease of m_0 and $P(\nu_\mu \to \nu_e)$. As follows from Fig. 3, a big region of parameters exists in which all the restrictions are satisfied. Furthermore, one of the charged Higgs bosons can be at the level of lower kinematical bound.

The model leads to the radiative decays of the muon $\mu \to e\gamma$ and neutrino $\nu_{3(2)} \to \nu_1 \gamma$ through the one-loop diagram with the Zee singlet.

The branching ratio for the $\mu \to e\gamma$ [4] equals

TABLE I. Input experimental data and constraints of our parameters.

Processes	Input experimental data	$f_{e\mu}^2 / \overline{M}^2$
$\overline{\nu_\mu} e \to \overline{\nu_\mu} e$, $\nu_\mu e \to \nu_\mu e$	$g_L^e = -0.528 \pm 0.026$ (CHARM II)	$< 3.6 \times 10^{-2} G_F$
$\mu \to \nu_\mu e \overline{\nu_e}$	$\Delta\Gamma_\mu / \Gamma_\mu < 10^{-3}$	$< 7 \times 10^{-4} G_F$
$e - \mu - \tau$ universality	$g_\tau / g_\mu = 1.0025 \pm 0.0060$ (OPAL)	$< 6 \times 10^{-3} G_F$

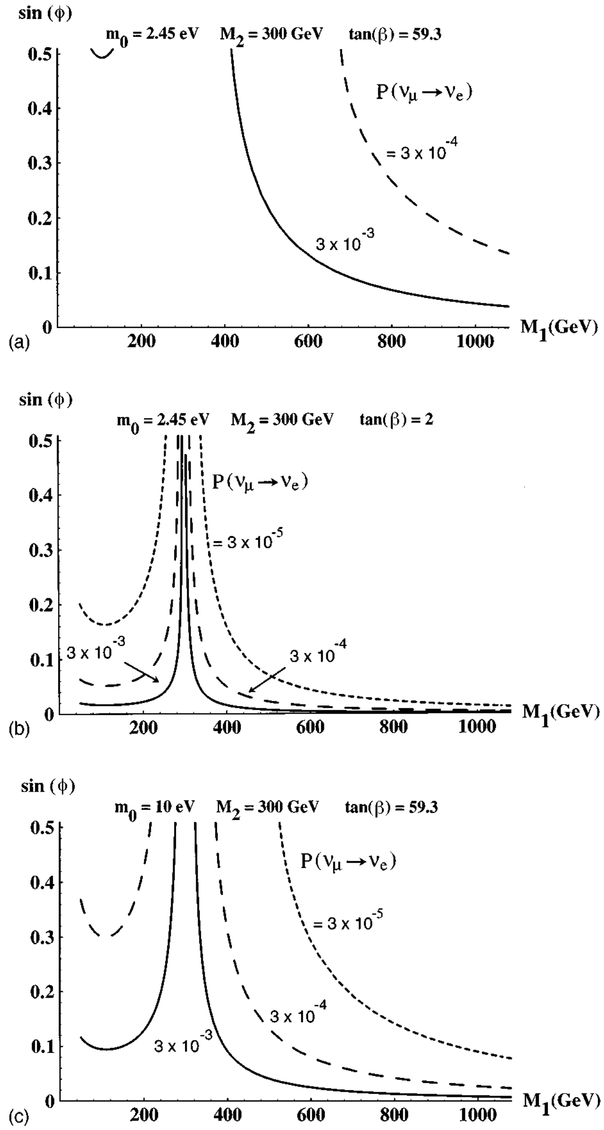

FIG. 3. The lower bound for $\sin\phi$ as the function of M_1 for $P(\nu_\mu \to \nu_e) = 3\times 10^{-3}$ (solid curve), 3×10^{-4} (long-dashed curve), and 3×10^{-5} (short-dashed curve). For other parameters we take (a) $\tan\beta = 59.3$, $m_0 = 2.45$ eV, (b) $\tan\beta = 2$, $m_0 = 2.45$ eV, and (c) $\tan\beta = 59.3$, $m_0 = 10$ eV. In all the cases $M_2 = 300$ GeV.

$$B(\mu \to e\gamma) = \left(\frac{\alpha}{48\pi}\right)\left(\frac{f_{e\tau}f_{\mu\tau}}{M_H^2 G_F}\right)^2. \qquad (30)$$

Using Eqs. (19), (20), and (27), we can express it as $B(\mu\to e\gamma) = A(\sin\phi, M_i, \tan\beta) m_0^4$. The branching ratio becomes smaller with increase of $\sin\phi$ and decrease of $\tan\beta$ (see Fig. 4). The present experimental upper bound, $B < 4.9\times 10^{-11}$ [20] (shown by the horizontal dashed line), can be strengthen up to 5×10^{-13} in the experiment at MEGA in LAMPF (Los Alamos) [21]. Future experiments will push the limit to 3×10^{-14} [21]. Results from these experiments combined with bounds from precision tests (Fig. 3) will cover essential parts of the parameter space of the model.

The lifetime of heavy neutrino with respect to decay $\nu_i \to \nu_1 \gamma (i = 2,3)$ equals [4]

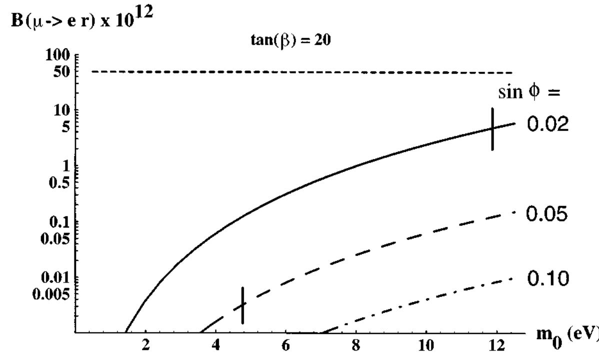

FIG. 4. The dependence of the branching ratio of $\mu\to e\gamma$ on m_0 for $\sin\phi = 0.02$ (solid curve), 0.05 (dashed curve), and 0.1 (dash-dotted curve). The values of other parameters are fixed as $\tan\beta = 20$, $M_1 = 500$ GeV, $M_2 = 300$ GeV, and $\sin^2 2\theta = 2\times 10^{-3}$. The short vertical lines indicate the lower bounds on m_0 which are given by the muon decay. The experimental upper bound on $B(\mu\to e\gamma)$ is shown by the horizontal dashed line.

$$\tau(\nu_i \to \nu_1) = \left\{ \alpha m_i^5 \left[2\frac{m_\mu^2}{m_\tau^2} C_\mu \left(1 - \frac{C_\tau}{C_\mu}\cos 2\theta \right)\right]^2 \times \left(1 - \frac{m_1}{m_i}\right)^3 \right\}^{-1}, \qquad (31)$$

where

$$C_\ell = \frac{1}{\ln(M_{H2}^2/M_{H1}^2)} \left\{ \frac{1}{M_{H2}^2}\left[\ln\left(\frac{M_{H2}^2}{m_\ell^2}\right) - 1 \right] - (2\to 1)\right\}, \quad \ell = \mu, \tau. \qquad (32)$$

The lifetime $\tau(\nu_i \to \nu_1)$ depends mainly on the charged Higgs scalar masses M_1 and M_2; $f_{\ell\ell'}$ and $\sin\phi$ enter only via the mass of neutrino. For $m_0 = 1$–10 eV the lifetime is in the interval 10^{22}–10^{29} years. This may have some cosmological implications.

In the limit of $f_{e\tau} = 0$ the corresponding anomalous magnetic moment of neutrino can be represented as [4]

$$\mu_\nu \simeq -4 e m_0 C_\tau, \qquad (33)$$

where C_τ is defined in Eq. (32). For $M_1 \simeq M_2 \simeq 300$ GeV and $m_0 = 2.65$ eV, we get $\mu_\nu \simeq 6\times 10^{-16} e/2m_e$.

VI. SOLAR NEUTRINOS

For solar neutrinos all oscillations are averaged and from Eq. (12) one gets a survival probability

$$P(\nu_e \to \nu_e) = \cos^4\theta + \tfrac{1}{2}\sin^4\theta + O(\epsilon^2). \qquad (34)$$

There is no dependence of suppression of the ν_e flux on energy and for $\epsilon, \sin^2\theta \ll 1$ the effect is small. Thus in the scenario under consideration there is no solution of the solar neutrino problem.

Let us suggest that apart from three known neutrinos also singlet (because of the CERN e^+e^- collider LEP bound) neutrino ν_s exists. This neutrino mixes with electron neutrino so that the resonance conversion $\nu_e \to \nu_s$ explains the deficit of the solar ν_e flux.[2] The explanation requires the mass squared difference and the mixing angle in the intervals [22,23]

$$\Delta m^2 = (4-10)\times 10^{-6}\ \text{eV}^2, \quad \sin^2 2\theta_{es} = 10^{-3} - 10^{-2}. \tag{35}$$

The singlet neutrino could be the right-handed counterpart of the known neutrino components or new very light fermion which comes from some other sector of theory.

The mass of the lightest neutrino in the Zee model (which is essentially the ν_e) equals

$$m_1 = m_0 \epsilon \sin 2\theta \approx \frac{\Delta m_{\text{atm}}^2}{2 m_{\text{HDM}}} \sim (1-5)\times 10^{-3}\ \text{eV}. \tag{36}$$

Squared mass m_1^2 is close to Δm^2 desired for solar neutrinos (35). This means that the mass of singlet neutrino, m_s, should be rather close to m_1 (recall that for the resonance conversion one needs $m_s > m_1$):

$$\frac{m_s - m_1}{m_1} \approx \frac{\Delta m^2}{2 m_1^2}. \tag{37}$$

For $m_1 > 4\times 10^{-3}$ eV one gets from this equation $\Delta m/m_1 < 0.2$.

Let us consider the simplest scheme with only one singlet neutrino. We extend the Lagrangian of the Zee model by adding the terms

$$f_l \overline{\Psi_l} \Phi \nu_s + m_{ss} \nu_s^T C \nu_s. \tag{38}$$

All couplings f_i can be of the same order. The first term leads to mixing of ν_s with the active neutrinos: $m_{ls} = f_l \langle \Phi \rangle$. Performing block diagonalization of 4×4 mass matrix we get the mass matrix for the $(\nu_s - \nu_e)$ system:

$$M \approx \begin{pmatrix} m_{ss} & m_{es} \\ m_{es} & m_1 \end{pmatrix}, \tag{39}$$

where $m_{es} = f_e \langle \Phi \rangle$ and m_1 is fixed in Eq. (36). The mixing angle is then

$$\sin 2\theta_{es} \approx \frac{2 m_{es}}{m_{ss} - m_1}. \tag{40}$$

If the mass m_{ss} is not too close to m_1, we get

$$m_s \sim (2-4)\times 10^{-3}\ \text{eV}, \quad m_{es} \sim 10^{-4}\ \text{eV}. \tag{41}$$

With increase of m_s (and consequently the degeneracy), m_{es} can be further diminished.

According to Eq. (41) a solution of the solar neutrino problem implies very small Yukawa coupling, $f_e < 10^{-15}$, which is of the order $m_{\text{EW}}/m_{\text{string}}$, where $m_{\text{string}} \sim 10^{18}$ GeV is the superstring scale. The mass of the singlet neutrino m_s is of the order $m_{3/2}^2/m_{\text{string}}$, which also may indicate the SUSY origin of the singlet.

Mixing of the singlet neutrino with high mass states ν_μ, ν_τ is of the order

$$\sin^2 2\theta_{\mu s} \sim \sin^2 2\theta_{es} \sin^2\theta \sim 10^{-7}, \tag{42}$$

so that the bound from the primordial nucleosynthesis [25] can be satisfied.

The influence of the singlet fermion on ''standard'' structure of the Zee model is negligibly small and the results of the previous sections are not changed.

VII. CONCLUSIONS

(1) The Zee model reproduces rather naturally the pattern of neutrino masses and mixing which solves the atmospheric neutrino problem, supplies a desired hot dark matter component in the Universe, and gives the $\overline{\nu_\mu} - \overline{\nu_e}$ oscillations in the range of sensitivity of existing experiments.

(2) The solar neutrino problem can be solved in extension of the model with an additional singlet fermion ν_s, so that solar neutrinos undergo $\nu_e \to \nu_s$ conversion. The introduction of ν_s does not destroy basic features of the Zee model.

(3) The data on oscillations of the solar and atmospheric neutrinos as well as the cosmological mass scale fix all parameters of the Zee mass matrix. In general the scenario implies inverse flavor hierarchy of the Zee boson couplings. There is a possibility of $f_{e\mu} \simeq f_{\mu\tau} \gg f_{e\tau}$ which may originate from certain horizontal symmetry.

(4) The masses of the charged scalar bosons are of the order 100–500 GeV, and in certain cases at least one of the bosons can be at the lower experimental bound.

(5) This scenario will be tested in forthcoming experiments: (i) The probability of $\nu_\mu \to \nu_\tau$ oscillations is expected to be very small; the discovery of these oscillations in CHORUS and NOMAD will rule out the scenario. (ii) On the contrary, the signal of $\nu_e \to \nu_\tau$ oscillations may be in the region of sensitivity of these experiments. (iii) The confirmation of the LSND positive result will further testify for the suggested scenario. (iv) One may observe deviations from the SM predictions in the $\mu \to \nu_\mu e \overline{\nu_e}$ decay, the $\overline{\nu_\mu} e^- \to \overline{\nu_\mu} e^-$ and $\nu_\mu e^- \to \nu_\mu e^-$ scatterings as well as the violation of $e - \mu - \tau$ universality, etc. (v) The probability of $\mu \to e\gamma$ decay can be close to the present experimental upper bound. (vi) The lifetime for the neutrino radiative decay $\nu_{3(2)} \to \nu_1 \gamma$ is expected to be 10^{22}–10^{29} years. The decay of the relic neutrinos may have observable astrophysical consequences.

ACKNOWLEDGMENTS

We would like to thank Z. Hioki, K. Hagiwara, and K. Hikasa for the helpful discussion of radiative corrections. We also thank H. Shibuya for experimental informations of $\nu_e \to \nu_\tau$ at CHORUS. This research was supported by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No. 07640413).

[2] Similar pattern of neutrino masses and mixings including the sterile neutrino for the solar neutrino, atomospheric neutrino, and hot dark matter (as well as LSND) has been discussed in the context of other mechanisms of neutrino mass generation as well as phenomenologically by several authors [24].

[1] A. Zee, Phys. Lett. **93B**, 389 (1980); **161B**, 141 (1985).

[2] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Freedmann (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in ''Proceedings of the Workshop on the Unified Theories and Baryon Number in the Universe,'' Tsukuba, Japan, 1979, edited by O. Sawada and A. Sugamoto, KEK Report No. 79-18, Tsukuba, 1979 (unpublished), p. 95.

[3] L. Wolfenstein, Nucl. Phys. **B175**, 93 (1980).

[4] S. T. Petcov, Phys. Lett. **115B**, 401 (1982); S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59**, 671 (1987).

[5] A. Y. Smirnov and Z. Tao, Nucl. Phys. **B426**, 415 (1994).

[6] K. S. Hirata *et al.*, Phys. Lett. B **205**, 416 (1988); **280**, 146 (1992); D. Casper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); NUSEX Collaboration, M. Aglietta *et al.*, Europhys. Lett. **8**, 611 (1989); **15**, 559 (1991); SOUDAN2 Collaboration, T. Kafka, in *TAUP 93*, Proceedings of the Third International Workshop on Theoretical and Phenomenological Aspects of Underground Physics, Assergi, Italy, edited by C. Arpesella *et al.* [Nucl. Phys. B (Proc. Suppl.) **35**, 427 (1994)]; M. C. Goodman, in *Neutrino 94*, Proceedings of the 16th International Conference on Neutrino Physics and Astrophysics, Eilat, Israel, edited by A. Dar *et al.* [*ibid.* **38**, 337 (1995)]; Fréjus Collaboration, K. Daum *et al.*, Z. Phys. C **66**, 417 (1995); MACRO Collaboration, S. Ahlen *et al.*, Phys. Lett. B **357**, 481 (1995); Y. Fukuda *et al.*, *ibid.* **335**, 237 (1994).

[7] R. N. Mohapatra and G. Senjanovic, Z. Phys. C **17**, 53 (1983); R. Holman, G. Lazarides, and Q. Shafi, Phys. Rev. D **27**, 995 (1983); Q. Shafi and F. W. Stecker, Phys. Rev. Lett. **53**, 1292 (1984); R. Schaefer, Q. Shafi, and F. W. Stecker, Astrophys. J. **347**, 575 (1989); J. A. Holtzman and J. R. Primack, *ibid.* **405**, 428 (1993); A. Klypin, J. Holtzman, J. Primack, and E. Regos, *ibid.* **416**, 1 (1993); Y. P. Jing, H. J. Mo, G. Borner, and L. Z. Fang, Astron. Astrophys. **284**, 703 (1994); J. R. Primack, J. Holtzman, A. Klypin, and D. O. Caldwell, Phys. Rev. Lett. **74**, 2160 (1995); K. S. Babu, R. K. Schaefer, and Q. Shafi, Phys. Rev. D **53**, 606 (1996); C. Y. Cardall and G. M. Fuller, *ibid.* **53**, 4421 (1996).

[8] E776 Collaboration, L. Borodovsky *et al.*, Phys. Rev. Lett. **68**, 274 (1992).

[9] KARMEN Collaboration, in *Neutrino 94* [6], p. 235.

[10] L. Wolfenstein, Carnegie Mellon University Report No. CMU-HEP95-05, 1995 (unpublished).

[11] A. Joshipura and A. Smirnov (unpublished).

[12] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995); Phys. Rev. C **54**, 2685 (1996); Phys. Rev. Lett. **77**, 3082 (1996); J. E. Hill, *ibid.* **75**, 2654 (1995).

[13] S. T. Petcov and A. Yu. Smirnov, Phys. Lett. B **322**, 109 (1994); D. O. Caldwell and R. N. Mohapatra, *ibid.* **354**, 371 (1995); H. Minakata, *ibid.* **356**, 61 (1995); Phys. Rev. D **52**, 6630 (1995); G. Raffelt and J. Silk, Phys. Lett. B **366**, 429 (1996).

[14] K. Winter, in *Neutrino 94* [6], p. 211; M. Baldo-Ceolin, in *TAUP 93* [6], p. 450; L. DiLella, in *Neutrino 92*, Proceedings of the XVth International Conference on Neutrino Physics and Astrophysics, Granada, Spain, 1992, edited by A. Morales [Nucl. Phys. B (Proc. Suppl.) **31**, 319 (1993)].

[15] L. Camilleri, ''Neutrino Oscillation Experiments at Accelerators,'' Report No. CERN-PPE/94-87, 1994 (unpublished); COSMOS Collaboration, R. Sidwell, invited talk at XVII Conference on Neutrino Physics and Neutrino Astrophysics, Helsinki, 1995 (unpublished).

[16] A. De Rújula, M. Lusignoli, L. Maiani, S. T. Petcov, and R. Petronzino, Nucl. Phys. **B168**, 54 (1980); S. M. Bilenky, M. Fabbrichesi, and S. T. Petcov, Phys. Lett. B **276**, 223 (1992); K. S. Babu, J. C. Pati, and F. Wilczek, *ibid.* **359**, 351 (1995).

[17] G. M. Fuller, R. Mayle, B. S. Meyer, and J. R. Wilson, Astrophys. J. **389**, 517 (1992); Y-Z. Qian, G. M. Fuller, G. J. Mathews, R. W. Mayle, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. **71**, 1965 (1993).

[18] CHARM II Collaboration, P. Vilain *et al.*, Phys. Lett. B **281**, 159 (1992).

[19] The OPAL Collaboration, Report No. CERN-PPE/95-142, 1995 (unpublished).

[20] Particle Data Group, R. Barnett *et al.*, Phys. Rev. D **54**, 21 (1996).

[21] Y. Kuno, Invited talk at KEK meeting ''CP violation and its origin,'' 1996 (unpublished).

[22] GALLEX Collaboration, P. Angelmann *et al.*, Phys. Lett. B **327**, 377 (1994); **342**, 440 (1995); T. Kirsten, invited talk at XVII Conference on Neutrino Physics and Neutrino Astrophysics, Helsinki, 1996 (unpublished); SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Lett. B **328**, 234 (1994); Homestake Collaboration, R. Davis *et al.*, Nucl. Phys. B (Proc. Suppl.) **38**, 47 (1995); Kamiokande Collaboration, K. S. Hirata *et al.*, *ibid.* **38**, 55 (1995); Y. Suzuki, invited talk at XXVIII International Conference for High Energy Physics, Warsaw, 1996 (unpublished).

[23] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; L. Wolfenstein Phys. Rev. D **17**, 2369 (1987); E. W. Kolb, M. S. Turner, and T. P. Walker, Phys. Lett. B **175**, 478 (1986); S. P. Rosen and J. M. Gelb, Phys. Rev. D **34**, 969 (1986); J. N. Bahcall and H. A. Bethe, Phys. Rev. Lett. **65**, 2233 (1990); N. Hata and P. Langacker, Phys. Rev. D **50**, 632 (1994); P. I. Krastev and A. Yu. Smirnov, Phys. Lett. B **338**, 282 (1994).

[24] J. T. Peltoniemi and J. W. F. Valle, Nucl. Phys. **B406**, 409 (1993); D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **48**, 3259 (1993); **50**, 3477 (1994); E. Ma and P. Roy, *ibid.* **52**, R4780 (1995); E. Chun, A. S. Joshipura, and A. Yu. Smirnov, Phys. Lett. B **357**, 608 (1995); Z. Berezhiani and R. N. Mohapatra, Phys. Rev. D **52**, 607 (1995); R. Foot and R. Volkas, *ibid.* **52**, 6595 (1995); J. J. Gomez-Cadenas and M. C. Gonzalez-Garcia, Z. Phys. C **71**, 443 (1996); S. Goswami, Phys. Rev. D (to be published); N. Okada and O. Yasuda, Report No. TMUP-HEL-9605, hep-ph/9606411, 1996 (unpublished); N. Okada, Report No. TUMP-HEL-9607, hep-ph/9606221, 1996 (unpublished); S. M. Bilenky, C. Giunti, and W. Grimus, Report No. UWThPh-1996-42, hep-ph9607372, 1996 (unpublished); M. Narayan *et al.*, Phys. Rev. D **53**, 2809 (1996).

[25] For recent discussion see, e.g., C. Y. Cardall and G. Fuller, Phys. Rev. D **54**, 1260 (1996).