Search for nonstandard model CP or T violation at the τ-charm factory

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We systematically investigate the possibility of finding CP or T violation in the τ sector at the τ-charm factory. CP or T violation may occur in the τ pair production process, expressed as an electric dipole moment, and in τ decay processes. By assuming that an electric dipole moment as large as 10^{-19} e cm and CP or T violation effects originating from τ decay as large as 10^{-3} are observable at the τ-charm factory, we study all the possible extensions of the SM which are relevant for generating CP or T violation in the τ sector. And we point out, there are a few kinds of models which are hopeful candidates for generating such CP or T violation. For these models we consider all the theoretical and current experimental constraints and find that there exists some parameter space which will result in a measurable CP or T violation. Therefore we conclude that the τ-charm factory is a hopeful place to discover CP or T violation in the τ sector. [S0556-2821(97)00703-3]

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I. INTRODUCTION

The origin of CP violation has remained an unsolved problem since the discovery of CP violation in the K meson system a quarter of a century ago [1]. Although the observed CP violation in the K meson system can be accommodated in the standard model (SM) of electroweak interactions by virtue of a physical complex phase in the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix [2], it is not clear if the CKM mechanism is really correct or the only source for CP or T violation [3]. To verify the CKM mechanism one needs not only information on the K meson mixing and decay, but also that from the B meson system or other systems. The main physical purpose of the B factory is to test the CKM mechanism. However, even if the CKM method is the correct mechanism to describe CP violation in K and B meson mixing and decay, it is not necessary that CKM matrix be the only source of CP or T violation in nature [4]. As pointed out by Weinberg [5], unless the Higgs sector is extremely simple, it would be unnatural for Higgs-boson exchange not to contribute to CP or T nonconservation. The CKM matrix may explain the observed CP violation in the K meson system and possibly CP violation in the B meson system, while other new sources of CP or T violation may occur everywhere they can. In fact there are some physical motivations for people to seek new sources of CP or T violation. One motivation is from the strong CP problem in the SM [6]. For most of the scenarios to solve this problem they need a more complex vacuum structure and therefore a new CP nonconservation origin. Another motivation is from cosmology; most astrophysical investigations show that additional sources of CP violation are needed to account for the baryon asymmetry of universe at present [7]. The third motivation is from supersymmetry. Even in the minimal supersymmetrical standard model (MSSM), there are some additional CP nonconservation sources beyond the CKM matrix [8]. Now the question is at what places the possible new CP or T violation effects may show up and what is the potential to search for those effects. In this work we are going to study systematically the possibility to find new CP or T violation effects at the τ-charm factory (TCF).

The TCF is a very good place to test the SM and search for new physics phenomena because of its high luminosity and precision [9]. Especially the τ sector is a good place to seek for non-SM CP or T violation effects because in the SM CP violation in the lepton sector occurs only at the multi loop level and is way below any measurable level in high energy experiments; only non-SM sources of CP or T nonconservation may contribute, and another reason is that τ has abundant decay channels with sizable branching ratios, which can be used to measure CP or T violation. Furthermore, production-decay sequences of τ pairs by electron-positron annihilation are also favored. The reason is that (i) τ pair production by electron-positron annihilation is a purely electroweak process and can be perturbatively calculated; (ii) for the unpolarized electron-positron collision, its initial state is CP invariant in the c.m. frame; (iii) when the electron and/or positron beams are longitudinally polarized, the initial state is still effectively CP even, which presents extra chances to detect possible CP violation. To detect possible CP or T violation, one can either compare certain decay properties of τ^- with the corresponding CP or T conjugations, or measure some CP- or T-odd correlation of momentum or spin of the final state particles from τ pair decay. These CP or T-violating observables can and should be constructed model independently, since normally these observables are not well predicted due to the complexity and many free parameters in non-SM. The sensitivity of the experimental measurement on possible CP or T violation is determined by the sensitivity of the measurement on momentum, spin, or other physical quantities of the final state particles, and from them the physical CP- or T-violating observables are constructed. The better one can measure these quantities, the momenta, for example, the smaller the CP violation phase can be reached. In the TCF, we can expect about 10^7 τ pairs in 1 year, and the precision of measurements on kinematic parameters at 10^{-3}. The statistical and systematic errors can be around or below this level. Therefore generally a CP or T violation phase as small as order 10^{-3} can be reached at the TCF [9]. In a non-SM the CP- or

T-violating phase may appear in various stages of the process of production-decay chain, $e^+e^- \to \tau^+\tau^- \to$ final particles. We sort them in three cases.

(i) CP or T violation is generated in the tree-level production process, $e^+e^- \to \gamma, Z, X \to \tau^+\tau^-$, where X is some new Higgs or gauge boson, a CP- or T-violating phase appears either in the propagator of X or in the coupling to lepton pairs, and the simplest possibility is X, being a neutral Higgs boson in two- or multi-Higgs-doublet models. In this case the size of CP or T violation is proportional to the interference between the X exchange and γ, Z exchange processes. Unfortunately, X, being a Higgs doublet this interference term is proportional to the initial and final state fermion masses $m_e m_\tau$ as a result of chirality conservation. This factor alone contributes a suppression factor $m_e/m_\tau \sim 3\times10^{-4}$ to all CP- or T-violating observables in these kinds of processes at the TCF besides other possible suppression factors, like the large mass of X, and small coupling between X and leptons. We conclude that it is hopeless to search for CP nonconservation from the tree-level production process at the TCF.

(ii) CP or T violation is also generated at the production stage, but through the loop level. The most hopeful cases are that there may exist large electric or weak dipole moments (EDM's or WDM's) for the τ lepton; i.e., there are sizable CP or T violation phases at the vertex $\tau^- - \gamma, Z - \tau^+$. For this situation the new physical particles beyond the SM only appear as virtual particles through loops and the size of CP violation is proportional to EDM's or WDM's and is not suppressed by other factors; so the point is just whether the EDM's or WDM's of τ are large enough to be observed. Generally the Lagrangian describing CP or T violation in τ pair production related to EDM's and WDM's is

$$L_{CP} = -1/2 i \overline{\tau \sigma}^{\mu\nu} \gamma_5 \tau [d_\tau^E(q^2) F_{\mu\nu} + d_\tau^W(q^2) Z_{\mu\nu}]. \quad (1)$$

$F_{\mu\nu}$ and $Z_{\mu\nu}$ are the electromagnetic and weak field tensors. The momentum transfer at the TCF is around 4 GeV, and in experiments at the CERN e^+e^- collider LEP it is around the mass of the Z boson. Therefore at the TCF we expect the contribution from WDM's to be a factor of $4m_\tau^2/M_Z^2 \simeq 2\times10^{-3}$ smaller than the contribution from EDM's, if EDM's and WDM's are at the same order of the magnitude. On the other hand, the EDM term is less important at LEP energy. That is the reason why the LEP data constrain more strictly on WDM's than EDM's of τ [10,11]. We will neglect the WDM contribution from now on in this work.

(iii) It is possible that the CP or T violation phase is small in the production process but it is relatively large in the τ pair decay processes. The processes like τ to neutrino plus light leptons or hadrons through some new boson exchange at the tree level can contribute significantly to CP or T violation observables. Obviously in this situation any CP or T violation effect from the loop level is negligible, since any loop effect is at least suppressed by a factor $(1/16\pi^2)(m_\tau^2/M^2)$, where M is the mass of some new heavy particles appearing in the loops. This factor is smaller than 10^{-4} if M is heavier than about 20 GeV.

Now let us recall that how one detects CP violation in K meson decays: One measures the partial widths for a decay channel and compares it with that for the corresponding CP-conjugate decay process. Underlying such a philosophy is the interference between a CP-violating phase and a CP-conserving strong interaction phase; i.e., the CP violation effect is only manifest in the process with a strong final state interaction. To observe possible non-CKM CP violation effects in τ decays, however, one has to invoke a new methodology in most cases. The basic reason is that both in the production vertex of a τ pair (EDM of τ) and in some τ decay channels (like pure leptonic decay, $\pi\nu$, $\rho\nu$ decay channels, etc.), there is no strong interaction phase, caused by a hadronic final state interaction, to interfere with a possible CP-violating phase. So far some efforts have been made to investigate the CP or T violation effects at the TCF. Mainly those works try to find various ways to measure possible CP or T violation. A simple and very useful method is to construct observables which are CP- or T-odd operators being made from momenta of final state particles coming from τ pair decay or polarization vector of the initial electron (or both electron and positron) beam [12]. These operators can be used very conveniently to test any CP or T violation from either EDM's of τ leptons or from the decay of the τ pair without much model dependence. Some of the operators are constructed by considering the reactions

$$e^+(p) + e^-(-p) \to \tau^+ + \tau^- \to A(q_-) + \overline{B(q_+)} + X \quad (2)$$

in the laboratory system, where A $(\overline{B})$ can be identified as a charged particle coming from τ^- (τ^+) decay. Some CP- or T-odd operators (so CPT even; we will not consider the CPT-odd operator in this work since it is certainly much smaller violation effect) can be expressed as [12]

$$O_1 = \hat{p} \cdot \frac{\hat{q}_+ \times \hat{q}_-}{|\hat{q}_+ \times \hat{q}_-|},$$

$$T^{ij} = (\hat{q}_+ - \hat{q}_-)^i \cdot \frac{(\hat{q}_+ \times \hat{q}_-)^j}{|\hat{q}_+ \times \hat{q}_-|} + (i \leftrightarrow j), \quad (3)$$

where $\hat{p}, \hat{q}$ denote the unit momenta. If the initial electron and/or positron beams are polarized, one can construct some more observables making use of the initial polarization vector. For example a T-violating operator

$$O_2 = \vec{\sigma} \cdot \frac{\hat{q}_+ \times \hat{q}_-}{|\hat{q}_+ \times \hat{q}_-|} \quad (4)$$

can be constructed from the electron polarization vector $\vec{\sigma}$ and momenta of final state particles. If there exists any sizable CP or T violation from EDM's of τ or in the τ pair decay vertex, in principle the experimental expectation values of these operators are nonzero. For EDM's of τ leptons, d_τ, the theoretical expectation values of these operators are worked out and expressed only as a function of d_τ [13]. Since at the TCF the precision of measurement for these operators is at the 10^{-3} level, one expects to probe d_τ as small as $10^{-3}/2m_\tau \simeq 10^{-17}$ e cm. An example is the measurement of d_τ or d_τ^W in LEP experiments. The expectation value of the T^{ij} operator is directly related to d_τ [10]:

$$\langle T^{ij}_{AB}\rangle = \frac{E_{\text{c.m.}}}{e} d_\tau C_{AB} \text{diag}(-1/6,-1/6,1/3). \qquad (5)$$

The term ''diag'' means a diagonal matrix with diagonal elements given above, and $E_{\text{c.m.}}$ is the energy at the c.m. frame. The proportional constants C_{AB} depend on the τ decay modes, but generally this constant is order 1 for all the decay models [12]. The decay channels, which can be measured in experiments, may be classified as l-l, l-h, and h-h classes, where l is the lighter leptons, and h is a charged hadron like π, ρ, and a_1. Very impressively, if the initial electron (or both electron and positron) is polarized, one may use the polarization-asymmetrized distribution. The distribution is defined as the differential cross section difference between two different polarizations. With this method, a d_τ as small as 10^{-19} e cm can be reached at the TCF [13]; this corresponds to a sensitivity of 10^{-5} of CP or T violation. Up to now the best experimental bound on d_τ is from LEP experimental data, which are used to exclude indirectly d_τ as large as 10^{-17} e cm [11], and so a two orders of magnitude improvement on d_τ measurements can be achieved at the TCF.

Besides the CP- or T-odd operator method, several other useful strategies were proposed to test these violations in τ decay.

(1) Nelson [14] investigated systematically the feasibility of using the so-called stage-2 spin-correlation functions to detect possible non-CKM CP violation in the τ-pair production-decay sequence and the corresponding CP-conjugate sequence. The two-variable energy-correlation distribution $I(E_A,E_B,\Psi)$, where Ψ is the opening angle between the final A and B particles, is essentially a kinematic consequence of the τ-pair spin correlation which depends on the dynamics of Z^0 or $\gamma^*\to\tau^-\tau^+$ amplitude, and of the $\tau^-\to A^-X_A$ and $\tau^+\to B^+X_B$ amplitudes. By including θ_e and ϕ_e which specify the initial electron beam direction relative to the final state A and B momentum directions in the c.m. frame of e^-e^+ system, one obtains the so-called beam-referenced stage-2 spin-correlation function $I(\theta_e,\phi_e,E_A,E_B,\Psi)$. For the $\gamma^*\to\tau^-\tau^+$ vertex, there are four complex helicity amplitudes. Hence, the beam-referenced stage-2 spin-correlation function constructs four distinct tests for possible CP violation in $e^-e^+\to\tau^-\tau^+$. To illustrate the discovery limit in using the beam-referenced stage-2 spin-correlation function, Goozovat and Nelson [15] calculated the ideal statistical errors corresponding to the four tests. An advantage of detecting CP violation by use of the stage-2 spin-correlation function is that the model independence and amplitude significance of the results are manifest. It is complementary to the greater dynamical information that can be obtained through other approaches, such as from higher-order diagrammatic calculations in the multi-Higgs-doublet extensions of the SM.

(2) Another strategy to test CP violation in the two-pion channels of τ decay is due to Tsai [16], the basic ingredient of which is to invoke a highly polarized τ pair. Consider the τ pair production by electron-positron annihilation near threshold. If the initial electron and positron beams are polarized longitudinally (along the same direction), the τ pair will be produced mainly in the S wave, resulting in polarizations of $\tau^\pm$ both pointing in the same direction as that of the initial beams. Such a polarization is independent of the production angle and the corresponding polarization vector supplies us with an important block to form products with the final particle momenta. By comparing such polarization-vector-momentum products for a specific τ decay channel with those for the corresponding CP-conjugate process, one can perform a series of tests for possible CP violation effects in τ decay. However, it is impossible to detect CP violation in $\tau\to\pi\nu_\tau$ decay without violating CPT symmetry. As for the two-pion channel, the existence of a complex phase due to the hadronic final state interactions, given by the Breit-Wigner formula for the P-wave resonance ρ, enables detecting possible non-CKM violation by measuring the asymmetry of $(\mathbf{w}\times\mathbf{q}_1)\cdot\mathbf{q}_2$ without violating the CPT symmetry, where $\mathbf{w}$ is the τ polarization vector and $\mathbf{q}_i$ $(i=1,2)$ are the final pion momenta. By limiting the weak interaction to be transmitted only by exchange of spin-1 and spin-0 particles, one can know that only the S-wave part of the amplitude for the exchange of the extra spin-1 particle makes contributions to CP-violating observables. A very generic conclusion is that unless two diagrams have different strong interaction phases, one cannot observe the existence of a weak phase using terms involving $\mathbf{w}\cdot\mathbf{q_1}$. Tsai [17] also points out that T violation cannot be detected in pure leptonic decay without detecting the polarization of the decay lepton, because it is impossible to construct a T-odd operator by the momenta of the initial and final state particles in pure leptonic three-body decays. This also implies that with CPT symmetry, one can not detect CP violation in τ decay processes with unpolarized τ. On the other hand, however, with polarized initial electron and positron beams, one can construct T-odd operators using the momenta and polarization vector of τ and the decay lepton. Therefore polarization of initial electron and positron is very desirable for detecting of CP or T violation at the TCF.

(3) As for the $\tau\to(3\pi)\nu_\tau$ decay, it can proceed either via $J^P=1^+$ resonance a_1 and the $J^P=0^-$ resonance π'. Choi, Hagiwara, and Tanabashi [18] investigated the possibility that the large width-mass ratios of these resonances enhance CP violation effects in the multi-Higgs-doublet extensions of the SM. To detect possible CP violation effects, these authors compare the differential decay width for the $\tau^-\to\pi^+\pi^-\pi^-\nu_\tau$ with that for the corresponding CP-conjugate decay process. To optimize the experimental limit, they suggested considering several CP-violating forward-backward asymmetries of differential decay widths, with appropriate real weight functions.

(4) To probe possible CP-violating effects in τ decay with $K^-\pi^-\pi^+$ or $K^-\pi^-K^+$ final states, Kilian *et al.* [19] partitioned the final state phase space into several sectors and constructed some asymmetries of the differential decay widths. As a result, they showed that T-odd triple momentum correlations are connected to certain asymmetries, and so their nonvanishing would indicate a possible non-CKM CP violation in exclusive semileptonic $\tau\to$three pseudoscalar-meson decays.

With this knowledge and the results obtained in the previous papers in mind, now the crucial question, which is also the motivation of this work, is whether for CP or T violation appearing in EDM's close to $d_\tau \sim 10^{-19}$ e cm and CP or T violation effects in τ decay as 10^{-3} are possible values theoretically. If for all possible extensions of the SM, which people can visualize now, with natural parameter choices, these values are much smaller than the theoretically predicted ones, then the effort to search for such small CP or T violation signals at the TCF would be not very meaningful, at least from the theoretical point of view. In this paper we are trying to answer this question by investigating various possible mechanisms for generating large EDM's of τ, CP or T violation in τ decay. This paper is organized as follows. In Sec. II we review the generation of EDM's of τ lepton's in various popular models beyond the standard model and stress what models can produce possible large EDM's of τ. Following the discussion of EDM's, in Sec. III we concentrate on CP or T violation effects from τ decay in the models beyond the standard model. The last section is reserved for some further discussion, and the conclusion on the possibility of finding CP or T violation at the TCF is given.

II. EDM'S OF τ LEPTONS

The EDM of the lepton d_l is a dimension-5 operator. It can only be generated from the loop level. Because this operator changes the chirality of the lepton, it must be proportional to a fermion mass. In the SM the EDM of the lepton is generated from three-loop diagrams and is proportional to the lepton mass itself; so d_l is very small [20]. However, generally, d_l can be produced from one-loop diagrams in models beyond the standard model. At the one-loop level, the d_l can be expressed as

$$d_l \sim \frac{e\lambda}{16\pi^2}\frac{M_F}{V^2}\sin\phi \sim 10^{-18}\left(\frac{\lambda}{1}\right)\left(\frac{M_F}{100\ \text{GeV}}\right) \times\left(\frac{100\ \text{GeV}}{V}\right)^2 \sin\phi\ e\ \text{cm}, \tag{6}$$

where M_F is some fermion mass, V is a large scale from intermediate states in the loops, and λ denotes other couplings. ϕ is a CP or T violation phase. In the following part we assume maximal CP or T violation phases, i.e., $\sin\phi \simeq 1$. From this equation one sees that d_l can be at most as large as 10^{-18}–10^{-19} e cm if λ is between 1.0 and 0.1. Since V is a scale around or larger than the weak scale, in order to obtain a large d_l, M_F must be a large fermion mass such as the t quark mass or new heavy fermion masses. For example, if M is the τ mass, then d_τ is smaller than 10^{-20} e cm which is not detectable at the TCF. The same is true for the scale V. If V is at the TeV scale, d_l is smaller than 10^{-20} e cm. Although, in principle, d_τ is possibly as large as 10^{-19} e cm , one has to avoid too large an EDM of the electron d_e at the same time. The current experimental upper limit on d_e is about 10^{-26} e cm. This is a very strong constraint especially when one is expecting a large d_τ. So in any model beyond the standard model, two requirements must be satisfied in order to obtain a measurable d_τ. The first one is that the model must provide d_τ at the one-loop level and that d_τ be not suppressed by a small fermion mass term; the fermion mass term should be a top quark mass, supersymmetric partner of bosons, or other exotic fermion masses. The second one is that the predicted d_e associated with large d_τ is below its current experimental bound. These two conditions altogether exclude most of the models beyond the standard model which can provide large enough d_τ observables for the TCF. We will see from the following discussion that many models beyond the standard model do not satisfy the two requirements.

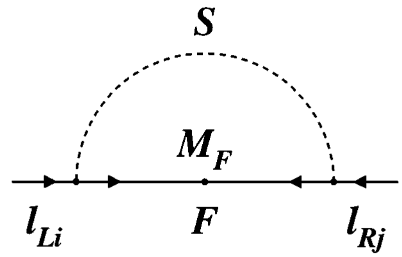

FIG. 1. One-loop diagram for lepton EDM generation, where F is the heavy fermion and S is the new boson. The photon line is attached to charged particles in the loop.

Usually the EDM of the lepton is generated from one-loop diagrams as an extension of the SM. Figure 1 is a typical one-loop diagram for the lepton EDM. The virtual particles are scalar or vector boson S and fermion F in the loop. A photon is attached to the charged intermediate particles. The d_l from this diagram is approximately proportional to the fermion mass M_F and it is divided by a scale V, which is larger or equal to M_F. Besides, there are two more couplings at the vertex l-S-F. In a practical model there could be many possible virtual bosons and fermions in the loop, but we only consider the dominant contribution here as an order of magnitude estimation. The diagram in Fig. 1 is evaluated as

$$d_i/Q \simeq \frac{|\lambda_i \lambda_i'^*|}{16\pi^2}\frac{M_F}{V^2}\xi\ \sin\phi, \tag{7}$$

where $i=e,\mu,\tau$ denotes three-generation leptons and Q is the electric charge of the virtual particles. ξ is an order of 1 factor from the loop integral. Equation (7) is true up to a factor of order 1. And there should be a logarithmic dependence on M_F/V in ξ, which is slowly varying.

In order to obtain measurable d_τ and avoid too large d_e, one needs a large M_F as discussed before and λ, λ' must be around order of 1 for τ but much smaller (smaller than about 10^{-3}) for electrons. We systematically investigate and review most of the popular extensions of the standard model and point out that the following types of models can fulfill the requirements.

Scalar leptoquark models [21]. The CP violation effects in τ sector for the models have been recently discussed extensively by some authors [18,22]. It is particularly interesting for generating a large d_l. These are the models which do not need to introduce additional fermions. Because the top quark mass is large, it is possible to generate a large d_τ through the coupling of the τ, top quark, and the corresponding leptoquark. d_e could be small enough due to the coupling of the electron, top quark, and leptoquark being independent of that for d_τ. As long as there is a relative large hierarchy for the couplings for different generations, the two requirements can be satisfied.

There are five types of scalar leptoquarks which can couple to leptons and quarks. We denote them by S_1, S_2, S_3, S_4, and $\vec{S}_5$. Their quantum numbers under a standard gauge group transformation are $(3,2,\frac{7}{3})$, $(3,1,-\frac{2}{3})$, $(3,2,\frac{1}{3})$, $(3,1,-\frac{7}{3})$, and $(3,3,-\frac{2}{3})$, respectively. The Yukawa coupling terms are therefore given by

$$L_1=(\lambda_1^{ij}\overline{Q_{Li}}i\tau_2E_{Rj}+\lambda_1'^{ij}\overline{U_{Ri}}l_{Lj})S_1+\text{H.c.},$$

$$L_2=(\lambda_2^{ij}\overline{Q_{Li}}i\tau_2l_{Lj}^c+\lambda_2'^{ij}\overline{U_{Ri}}E_{Rj}^c)S_2+\text{H.c.},$$

$$L_3=\lambda_3^{ij}\overline{D_{Ri}}l_{Lj}S_3+\text{H.c.},$$

$$L_4=\lambda_4^{ij}\overline{D_{Ri}}E_{Rj}^CS_4+\text{H.c.},$$

$$L_5=(\lambda_5^{ij}\overline{Q_{Li}}i\tau_2\vec{\tau}l_{Lj}^c)\cdot\vec{S}_5+\text{H.c.}$$

Here l_L and Q_L are lepton and quark doublets, respectively, and U_R, D_R, and E_R are singlet quarks and leptons, respectively. Individually only S_1 and S_2 contribute to the EDM of leptons.

The ξ factor in Eq. (7) is evaluated as $\xi=\frac{2}{3}\ln(M_F^2/V^2)+\frac{11}{6}$ [22]. Currently the constraints on mass and coupling of leptoquarks are relatively weak [23]. For leptoquarks coupled only to a third generation, its lower mass bound is about 45 GeV with order of unit coupling [23]. This bound is from a leptoquark pair production from LEP experiments. On the other hand, with the leptoquark mass at the weak scale, the coupling is very weakly bounded too. In fact the coupling could be as large as order of 1. If we take λ^{33} and λ'^{33} as 0.5 and the mass of leptoquark as 200 GeV and assume a maximal CP or T violation phase, we estimate that $d_\tau\simeq2\times10^{-19}$ e cm, while d_e is determined by other coupling components, and so a small d_e is not necessary in conflict with a large d_τ in this model.

Models with the fourth generation or other exotic leptons. The SM with a fourth generation is another possible model to generate a large d_τ. The heavy fourth-generation leptons may play a role of the heavy fermion F in the loop. However, it is well known that if the fourth generation exists, it must satisfy the constraints from LEP experiments [24]. Here we propose a realistic model for this purpose.

Besides the fourth-generation fermions, we also introduce a right-handed neutrino ν_R and a singlet scalar η^- with one unit electric charge [25]. The new interaction terms are

$$L=\lambda_{ij}l_i^Ti\tau_2l_j\eta^-+\lambda_i'E_{Ri}^T\nu_R\eta^-+M_R\nu_R^T\nu_R+M_i^D\overline{\nu_{Li}}\nu_R+\text{H.c.},\tag{8}$$

where λ_{ij} is antisymmetric due to the Fermi statistics. M^D is the Dirac neutrino mass from the standard Higgs vacuum expectation value. In this model three light neutrinos remain massless and the fourth neutrino is massive [26]. The constraints from LEP experiments and other low-energy data can be satisfied provided M_R is at weak scale or higher and M_i^D is not much smaller than M_R. In the one-loop diagram contribution to d_τ, η^- appears as the scalar S. The fermion line is two massive neutrinos ν_4 and ν_H in the mass basis and they are related to each other:

$$\nu_{L4}=\cos\theta\nu_4-\sin\theta\nu_H,$$
$$\nu_R=\sin\theta\nu_4+\cos\theta\nu_H.\tag{9}$$

We assume that ν_4 is the lighter neutrino and that the dominant contribution is from either ν_H or ν_4 depending on whether ν_H is heavier than the mass of η, M_η. Here d_τ is evaluated as in Eq. (7) with $M_F=M_H\cos\theta\sin\theta$ and $V\simeq M_\eta$ if $M_\eta\geqslant M_H$ and with $M_F=M_{\nu_4}\cos\theta\sin\theta$ and $V\simeq M_H$ if $M_\eta\leqslant M_H$. Choosing $\lambda_{34}=\lambda_3'=1.0$ and $M_F=50$ GeV, $V=200$ GeV, we have the numerical result $d_\tau\simeq10^{-19}$ e cm. Also in this model a hierarchy on the coupling λ and λ' for different generations is needed to keep a small enough d_e, i.e., $\lambda_{34}\gg\lambda_{14}$ and $\lambda_3'\gg\lambda_1'$.

The existence of exotic leptons provides another possibility to generate a measurable d_τ. It can be realized in horizontal models [27]. With only three standard leptons, it is impossible to obtain a large enough d_τ, because the largest fermion mass in the loop is m_τ. However, with some new heavy leptons this model can provide a large d_τ. The constraints from low-energy data can be avoided if one assumes that the horizontal interaction is strong between τ and the exotic lepton, but it is much weaker in other sectors. A similar result on d_τ as for the case with the fourth generation can be obtained.

Finally, we should point out that for our purpose it is clear that some new exotic heavy leptons are needed in the new physics models; however, even though there exist some kinds of models with some new heavy leptons, they are able to generate d_l only from two-loop diagrams [28], and so they may result in interesting d_e but not d_τ.

Generic MSSM. The generic MSSM contains 63 parameters not including the parameters in the non-SUSY SM. Fermionic superpartners of the ordinary bosons can be the heavy fermions in the loop diagrams for d_l. It provides some new sources for CP or T violation. It is well known that the electron and neutron can acquire large EDM's [29] in this model. In fact, in order to obey the experimental bounds on d_n and d_e, some parameters in the model are strongly restricted [30]. For d_l generation, it is dominated by a photino-mediated one-loop diagram. Both left- and right-handed sleptons also appear in the loop. The contribution to d_l from this diagram is proportional to left- and right-handed slepton mixing matrices $M_{LR}=(A_l-\mu\ \tan\beta)M_l$. A_l is the matrix of soft-SUSY-breaking parameters that appears in the SUSY Yukawa terms of slepton coupling to Higgs doublets. Here M_l is diagonal mass matrix of lepton mass. Usually it is assumed that A_l is diagonal and the diagonal elements are not much different for different generation; for example, in supergravity-inspired models A_l is universal for three generations [8], and therefore one can get $d_\tau/d_e\simeq m_\tau/m_e$. Using the experimental limit $d_e\leqslant10^{-26}$ e cm, one concludes that $d_\tau\leqslant4\times10^{-23}$ e cm [31]. However, in the generic MSSM all the elements of A_l are free parameters, and so the above constraint is not necessarily true. For example, if for some unknown reason the 33 component of A_l is much larger than other elements, and the μ term is much smaller than the SUSY-breaking scale, then d_τ still can be larger than 10^{-22}

e cm and d_e is in the allowed region. In this case d_τ can also be expressed as Eq. (7), but with $M_F = \tilde{m}_\gamma$, $V = \tilde{m}_\tau^2 / M_{LR}$, $\lambda_{33} = \lambda'_{33} = e$, and $\phi = \arg(M_{LR}^2 \tilde{m}_\gamma)$. The loop integral ξ was 4 times the function calculated some years ago in dealing with d_e in the MSSM known as the Polchinski-Wise function [32]. Here $\tilde{m}_\gamma$ and $\tilde{m}_\tau$ are photino and the third slepton masses, respectively. We estimate that $d_\tau \simeq 10^{-19}$ e cm with $\tilde{m}_\gamma = 100$ GeV and $V = 200$ GeV.

As for other popular extensions of the SM, we would like to point out here that, although they have some new sources of CP or T violation, they cannot offer an observable d_τ at the TCF. These include multi-Higgs doublet models (including a two-Higgs-doublet model) [3,33], left-right symmetric models [34], mirror fermion models [35], universal soft breaking SUSY models [8], etc. In multi-Higgs-doublet models electrons [36] and neutrons [5] may obtain a large EDM close to current experimental bounds through two-loop diagrams, but d_τ generated in the model is quite below the TCF observable value. The reason is that d_τ is proportional m_τ instead of a large fermion mass. We estimate $d_\tau \leq 4 \times 10^{-21}$ e cm [37] in this model. For left-right symmetric models, Nieves, Chang, and Pal [38] find that the upper bound for d_τ is 2.4×10^{-22} e cm. It is the right- or left-handed gauge boson in the loop as the role of the S particle, while the right-handed neutrino is the virtual fermion particle in the loop. d_τ in this model is proportional to left- and right-handed gauge boson mixing angle. Though it is not suppressed by the small fermion mass (M_F is a large right-handed neutrino mass), the mixing angle is constrained to be smaller than 0.004 [Don2] from purely nonleptonic strange decays. It leads to about a three orders of magnitude suppression. In the mirror fermion model, standard gauge bosons couple to ordinary leptons and the mirror lepton with a mixing angle. It is Z and W bosons in the one-loop diagrams, and the heavy fermion line is the mirror lepton. However, the mixing angle in this model is constrained by various experiments [40] and, most stringently, by LEP data on $Z \to \tau^+ \tau^-$ [41]. The constraint from LEP data on the mixing angle is less than about 0.3. The resulting bound is $d_\tau \leq 2.1 \times 10^{-20}$ e cm, which is a few times smaller than the TCF measurable value. As we have mentioned above in the universal soft breaking SUSY model, $d_\tau \leq 4 \times 10^{-23}$ e cm due to the constraint on d_e. The only alternative situation is discussed above on the generic MSSM in this section.

III. *CP*- OR *T*-VIOLATED τ DECAYS

As we have pointed out in the Introduction, CP or T violation effects in τ decays, if observed, must occur at tree-level diagrams. That is the interference between the SM τ decay processes and new tree-level processes of τ decays, in which CP or T violation phases appear at the interaction vertexes, provides the information of CP or T violation in the τ sector. Feynman diagrams of these processes can be shown as in Fig. 2, where f_i, f_j, and f_k are light fermions. X is a new particle (scalar or vector boson) which mediates CP- or T-violating interactions. The size of CP or T violation is always proportional to the interference of the tree-level diagrams. We denote the amplitudes for these diagrams as A_1 for the W boson exchange diagram, A_2 for the other X boson exchange diagrams. The size of CP or T violation

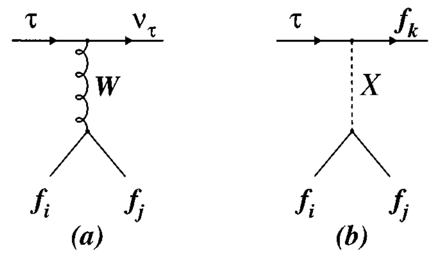

FIG. 2. The diagrams for τ decay. (a) is the contribution from the SM and (b) is the contribution from new boson exchange.

in the τ decay can be characterized by a dimensionless quantity

$$\epsilon = \frac{\mathrm{Im}(A_1^* A_2)}{|A_1|^2 + |A_2|^2}. \tag{10}$$

Practically, physical quantity expectation values which are used to reflect CP or T violation, like the expectation values of CP- or T-odd operators, the difference between a partial decay width of a τ^- decay channel and its conjugate τ^+ decay channel, are model dependent and generally quite complicated. It needs detailed information on the new physics model and a lot of parameters enter into the expression. This makes it a very much involved task to write down these quantities in a specific model beyond the SM. And the exact CP or T violation quantity expression written down from a model should be different from the ϵ defined above. However, as a simple and reasonable estimation, the quantity ϵ in Eq. (11) can be used as an indication of how large a CP or T violation may happen at various τ decays. Moreover, the amplitude A_2 is usually much smaller than A_1 because so far all the experimental data agree with the SM prediction very well. So an A_2 term in the denominator can be neglected. Using A_1 as the amplitude from W boson exchange and A_2 from the new boson X exchange, we estimate its size,

$$\epsilon \sim (4\sqrt{2} G_F)^{-1} \frac{\mathrm{Im}(\lambda \lambda'^*)}{M_X^2}. \tag{11}$$

Here G_F is Fermi constant and λ, λ' are couplings in A_2. From Eq. (12) one sees that the size of CP or T violation is determined by the parameter $\mathrm{Im}(\lambda\lambda'^*)/M_X^2$. For different models, this parameter is constrained by some other physical processes. So the possible size of CP or T violation depends on the parameter region which is restricted in a specific model.

In Fig. 2 the final state fermions can be a pair of leptons and quarks besides ν_τ. It corresponds to pure leptonic and hadronic decays respectively. At the quark level, the diagrams with a pair of quarks in the final states denote an inclusive process; it includes all possible hadronic channels originating from quark pair hadronization. Some useful hadronic final states such as 2π, 3π, $K\pi$, $K\pi\pi$, $KK\pi$, and ρ, a_1 can be used to measure the properties of τ. However, it is often difficult to make a reliable quantitative prediction for CP or T violation in exclusive hadronic decay modes, because of the uncertainty in the hadronic matrix elements. On the other hand, for the inclusive cases, one may make a more reliable quantitative estimation due to the fact that one has no need to deal with the hadronization of quarks in this case. In addition, a QCD correction should not change the order of the tree-level diagram evaluation as the energy scale for τ decay processes is around 1 GeV. In this section we only deal with the diagrams containing quark pairs inclusive. So the CP or T violation size we estimate below is for all the possible hadronic decay channels. In the last section we will comment on our results in exclusive processes. Because of the scale of the τ mass, its decay products can only be neutrinos, electrons, muons, and hadrons containing only light u, d, and s quarks as other heavy quarks are kinematically forbidden. Therefore there are not many possibilities for the X particle being the candidate for mediating CP or T violation in Fig. 2. In fact all possible choices are the following: X being a leptoquark, charged Higgs singlet, doublet and triplet, and double charged singlet. Now we come to discuss these different cases separately.

Scalar leptoquark models. At the tree level it is obvious that only S_1, S_2, and $\vec{S}_5$ contribute to τ decays. There are two types of decay processes at the quark level, $\tau\to\nu_\tau\overline{ud}$ and $\tau\to\nu_\tau\overline{us}$. The ϵ parameter is determined by $\lambda^{31}\lambda'^{31*}$ and $\lambda^{32}\lambda'^{31*}$ for these two types of decays, respectively, in models 1 and 2 in Eq. (8). For model 5 there is CP or T violation effect only in the second type process, which is determined by $\lambda^{32}\lambda'^{31*}$. A direct constraint on these parameters can be obtained through comparing the theoretical value $\Gamma^{\rm th}(\tau\to\pi\nu_\tau)=(2.480\pm0.025)\times10^{-13}$ GeV and the measurement value of $\Gamma^{\rm expt}(\tau\to\pi\nu_\tau)=(2.605\pm0.093)\times10^{-13}$ GeV [42]. Assuming that the real and imaginary parts of the coupling $\lambda\lambda'^{*}$ are approximately equal, one has, from $\tau\to\pi\nu_\tau$ [18],

$$\frac{|\mathrm{Im}(\lambda^{31}\lambda'^{31*})|}{M_X^2}\sim\frac{|\mathrm{Re}(\lambda^{31}\lambda'^{31*})|}{M_X^2}<3\times10^{-6}\ \text{GeV} \tag{12}$$

at the 2σ level for models 1 and 2. And from $\tau\to K\nu_\tau$ a similar result can be obtained for all three models. Using the theoretical value $\Gamma^{\rm th}(\tau\to K\nu_\tau)=(0.164\pm0.036)\times10^{-13}$ GeV [42,43] and the measurement value $\Gamma^{\rm expt}(\tau\to K\nu_\tau)=(0.149\pm0.051)\times10^{-13}$ GeV for the $\tau\to K\nu_\tau$ decay width we obtain

$$\frac{|\mathrm{Im}(\lambda^{32}\lambda'^{31*})|}{M_X^2}\sim\frac{|\mathrm{Re}(\lambda^{32}\lambda'^{31*})|}{M_X^2}<7\times10^{-6}\ \text{GeV} \tag{13}$$

at the 2σ level. This constraint is less stringent due to the large uncertainties in $\Gamma^{\rm expt}(\tau\to K\nu_\tau)$. With these constraints, one estimates the upper bound of the ϵ value for the two types of processes as

$$\epsilon(\tau^-\to\nu_\tau\overline{ud})\simeq(4\sqrt{2}G_F)^{-1}\frac{\mathrm{Im}(\lambda^{31}\lambda'^{31*})}{M_X^2}\leqslant4\times10^{-2} \tag{14}$$

and

$$\epsilon(\tau^-\to\nu_\tau\overline{us})\simeq(4\sqrt{2}G_F)^{-1}\sin\theta_C\frac{\mathrm{Im}(\lambda^{32}\lambda'^{31*})}{M_X^2}\leqslant2\times10^{-2}, \tag{15}$$

where θ_C is the Cabibbo angle. $\epsilon(\tau^-\to\nu_\tau\overline{us})$ is proportional to $\sin\theta_C$ and is smaller than $\epsilon(\tau\to\nu_\tau\overline{ud})$ because this process is Cabibbo suppressed, even though the coupling is less constrained than that of the Cabibbo-unsuppressed process. From this estimation we expect that CP or T violation in these models could be large enough for the TCF or in the other words TCF data can put a stronger direct restriction on the parameters of the model. However, if one assumes that all the couplings λ and λ' are at the same size irrespective of the generation indexes, then much more stringent bounds exist. These bounds are obtained from experimental bounds of $B(K_L\to\mu e)$, $B(\pi\to e\nu_e(\gamma))$, $B(\pi\to\mu\nu_\mu(\gamma))$, and $\Gamma(\mu Ti\to eTi)/\Gamma(\mu Ti\to\text{capture})$ [18]. They are generally about five orders of magnitude smaller than the direct bounds. Therefore the size of the CP or T violation is $\epsilon\leqslant4\times10^{-7}$ which is far below the capability of the TCF.

Multi-Higgs doublet (MHD) models. With the natural suppression of flavor-changing neutral current, it is necessary to have more than two Higgs doublets, so that there are at least two physical charged Higgs particles. CP or T violation may generally happen through the mixing of these charged Higgs particles. We consider a multi-Higgs-doublet model, say, n Higgs doublets. In this model there are $2(n-1)$ charged and $(2n-1)$ neutral physical scalars, since only the Yukawa interactions of the charged scalars with fermions are relevant for our purpose. Following Grossman [44] we write down the Yukawa interactions in fermion mass eigenstates as

$$L_{\rm MHD}=\sqrt{2\sqrt{2}G_F}\sum_{i=2}^{n}[X_i(\overline{U_L}VM_DD_R)+Y_i(\overline{U_R}M_UVD_L)+Z_i(\overline{l_L}M_EE_R)]H_i^{+}+\text{H.c.} \tag{16}$$

Here M_U, M_D, and M_E denote the diagonal mass matrices of up-type quarks, down-type quarks, and charged leptons, respectively. V is the Kobayashi-Maskawa (KM) matrix. X, Y, and Z are complex couplings which arise from the mixing of the charged scalars and CP or T violation in τ decay processes is due to these couplings. How large the ϵ is for various τ decay channels depends on the values of these parameters. More precisely, in the pure leptonic decays the size of CP or T violation is determined by $\mathrm{Im}(Z_iZ_j^*)$ with $i\neq j$ and in hadronic decays it is determined by $\mathrm{Im}(X_iZ_j^*)$ and $\mathrm{Im}(Y_iZ_j^*)$. The three combinations of parameters are constrained by various experiments [44]. The strongest constraint on Z is from e-μ universality in τ decay, which gives $|Z|\leqslant1.93M_H$ GeV^{-1} for a Higgs boson mass M_H of around 100 GeV. $\mathrm{Im}(XZ^*)$ is bounded from above from the measurement of the branching ratio $B(B\to X\tau\nu_\tau)$, $\mathrm{Im}(XZ^*)\leqslant|XZ|\leqslant0.23M_H^2$ GeV^{-2} if $M_H\leqslant440$ GeV. Finally an upper bound is given as $\mathrm{Im}(YZ^*)\leqslant|YZ|\leqslant110$ from the experimental data of the process $K^+\to\pi^+\nu\overline{\nu}$. This bound is obtained for t quark mass at 140 GeV [44] and $M_H=45$ GeV; however, for a different M_H, say, 100 GeV, this bound is expected not to change much. With these bounds we can estimate the CP or T violation size of the τ leptonic and hadronic decays. For the leptonic decay $\tau\to\mu\nu\overline{\nu}$, we have the quantity

$$\epsilon\simeq\frac{1}{2}\frac{\mathrm{Im}(ZZ^*)m_\mu m_\tau}{M_H^2}\frac{m_\mu}{m_\tau}=\frac{1}{2}\frac{m_\mu^2}{M_H^2}\mathrm{Im}(ZZ^*)\leqslant 2\times10^{-2}. \tag{17}$$

Here the additional factor m_μ/m_τ comes from the interference of left- and right-handed muon lines in the final states. So we expect that the CP or T violation effect in the process $\tau\to e\nu\overline{\nu}$ is suppressed by a factor m_e/m_μ and is negligible. For the hadronic decay $\tau\to\overline{u}d\nu$ we have

$$\epsilon\simeq\frac{1}{2}\frac{m_d\overline{m_d}}{M_H^2}\mathrm{Im}(XZ^*)\leqslant 3\times10^{-4}. \tag{18}$$

With the current d quark mass $m_d=7$ MeV and the dynamical d quark mass $\overline{m_d}=300$ MeV. For hadronic decay $\tau\to\overline{u}s\nu$ a similar result is obtained:

$$\epsilon\simeq\frac{1}{2}\frac{m_s\overline{m_s}}{M_H^2}\mathrm{Im}(XZ^*)\leqslant 1.5\times10^{-3}. \tag{19}$$

Here we use current and dynamical s quark masses as 150 MeV and 400 MeV, respectively. In summary, in the multi-Higgs-doublet model CP or T violation effect is possibly as large as the order of 10^{-3} for exclusive hadronic decays and it could be even close to 10^{-2} in pure leptonic decay to μ and neutrinos.

Other extensions of the SM for pure leptonic decays. Besides leptoquarks and Higgs doublets, there are three other kinds of scalars which can couple to leptons. We denote l as a lepton doublet and E as a singlet lepton. Two l can combine to a charged singlet or a triplet. Two E can combine to a double charged singlet. Corresponding to these three cases one can introduce a charged singlet scalar h^-, triplet scalar Δ, and double charged scalar K^{--}. However, K^{--} only induces a lepton family-number-violating process $\tau\to3l$. There is no diagram corresponding SM contribution, and so there is no CP or T violation mediated by this particle. Also the branching ratio ($\leqslant10^{-5}$) for this decay is much smaller than the TCF reachable CP or T violation precision 10^{-3}. In principle, if there exists more than one h or Δ, CP or T violation can be induced by the interference of the W exchange diagram and h or Δ exchange diagram in the process $\tau\to l\overline{\nu}\nu$ with $l=e,\mu$. Now let us discuss these two possibilities in detail. We can write down the new interaction terms which couple the new scalar particles to leptons as

$$L_h=\tfrac{1}{2}f_{ij}l^T{}_iCi\tau_2l_jh+\mathrm{H.c.}, \tag{20}$$

$$L_\Delta=\tfrac{1}{2}g_{ij}l^TiCi\tau_2\vec{\tau}l_j\vec{\Delta}+\mathrm{H.c.}, \tag{21}$$

where C is the Dirac charge conjugation matrix, f_{ij} is antisymmetric, and g_{ij} is symmetric due to Fermi statistics. The ϵ parameters for these singlet and triplet models are given by

$$\epsilon_h\simeq(4\sqrt{2}G_F)^{-1}\frac{\mathrm{Im}(f_{\tau l}f_{l\tau}^*)}{M_h^2} \tag{22}$$

in the singlet model and

$$\epsilon_\Delta\simeq(4\sqrt{2}G_F)^{-1}\frac{\mathrm{Im}(g_{\tau l}g_{l\tau}^*)}{M_\Delta^2} \tag{23}$$

in the triplet model, respectively.

For the singlet model we assume that $f_{e\mu}$ is considerably smaller than $f_{\tau l}$, so that one does not need to readjust the Fermi constant G_F. This assumption is also consistent with the constraint set by the universality between β and μ decay [25,45]. The parameter $\mathrm{Im}(f_{\tau l}f_{l\tau}^*)/M_h^2$ is constrained only by the measurement of τ leptonic decays. At the 2σ level (which is about 2–3 % precision) we estimate approximately $\mathrm{Im}(f_{\tau l}f_{l\tau}^*)/M_h^2\leqslant10^{-6}$ GeV^{-2} [46]. It implies that

$$\epsilon_h\simeq(4\sqrt{2}G_F)^{-1}\frac{\mathrm{Im}(f_{\tau l}f_{l\tau}^*)}{M_h^2}\leqslant1.4\times10^{-2}, \tag{24}$$

with $M_h=100$ GeV. Therefore in this model there is a possibility that the CP or T violation effect may show up with a size reachable at the TCF in pure leptonic decay channels.

For the triplet model the direct constraint is also from the measurement of pure leptonic decays. The same result is obtained as that in the singlet model, i.e., $\mathrm{Im}(g_{\tau l}g_{l\tau}^*)/M_h^2\leqslant10^{-6}$ GeV^{-2}. As a result of this constraint one has

$$\epsilon_h\simeq(4\sqrt{2}G_F)^{-1}\frac{\mathrm{Im}(g_{\tau l}g_{l\tau}^*)}{M_\Delta^2}\leqslant1.4\times10^{-2}, \tag{25}$$

with $M_\Delta=100$ GeV. However, in this model the new interactions will induce lepton family-number-violating decay $\tau\to3l$ and $\mu\to3e$ through exchange of the double charged scalar particle Δ^{--}. Without seeing any signal, one obtains some approximate bounds on the coupling constants as [47]

$$\frac{|g_{\mu e}g_{ee}^*|}{M_\Delta^2}\leqslant5\times10^{-12} \tag{26}$$

and

$$\frac{|g_{\tau l}g_{ll}^*|}{M_\Delta^2}\leqslant10^{-8} \tag{27}$$

for $M_\Delta=100$ GeV. If one assumes that all the couplings g_{ij} are at the same order of magnitude, then these bounds will restrict the CP or T violation size far below the ability of the TCF. Again we see that some hierarchies on the couplings are needed for this model to give rise to observable CP or T violation effects. Additionally in the triplet model one has to avoid the restriction from neutrino mass generation [48]. If a neutrino develops a mass at the tree level, either the coupling or the vacuum expectation value of the neutral component of the triplet Δ^0 is extremely small. The natural way to deal with this problem is to impose some symmetry on this model. An example is to introduce a discrete symmetry:

$$l\to il,\quad E\to iE,\quad \Delta\to-\Delta. \tag{28}$$

With this symmetry, Δ^0 will never develop a nonzero vacuum expectation value; therefore, the couplings are not constrained by the neutrino mass generation.

IV. DISCUSSION AND CONCLUSION

In this work we systematically investigated the possibility of finding CP or T violation in the τ sector with the TCF. The origin of CP or T violation is from the extensions of the SM. We discuss most of the popular models beyond the SM and present the models which may give rise large CP or T violation in the τ sector through either EDM's or decay of the τ lepton. Before making our conclusion, some interesting points should be further discussed or emphasized.

(1) Polarization of initial electron and/or positron is very desired for our purpose. First, with polarization the precision of measurement of EDM's will be increased by about two orders of magnitude, as 10^{-19} e cm, which is used through this work. Without polarization, from our above discussion one sees that we have no hope to expect a detectable EDM of τ at the TCF. Second, in some decay channels without a final state interaction, such as pure leptonic decays, two-body decays $\pi\nu_\tau$, etc., polarization is needed to search for CP or T violation occurring at the τ decay vertex. With unpolarized electron and positron beams the CP or T violation could only be detected using channels with a final state interaction phase, such as $2\pi\nu_\tau$, etc.

(2) For hadronic decay we only consider inclusive processes. The advantage of inclusive processes is that one does not need not to consider the hadronization of quarks, which may bring in large uncertainties in the estimation. And the event number in inclusive processes is larger than that in certain exclusive processes. However, we should mention that for certain exclusive decays the CP or T violation parameter ϵ can be larger than that in inclusive decay. One example is from the multi-Higgs-double model. We estimate that $\epsilon \leqslant 3\times10^{-4}$ for the decay $\tau\to\overline{u}\overline{d}\nu_\tau$. Here we may also consider the exclusive decay $\tau\to3\pi\nu_\tau$ contributed by a_1 and π' resonances. Compared to inclusive decay, the ϵ parameter is larger by a factor of (using current algebra relation)

$$\frac{\langle o|\overline{u_L}d_R|\pi'\rangle}{\langle o|\overline{u_L}\gamma_0 d_L|\pi'\rangle}\simeq\frac{m_{\pi'}}{m_u+m_d}\simeq100. \tag{29}$$

So $\epsilon\leqslant3\times10^{-2}$ is obtained. However, on the other hand, the event number decreases by a factor of

$$\frac{f'_\pi}{f_\pi}\frac{B(\tau\to\pi\nu_\tau)}{B(\tau\to\text{hadron}+\nu_\tau)}\simeq10^{-2}. \tag{30}$$

Here $f_{\pi'}=5\times10^3$ GeV is used. Therefore the statistical error increases by about 10 times. In other words the measurement precision at the TCF for this channel is about 10^{-2}. As a result, at the 2σ level $\epsilon\simeq3\times10^{-2}$ is observable. This estimation agrees with the exact result of Ref. [18].

(3) Obviously the numerical result we obtained above is quite crude. A more accurate estimation is necessary in the future. For instance, through this paper we assume that EDM's as large as 10^{-19} e cm and ϵ as large as 10^{-3} can be observed. This of course is a rough estimation. To be more precise, a Monte Carlo simulation is needed, which will tell us more confidently how large CP or T violation is able to be observed at the TCF. Especially the Monte Carlo simulation on EDM's of τ will give us a quite clear result, because in this case the d_τ is the only parameter we should treat. All the model dependence is included in it. Recently a group of people analyzed the data from BEPC experiments to set bounds on the T-violating effect for the τ system [49]. Following a suggestion by Lee, they considered the pure leptonic $\tau^\pm$ decays to $e^\pm\mu^\mp$ plus neutrinos in the final states. The T-violating amplitude

$$A=\langle\hat{p}_e\cdot(\hat{p}_1\times\hat{p}_2)\rangle_{\text{average}} \tag{31}$$

is measured, where $\hat{p}_e$ is the unit momentum vector of the initial electron beam, and $\hat{p}_1$ and $\hat{p}_2$ are the unit momenta of the final state electron and muon, respectively. In total, 432 events are analyzed and it results in

$$A=-0.027\pm0.031\pm0.006. \tag{32}$$

This result agrees with no T violation as expected from our previous discussion on pure leptonic τ decays.

(4) In order to generate detectable large CP or T violation effects, we know from our investigation that there must exist new physics and the new physics scale is not far above the weak scale. Therefore, if there is an observable CP or T violation effect in the τ sector at the TCF, the associated new physics phenomena should be observed at high-energy experiments, such as LHC and LEP II experiments. It is interesting to see if the new particles predicted by the various models we have discussed in this paper are indeed detectable in these high-energy experiments.

(5) Precise measurement of the pure leptonic decay is another way to test the new physics responsible for CP or T violation. Since if there is a CP or T violation effect at the level of 10^{-3}, the τ leptonic decay width must deviate from the SM prediction at the same level. So we expect to observe the deviation by measuring the branching ratio of the pure leptonic decay. However, it is not true *vice versa*, since a deviation of the leptonic branching ratio from that of the SM does not necessarily indicate CP or T violation.

Finally we come to our conclusion. There exists the possibility that CP or T violation in the τ sector is large enough to be discovered at the TCF, although for this large violation effect some specific new physics phenomena beyond the SM are needed and the parameter spaces of the models are strongly restricted.

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[1] J. H. Christension *et al.*, Phys. Rev. Lett. **13**, 138 (1964).
[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[3] T. D. Lee, Phys. Rep. **9C**, 144 (1974); S. Weinberg, Phys. Rev. Lett. **37**, 657 (1976).
[4] For recent review on *CP* violation, see *CP Violation*, edited by C. Jarlskog (World Scientific, Singapore, 1989).
[5] S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989); Phys. Rev. D **42**, 860 (1990).
[6] See, for example, R. D. Peccei, in *CP Violation* [4]; J. Kim, Phys. Rep. **150**, 1 (1987); H. Y. Chen, *ibid.* **158**, 1 (1988).
[7] M. E. Shaposhnikov, JETP Lett. **44**, 465 (1986); Nucl. Phys. **B287**, 757 (1987); **B299**, 797 (1988); A. I. Bochkrarev and M. E. Shaposhnikov, Mod. Phys. Lett. A **2**, 417 (1987); M. Dine, P. Huet, R. G. Leigh, A. Linde, and D. Linde, Phys. Lett. B **283**, 319 (1992); Phys. Rev. D **46**, 550 (1992); J. R. Espinosa, M. Quirós, and F. Zwirner, Phys. Lett. B **314**, 206 (1996).
[8] H. P. Nilles, Phys. Rep. **110**, 1 (1984); M. Sohnius, *ibid.* **128**, 39 (1985); H. E. Haber and G. L. Kane, *ibid.* **117**, 75 (1987).
[9] Beijing Tau-Charm Factory Workshop'96, IHEP-BTCF, Report No. -02, 1996 (unpublished); T. Huang, in *Proceedings of Workshop on the Tau/Charm Factory*, Argonne, 1995, edited by J. Repond (AIP, New York, 1996), p. 89.
[10] ALEPH Collaboration, Phys. Lett. B **281**, 405 (1992); **281**, 459 (1992).
[11] ALEPH Collaboration, Phys. Lett. B **272**, 411 (1991); R. Escribano and E. Masso, *ibid.* **301**, 419 (1993).
[12] W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. **63**, 2787 (1989); Phys. Rev. D **48**, 78 (1993); W. Bernreuther, O. Natchmann, G. W. Botz, and P. Overmann, Z. Phys. C **52**, 567 (1991).
[13] B. Ananthanarayan and S. D. Rindani, Phys. Rev. D **51**, 5996 (1995); **50**, 4447 (1994); Phys. Rev. Lett. **73**, 1215 (1994).
[14] C. A. Nelson, Phys. Rev. D **41**, 2805 (1990); **43**, 1465 (1991); **50**, 4544 (1994).
[15] S. Goozovat and C. A. Nelson, Phys. Lett. B **267**, 128 (1991).
[16] Y. S. Tsai, Phys. Rev. D **51**, 3172 (1995).
[17] Y. S. Tsai, Phys. Lett. B **378**, 272 (1996).
[18] S. Y. Choi, K. Hagiwara, and M. Tanabashi, Phys. Rev. D **52**, 1614 (1995).
[19] U. Kilian, J. G. Körner, K. Schilcher, and Y. L. Wu, Z. Phys. C **62**, 413 (1994).
[20] I. B. Khriplovich and M. Pospelov, Yad. Fiz. **53**, 1030 (1991) [Sov. J. Nucl. Phys. **53,** 638 (1991)]; M. J. Booth, University of Chicago Report No. EFI-93-01, hep-ph/9301293 (unpublished).
[21] W. Buchmüller, R. Rückl, and D. Wyler, Phys. Lett. B **177**, 377 (1986); **191**, 44 (1987); A. J. Davies and X.-G. He, Phys. Rev. D **43**, 225 (1991).
[22] S. M. Barr and A. Masiero, Phys. Rev. Lett. **58**, 187 (1987); U. Mahanta, Phys. Rev. D **54**, 3377 (1996).
[23] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994), p. 1375.
[24] L. Montanet *et al.* [23], p. 1417.
[25] This particle was invented for other purpose by Zee some years ago, it has some very interesting application. See A. Zee, Phys. Lett. **93B**, 389 (1980); **161B**, 141 (1985); M. Fukugita and T. Yanagida, Phys. Rev. Lett. **58**, 1807 (1987); S. M. Barr, E. M. Freire, and A. Zee, *ibid.* **65**, 2626 (1990); R. Barbieri and L. Hall, Nucl. Phys. **B363**, 27 (1991); Z. Tao, Phys. Rev. D **48**, 3221 (1993).
[26] X. Q. Li and Z. Tao, Phys. Rev. D **43**, 3691 (1991).
[27] S. Barr and A. Zee, Phys. Rev. D **17**, 1854 (1978); M. B. Gavela and H. Georgi, Phys. Lett. **119B**, 141 (1982).
[28] See, for example, M. Fabbrichesi, P. M. Fishbane, and R. E. Norton, Phys. Rev. D **37**, 1942 (1988).
[29] W. Buchmuller and D. Wyler, Phys. Lett. **121B**, 321 (1983); J. Polchinski and M. B. Wise, *ibid.* **125B**, 393 (1983); F. del Aguila, M. Gavela, J. Grifols, and A. Mendez, *ibid.* **126B**, 71 (1983); E. Franco and M. Mangano, *ibid.* **135B**, 445 (1984); M. Dugan, B. Grinstein, and L. Hall, Nucl. Phys. **B255**, 413 (1985); P. Nath, Phys. Rev. Lett. **66**, 2565 (1991).
[30] E. Ma and D. Ng, Phys. Rev. Lett. **65**, 2499 (1990); K. Choi, *ibid.* **72**, 1592 (1994); R. Garisto, Phys. Rev. D **49**, 4820 (1994); Nucl. Phys. **B419**, 279 (1994).
[31] See Mahanta [22].
[32] See Polchinski and Wise [29].
[33] S. Weinberg, Phys. Rev. D **7**, 1068 (1973); G. C. Branco and M. N. Rebelo, Phys. Lett. **160B**, 117 (1985); J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987); Y. L. Wu and L. Wolfenstein, Phys. Rev. Lett. **73**, 1762 (1994).
[34] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975); G. Senjanović and R. N. Mohapatra, *ibid.* **12**, 1502 (1975).
[35] J. F. Donoghue, Phys. Rev. D **18**, 1632 (1972); J. Maalampi, Phys. Lett. B **214**, 609 (1988).
[36] S. M. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990)
[37] We follow the work by A. Soni and R. M. Xu, Phys. Rev. Lett. **69**, 33 (1992). They discussed generation of EDM's of top quarks. Their calculation is also true for τ leptons. We estimate that $d_\tau \leqslant \sqrt{2}/(8\pi^2) G_F m_\tau e \tan^2\beta (m_\tau^2/M_H^2)[\ln(M_H^2/m_\tau^2) - \frac{3}{2}]$, and taking $M_H = 100$ GeV (Higgs boson mass) and $\tan\beta \leqslant 40$, we obtain this result.
[38] J. F. Nieves, D. Chang, and B. Pal, Phys. Rev. D **33**, 3324 (1986).
[39] J. Donoghue and B. Holstein, Phys. Lett. **113B**, 382 (1982); I. L. Bigi and J. M. Frere, *ibid.* **110B,** 255 (1982).
[40] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988); **38**, 907 (1988).
[41] G. Bhatacharrya *et al.*, Phys. Rev. Lett. **64**, 2870 (1990); Phys. Rev. D **42**, 268 (1990).
[42] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **71**, 3629 (1993).
[43] W. J. Marciano, Phys. Rev. D **45**, R721 (1992).
[44] Y. Grossman, Nucl. Phys. **B426**, 355 (1994), and references therein.
[45] D. I. Britton *et al.*, Phys. Rev. Lett. **68**, 3000 (1992).
[46] See Tao [25].
[47] Montanet *et al.* [23], p. 1194.
[48] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).
[49] N. D. Qi *et al.*, presented at the 4th International Workshop on τ physics, Stanley, Colorado, 1996 (unpublished).