# Could the MSSM have no *CP* violation in the CKM matrix?

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We show that all $CP$ violation in the MSSM could originate in the supersymmetry-breaking sector rather than the CKM matrix, and discuss the important consequences for $B$ physics. [S0556-2821(97)02503-4]

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## I. INTRODUCTION

When discussing supersymmetric extensions to the standard model (SM), most authors choose to incorporate the Kobayashi-Maskawa model of $CP$ violation [1]. In the minimal supersymmetric standard model (MSSM), as in the SM, this can successfully explain the experimental observations of $CP$ violation (which admittedly are in rather short supply). However, there are many other possible sources of $CP$ violation in the MSSM, such as phases on trilinear $A$ couplings and bilinear $B$ couplings. In fact writing the superpotential of the MSSM as

$$W = h_U Q_L H_2 U_R + h_D Q_L H_1 D_R + h_E L H_1 E_R + \mu H_1 \varepsilon H_2, \tag{1}$$

where generation indices are implied [and where the left-handed superfields contain the antiparticles, with the vacuum expectation values (VEVs) of the Higgs fields ($v_1$ and $v_2$) defined such that $m_u = h_U v_2$, $m_d = h_D v_1$, and $m_e = h_E v_1$], $CP$ violation can appear in any of the soft supersymmetry-breaking terms which consist of mass-squared scalar terms, gaugino masses, and scalar couplings of the form

$$-\delta\mathcal{L} = m_{ij}^2 z_i z_j^* + \tfrac{1}{2} M_A \lambda_A \lambda_A + A_U \tilde{q}_L^* h_2 \tilde{u}_R + A_D \tilde{q}_L^* h_1 \tilde{d}_R + A_E \tilde{l}^* h_1 \tilde{e}_R + B\mu h_1 \varepsilon h_2 + \text{H.c.}, \tag{2}$$

where again, generation indices are suppressed, the $\lambda_A$ are the gauginos, and the $z_i$ are generic scalar fields. In the case that the couplings $A$, $M_A$, and $m_{ij}^2$ are all degenerate at the grand unified theory (GUT) scale,

$$A_{U_{ij}} = A h_{U_{ij}},$$
$$A_{D_{ij}} = A h_{D_{ij}},$$
$$A_{E_{ij}} = A h_{E_{ij}},$$
$$m_{ij}^2 = \delta_{ij} m_0^2,$$
$$M_A = m_{1/2}, \tag{3}$$

there are four physical phases describing $CP$ violation which were ennumerated in Refs. [2,3]. Two of these are the usual $\theta$ angle and Cabibbo-Koboyashi-Maskawa (CKM) phase. As pointed out in Ref. [2], only the relative phases between $A$ and $B$ and $m_{1/2}$ are physically significant since the phase on $m_{1/2}$ may be removed by a suitable $R$ rotation. Thus, the other two $CP$ phases are those on $(Am_{1/2}^*)$ and $(Bm_{1/2}^*)$ (denoted $\phi_A$ and $\phi_B$, respectively).

Thus, a scenario which is complimentary to the one usually considered, is one in which $CP$ violation arises *only* in the soft-supersymmetry-breaking terms, with the CKM matrix being entirely real. In fact this possibility had earlier been considered in Ref. [2] for degenerate $A$, $M_A$, and scalar masses at the GUT scale. Here it was found that the direct $CP$ violation parameter, $\varepsilon'$, was generally too large. The subsequent work by Dugan *et al.* discouraged any further attempts in this direction, since they placed quite severe limits on the values of $\phi_A$ and $\phi_B$ by using experimental bounds on the electric dipole moments (EDMs) of the neutron and electron. Typically one imposes

$$\phi_A, \phi_B \lesssim \text{few} \times 10^{-3}. \tag{4}$$

Such small phases are unable (by themselves) to generate the $CP$ violation parameters ($\varepsilon$ and $\varepsilon'$) of the $K$-$\bar{K}$ system. The usual choice is to take these phases instead to be exactly zero, in which case $CP$ violation leaks into the scalar couplings only through the running of the renormalization group equations. The resulting dipole moments are enhanced over those in the SM, although probably not measurably so [4,5].

More recently, it has been demonstrated that, with the commonly adopted set of supersymmetry parameters, $\phi_A$ is far less constrained than $\phi_B$ [6] (and we independently reproduce these findings). This might give hope that the $CP$ violation in the $K$ system could arise purely from phases on the $A$ terms. The purpose of this paper, therefore, is to reexamine whether the $CP$ violation could reside *only* in the soft-supersymmetry-breaking terms, and to what extent such a scenario would be ''fine tuned.'' In the next section we show that, with *degenerate A* terms at the GUT scale, it is if fact not possible to generate sufficiently large values of $\varepsilon$ because of cancellations that occur.

We then go on to consider more general forms for the soft-supersymmetry breaking. Since in this context EDMs are generated from flavor diagonal terms, and $\varepsilon$ from off diagonal terms, one might expect that is possible to avoid bounds from EDMs [such as those in Eq. (4)] whilst at the same time generating reasonable values of $\varepsilon$, if rather than being degenerate, the $A$ parameters have an off-diagonal ''texture.'' In the light of recent work on supersymmetry breaking in string theory, this is a reasonably well-motivated assumption. In fact, one of the properties of the supersymmetry breaking in these theories is that there are only large,

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nontrivial phases on the $A$ terms, precisely when one expects there to be a high degree of nondegeneracy (that is when supersymmetry breaking is dominated by the moduli rather than the dilaton). (In addition, since $CP$ is an exact (discrete gauge) symmetry of the string theory, its appearance in the Yukawa couplings is not particularly easy to explain.)

We shall see that one can indeed explain the $CP$ violation observed in the $K$-$\bar{K}$ system with a rather simple nondegenerate structure for the soft-supersymmetry breaking. We then go on to discuss the expected pattern of $CP$ violation in the $B$-$\bar{B}$ system in this picture.

First let us discuss the procedure we have used. This is based on the very complete analyses of the ''constrained'' MSSM by Kane *et al.* [7] and Barger *et al.* [8]. As in Ref. [5], we have used two-loop RGE evaluation of gauge and Yukawa couplings and have minimized the full one-loop Higgs potential to determine the parameters $\mu$ and $B$, including contributions from matter and gauge sectors, but retaining the full flavor dependence in the RGEs. The process is as described in Ref. [5] except here, of course, we must allow for more general choices of supersymmetry-breaking parameters at the GUT scale. This requires a few modifications.

The first is to the equations for the electric dipole moments, which now receive significant *left-left* contributions from diagrams involving one Higgs vertex and one gauge vertex [9]. Let us define the diagonalizations of the mass matrices as

$$\text{squarks}: V^\dagger_{\tilde q} M^2_{\tilde q} V_{\tilde q} = m^2_{\tilde q},$$

$$\text{neutralinos}: V^\dagger_N M_N V_N = m_{\chi^0},$$

$$\text{charginos}: U^\dagger_C M_C V_C = m_{\chi^\pm}, \tag{5}$$

where the squark mass-squared term is of the form

$$(\tilde q^\dagger_L, \tilde q^\dagger_R) M^2_{\tilde q} \begin{pmatrix} \tilde q_L \\ \tilde q_R \end{pmatrix}, \tag{6}$$

and

$$M^2_{\tilde u} = \begin{pmatrix} m^2_U + K\delta m^2_{\tilde U_{LL}} K^\dagger & v_2 K A_U + m_U \mu v_1/v_2 \\ v_2 A^\dagger_U K^\dagger + m_U \mu v_1/v_2 & m^2_D + \delta m^2_{\tilde U_{RR}} \end{pmatrix},$$

$$M^2_{\tilde d} = \begin{pmatrix} m^2_D + \delta m^2_{\tilde D_{LL}} & v_1 A_D + m_D \mu v_2/v_1 \\ v_1 A^\dagger_D + m_D \mu v_2/v_1 & m^2_D + \delta m^2_{\tilde D_{RR}} \end{pmatrix},$$

and where the $\delta m^2$ contain the renormalized squark mass-squared terms and also generation independent contributions from the $D$ terms. We are using the down-quark diagonal basis, and $K$ is the CKM matrix [$m_U = \mathrm{diag}(m_u, m_c, m_t) = K h_U v_2$ ]. We find the following chargino contributions to the quark electric dipole moments:[1]

$$d_d = -\frac{1}{3}\frac{e}{32\pi^2}\sum_i^2 \frac{(V_C)_{2i}(U_C)^*_{\alpha i}}{m_{\chi_i^\pm}} \times \mathrm{Im}\left(\delta_{\alpha 2} h^\dagger_D K^\dagger \left[V_{\tilde u} F_d\left(\frac{m^2_{\tilde u}}{m^2_{\chi_i^\pm}}\right) V^\dagger_{\tilde u}\right]_{LR} h^\dagger_U - \delta_{\alpha 1} h^\dagger_D K^\dagger \left[V_{\tilde u} F_d\left(\frac{m^2_{\tilde u}}{m^2_{\chi_i^\pm}}\right) V^\dagger_{\tilde u}\right]_{LL} K g_2\right)_{11},$$

$$d_u = \frac{2}{3}\frac{e}{32\pi^2}\sum_i^2 \frac{(V_C)_{\alpha i}(U_C)^*_{2i}}{m_{\chi_i^\pm}} \times \mathrm{Im}\left(\delta_{\alpha 2} h^\dagger_U \left[V_{\tilde d} F_u\left(\frac{m^2_{\tilde d}}{m^2_{\chi_i^\pm}}\right) V^\dagger_{\tilde d}\right]_{LR} h^\dagger_D K^\dagger - \delta_{\alpha 1} h^\dagger_U \left[V_{\tilde d} F_u\left(\frac{m^2_{\tilde d}}{m^2_{\chi_i^\pm}}\right) V^\dagger_{\tilde d}\right]_{LL} K^\dagger g_2\right)_{11}, \tag{7}$$

where we have defined the functions

$$F_d = \frac{1}{(1-x)^3}[5 - 12x + 7x^2 + 2x(2-3x)\ln x],$$

$$F_u = \frac{1}{(1-x)^3}[2 - 6x + 4x^2 + x(1-3x)\ln x]. \tag{8}$$

For the case we are considering, the CKM matrix will of course be real. For the gluino contributions we find

$$d_d = -\frac{e\alpha_s}{9\pi m_{\tilde g}} \mathrm{Im}\left(\left[V_{\tilde d} G\left(\frac{m^2_{\tilde d}}{m^2_{\tilde g}}\right) V^\dagger_{\tilde d}\right]_{LR}\right)_{11},$$

$$d_u = \frac{2e\alpha_s}{9\pi m_{\tilde g}} \mathrm{Im}\left(\left[V_{\tilde u} G\left(\frac{m^2_{\tilde u}}{m^2_{\tilde g}}\right) V^\dagger_{\tilde u}\right]_{LR}\right)_{11}, \tag{9}$$

where we have defined the function

$$G = \frac{1}{(1-x)^3}[1 - x^2 + 2x\ln x]. \tag{10}$$

The neutralino contributions, which were also included, were found always to be small.

The second modification is to the conditions which indicate whether the minimum obtained is global, or whether there are other minima which may have broken color or charge (CCB), or directions in which the potential is unbounded from below (UFB). Necessary conditions were deduced in Refs. [10], and have been exhaustively generalized in Refs. [11,12]. Since here we are considering the possibility of large nondegeneracy in the $A$ terms, it is especially important to use the flavor violating conditions of Ref. [12] which take a particularly simple form. The CCB conditions are

[1]This corrects Eq. (23) of Ref. [5] in which the quark charges were omitted.

$$|A_{U_{ij}}|^2 \leq |h_{U_{kk}}|^2 (m^2_{uL_i} + m^2_{uR_j} + m^2_2 + \mu^2),$$

$$|A_{D_{ij}}|^2 \leq |h_{D_{kk}}|^2 (m^2_{dL_i} + m^2_{dR_j} + m^2_1 + \mu^2),$$

$$|A_{E_{ij}}|^2 \leq |h_{E_{kk}}|^2 (m^2_{eL_i} + m^2_{eR_j} + m^2_1 + \mu^2), \quad (11)$$

where $i \neq j$, $k = \mathrm{Max}(i,j)$, and $m_1^2$ and $m_2^2$ are the scalar mass-squared terms for the Higgs, and the UFB conditions are

$$|A_{U_{ij}}|^2 \leq |h_{U_{kk}}|^2 (m^2_{uL_i} + m^2_{uR_j} + m^2_{eL_p} + m^2_{eR_q}),$$

$$|A_{D_{ij}}|^2 \leq |h_{D_{kk}}|^2 (m^2_{dL_i} + m^2_{dR_j} + m^2_{\nu_m}),$$

$$|A_{E_{ij}}|^2 \leq |h_{E_{kk}}|^2 (m^2_{eL_i} + m^2_{eR_j} + m^2_{\nu_m}), \quad (12)$$

where $p \neq q$ and $m \neq i \neq j$. For the diagonal terms we used the more complete expressions given in Ref. [11].

The $\varepsilon$ parameter was calculated using the expressions for the MSSM of Refs. [13,14]. Since the SM contributions are insignificant here (see below), the main contributions are from chargino and gluino box diagrams. To demonstrate our nomenclature, we shall present the full chargino terms for left-handed external quarks here. The contributions to the mixing matrix elements are

$$M_{12}(K) = \frac{B_K \eta_K f_K^2 M_K}{384\pi^2} [A_{SM} + A_{H^\pm} + A_{\chi^\pm} + A_{\tilde{g}}],$$

$$A_{\chi^\pm} = \sum_{\alpha\beta}^{2} \sum_{ij}^{6} \frac{g_2^4}{m^2_{\chi_\alpha}} [g_2 (V^\dagger_{\tilde{u}L} K)^i_2 (V_C)^1_\alpha - (V^\dagger_{\tilde{u}R} h^\dagger_U)^i_2 (V_C)^2_\alpha]$$
$$\times [g_2 (V^\dagger_{\tilde{u}L} K)^j_2 (V_C)^1_\beta - (V^\dagger_{\tilde{u}R} h^\dagger_U)^j_2 (V_C)^2_\beta]$$
$$\times [g_2 (K^\dagger V_{\tilde{u}L})^1_i (V^\dagger_C)^1_\beta - (h_U V_{\tilde{u}R})^1_i (V^\dagger_C)^2_\beta]$$
$$\times [g_2 (K^\dagger V_{\tilde{u}L})^1_j (V^\dagger_C)^1_\alpha - (h_U V_{\tilde{u}R})^1_j (V^\dagger_C)^2_\alpha]$$
$$\times \hat{F}(m_i^2/m^{\alpha 2}_{\chi^\pm}, m_j^2/m^{\alpha 2}_{\chi^\pm}, m^{\beta 2}_{\chi^\pm}/m^{\alpha 2}_{\chi^\pm}), \quad (13)$$

where we have defined the $6\times 3$ matrices $(V_{\tilde{q}L})^a_i = (V_{\tilde{q}})^a_i$, and $(V_{\tilde{q}R})^a_i = (V_{\tilde{q}})^{a+3}_i$, and where $\hat{F}$ represents combinations of Inami-Lim functions [13]. (The terms with right-handed quarks are expected to be insignificant for the charginos since they are suppressed by Yukawa couplings.) For the gluino contribution $A_{\tilde{g}}$, we used the approximations of Ref. [14] which include all chiralities of external quarks.

The mass-insertion approximation was also used for the $\varepsilon'$ parameter (see Ref. [14] and references therein). In view of other uncertainties, this was sufficient for the present analysis. Other possible FCNC effects were also checked using the expressions of Ref. [14], except for $b \to s\gamma$, for which the full expressions of Ref. [15] were used.

## II. THE DEGENERATE MSSM

Before considering more general soft-supersymmetry breaking, we shall first discuss the effect of having degenerate boundary conditions as in Eq. (3), but following Ref. [6] allow the phases $\phi_A$ and $\phi_B$ to be nonzero. In this case it is not possible to generate the experimentally observed *CP* violation if there is no *CP* violation in the CKM matrix.

The reason why becomes apparent when one considers the leading supersymmetric box diagrams. Consider for example the potentially significant contribution to $\varepsilon$ from the chargino/up squark box with external left-handed quarks. This diagram may be approximated by the box diagram with a single mass insertion $M^2_{\tilde{u}LR}$ on the squark lines, and top-quark Yukawa couplings, $h_U$ on two vertices. The contribution is of the form

$$\varepsilon \propto \mathrm{Im}[(h_U M^2_{\tilde{u}RL} K)^2_{12} + (K^\dagger M^2_{\tilde{u}LR} h^\dagger_U)^2_{12}]. \quad (14)$$

This corresponds to the cross term in $A_{\chi^\pm}$ of Eq. (13) when the Inami-Lim functions are expanded and the leading linear terms taken. Since we are assuming no *CP* violation in the CKM matrix, then $K^\dagger = K^T$ and $h^\dagger_U = h^T_U$. It is convenient to define the matrices $\mathbf{A}_U$ such that $A_U = h_U \cdot \mathbf{A}_U$. The degenerate boundary condition corresponds to $\mathbf{A}_{Uij} = A\delta_{ij}$, and

$$\varepsilon \propto [(h_U \mathbf{A}^\dagger_U h^T_U)^2_{12} - (h_U \mathbf{A}^*_U h^T_U)^2_{12} + (h_U \mathbf{A}_U h^T_U)^2_{12} - (h_U \mathbf{A}^T_U h^T_U)^2_{12}]. \quad (15)$$

In the event that the $\mathbf{A}_U$ matrix is symmetric, this contribution completely vanishes. Inspection of the renormalization group equations (see for example Ref. [15]) shows that for degenerate boundary conditions this is the case to leading order. The matrix $\mathbf{A}_U$ is in fact found to be symmetric to typically one part in $10^4$ at the weak scale.

This greatly suppresses any contribution to $\varepsilon$ from chargino box diagrams and similar arguments apply to the other box diagrams too. In order to demonstrate this, we shall consider a ''typical'' point in parameter space where $A = 500$ GeV, $m_0 = 300$ GeV, $m_{1/2} = 100$ GeV, $\tan\beta = 5$, and $\mu + ve$. Minimizing the effective potential gave the values[2] $B = -116$ GeV and $\mu = 187$ GeV. The dependence of the EDM of the neutron on $\phi_A$ and $\phi_B$ is shown in Fig. 1. The contour $1.1\times 10^{-25}$ clearly agrees with the results in Ref. [6]. In the region which is shown in the plot, the value of $\varepsilon$ was never found to exceed $2\times 10^{-11}$. [Note that this suppression occurs because the CKM matrix is real; if one allows the usual *CP* phase into the CKM matrix, the supersymmetric contribution to $\varepsilon$ is $O(10^{-4})$].

## III. MORE GENERAL PARAMETERS

Before presenting some more general patterns of soft-supersymmetry breaking, let us say a little about how non-degenerate supersymmetry breaking can arise in string theory. Recent progress in this area has shown that the $A$ term for a coupling, $h_{ijk}$, between three superfields, $ijk$, may at tree level be written schematically in the form [16–18]

$$A_{ijk} \sim -m_{1/2}(1 + e^{i(\gamma_T)} \cot\theta F_{ijk}) h_{ijk}, \quad (16)$$

[2]These results were verified using a different minimization routine to within $\pm 10$ GeV by P. L. White.

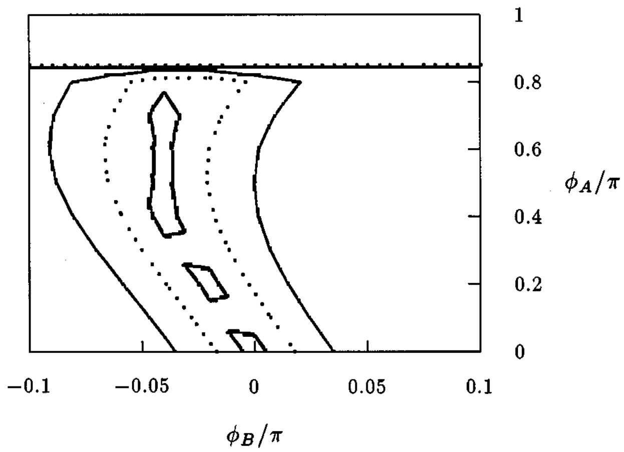


FIG. 1. The EDM of the neutron for the degenerate case with $A=500$ GeV, $m_0=300$ GeV, $m_{1/2}=100$ GeV, and $\tan\beta=5$ with $\mu=+ve$. The contours are $1.1\times10^{-25}$ (thick solid), $5\times10^{-25}$ (dotted), and $10^{-24}e$ cm (solid). The jagged line dilineates the region above which one cannot find a minimum. The other constraints were not imposed for this diagram.

where the angle $\theta$ describes the Goldstino direction, and where the VEV of the dilaton is assumed to be real. When $\theta=\pi/2$ the supersymmetry breaking is along the dilaton direction, and when $\theta=0$ it is in the direction of moduli describing the size and shape of the compactification. The phase on the second term is the putative source of $CP$ violation and represents $CP$ violation in the VEVs of the moduli. Such spontaneous breaking of $CP$ by moduli has been discussed for orbifolds in Ref. [18]. The function $F_{ijk}$ is a function of the moduli VEVs and vanishes in a number of interesting cases outlined in Ref. [17]. The first case is obvious when supersymmetry breaking is dominated by the dilaton and $\cot\theta=0$. However, it is also clear that in this case $\phi_A=0$ and the soft-supersymmetry breaking cannot be the source of $CP$ violation. The moduli dependent term also vanishes for renormalizable couplings in which all the fields come from untwisted sectors, or have weight $-1$ under certain duality transformations (for instance all renormalizable couplings in the $Z_2\times Z_2$ orbifold satisfy this criterion). Thus, one can identify a number of possibilities for generating an off-diagonal structure in the $A$ terms, all of which require supersymmetry breaking to be dominated by the moduli with $\cot\theta\gg1$.

The off-diagonal Yukawa couplings come from nonrenormalizable terms whereas the diagonal ones are renormalizable.

The nondegeneracy is generated by one-loop corrections, with the $A$ terms being zero at tree level. This possibility has been discussed recently in the context of FCNCs in Ref. [19].

The nondegeneracy is generated for couplings involving fields with weights other than $-1$ (for example in the third generation only).

These possibilities, together with the recent observation that the pure dilaton breaking scenario breaks charge and color [20], make the assumption of nondegeneracy a reasonable one.

It is beyond the scope of this paper to discuss soft-supersymmetry breaking in string theory in any great depth, and we shall instead select a number of ''textures'' to analyze. Here our aim will be merely to demonstrate the possibility that $CP$ violation comes only from the soft-supersymmetry breaking. As we shall see in the next section, the experimental signatures of this scenario are quite striking, so that for the moment they are of more immediate interest.

In order to anticipate the effect of various patterns of soft-supersymmetry breaking, it is useful to think in terms of the leading mass insertion approximations. It is customary to consider the parameters

$$\delta^q_{ij}=\frac{M^2_{\tilde q_{ij}}}{\tilde m^2}, \tag{17}$$

where $\tilde m$ is an ''average'' sfermion mass. From the limits derived in Ref. [14], it is clear which are the important elements corresponding to each process provided that the gluino diagrams are the dominant contribution. The EDMs of the neutron and electron impose quite severe limits on the imaginary diagonal components in the left-right blocks, $(\delta^d_{11})_{LR}$, $(\delta^u_{11})_{LR}$, and $(\delta^e_{11})_{LR}$. The flavor changing neutral currents on the other hand impose bounds on the off-diagonal components; $b\to s\gamma$ constrains $(\delta^d_{23})_{LR}$ and $(\delta^d_{23})_{LL}$ (weakly) and $\Delta m_K$ depends on $(\delta^d_{1i})_{LL}$, $(\delta^d_{1i})_{LR}$, and $(\delta^d_{1i})_{RR}$ where $i\neq1$. Large values of these should be avoided, although $\Delta m_K$ must inevitably be affected. The parameters $\varepsilon$ and $\varepsilon'$ depend most strongly on $(\delta^d_{12})_{LL}$ and $(\delta^d_{13})_{LR}$. There are, however, relatively few constraints on $(\delta^u_{i\neq j})$ and in addition $h_D$ is almost diagonal at the GUT scale. If we maintain the assumption that the $A$ terms include factors of the Yukawa couplings, this suggests that in the basis we are using, off-diagonal terms in $M^2_{\tilde d}$ should be generated radiatively from terms in $A_U$.

We shall, therefore, consider the following ''textures'' for the $A$ matrices and the squark masses at the GUT scale:

$$A_{U_{ij}}=Ah_{U_{ij}}+\begin{pmatrix} 0 & \delta A_{12}h_{U_{12}} & \delta A_{13}h_{U_{13}} \\ \delta A_{21}h_{U_{21}} & 0 & \delta A_{23}h_{U_{23}} \\ \delta A_{31}h_{U_{31}} & \delta A_{32}h_{U_{32}} & 0 \end{pmatrix},$$

$$A_{D_{ij}}=Ah_{D_{ij}},$$

$$A_{E_{ij}}=Ah_{E_{ij}},$$

$$m^2_{ij}=\delta_{ij}m_0^2+\delta m^2,$$

$$M_A=m_{1/2},$$

$$\phi_B=0. \tag{18}$$

The parameter $\delta m^2$ represents off diagonal terms which may also be generated in the mass-squared matrices. From now on we shall impose $\phi_B=0$ to avoid large EDMs, assuming that an explanation for this lies in the mechanism which generates the $\mu$ term [16,18].

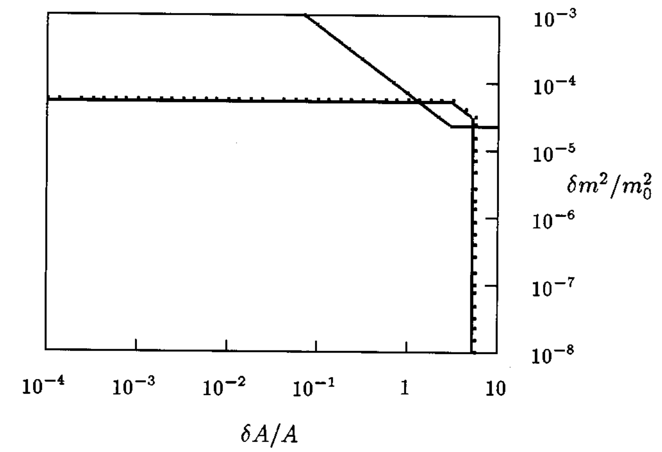


FIG. 2. The allowed ($\delta m^2$, $\delta A$) parameter space for Eq. (19). The allowed region is below the jagged line. The solid line is the contour $\varepsilon = 2.3\times 10^{-3}$.

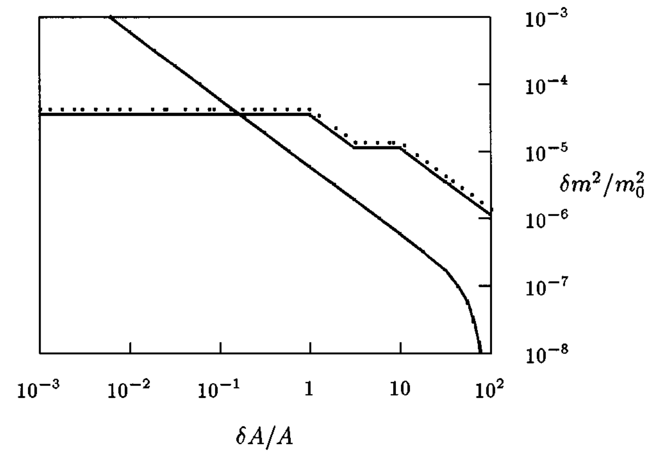


FIG. 3. The allowed ($\delta m^2$, $\delta A$) parameter space for Eq. (20).

For simplicity we shall introduce the real parameters $\delta A$, and $\phi_{\delta A}$, and consider the following three symmetric structures:

$$A_{U_{ij}} = A h_{U_{ij}} + \delta A e^{i\phi_{\delta A}} \begin{pmatrix} 0 & h_{U_{12}} & 0 \\ h_{U_{21}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad (19)$$

$$A_{U_{ij}} = A h_{U_{ij}} + \delta A e^{i\phi_{\delta A}} \begin{pmatrix} 0 & 0 & h_{U_{13}} \\ 0 & 0 & 0 \\ h_{U_{31}} & 0 & 0 \end{pmatrix}, \qquad (20)$$

$$A_{U_{ij}} = A h_{U_{ij}} + \delta A e^{i\phi_{\delta A}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & h_{U_{23}} \\ 0 & h_{U_{32}} & 0 \end{pmatrix}. \qquad (21)$$

For each of these possibilities there is a seven-dimensional parameter space consisting of ($A$, $m_0$, $m_{1/2}$, $\tan\beta$, $\delta m^2$, $\delta A$, $\phi_{\delta A}$) in addition to the sign of $\mu$. The results are shown in Figs. 2–4 for $\phi_{\delta A} = \pi/4$.[3] The vertical bounds in these figures are from CCB and UFB constraints, and the horizontal bounds are from $\Delta M_K$ constraints.

As one might expect, the first texture is not very efficient at generating $\varepsilon$ (since the relevant contribution in the RGEs is Cabibbo suppressed), however, there is a large region in each of the remaining two cases which can successfully explain the observed value of $\varepsilon$ whilst avoiding all other experimental constraints. In addition the value of $\varepsilon'$ was in each case found to be very small:

$$\left|\frac{\varepsilon'}{\varepsilon}\right| \lesssim 10^{-6} \qquad (22)$$

along the $\varepsilon = 2.3\times 10^{-3}$ contour. In this sense the experimental signatures are expected to be ''superweak'' with no observable direct $CP$ violation. The picture of $CP$ violation here is, therefore, more consistent with the results on $\varepsilon'$ from E731 than those from NA31 (see for example Ref. [21] and references therein).

For $B$ physics the picture is rather unusual. In $B$ physics, because of the similar decay times of the two eigenstates, one cannot disentangle the direct and indirect $CP$ violation using just one process. Instead one compares $CP$ violation for different processes using the parameters [22]

$$\Phi_{CPV}(f) = \arg\left(\frac{q}{p}\overline{\rho(f)}\right), \qquad (23)$$

where $q/p$ is associated with the mixing between $B^o$-$\overline{B^o}$ given by

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*}{M_{12} - i\Gamma_{12}}}, \qquad (24)$$

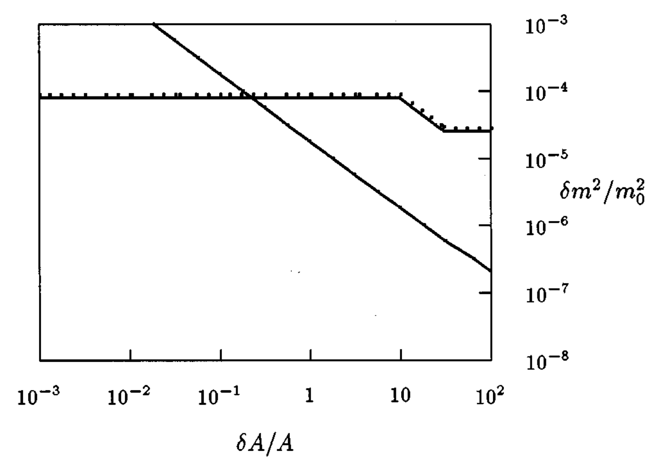


FIG. 4. The allowed ($\delta m^2$, $\delta A$) parameter space for Eq. (21).

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[3]This is the maximal case. Smaller values of $\phi_{\delta A}$ may be compensated by larger values of $\delta A$. For [** em **] very small values such as those considered in Ref. [18], this may be considered to be a fine-tuning in the sense that for the example of string derived soft terms, one requires the Goldstino to have almost no dilaton component.

where $|q/p|\approx 1$. The parameter $\overline{\rho(f)}$ is related to the direct $CP$ violation in the decay $B\to f$. Neither $q/p$ nor $\overline{\rho}$ is phase-reparametrization invariant, and thus cannot be independently observed.

In the SM, the $\overline{\rho(f)}$ receive contributions from tree-level $W$-exchange diagrams, and different phases from the KM matrix appear according to the channel considered, leading to a determination of the angles in the unitarity triangle [22]. The pattern of $CP$ violation here is in sharp contrast, since contributions to direct $CP$ violation arise only through penguin diagrams which in addition to being one-loop, are suppressed by factors of Yukawa couplings. The *relative* phases of the various $\overline{\rho(f)}$ are thus small with respect to the SM, and the picture of $CP$ violation is close to that of the ''superweak'' models in the tree-level approximation. (One-loop penguin diagrams may be significant for processes which are Cabibbo suppressed at tree level.) There is therefore a basis (i.e., the one which we are using) in which all the $\overline{\rho(f)}$ are approximately real for every process and hence all the $\Phi_{CPV}$ are given by

$$\Phi_{CPV}\approx -\ \arg(M_{12}). \tag{25}$$

Moreover we find that, for the three examples studied here, this phase is insignificant, in accord with previous analyses of the $B$ system in the constrained MSSM [13]. Thus one concludes that *for the B system there is little detectable CP violation*. (Some higher-loop contributions such as finite contributions to the Yukawa couplings, may be detectable for some Cabibbo suppressed processes.)

## IV. CONCLUSION

We have shown that the experimentally observed $CP$ violation could be generated in the soft-supersymmetry breaking sector of the MSSM rather than the Yukawa couplings. It is possible to avoid constraints from EDMs and FCNCs by choosing an off-diagonal texture for the trilinear couplings. The experimental signatures of this type of $CP$ violation are markedly different from those in the SM or the ''constrained'' MSSM. Generally the $CP$ violation is expected to be of the ''superweak'' variety, arising only through mixing, with little direct $CP$ violation. For the $B$ system the relatively small contribution to mixing means that there will be no detectable $CP$ violation at all (modulo possible one-loop effects).

This picture seems an attractive prospect for a number of reasons. For example, if this scheme is correct, then in conjunction with other FCNC processes, one has access to rather direct information about physics occuring at the Planck scale, specifically the nature of the supersymmetry breaking fields and their VEVs.

Another promising aspect is that of baryogenesis. In order to generate a sufficient baryon number in the SM and even the MSSM, one generally requires additional $CP$ violation beyond that in the CKM matrix. Here however the $CP$ violation responsible for the value of $\varepsilon$ could easily be sufficient to generate the observed baryon number since it is a ''hard'' violation; $CP$ violation in the SM for example is typically suppressed at $T\gg m_t$ by a factor $O(m_{\text{quark}}^{12}/T^{12})$ whereas here the suppression need only be $O(\widetilde{m}^2/T^2)$. This will be the subject of future work.

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