Finite W-width and initial-state radiative corrections to the reaction $e^+e^- \to W^+W^- \to l^+l^-\nu\nu$

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Lepton-inclusive cross sections for the process $e^+e^- \to W^+W^- \to l^+l^-\nu\nu$ are calculated. The influence of the radiative corrections and a finite decay W width is considered. Its large contribution to the total and, especially, differential cross sections of the final decay leptons is shown. The method to extract Γ_W, using the leptonic channel of W decay, is proposed. [S0556-2821(97)04401-9]

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I. INTRODUCTION

The standard model (SM) describes very well all known phenomena of electroweak origin. But up to now all experiments including those at the CERN e^+e^- collider LEP1 and SLAC Linear Collider (SLC) have tested the mass parameters of gauge bosons and boson-fermion couplings. The self-interactions of gauge bosons predicted by the SM have still not been confirmed. It is also necessary to check the structure of gauge boson couplings. These questions are the critical point of the SM.

The best process for checking triple gauge boson couplings is W pair production in e^+e^- annihilation. It contains non-Abelian couplings γWW and ZWW in the lowest order and has a rather large cross section $\sim$18 pb at $\sqrt{s}=200$ GeV and $\sim$7 pb at $\sqrt{s}=500$ GeV.

The main aims of the experiments at LEP 200 are to gain a precise measurement of the W-boson mass, indirect determination of the Higgs boson mass, and the possibility to investigate the triple gauge couplings.

We would like to present very shortly some of our results in the framework of the SM dealing with the problems of a finite W width and radiative corrections. These questions are also very important for the performance of very high-precision measurements of SM parameters. Moreover, they can imitate phenomena far beyond the SM. We have also proposed a method to extract the total W width. All these problems and our results can be found in detail in our collaboration works [1–4].

So let us consider the process

$$e^+e^- \to W^+W^- \to l^+l^-\nu\nu \tag{1}$$

now intensively investigated [5,6]. It can easily be detected and is the clearest signal of W-pair production [7]. But all the formulas given below do not depend on the concrete final states of W decay.

It is well known that the radiative corrections (RC's) as well as accounts of the produced W decay widths are significant in this process. The complete one-loop RC's to the on-shell W production were calculated by several authors [8–11] many years ago. The calculation of RC's to off-shell W production is much more difficult and the complete one-loop result has not been obtained yet. There is also the problem of the violation of gauge invariance by the introduction of the W-boson decay width. This problem has not been solved either. For a detailed discussion we refer to the paper by Aeppli *et al.* in [6], part C, and to a report by Beenakker and Denner [12].

The higher-order corrections should also be included in the calculation especially at very high energies of initial particles. Now there is only one method for counting them. It is the structure function method of calculating the large-logarithmic higher-order RC's [13,14].

The pure QED corrections in the leading logarithmic approximation are factorized and do not depend on the process in question. So we can use the structure function method [13,14] that allows us to take into account infrared and collinear singularities. In [1], a detailed description of this method including the case of polarized initial particles is also given. This approach to the calculation of RC's was developed independently by other groups (see [6,10–12]).

Treating the processes of massive particle production, we can restrict ourselves to considering the initial-state radiation (ISR) due to the dominance of its contribution to the total and differential cross sections. It should be noted that the ISR effect is important for both the total cross sections (in the energy region its contribution comprises up to 20%) and the differential ones (for some kinematical regions the RC contribution is several times larger than the Born term) [1,2].

The effect of the total decay width is very significant in the near-threshold energy region of the reaction (1) and decreases with energy growth but does not disappear and remains finite at infinitely large energies. For the total cross section this effect is not so large (less than 5%) [3]. The effect of the final width is much greater in the W decay product distributions [4].

II. THE BORN CROSS SECTIONS AND METHOD TO EXTRACT Γ_W

Taking into account the W-boson decay width, one can write the Born cross section of the process (1) as [15]

$$\frac{d\sigma_{\rm off}}{d\Omega} = \int_0^s ds_1 \rho(s_1) \int_0^{(\sqrt{s}-\sqrt{s_1})^2} ds_2 \rho(s_2)\, \frac{d\sigma}{d\Omega}, \tag{2}$$

where we consider off-shell kinematics

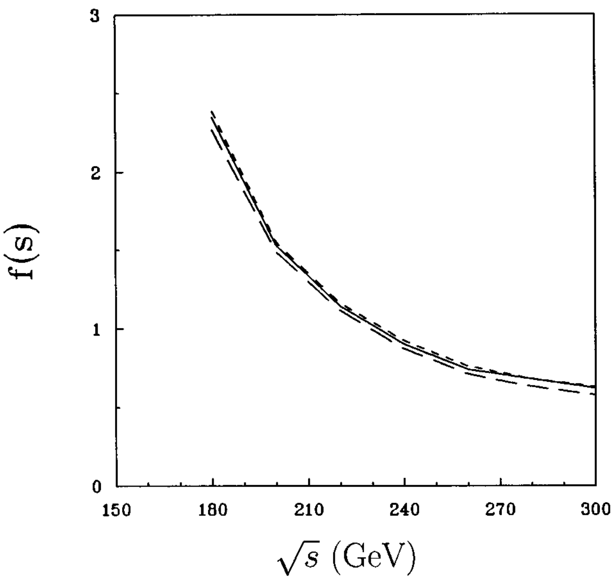

FIG. 1. $f(s)$ (5) as function of energy. Solid, dashed, and long-dashed lines correspond to $\Gamma=2.25$, 3, and 1.75 GeV.

$$\frac{d\sigma}{d\Omega}=\frac{v}{64\pi^2 s}\{|\mathcal{M}_\nu(-)+\mathcal{M}_{\gamma Z}(-)|^2+|\mathcal{M}_{\gamma Z}(+)|^2\}\times\frac{1}{4}\left[\frac{3}{8\pi M_W^2}\right]^2 d\Omega_-^* d\Omega_+^*, \tag{3}$$

chiral amplitudes are given in the Appendix, v is the velocity of the W boson, $d\Omega_\pm^*$ are the solid angles of the final fermions in the $W^\pm$ rest frames,

$$\rho(s_i)=\frac{1}{\pi}\frac{M_W\Gamma_W}{|s_i-M_W^2+iM_W\Gamma_W|^2}B \tag{4}$$

is the Breit-Wigner propagator, and B is the branching ratio. Introduction of the Γ_W violates the gauge invariance even if we include other diagrams giving the same final state. The reason is that Γ_W includes higher-order corrections not included in order-by-order calculation. But evidently the terms violating the gauge invariance are of an order of $(\Gamma/M_W)^2$ and can be omitted in the calculation with precision of $\sim$1%. Detailed analyses of different schemes accounting for the finite W width are presented in the paper by Aeppli *et al.* in [6].

It is convenient to describe the difference between total Born cross sections with $\Gamma_W=0$ and with $\Gamma_W\neq 0$ using the following function weakly dependent upon Γ_W [3]:

$$f(s)=\left(\frac{\sigma_0(s)-\tilde{\sigma}(s)}{\sigma_0(s)}\frac{M_W}{\Gamma_W}\right)_{\Gamma_{W=0}}=\left(\frac{M_W}{\Gamma_W}\frac{\Delta\sigma(s)}{\tilde{\sigma}(s)}\right)_{\Gamma_{W=0}}, \tag{5}$$

where $\sigma_0(s)$ is the cross section with on-shell W boson and $\tilde{\sigma}$ (s) the off-shell one. This function is depicted in Fig. 1 for three different values of Γ_W. It is clearly seen that in the total cross sections the effect of the finite W decay width is significant in the near-threshold energies of the process (1) and decreases with energy growth.

The replacement in (4)

$$M_W\Gamma_W\to s_i\Gamma_W/M_W^2 \tag{6}$$

is often used in calculations (see, for example, [15,16]). This replacement qualitatively changes the behavior of the width-corrected total cross section. In the case of (4) the width-corrected total cross section is never higher than the on-shell one. This can be explained by the influence of the integral limits in (2):

$$\frac{1}{\pi}\int_{-a}^{b}\frac{\varepsilon dx}{x^2+\varepsilon^2}=\frac{1}{\pi}\left(\arctan\frac{b}{\varepsilon}+\arctan\frac{a}{\varepsilon}\right)=1-\frac{\varepsilon}{\pi}\left(\frac{1}{a}+\frac{1}{b}\right), \tag{7}$$

$$a,b>0,\quad \varepsilon\to 0.$$

Having in mind the integration in (7) over x from $-a$ to b, we present the integrand in Eq. (7) as follows:

$$\frac{1}{\pi}\frac{\varepsilon}{x^2+\varepsilon^2}=\delta(x)-\frac{\varepsilon}{\pi ab},\quad \varepsilon\to 0,\quad -a<x<b. \tag{8}$$

After the replacement (6) there is the additional term $s_i/M_W^2>1$ in the numerator of (4) leading to a corrected cross section. This becomes larger than the on-shell cross section and their difference increases logarithmically with energy growth. Moreover, the new Breit-Wigner weight is now not normalized to 1. Therefore we prefer to use the standard form (4).

Consider now the effect of the finite Γ_W in the distribution of final leptons. Let us introduce the energy fractions of the final-state leptons and the angle between them:

$$x=\frac{2\varepsilon_{l^+}}{\varepsilon},\quad y=\frac{2\varepsilon_{l^-}}{\varepsilon},$$

$$\theta=\widehat{\vec{r}_-,\vec{r}_+},\quad \varepsilon=\sqrt{s}/2.$$

In the case of $\Gamma_W=0$ these variables are connected via the equation [4]

$$D(x,y,\theta)>0, \tag{9}$$

$$D(x,y,\theta)=\sin^2\theta x^2y^2v^2-2(1+\cos\theta)(1-v^2-x)\times(1-v^2-y)xy-(1-v^2)^2(x-y)^2, \tag{10}$$

where $v=\sqrt{1-4M_W^2/s}$ is the velocity of the produced W bosons. In Fig. 2 the sections of the body (10) formed by the planes θ=30°, 50°, 90°, and 120° are depicted.

The effect of the finite total decay width leads to the "smearing" of the region $D>0$. The ratio of the point number ΔN associated with the values of the parameters x, y, θ beyond the region $D>0$ to the total number of points N is a value of the order of Γ_W/M_W. By analogy with Eq. (5) we can introduce the quantity

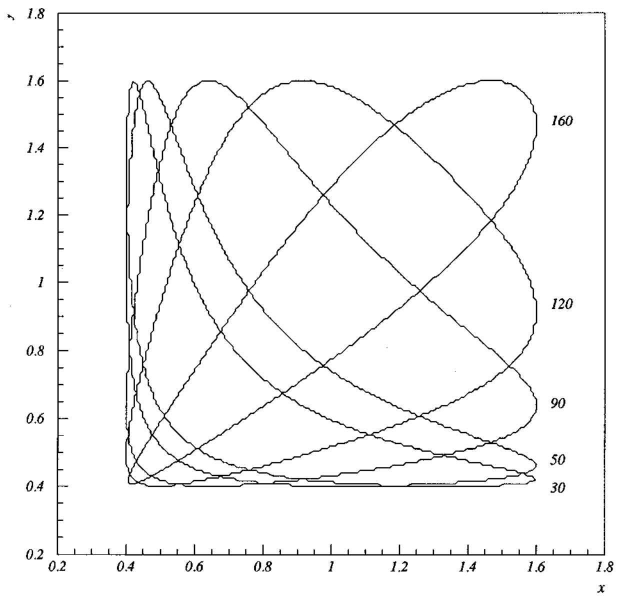

FIG. 2. Sections of the body $D=0$ (10) formed by the planes $\theta=30°$, 50°, 90°, 120°.

$$\left(\frac{\Delta N}{N}\bigg/\frac{M_W}{\Gamma_W}\right)\bigg|_{\Gamma_W\ll M_W}=C \qquad (11)$$

which is weakly dependent on Γ_W. Numerical calculation gives $C=5.25$ for $\sqrt{s}=200$ GeV.

This value is rather large to use for total decay width measurement. Calculating beforehand the value C with high accuracy and measuring the values ΔN and N we can determine Γ_W from Eq. (11).

Below we will discuss the factors that will influence the accuracy of the determination of Γ_W by this method.

III. BACKGROUND

The main background to the process (1) is the process of two-pair charged lepton production. Its cross section (if, for example, pairs of μ and τ are produced) [17] is

$$\sigma^{e^+e^-\to\mu^+\mu^-\tau^+\tau^-}\sim\frac{\alpha^4}{s}=(10^{-3}\ \text{pb})\left(\frac{200}{\sqrt{s}\ \text{GeV}}\right)^2. \qquad (12)$$

The cross section of the investigated process in the near-threshold region due to the resonant nature of W bosons has the behavior

$$\sigma^{\nu\tau}\sim\frac{\alpha^4}{s}\left(\frac{m}{\Gamma}\right)^2\sim(1\ \text{pb})\left(\frac{200}{\sqrt{s}}\right)^2 \qquad (13)$$

and exceeds significantly the cross section of this background process.

The other background process having the same near-threshold behavior is $e^+e^-\to ZZ$. The total cross section of this process is smaller by an order of magnitude than the main one and also the probability to ''waste'' leptons by a detector is small, so we can conclude that this background is negligible.

There are other processes giving the same four-fermion final state. They are described in the paper by Aeppli *et al.* in [6], and the conclusion about the smallness of this background is made there. Note here once more that the addition of these diagrams to the three main ones does not restore the gauge invariance because of the inclusion of diagrams with different orders of perturbation theory at the same time.

IV. INITIAL-STATE RADIATIVE CORRECTIONS

Using the structure function technique [13,14,1], the radiative correction can be taken into account by convolution of the (possibly improved) Born cross section with structure functions (see also the results of [6,10–12]):

$$\frac{d\sigma^{A+B\to C+D}}{d\Omega}(s)=\sum_{a,b,c,d}\int_{\theta(\text{cuts})}dx_1\,dx_2\,dx_3\,dx_4\,D_{A\to a}(x_1,s)D_{B\to b}(x_2,s)\frac{d\hat{\sigma}^{a+b\to c+d}}{d\hat{\Omega}}(x_1x_2s)|J|$$
$$\times D_{c\to C}(x_3,x_1x_2s)D_{d\to D}(x_4,x_1x_2s), \qquad (14)$$

where $d\hat{\sigma}/d\hat{\Omega}(x_1,x_2,s)$ is the Born cross section, x_1 and x_2 are momenta fractions of the initial leptons, $\hat{\Omega}$ is the solid angle of the W-boson scattering in the c.m. system (c.m.s.) of the hard subprocess, and $|J|$ is the Jacobian of transformation of the hard subprocess from the c.m. system to the lab frame connected with the initial electron and positron. θ(cuts) is the theta function defining integration area. $D_{A\to a}(x,Q^2)$ is the structure function describing the ISR and presenting the probability of finding ''inside'' initial particle A the parton a having momentum fraction x and virtuality up to Q^2. These functions include the leading and, partly, next-to-leading corrections to soft photon radiation in all orders of perturbation theory and leading corrections to hard photon emission up to the order of $O(\alpha^2)$. The function $D(x,Q^2)=D_{e\to e}(s,Q^2)$ has the next form

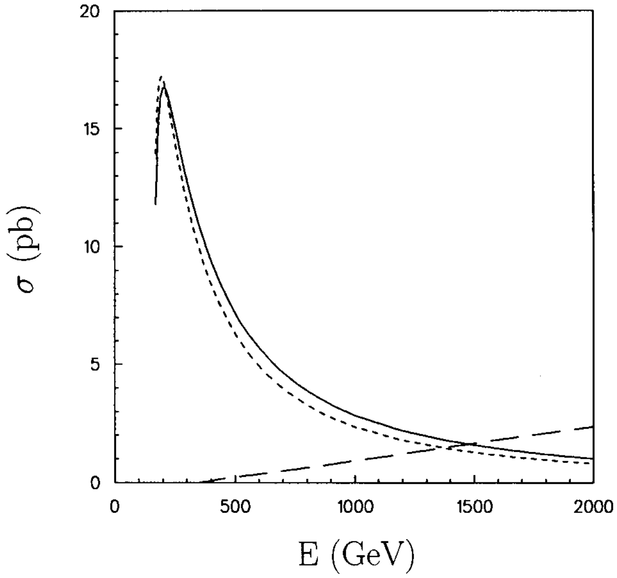

FIG. 3. Total cross sections for the process $e^+e^-\rightarrow W^+W^-$ as a function of energy; long-dashed curve corresponds to the contribution from $\gamma\gamma\rightarrow W^+W^-$ subprocess.

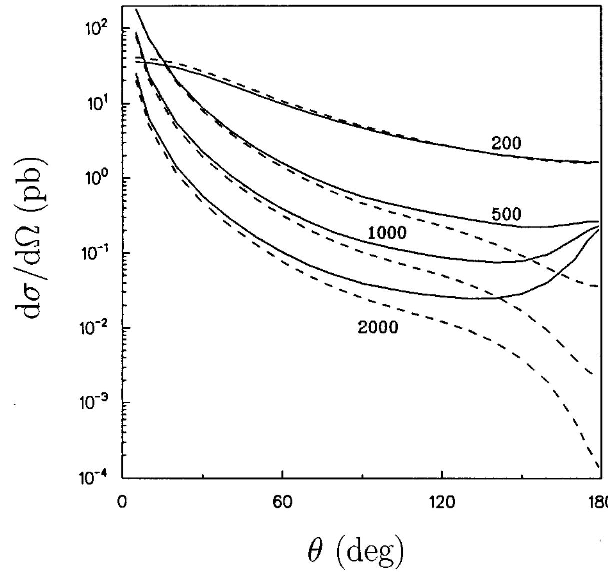

FIG. 4. Differential cross sections for the process $e^+e^-\rightarrow W^+W^-$ for different energies. Solid lines correspond to $O(\alpha^2)$ calculation and dashed lines to Born approximation.

$$D(x,Q^2)=\frac{\beta}{2}\left\{\left[1+\frac{3}{8}\beta+\beta^2\left(\frac{9}{128}-\frac{\pi^2}{48}\right)\right](1-x)^{\beta/2-1}-\frac{1}{2}(1+x)\right\}+\left(\frac{\beta}{4}\right)^2\left[2(1+x)\ln\frac{1}{1-x}-\frac{1+3x^2}{2(1-x)}\ln x-\frac{5+x}{2}\right], \tag{15}$$

where

$$\beta=2\alpha\pi[\ln(Q^2/m^2)-1],\quad \alpha(t)=\alpha(1-\alpha t/3\pi)^{-1},$$

$$\alpha=1/137,$$

and, m is the radiated particle mass (m_e for our process).

It is evident that only the ISR term is not gauge invariant again. We have to add also at the same level of perturbation theory (PT) the final-state corrections and initial-final interference. In the case of diagrams of annihilation type, the ISR only in the first order of PT is gauge invariant, but in general cases it is not. The direct calculation by Bardin *et al.* [16] gives us a good example of the violation of the gauge invariance in the case of the existence of t-channel diagrams. It can be restored by addition of some fictitious part. As we are to add the same part with the opposite sign to the final-state corrections, this part has no effect to the cross sections. In [2] we have shown that the initial-final interference does not contain large logarithms and hence we can use the structure function method giving accuracy at the 1% level.

The choice of the scale Q^2 is nontrivial because the contributions to the process $e^+e^-\rightarrow W^+W^-$ originate from the diagrams in both t and s channels. Nevertheless, because of the consideration of only the ISR, the scale Q^2 is s, the invariant mass of the initial leptons. This result is confirmed by our calculations in one-loop order [2] (see also the paper by Nicrosini *et al.* in [6], part A, and Ref. [12]).

Finally, θ (cuts) in Eq. (14) include different experimental cuts needed for realistic calculations. In all calculations given below we calculated cross sections without any cuts except the kinematical one:

$$\theta(\text{cuts})=\theta[x_1x_2s-(\sqrt{s_1}+\sqrt{s_2})^2], \tag{16}$$

where s_1 and s_2 are the virtualities squared of the produced W bosons. We have checked our calculations by comparison with the results by Nicrosini *et al.* in [6], part A.

In Fig. 3 the dependence of the total cross section on energy is shown. The solid lines correspond to the corrected cross sections and the dashed lines to the Born cross sections. It is very interesting to note the contribution of the process $\gamma\gamma\rightarrow W^+W^-$ when leptons producing photons are not detected. This contribution is shown by a long-dashed curve. We can see that for energies beginning from 1.5 TeV the cross section of the subprocess is more than that of the main process. The lepton collider becomes a photon one.

The influence of the RC's on differential cross sections is much greater. In Fig. 4 the differential cross sections for different energies are shown.

We can see that at large angles the corrected cross sections are more than the Born ones by several times. This fact is connected with the possibility of hard photon emission from one initial lepton. The large Lorentz boost in this case ''reflects'' the peak at small angles to the backward direction.

Let us consider now the effect of the ISR on the decay product distribution. In Table I the results of $\Delta N/N$ calculation are presented including (1) the width Γ_W, (2) $\Gamma_W+X^i_{\rm RC}$, where $X^i_{\rm RC}$ are the radiative corrections related to initial particles, and (3) $\Gamma_W+X^i_{\rm RC}+X^f_{\rm RC}$, where $X^f_{\rm RC}$ are the radiative corrections related to the final leptons of W decay, which result in independence in the lepton four-momenta measurement.

TABLE I. Results of the simulation of the process $e^+e^-\to W^+W^-$ with subsequent leptonic decays.

$\sqrt{s}=200$ GeV	ΔN	$\delta\Gamma/\Gamma$ (%)
Born	0	
Born+Γ	179	3.5
Born+Γ+$X^i_{\rm RC}$	142	4.0
Born+Γ+$X^i_{\rm RC}$+$X^f_{\rm RC}$	198	3.5

The estimations of the width measurement error $\delta\Gamma$ (for $N\sim 1000$, where N is the average number of W pairs with leptonic decays in four LEP 200 experiments)

$$\frac{\delta\Gamma}{\Gamma}\sim\frac{\delta(\Delta N)}{N}\frac{m}{\Gamma C}\tag{17}$$

are also given in Table I. Thus the errors in determination of the four-momenta of the final lepton decay products as well as the RC's related to the initial particles will lead to inaccuracies of $\sim$3.5–4 % against the current experimental value of 5%.

In conclusion, we would like to note the influence of final-state rescattering on the cross section of two-lepton production. The cross section of produced lepton scattering is large in the case of a small angle of scattering. However, as follows from the definition of the allowed region, its phase volume tends to zero. So this effect is negligible.

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APPENDIX

Based on the Kleiss-Stirling [18] technique we can write the chiral amplitudes (the chirality of the initial electrons are given in parentheses):

$$\begin{aligned}\mathcal{M}_\nu(-)&=\frac{-2ie^2}{\xi t}\,t(p_+,r_2)s(r_1,p_-)\times[s(r_+,p_-)t(p_-,r_-)\\&\quad-s(r_+,r_1)t(r_1,r_-)],\\
\mathcal{M}_{\gamma Z}(-)&=4ie^2\left\{\frac{1}{s}-\frac{(-\frac{1}{2}+\xi)}{\xi(s-M_Z^2)}\right\}\{t(p_+,r_-)s(r_1,p_-)\\&\quad\times[t(r_2,p_+)s(p_+,r_+)+t(r_2,p_-)s(p_-,r_+)]\\&\quad-t(p_+,r_2)s(r_+,p_-)[t(r_-,p_+)s(p_+,r_1)\\&\quad+t(r_-,p_-)s(p_-,r_1)]+t(r_2,r_-)s(r_1,r_+)\\&\quad\times[t(p_+,r_+)s(r_+,p_-)+t(p_+,r_2)s(r_2,p_-)]\},\\
\mathcal{M}_{\gamma Z}(+)&=4ie^2\left\{\frac{1}{s}-\frac{1}{s-M_Z^2}\right\}\{t(r_-,p_-)s(p_+,r_1)\\&\quad\times[t(r_2,p_+)s(p_+,r_+)+t(r_2,p_-)s(p_-,r_+)]\\&\quad-t(r_2,p_-)s(p_+,r_+)[t(r_-,p_+)s(p_+,r_1)\\&\quad+t(r_-,p_-)s(p_-,r_1)]+t(r_2,r_-)s(r_1,r_+)\\&\quad\times[t(r_+,p_-)s(p_+,r_+)+t(r_2,r_-)s(p_+,r_2)]\}.\end{aligned}\tag{A1}$$

Here we introduced the next momenta definition

$$\begin{aligned}e^-(p_-)+e^+(p_+)&\to W^+(q_+)\\&+W^-(q_-)\to l_1^+(r_+)l_2^-(r_-)\nu_2(r_2)\nu_1(r_1)\end{aligned}$$

and $\xi=\sin^2\theta_W$.

The functions s and t have the form

$$\begin{aligned}s(p_1,p_2)&=\overline{u_+}(p_1)u_-(p_2)\\&=(p_1^y+ip_1^z)\sqrt{\frac{p_2^0-p_2^x}{p_1^0-p_1^x}}-(p_2^y+ip_2^z)\sqrt{\frac{p_1^0-p_1^x}{p_2^0-p_2^x}}\end{aligned}\tag{A2}$$

and

$$t(p_1,mp_2)=\overline{u_-}(p_1)u_+(p_2)=[s(p_2,p_1)]^*.\tag{A3}$$

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