Unitarity triangle and quark mass matrices on the nearest-neighbor interaction basis

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We examine the unitarity triangle of the KM matrix, which is derived from the general quark mass matrices in the NNI basis. The Fritzsch *Ansätze* are modified by introducing four additional parameters. The KM matrix elements are expressed in terms of quark mass ratios, two phases, and four additional parameters. It is found that the vertex of the unitarity triangle is predicted to be almost in the second quadrant on the ρ-η plane as far as $V_{us}\simeq -\sqrt{m_d/m_s}e^{ip}+\sqrt{m_u/m_c}e^{iq}$. [S0556-2821(97)05303-4]

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I. INTRODUCTION

One of the most important unsolved problems of flavor physics is the understanding of flavor mixing and the fermion masses, which are free parameters in the standard model. The observed values of those mixing and masses provide us with clues of the origin of the fermion mass matrices. One of the most strict methods to test quark mass matrices is given by investigating the so-called unitarity triangle of the Kobayashi-Maskawa (KM) matrix [1]. At present, the unitarity triangle in the ρ-η plane [2] is determined by the experimental data of $B\to X\ell\overline{\nu_\ell}$, ϵ parameter in the neutral K meson system and B_d-$\overline{B}_d$ mixing. However, the experimentally allowed region is too wide to determine the position of the vertex point in the ρ-η plane. The unitarity triangle is expected to be determined precisely by the experimental data from the B factory at KEK and SLAC in the near future. On the other hand, one needs the experimental information of the six quark masses to estimate the KM matrix elements in the quark mass matrix models. The discovery of the top quark [3,4] provides us with the chance of the precise study of quark mass matrices phenomenologically. Thus, we are now in the epoch of examination of quark mass matrices by focusing on the unitarity triangle.

In this paper, we examine the unitarity triangle of the KM matrix, which is derived from the quark mass matrices in the nearest-neighbor interactions (NNI) basis [5]. Any up- and down- 3×3 quark mass matrices can always be transformed to those in this basis by a weak-basis transformation. We present general discussions of the unitarity triangle in context with quark mass matrices in the NNI basis. It is likely that the vertex of the unitarity triangle almost lies on the second quadrant of the ρ-η plane.

In Sec. II, we give the expression of the KM matrix obtained from general quark mass matrices in the NNI basis. The KM matrix elements are expressed in terms of quark mass ratios, two phases, and four additional parameters which stand for the discrepancy of our matrix elements from those of the Fritzsch *Ansätze* [6]. We show the vertex of the unitarity triangle on the ρ-η plane related with the parameters of the quark mass matrices in Sec. III. A summary is given in Sec. IV.

II. QUARK MASS MATRIX IN THE NNI BASIS AND KM MATRIX

As presented by Branco, Lavoura, and Mota, both up and down quark mass matrices could always be transformed to the non-Hermitian matrices in the NNI basis by a weak-basis transformation for the three and four generation cases [5]. In this basis, the KM matrix elements are expressed generally in terms of mass matrix parameters due to eight texture zeros. In particular, phases of the mass matrices can be easily isolated as shown later. The famous Fritzsch *Ansätze* are the special ones in the NNI basis. Therefore, the discrepancy from the Fritzsch *Ansätze* can be simply estimated in this basis. We begin with discussing general quark mass matrices in the NNI basis as follows:

$$M_u=\begin{pmatrix} 0 & A & 0\\ B & 0 & C\\ 0 & D & E\end{pmatrix},\quad M_d=\begin{pmatrix} 0 & F & 0\\ G & 0 & H\\ 0 & I & J\end{pmatrix},\qquad (1)$$

where $A\sim J$ are c numbers. The matrices U_u and U_d are defined as the unitarity matrices which diagonalize the Hermitian matrices $H_u=M_uM_u^\dagger$ and $H_d=M_dM_d^\dagger$, respectively:

$$U_u^\dagger H_uU_u=D_u,\quad U_d^\dagger H_dU_d=D_d,\qquad (2)$$

where

$$D_u=\begin{pmatrix} m_u^2 & 0 & 0\\ 0 & m_c^2 & 0\\ 0 & 0 & m_t^2\end{pmatrix},\quad D_d=\begin{pmatrix} m_d^2 & 0 & 0\\ 0 & m_s^2 & 0\\ 0 & 0 & m_b^2\end{pmatrix}.\qquad (3)$$

In the NNI basis, we can extract phases from each quark mass matrix by the use of the diagonal phase matrices. Then, we can write

$$U_u=\phi_uO_u,\quad U_d=\phi_dO_d,\qquad (4)$$

where $\phi_u=\mathrm{diag}(e^{ip_u},e^{iq_u},1)$, $\phi_d=\mathrm{diag}(e^{ip_d},e^{iq_d},1)$ and O_u, O_d are orthogonal matrices. We define the phase matrix Φ as

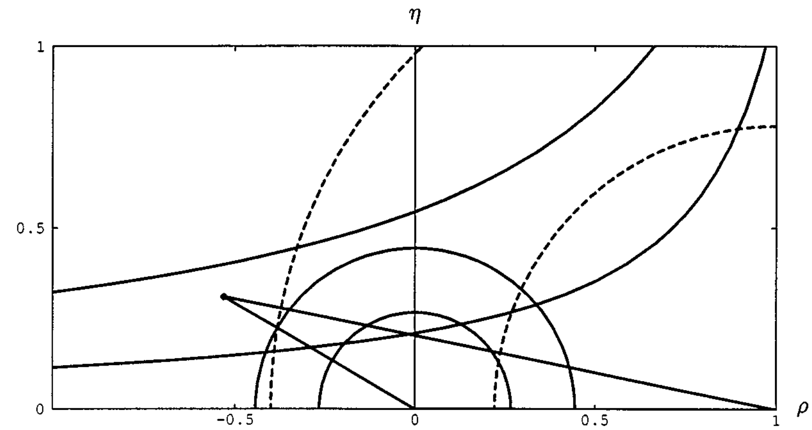

FIG. 1. The unitarity triangle in the *Ansatz* by Branco *et al.*

$$\Phi = \phi_u^* \phi_d \tag{5}$$

$$= \begin{pmatrix} e^{ip} & 0 & 0 \\ 0 & e^{iq} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

where $p = p_d - p_u$ and $q = q_d - q_u$. So the KM matrix is given by

$$V_{\rm KM} = O_u^T \Phi O_d . \tag{7}$$

The Fritzsch *Ansatz* of the quark mass matrix is to take the simplest form in the NNI basis:

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & a_d & 0 \\ a_d & 0 & b_d \\ 0 & b_d & c_d \end{pmatrix}. \tag{8}$$

Although this *Ansatz* is successful for the V_{us} element like

$$V_{us} \simeq -\sqrt{\frac{m_d}{m_s}} e^{ip} + \sqrt{\frac{m_u}{m_c}} e^{iq}, \tag{9}$$

it fails for V_{cb},

$$V_{cb} \simeq \sqrt{\frac{m_s}{m_b}} e^{iq} - \sqrt{\frac{m_c}{m_t}}, \tag{10}$$

as far as $m_t \geqslant 100$ GeV. So this simplest *Ansatz* seems to be ruled out since the observed top-quark mass is larger than 160 GeV [3,4]. On the other hand, another *Ansatz* proposed by Branco *et al.* [7],

$$M_u = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & c_u & c_u \end{pmatrix}, \quad M_d = \begin{pmatrix} 0 & a_d & 0 \\ a_d & 0 & b_d \\ 0 & c_d & c_d \end{pmatrix}, \tag{11}$$

is successful not only for the V_{us} element,

$$V_{us} \simeq -\frac{1}{\sqrt[4]{2}} \sqrt{\frac{m_d}{m_s}} e^{ip} + \frac{1}{\sqrt[4]{2}} \sqrt{\frac{m_u}{m_c}} e^{iq}, \tag{12}$$

but also for V_{cb}:

$$V_{cb} \simeq \frac{m_s}{m_b} e^{iq} - \frac{m_c}{m_t}. \tag{13}$$

Although this *Ansatz* overcomes the fault of the Fritzsch *Ansatz*, it cannot reproduce the observed ratio $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$ [8]. If we describe the unitarity triangle which is obtained by using the *Ansatz* by Branco *et al.*, the vertex

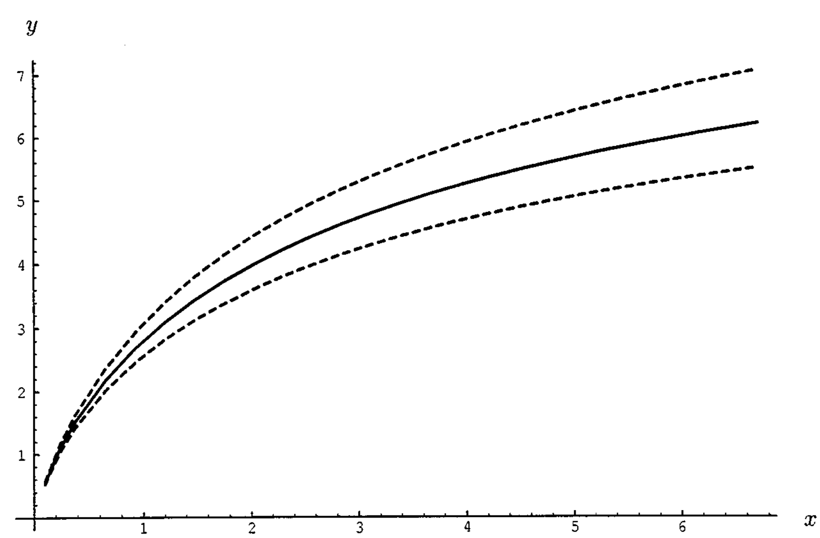

FIG. 2. The relation between x and y at $q = 0°$. The solid line is the case of $|V_{cb}| = 0.040$. The upper (lower) dashed line is the case of $|V_{cb}| = 0.035$ ($|V_{cb}| = 0.045$).

of the triangle is plotted outside of the experimentally allowed region on the ρ-η plane, as shown in Fig. 1. Here, in order to describe the experimental allowed region, we used the recent JLQCD result [9] of the lattice calculation for the theoretical parameters $\hat{B}_K$ and f_{B_d} as follows:

$$\hat{B}_K=0.76\pm0.04,\quad f_{B_d}=0.19\pm0.01\ \text{GeV}. \tag{14}$$

Therefore we should consider more general mass matrices in order to study the unitarity triangle. We adopt a weak hypothesis for the matrix in Eq. (1): the generation hierarchy of the matrix elements, which was guaranteed in the Fritzsch *Ansatz* due to the observed mass hierarchy,

$$M_{12}\approx M_{21}\ll M_{23}\approx M_{32}\ll M_{33}, \tag{15}$$

for both up- and down-quark sectors. The quark mass matrices are parametrized in general as

$$M_u=\begin{pmatrix} 0 & a_u & 0\\ a_u z & 0 & b_u\\ 0 & b_u x & c_u\end{pmatrix},\quad M_d=\begin{pmatrix} 0 & a_d & 0\\ a_d w & 0 & b_d\\ 0 & b_d y & c_d\end{pmatrix}, \tag{16}$$

where x, y, z, and w are parameters that stand for discrepancies from the Fritzsch mass matrix. Because of the hierarchy of mass matrix elements, we may restrict the parameter region:

$$0.1\le x,y,z,w\le 10. \tag{17}$$

We comment on the number of parameters. There are twelve parameters in the mass matrices because there are two phase parameters in addition to the parameters in Eq. (16). Since the number of the physical parameters is ten (six masses and four KM mixing parameters), the basis in Eq. (16) is the general one.

In order to obtain the elements of $O_{u(d)}$ in Eq. (4), we express $a_{u(d)}$, $b_{u(d)}$, and $c_{u(d)}$ in terms of quark masses from eigenvalue equations of $H_{u(d)}$. For the parameter c_u, the equation is given as follows: c_u is written as

$$c_u^4-2M_1c_u^2+\frac{(1+x^2)^2(1+z^2)}{x^2z}\sqrt{M_3}c_u+M_1^2-\frac{(1+x^2)^2}{x^2}M_2=0, \tag{18}$$

where

$$M_1=m_u^2+m_c^2+m_t^2, \tag{19}$$

$$M_2=m_u^2m_c^2+m_c^2m_t^2+m_t^2m_u^2, \tag{20}$$

$$M_3=m_u^2m_c^2m_t^2. \tag{21}$$

Neglecting the third term of Eq. (18), which is very small, we obtain c_u:

$$c_u\simeq m_t\left(1-\frac{1+x^2}{2x}\frac{m_c}{m_t}\right). \tag{22}$$

For a_u and b_u, we get

$$a_u\simeq\sqrt{\frac{m_um_c}{z}}\left(1+\frac{1+x^2}{4x}\frac{m_c}{m_t}\right), \tag{23}$$

$$b_u\simeq\sqrt{\frac{m_cm_t}{x}}\left(1-\frac{1+z^2}{4z}\frac{m_u}{m_c}\right). \tag{24}$$

Then the matrix elements of O_u are written as

$$(O_u)_{11}\simeq 1/N_{u_1}, \tag{25}$$

$$(O_u)_{12}\simeq-\frac{1}{\sqrt{z}}\sqrt{\frac{m_u}{m_c}}(1+R_u^{ct}+r_u^{ct}+r_u^{uc})/N_{u_2}, \tag{26}$$

$$(O_u)_{13}\simeq\sqrt{\frac{x}{z}\frac{m_c}{m_t}}\sqrt{\frac{m_u}{m_t}}(1+R_u^{ct}-R_u^{uc})/N_{u_3}, \tag{27}$$

$$(O_u)_{21}\simeq\frac{1}{\sqrt{z}}\sqrt{\frac{m_u}{m_c}}(1-R_u^{ct}+r_u^{uc})/N_{u_1}, \tag{28}$$

$$(O_u)_{22}\simeq 1/N_{u_2}, \tag{29}$$

$$(O_u)_{23}\simeq\frac{1}{\sqrt{x}}\sqrt{\frac{m_c}{m_t}}(1-R_u^{uc}+r_u^{ct})/N_{u_3}, \tag{30}$$

$$(O_u)_{31}\simeq-\frac{1}{\sqrt{xz}}\sqrt{\frac{m_u}{m_c}}(1+R_u^{ct}-R_u^{uc}+r_u^{uc})/N_{u_1}, \tag{31}$$

$$(O_u)_{32}\simeq-\frac{1}{\sqrt{x}}\sqrt{\frac{m_c}{m_t}}(1-R_u^{uc}+r_u^{ct})/N_{u_2}, \tag{32}$$

$$(O_u)_{33}\simeq 1/N_{u_3}, \tag{33}$$

where $N_{u_i}(i=1,2,3)$ are normalization factors, and

$$R_u^{uc}=\frac{1+z^2}{4z}\frac{m_u}{m_c},\quad R_u^{ct}=\frac{1+x^2}{4x}\frac{m_c}{m_t}, \tag{34}$$

$$r_u^{uc}=\frac{1-z^2}{2z}\frac{m_u}{m_c},\quad r_u^{ct}=\frac{1-x^2}{2x}\frac{m_c}{m_t}. \tag{35}$$

The matrix elements of O_d are given analogously. Here we neglect the terms of order of m_u/m_t, m_d/m_b, and higher order corrections. Thus the KM matrix elements are given in terms of quark mass ratios, two phases, and four additional parameters x, y, z, and w as

$$V_{ud}\simeq\frac{1}{N_{ud}}e^{ip}, \tag{36}$$

$$V_{us}\simeq\frac{1}{N_{us}}\left[-\sqrt{\frac{m_d}{wm_s}}(1+R_d^{sb}+r_d^{sb}+r_d^{ds})e^{ip}+\sqrt{\frac{m_u}{zm_c}}(1-R_u^{ct}+r_u^{uc})e^{iq}\right], \tag{37}$$

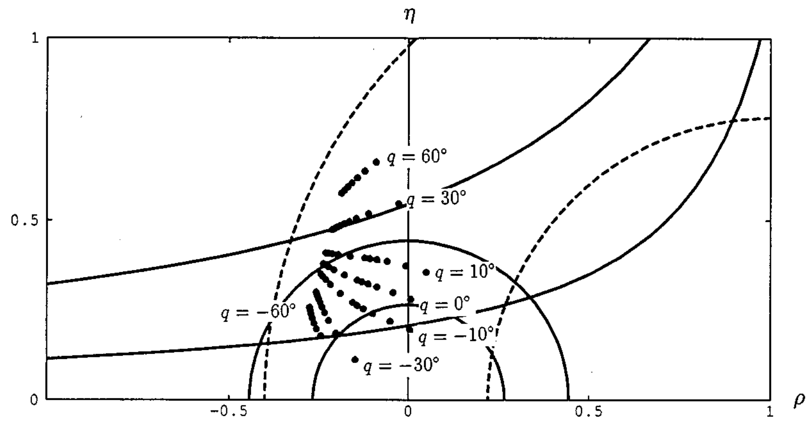

FIG. 3. The vertices of the unitarity triangle in the case of $z=w=1$. Each dotted line is the case of $q=+60°$, $+30°$, $+10°$, $0°$, $-10°$, $-30°$, $-60°$ from the upper side, respectively. The dots of $q=0°$ and $\pm 10°$ lines correspond to $x=0.5$, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0 in order from the right hand side. The dots of $q=\pm 30°$ lines correspond to $x=1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right hand side. The dots of $q=\pm 60°$ lines correspond to $x=4.0$, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right hand side.

$$V_{ub}\simeq\frac{1}{N_{ub}}\left[\sqrt{\frac{m_u}{xzm_t}}(1+R_u^{ct}-R_u^{uc}+r_u^{uc})\right], \tag{38}$$

$$V_{cd}\simeq\frac{1}{N_{cd}}\left[-\sqrt{\frac{m_u}{zm_c}}(1+R_u^{ct}+r_u^{ct}+r_u^{uc})e^{ip}+\sqrt{\frac{m_d}{wm_s}}(1-R_d^{sb}+r_d^{sb})e^{iq}\right], \tag{39}$$

$$V_{cs}\simeq\frac{1}{N_{cs}}e^{iq}, \tag{40}$$

$$V_{cb}\simeq\frac{1}{N_{cb}}\left[\sqrt{\frac{m_s}{ym_b}}(1-R_d^{sb}+r_d^{sb})e^{iq}-\sqrt{\frac{m_c}{xm_t}}(1-R_u^{uc}+r_u^{ct})\right], \tag{41}$$

$$V_{td}\simeq\frac{1}{N_{td}}\left[\sqrt{\frac{m_d}{ywm_b}}(1+R_d^{sb}-R_d^{ds}+r_d^{ds})\right], \tag{42}$$

$$V_{ts}\simeq\frac{1}{N_{ts}}\left[\sqrt{\frac{m_c}{xm_t}}(1-R_u^{uc}+r_u^{ct})e^{iq}-\sqrt{\frac{m_s}{ym_b}}(1-R_d^{ds}+r_d^{sb})\right], \tag{43}$$

$$V_{tb}\simeq\frac{1}{N_{tb}}, \tag{44}$$

where N's are normalization factors such as $N_{us}=N_{u_1}N_{d_2}$, etc.

III. THE UNITARITY TRIANGLE AND PARAMETERS OF QUARK MASS MATRICES

In order to estimate the absolute values of KM matrix elements $|V_{ij}|$, we use the values of the masses of six quarks on the same energy scale. There are many studies for the values of quark masses [10–16]. Since Koide [17] has discussed those works recently, we follow his summary. For the light quark masses, we take the results by Dominguez and Rafael [11] since the m_s/m_d ratio is in good agreement with the one in Ref. [13]:

$$m_u=0.0056\pm 0.011,\quad m_d=0.0099\pm 0.0011,$$
$$m_s=0.199\pm 0.033\ \ (\mathrm{GeV}), \tag{45}$$

at 1 GeV scale. For the heavy quarks, we adopt the running masses [3,4,16]

$$m_c(m_c)=1.237\pm 0.018,\quad m_b(m_b)=4.273\pm 0.064,$$
$$m_t(m_t)=166.2\pm 12.4\ \ (\mathrm{GeV}), \tag{46}$$

which correspond to the following masses at 1 GeV scale:

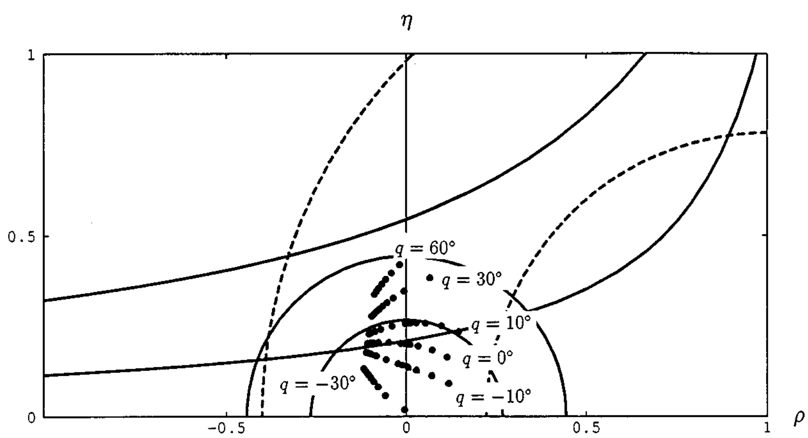

FIG. 4. The vertices of the unitarity triangle in the case of $z=w=1.3$. Each dotted line is the case of $q=+60°$, $+30°$, $+10°$, $0°$, $-10°$, $-30°$ from the upper side, respectively. The dots of $q=0°$ and $\pm 10°$ lines correspond to $x=0.5$, 0.6, 0.7, 0.8, 0.9, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0 in order from the right hand side. The dots of $q=\pm 30°$ lines correspond to $x=1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right hand side. The dots of $q=60°$ lines correspond to $x=4.0$, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right hand side.

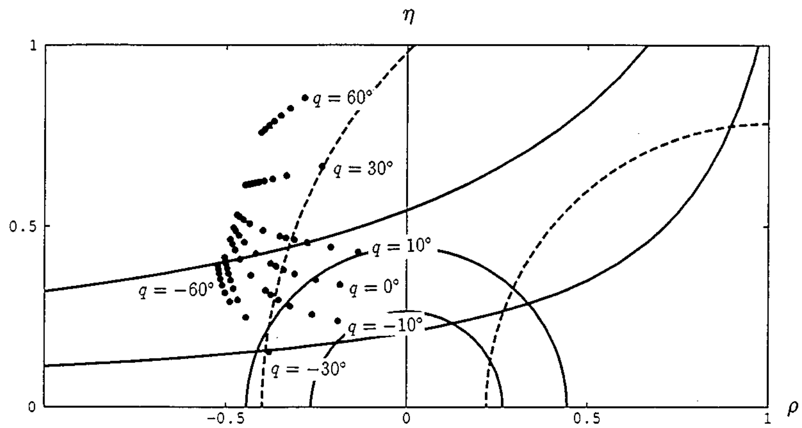

FIG. 5. The vertices of the unitarity triangle in the case of $z=w=0.7$. Each dotted line is the case of $q=+60°$, $+30°$, $+10°$, $0°$, $-10°$, $-30°$, $-60°$ from the upper side, respectively. The dots of $q=0°$ and $\pm 10°$ lines correspond to $x=0.7$, 0.8, 0.9, 1.0, 2.0, 4.0, 6.0, 8.0, 10.0 in order from the right-hand side. The dots of $q=\pm 30°$ lines correspond to $x=1.0$, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right-hand side. The dots of $q=\pm 60°$ lines correspond to $x=4.0$, 5.0, 6.0, 7.0, 8.0, 9.0, 10.0 in order from the right-hand side.

$$m_c=1.316\pm 0.024, \quad m_b=5.934\pm 0.101,$$

$$m_t=349.5\pm 27.9 \;\; (\text{GeV}). \tag{47}$$

Here, we used 2-loop renormalization group equations with $\Lambda^{(5)}_{\overline{\text{MS}}}=0.195$ GeV.

Now we can estimate the KM matrix elements. At first, we set $z=w=1$, which corresponds to the Fritzsch *Ansatz* for the first- and second-generation mixing sector, since this *Ansatz* works well for $|V_{us}|$. It is remarkable that the following results are available for all models which express V_{us} as $-\sqrt{m_d/m_s}e^{ip}+\sqrt{m_u/m_c}e^{iq}$, because our KM matrix elements in Eq. (37) are most generally expressed.

Since we study the unitarity triangle which is normalized by $V^*_{cb}V_{cd}$, parameters should be constrained to reproduce the values of $|V_{cb}|$ and $|V_{cd}|$ in PDG [8], as well as $|V_{ud}|$, $|V_{us}|$, and $|V_{cs}|$, which do not appear explicitly in the unitarity triangle. We use the values [8]

$$|V_{ud}|=0.9753\pm 0.0006, \quad |V_{us}|=0.221\pm 0.003,$$

$$|V_{cd}|=0.221\pm 0.003, \quad |V_{cb}|=0.040\pm 0.008,$$

$$|V_{cs}|=0.9745\pm 0.0007. \tag{48}$$

In the numerical analyses, we take the central values for $|V_{cd}|$ and $|V_{cb}|$ in Eq. (48). The ambiguity of our predictions due to experimental errors is rather small. For the given values of x and q, we can obtain the value of parameter y by using Eq. (41). In order to reproduce $|V_{cb}|$, we cannot set $x=y=1$, which leads us to the unsuccess of the Fritzsch *Ansatz*. In Fig. 2, the allowed lines for $|V_{cb}|$ are shown on the $x-y$ plane in the case of $q=0$. We find that the phase q is restricted to be $-60°\leqslant q\leqslant 60°$ due to the condition $0.1\leqslant y\leqslant 10$. Next we obtain the value of the phase p from Eq. (39). Since p is a real number, the value of x is also restricted for each value of q. Thus if x and q are fixed, we can calculate the KM matrix elements. Furthermore, we restrict the parameter region of x for each q value due to fitting with experimental data of $|V_{ud}|$, $|V_{us}|$, and $|V_{cs}|$:

$$q=\pm 60°:2.0\leqslant x\leqslant 10, \quad q=\pm 30°:1.0\leqslant x\leqslant 10,$$

$$q=0°:0.5\leqslant x\leqslant 10.$$

Let us present the unitarity triangle. If the phase q is fixed, the vertex point of the triangle moves on the $\rho-\eta$ plain according to the change of parameter x. We show in Fig. 3 the changes of the vertex points by the dotted lines for the fixed values of q such as $q=-60°$, $-30°$, $-10°$, $0°$, $10°$, $30°$, and $60°$. It is found that the triangle vertex is sensitive to the value of x in the $x\leqslant 1$ region, but insensitive to x in the large x region. If $-60°\leqslant q\leqslant 15°$, a part of the dotted line comes into the experimentally allowed region. All of the triangle vertices are found to be on the second quadrant of the $\rho-\eta$ plane. It is to be emphasized that, if we assume the generation hierarchy of the quark mass matrices, the vertex of the unitarity triangle is on the second quadrant as far as $z=w=1$, namely, as for as $V_{us}\simeq -\sqrt{m_d/m_s}e^{ip}+\sqrt{m_u/m_c}e^{iq}$.

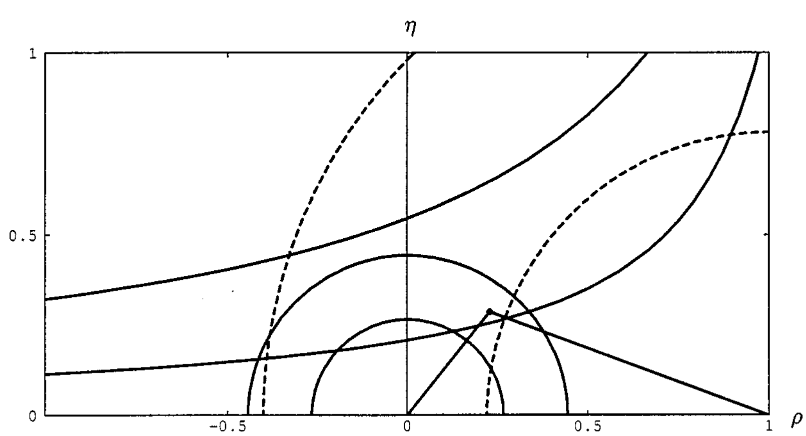

FIG. 6. One case in which the vertex of the unitarity triangle comes on the center of the first quadrant of the $\rho-\eta$ plane.

Next, we consider the cases of $z=w\neq 1$. Due to the condition that p is a real number, z and w are restricted to be $z=w\leqslant 1.3$. In the case of $1\leqslant z=w\leqslant 1.3$, each dotted line of Fig. 3 moves down to the right side. So, in this case, the possible predicted region of the vertex in the first quadrant appears unlike the case of $z=w=1$. While in the case of $z=w\leqslant 1$, each line of Fig. 3 moves up to the left side. In the case of $z=w\leqslant 1.1$, the triangle vertices never come in the allowed region on the first quadrant. Figures 4 and 5 show the cases of $z=w=1.3$ and $z=w=0.7$, respectively. Thus, we conclude that the vertex of the unitarity triangle almost appears in the second quadrant under the assumption $z=w$.

What is the condition that the vertex of the unitarity triangle appears in the first quadrant? As seen in Eqs. (38) and (42), V_{ub} and V_{td} are written roughly as

$$V_{ub}\simeq\sqrt{\frac{m_u}{xzm_t}},\quad V_{td}\simeq\sqrt{\frac{m_d}{ywm_b}}. \tag{49}$$

Due to $m_u\sim m_d$ and $m_t\gg m_b$, $|V_{td}|$ is expected to be larger than $|V_{ub}|$ in the case of $x\sim y$ as well as $w\sim z$. This fact suggests that the conditions $w>1$ and $z<1$ are necessary in order to move the vertex of the unitarity triangle into the first quadrant. Actually, we get the vertex in the first quadrant for some cases of $w>1$ and $z<1$. For example, we show one case in Fig. 6 in which $x=1.0$, $y=2.8$, $z=0.4$, $w=2.0$, $p=128.2°$, and $q=0°$ are taken. Then, V_{us} is no longer expressed in the simple form as Eq. (9).

IV. SUMMARY

In this paper, we examined the unitarity triangle, which is derived from general quark mass matrices in the NNI basis. For quark mass matrices, we introduced four additional parameters x, y, z, and w, which are restricted to be $0.1\leqslant x,y,z,w\leqslant 10$ due to the assumed generation hierarchy. Then the KM matrix elements have been written in terms of quark mass ratios, two phases, and four parameters. In the case of $z=w$, the vertex of the unitarity triangle almost comes on the second quadrant of the $\rho-\eta$ plane as shown in Figs. 3, 4, and 5. If the vertex of the unitarity triangle is determined to be in the neighborhood of the vertex of Fig. 6 by the B factory experiment, the quark mass matrix models which predict $V_{us}\simeq-\sqrt{m_d/m_s}e^{ip}+\sqrt{m_u/m_c}e^{iq}$ should be ruled out as far as the generation hierarchy of the quark mass matrices is assumed. Thus, the determination of the unitarity triangle will give the important impact on the study of the quark mass matrix model.

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