Theory of τ mesonic decays

Bing An Li
Department of Physics and Astronomy, University of Kentucky, Lexington, Kentucky 40506
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Studies of τ mesonic decays are presented. A mechanism for the axial-vector current at low energies is proposed. The VMD is used to treat the vector current. All the meson vertices of both normal parity and abnormal parity (Wess-Zumino-Witten anomaly) are obtained from an effective chiral theory of mesons. a_1 dominance is found in the decay modes of the τ lepton: 3π, $f(1285)\pi$. Both the ρ and the a_1 meson contribute to the decay $\tau\to K^*K\nu$; it is found that the vector current is dominant. CVC is tested by studying $e^+e^-\to\pi^+\pi^-$. The branching ratios of $\tau\to\omega\pi\nu$ and $K\bar{K}\nu$ are calculated. In terms of a similar mechanism the $\Delta s=1$ decay modes of the τ lepton are studied and K_a dominance is found in $\tau\to K^*\pi\nu$ and $K^*\eta\nu$. The suppression of $\tau\to K\rho\nu$ is revealed. The branching ratio of $\tau\to\eta K\nu$ is computed. As a test of this theory, the form factors of $\pi\to e\gamma\nu$ and $K\to e\gamma\nu$ are determined. The theoretical results agree with data reasonably well. [S0556-2821(97)05603-8]

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I. INTRODUCTION

The τ mesonic decays were studied by Tsai [1] before the discovery of the τ lepton. All the hadrons in τ hadronic decays are mesons; therefore, τ mesonic decays provide a test ground for all meson theories. The mesons produced in τ hadronic decays are made of light quarks. Therefore, chiral symmetry plays an important role in studying τ mesonic decays. In Ref. [2] τ mesonic decays are associated with chiral dynamics. It is pointed out [2] that ρ dominance is necessary to be introduced and the chiral limits of the hadronic matrix elements at low energies are set up. In Ref. [3] τ mesonic decays are studied by using SU(3)$\times$ SU(3) chiral dynamics with the resonances phenomenologically introduced. In Ref. [4] a Lagrangian for psedoscalar and vector mesons has been constructed to investigate τ physics. In Ref. [5] in studying three pseudoscalar meson decays of the τ lepton the form factors of these decays is constructed by chiral symmetry and dominated by the lowest resonances. Abnormal decays have also been studied [6–10]. Vector meson dominance (VMD) [11] has been applied to study τ decays in which the meson states have even G parity [2–10].

In Ref. [12] an effective Lagrangian of three nonets of pseudoscalar, vector, and axial-vector mesons with $U(3)_L\times U(3)_R$ symmetry is obtained. The chiral symmetry-breaking scale Λ determined in Ref. [12] is 1.6 GeV; therefore, this theory is suitable to be applied to study τ mesonic decays. VMD is a natural result of this theory. The Wess-Zumino-Witten action (WZW) [13] is obtained from the leading terms of the imaginary part of the effective Lagrangian. This theory has been applied to study the form factors of K_{l3}, $\tau\to\rho\nu$, and $\tau\to K^*\nu$, and theoretical results are in good agreement with the data [12]. Based on this effective chiral theory of mesons [12], a theory of τ mesonic decays is developed and a unified study of τ mesonic decays is presented in this paper.

In the standard model the W bosons are coupled to both the vector and the axial-vector currents of ordinary quarks. The hadronization of the quark currents is a problem of nonperturbative QCD. In the dynamics of ordinary quarks both chiral symmetry and chiral symmetry breaking are important. VMD and conservation of vector current (CVC) are successful in studying the matrix elements of vector currents [14]. VMD is a natural result of the effective chiral theory of mesons [12]. In this paper VMD is exploited to treat the matrix elements of the vector currents of τ decays. Before this paper VMD was already exploited to study τ mesonic decays [2–10]. The difference between this paper and others in the case of two flavors is that the coupling of $\rho\pi\pi$, $f_{\rho\pi\pi}$, derived in Ref. [12], is no longer a constant, but a function of q^2 (q is the momentum of the ρ meson). It means that the vertex $\rho\pi\pi$ has a form factor. A detailed discussion is presented in Sec. VII.

It is well known that in the chiral limit, the axial-vector currents and the vector currents of ordinary quarks form a $SU(2)_L\times SU(2)_R$ algebra which leads to Weinberg's sum rules [15]. On the other hand, the a_1 meson is the chiral partner of the ρ meson [16]. However, the a_1 meson is much heavier than the ρ meson. In Ref. [12] this mass difference refers to spontaneous chiral symmetry breaking and this effect should be taken into account in bosonizing the axial-vector currents. The effect of spontaneous chiral symmetry breaking on the bosonization of the axial-vector currents at low energies is studied in this paper. The axial-vector currents contribute to the meson states of τ decays, which have negative G parity. The hadronic matrix elements of the axial-vector currents derived in this paper are different from other studies. The Breit-Wigner formula for a_1 resonance is [see Eq. (35)]

$$\frac{-g^2f_a^2m_\rho^2+i\sqrt{q^2}\Gamma_a(q^2)}{q^2-m_a^2+i\sqrt{q^2}\Gamma_a(q^2)}.$$

The difference originates in spontaneous chiral symmetry breaking which is responsible for the mass difference of the ρ and the a_1 mesons. Because of chiral symmetry breaking, at $q^2=0$ this formula is equal to $g^2f_a^2m_\rho^2/m_a^2$ and the mass of the a_1 meson is still there.

There are two kinds of meson vertices in τ mesonic decays: the vertices of normal parity and the ones of abnormal parity. The later are from the Wess-Zumino-Witten Lagrangian. In Ref. [12] it is shown that the WZW Lagrangian is the leading term of the imaginary part of the effective meson Lagrangian and the fields in the WZW Lagrangian are normalized to physical meson fields. The normalization of the fields of the WZW action is very important. The normalization constants for the vector and axial-vector fields are different [12], and as a result of mixing effects, the axial-vector fields are always associated with $\partial_\mu P$, where P is the corresponding pseudoscalar field.

All the meson vertices are obtained from Ref. [12] and they are fixed completely; most of them have been tested already [12]. It is necessary to point out that the vertices of VPP obtained in Ref. [12] are functions of momentum [see Eq. (37) $f_{\rho\pi\pi}$ and see Sec. VII for details] and the vertices of AVP depend on momentum strongly [see Eq. (27), for example], and as a result of the cancellation in the vertices of AVP, the dependence of momentum is very important in understanding $\tau\to\nu(\text{mesons})_{G=-}$.

In the chiral limit, the theory used to study τ mesonic decays consists of three parts: VMD for vector currents, a new expression of axial-vector currents, and vertices of mesons. All three parts are determined by the effective chiral theory of mesons [12]. All parameters have been fixed.

In many studies [5,6,8,17], besides the ρ meson, the exicted $\rho(\rho'$ and $\rho'')$ are taken part in. In this paper only the ρ meson is included. In the region of higher q^2 the effects of the form factor of the VPP vertex and other decay channels of ρ (such as $K\bar{K}$, KK^*, . . .) are taken into account in calculating the decay widths. So far, theoretical results agree with the data reasonably well. In this paper only the lowest resonances are taken into consideration.

It has been shown in Ref. [12] that diagrams at the tree level are at order of N_C and loop diagrams of mesons are at higher order in the large N_C expansion. In Ref. [12] a large N_C expansion is invoked to argue the success of the effective theory. Following the same argument, all calculations are done at the tree level in this paper.

The paper is organized as Introduction (Sec. I), general expression of the axial-vector currents (Sec. II), determination of $\mathcal{L}^{V,A}$ (Sec. III), a_1 dominance in $\tau\to\pi\pi\pi\nu$ decay (Sec. IV), a_1 dominance in $\tau\to f_1(1285)\pi\nu$ decay (Sec. V), $\tau\to K^*K\nu$ (Sec. VI), CVC and $e^+e^-\to\pi^+\pi^-$ (Sec. VII), $\tau\to\omega\pi\nu$ (Sec. VIII), $\tau\to K\bar{K}\nu$ (Sec. IX), the form factors of $\pi\to e\gamma\nu$ (Sec. IX), effective Lagrangian of $\Delta s=1$ weak interactions (Sec. XI), K_a dominance in $\tau\to K^*(892)\pi\nu$ (Sec. XII), $\tau\to K^*\eta\nu$ (Sec. XIII), $\tau\to\eta K\nu$ (Sec. XIV), the form factors of $K\to e\gamma\nu$ (Sec. XV), and conclusions (Sec. XVI).

II. GENERAL EXPRESSION OF THE AXIAL-VECTOR CURRENTS

In the case of two flavors the expression of VMD [11] is written as

$$\frac{e}{2f_v}\{-\tfrac{1}{2}F^{\mu\nu}(\partial_\mu v_\nu-\partial_\nu v_\mu)+A^\mu j^v_\mu\}, \tag{1}$$

where $v=\rho^0,\omega,\phi$, f_v is the decay constant of these vector mesons, respectively, and j^v_μ are the appropriate currents determined by the substitution

$$v_\mu\to\frac{e}{2f_v}A_\mu \tag{2}$$

in the vertices involving neutral vector mesons. CVC works very well in the weak interactions of hadrons and in τ mesonic decays [14]. In the chiral limit, the vector part of the weak interaction of ordinary quarks is determined by CVC:

$$\mathcal{L}^V=\frac{g_W}{4}\cos\theta_C\frac{1}{f_\rho}\{-\tfrac{1}{2}(\partial_\mu A^i_\nu-\partial_\nu A^i_\mu)(\partial_\mu\rho^i_\nu-\partial_\nu\rho^i_\mu)+A^i_\mu j^{i\mu}\}, \tag{3}$$

where $i=1,2$ and A^i_μ are W boson fields. In the vector part of the weak interaction there is ρ dominance (two-flavor case). j^i_μ is derived by the substitution

$$\rho^i_\mu\to\frac{g_W}{4f_\rho}\cos\theta_C A^i_\mu \tag{4}$$

in the vertices involving ρ mesons. At low energies the matrix elements of the vector currents go back to the chiral limit [2].

Chiral symmetry is one of major features of QCD. It has been known for a long time that the a_1 meson is the chiral partner of the ρ meson [16] and both are treated as non-Abelian chiral gauge fields [16]. On the other hand, it is well known that in the chiral limit, the vector and axial-vector currents form a $SU(2)_L\times SU(2)_R$ algebra which leads to Weinberg's sum rules [15]. Based on chiral symmetry it is reasonable to think that in the axial-vector part of the weak interaction of ordinary quarks there is a term which is similar to VMD:

$$-\frac{g_W}{4}\cos\theta_C\frac{1}{f_a}\{-\tfrac{1}{2}(\partial_\mu A^i_\nu-\partial_\nu A^i_\mu)(\partial_\mu a^i_\nu-\partial_\nu a^i_\mu)+A^{i\mu}j^{iW}_\mu\}, \tag{5}$$

where a^i_μ is the a_1 meson field, f_a is a constant, and j^{iW}_μ is the appropriate current obtained by substituting

$$a^i_\mu\to-\frac{g_W}{4f_a}\cos\theta_C A^i_\mu \tag{6}$$

into the Lagrangian in which an a_1 meson is involved. On the other hand, a pion can couple to a W boson directly. The second term in the axial-vector part of the weak interaction of the ordinary quarks is

$$-\frac{g_W}{4}\cos\theta_C f_\pi A^i_\mu\partial^\mu\pi^i. \tag{7}$$

As a matter of fact, the a_1 meson is much heavier than the ρ meson. Spontaneous chiral symmetry breaking is responsible for the mass difference. Therefore, because of the effect of spontaneous chiral symmetry breaking in the Lagrangian of meson theory, there should be an additional mass term for the a_1 meson:

$$\tfrac{1}{2}\Delta m^2 f_a^2 a^i_\mu a^{i\mu}. \tag{8}$$

Using the substitution (6), a new coupling between W bosons and a_1 mesons is revealed:

$$-\frac{g_W}{4}\cos\theta_C \Delta m^2 f_a A^i_\mu a^{i,\mu}. \quad (9)$$

Adding these three terms [Eqs. (5), (7), and (9)] together, the axial-vector part of the effective Lagrangian of the weak interaction of ordinary quarks is obtained:

$$\mathcal{L}^A = -\frac{g_W}{4}\cos\theta_C \frac{1}{f_a}\{-\tfrac{1}{2}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial_\mu a^i_\nu - \partial_\nu a^i_\mu) + A^{i\mu} j^{iW}_\mu\} - \frac{g_W}{4}\cos\theta_C \Delta m^2 f_a A^i_\mu a^{i\mu} - \frac{g_W}{4}\cos\theta_C f_\pi A^i_\mu \partial^\mu \pi^i. \quad (10)$$

In Eq. (10) there are two parameters f_a and Δm^2 which are necessary to be determined. Weinberg's first sum rule

$$\frac{g_\rho^2}{m_\rho^2} - \frac{g_a^2}{m_a^2} = f_\pi^2 \quad (11)$$

is derived by using $SU(2)_L \times SU(2)_R$ chiral symmetry, current algebra, and VMD, where g_ρ and g_a are defined by the formulas

$$\langle 0|\bar{\psi}\tau_i\gamma_\mu\psi|\rho^\lambda_j\rangle = g_\rho \delta_{ij}\epsilon^\lambda_\mu,$$
$$\langle 0|\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi|a^\lambda_j\rangle = g_a \delta_{ij}\epsilon^\lambda_\mu. \quad (12)$$

Using Eqs. (3) and (10), we obtain

$$g_\rho = -\frac{m_\rho^2}{f_\rho}, \quad g_a = -\frac{m_a^2}{f_a} + \Delta m^2 f_a. \quad (13)$$

It can be seen from Eqs. (4) and (6) that the ρ fields are associated with f_ρ and a_1 with f_a. After spontaneous chiral symmetry breaking the effective mass terms of ρ and a_1 mesons are written as

$$\tfrac{1}{2}(\Delta m^2 + m_0^2) f_a^2 a^i_\mu a^{i\mu} + \tfrac{1}{2} m_0^2 f_\rho^2 \rho^i_\mu \rho^{i\mu} \quad (14)$$

and

$$m_\rho^2 = m_0^2 f_\rho^2, \quad m_a^2 = f_a^2(\Delta m^2 + m_0^2). \quad (15)$$

Equation (15) leads to

$$\Delta m^2 = \frac{m_a^2}{f_a^2} - \frac{m_\rho^2}{f_\rho^2}. \quad (16)$$

From Eqs. (13) and (16) we obtain

$$g_a = -\frac{f_a}{f_\rho^2} m_\rho^2. \quad (17)$$

Substituting Eqs. (13) and (17) into Eq. (11), we determine

$$f_a^2 = f_\rho^2\left(1 - \frac{f_\pi^2 f_\rho^2}{m_\rho^2}\right)\frac{m_a^2}{m_\rho^2}, \quad \Delta m^2 = f_\pi^2\left(1 - \frac{f_\pi^2 f_\rho^2}{m_\rho^2}\right)^{-1}. \quad (18)$$

The values of f_a and Δm^2 are determined by f_π, f_ρ, m_ρ, and m_a. In general, $f_a \neq f_\rho$. Therefore, $\mathcal{L}^A$ is fixed. The vector current is conserved in the limit of $m_q = 0$. The axial-vector current derived from Eq. (10) must satisfy PCAC. It will be shown that it is necessary to have all the terms in Eq. (10) to satisfy PCAC.

III. DETERMINATION OF $\mathcal{L}^{V,A}$

An effective chiral theory of pseudoscalar, vector, and axial-vector mesons has been proposed [12]. In this theory both the physical processes of normal parity and abnormal parity are studied by one Lagrangian. Theoretical results agree with the data well. In the limit of $m_q = 0$, the Lagrangian is

$$\mathcal{L} = \bar{\psi}(x)[i\gamma\cdot\partial + \gamma\cdot v + \gamma\cdot a\gamma_5 - mu(x)]\psi(x) + \tfrac{1}{2} m_0^2(\rho^\mu_i \rho_{\mu i} + \omega^\mu\omega_\mu + a^\mu_i a_{\mu i} + f^\mu f_\mu), \quad (19)$$

where $a_\mu = \tau_i a^i_\mu + f_\mu$, $v_\mu = \tau_i \rho^i_\mu + \omega_\mu$, and $u = \exp\{i\gamma_5(\tau_i\pi_i + \eta)\}$; m is a parameter. The fields in the Lagrangian (19) are not physical and the physical meson fields have been defined in Ref. [12]. This Lagrangian is global $SU(2)_L \times SU(2)_R$ chiral symmetric. On the other hand, spontaneous chiral symmetry breaking is revealed in this theory when $\pi^i, \eta = 0$ are taken. This theory has both explicit chiral symmetry breaking (PCAC) by adding the current quark masses $-\bar{\psi}M\psi$ (M is the quark matrix) to the Lagrangian and dynamical chiral symmetry breaking (quark condensate) [12].

The explicit expression of VMD, Eq. (1), has been found in Ref. [12]. The expressions of $\mathcal{L}^{V,A}$ Eqs. (3) and (10), have been derived by this effective theory [see Eqs. (76)–(78) of Ref. [12]] too. The following expressions are revealed from this effective chiral theory:

$$g_\rho = -g m_\rho^2, \quad (20)$$

$$g_a = -g\left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1/2} m_\rho^2, \quad (21)$$

$$f_\rho = g^{-1}, \quad (22)$$

$$f_a = g^{-1}\left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1/2}, \quad (23)$$

$$\left(1 - \frac{1}{2\pi^2 g^2}\right) m_a^2 = 6m^2 + m_\rho^2, \quad (24)$$

$$\Delta m^2 = 6m^2 g^2 = f_\pi^2\left(1 - \frac{f_\pi^2}{g^2 m_\rho^2}\right)^{-1}, \quad (25)$$

where g is a universal coupling constant and m is a parameter related to the quark condensate. The term $6m^2$ in Eq. (24) and the factor $(1 - 1/2\pi^2 g^2)$ in Eqs. (23) and (24) are from spontaneous chiral symmetry breaking of this theory. In this paper we choose $g = 0.39$. Using this value, the theoretical results obtained in Ref. [12] agree with the data well. For example, we obtain $\Gamma_\rho = 142$ MeV and $m_a = 1.20$ GeV. It is necessary to point out that Weinberg's first sum rule is satisfied analytically in this effective chiral theory. Equation (18) are satisfied too. Therefore, all the parameters in the Lagrangians ($\mathcal{L}^{V,A}$) are fixed.

Besides the Lagrangians (3) and (10), appropriate meson (pseudoscalar, vector, and axial-vector) vertices are needed in studying τ mesonic decays and all these vertices can be derived from the Lagrangian (19) and they are fixed [12]. Most of these vertices have been tested by calculating appropriate decay widths and the results agree with the data well. Therefore, the Lagrangians of the weak interactions of mesons are completely determined. There are no other undetermined parameters in studying τ mesonic decays. This effective theory makes definite predictions for τ mesonic decays.

IV. a_1 DOMINANCE IN $\tau \to \pi\pi\pi\nu$

The a_1 meson has a long history. In determining the parameters of this meson the process of a_1 production in τ decays play an important role [18,19]. In Ref. [19] the flux tube quark model has been exploited to study $\tau \to \pi\pi\pi\nu$. The decay rate of $\tau \to a_1\nu$ has been calculated by the effective chiral theory [12]. However, the effect of wide resonance should be taken into account. On the other hand, experimental observations [20–25] have reported a_1 dominance in $\tau \to 3\pi\nu$.

Only the axial-vector part of the Lagrangian, $\mathcal{L}^A$, takes part in this process. There are five diagrams contributing to this decay: a_1 couples to $\rho\pi$, the W boson couples to $\rho\pi$ directly, π couples to $\rho\pi$, a_1 directly couples to three pions, and π directly couples to three pions. The study done in Ref. [12] indicates that the contribution of the four π couplings to $\pi\pi$ scattering is smaller than the contribution of ρ exchange by two orders of magnitude. From Ref. [12] it is learned that the contribution of the vertex of $a_1\pi\pi\pi$ to a_1 decay is very small and this result agrees with the data. Therefore, we omit the contributions of both the vertices of the four π and the $a_1\pi\pi\pi$. The remaining three diagrams indicate the existence of ρ resonances in the final states of $\tau \to 3\pi\nu$ and this result is in agreement with the data [25,26]. From these three vertices $a_1\rho\pi$, $W\rho\pi$, and $\pi\rho\pi$ ($\rho\pi\pi$ is included too), it is not obvious why a_1 dominates this decay. This is a crucial test of this theory. The vertices derived in Ref. [12] contribute to $\tau \to \pi\pi\pi\nu$:

$$\mathcal{L}^{a_1\rho\pi} = \epsilon_{ijk}\{Aa^i_\mu \rho^{j\mu}\pi^k - Ba^i_\mu \rho^j_\nu \partial^{\mu\nu}\pi^k + Da^i_\mu \partial^\mu(\rho^j_\nu \partial^\nu \pi^k)\}, \tag{26}$$

$$A = \frac{2}{f_\pi} g f_a \left\{ \frac{m_a^2}{g^2 f_a^2} - m_\rho^2 + p^2\left[\frac{2c}{g} + \frac{3}{4\pi^2 g^2}\left(1 - \frac{2c}{g}\right)\right] + q^2\left[\frac{1}{2\pi^2 g^2} - \frac{2c}{g} - \frac{3}{4\pi^2 g^2}\left(1 - \frac{2c}{g}\right)\right]\right\}, \tag{27}$$

$$c = \frac{f_\pi^2}{2 g m_\rho^2}, \tag{28}$$

$$B = -\frac{2}{f_\pi} g f_a \frac{1}{2\pi^2 g^2}\left(1 - \frac{2c}{g}\right), \tag{29}$$

$$D = -\frac{2}{f_\pi} f_a \left\{ 2c + \frac{3}{2\pi^2 g}\left(1 - \frac{2c}{g}\right)\right\}, \tag{30}$$

$$\mathcal{L}^{\rho\pi\pi} = \frac{2}{g}\epsilon_{ijk}\rho^i_\mu \pi^j \partial^\mu \pi^k - \frac{2}{\pi^2 f_\pi^2 g}\left\{\left(1 - \frac{2c}{g}\right)^2 - 4\pi^2 c^2\right\} \times \epsilon_{ijk}\rho^i_\mu \partial_\nu \pi^j \partial^{\mu\nu}\pi^k - \frac{1}{\pi^2 f_\pi^2 g} \times \left\{3\left(1 - \frac{2c}{g}\right)^2 + 1 - \frac{2c}{g} - 8\pi^2 c^2\right\}\epsilon_{ijk}\rho^i_\mu \pi_j \partial^2 \partial_\mu \pi_k, \tag{31}$$

where p is the momentum of the ρ meson and q is the momentum of a_1. Because the mesons of the vertices are not necessary to be on mass shell, in Eqs. (26) and (31) the divergence of a_μ and $\partial^2\pi_k$ are kept. In the chiral limit, these new terms do not contribute to the decays of ρ or a_1; however, they are important in keeping the axial-vector current conserved in $\tau \to 3\pi\nu$ in the chiral limit. The vertex $\mathcal{L}^{W\rho\pi}$ is derived by using the substitution (6) in Eq. (26).

The two pions in $\tau \to 3\pi\nu$ are from the decays of the ρ meson; therefore, we only need to show that the axial-vector current is conserved (in the limit of $m_q \to 0$) in $\tau \to \rho\pi\nu$, and then this conservation is satisfied in $\tau \to 3\pi\nu$.

Using $\mathcal{L}^A$, Eq. (10), and three vertices $\mathcal{L}^{a_1\rho\pi}$, Eq. (26), $\mathcal{L}^{W\rho\pi}$, and $\mathcal{L}^{\rho\pi\pi}$, Eq. (31), the matrix element of the axial-vector current of $\tau^- \to \rho^0\pi^-$ is obtained as

$$\begin{aligned}\langle \rho^0\pi^-|\overline{\psi\tau_+}\gamma_\mu\gamma_5\psi|0\rangle = \frac{i}{\sqrt{4\omega E}}\Bigg(& \frac{1}{f_a(q^2 - m_a^2)}(q_\mu q_\nu - q^2 g_{\mu\nu})(Ag_{\lambda\nu} + Bk_\lambda k_\nu)\epsilon^{*\lambda}_\sigma \\ & - \frac{\Delta m^2 f_a}{q^2 - m_a^2}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)(Ag_{\lambda\nu} + Bk_\lambda k_\nu)\epsilon^{*\lambda}_\sigma - \frac{\Delta m^2 f_a}{m_a^2}\frac{q_\mu}{q^2}(A + k\cdot qB)k\cdot\epsilon^*_\sigma + \frac{1}{f_a}(Ag_{\mu\nu} + Bk_\mu k_\nu)\epsilon^{*\nu}_\sigma \\ & - \left(\frac{1}{f_a} - \frac{\Delta m^2 f_a}{m_a^2}\right)Dk\cdot\epsilon^*_\sigma q_\mu - \frac{4f_\pi}{g}\frac{q_\mu}{q^2}\left\{1 + \frac{p^2}{2\pi^2 f_\pi^2}\left[\left(1 - \frac{2c}{g}\right)^2 - 4\pi^2 c^2\right] \right. \\ & \left. + \frac{q^2}{2\pi^2 f_\pi^2}\left(1 - \frac{2c}{g}\right)\left(1 - \frac{c}{g}\right)\right\}k\cdot\epsilon^*_\sigma\Bigg),\end{aligned} \tag{32}$$

where k, p, and q are the momenta of the pion, ρ, and a_1, respectively. In the effective Lagrangian of mesons [12] derived from the Lagrangian (19) there is mass term of the a_1 meson; therefore, the propagator of a_1 field is taken to be

$$\frac{i}{(2\pi)^4}\frac{1}{q^2-m_a^2}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_a^2}\right). \tag{33}$$

There are cancellations in Eq. (32). Using Eqs. (24), (25), and (27)–(30), it is proved that

$$-\frac{\Delta m^2 f_a}{m_a^2}(A+k\cdot qB)+\frac{1}{f_a}(A+Bk\cdot q)-\left(\frac{1}{f_a}-\frac{\Delta m^2 f_a}{m_a^2}\right)Dq^2-\frac{4f_\pi}{g}\left\{1+\frac{p^2}{2\pi^2 f_\pi^2}\left[\left(1-\frac{2c}{g}\right)^2-4\pi^2c^2\right]\right.$$
$$\left.+\frac{q^2}{2\pi^2 f_\pi^2}\left(1-\frac{2c}{g}\right)\left(1-\frac{c}{g}\right)\right\}=0. \tag{34}$$

Equation (34) leads to the conservation of axial-vector current in the limit of $m_q=0$:

$$q^\mu\langle\rho^0\pi^-|\overline{\psi}\tau_+\gamma_\mu\gamma_5\psi|0\rangle=0.$$

From this discussion it is learned that in order to have the axial-vector conserved (in the limit of $m_q=0$) the new term (9) is necessary to be included in Eq. (10). Using Eq. (34), the matrix element (32) is rewritten as

$$\langle\rho^0\pi^-|\overline{\psi}\tau_-\gamma_\mu\gamma_5\psi|0\rangle=\frac{i}{\sqrt{4\omega E}}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)(Ag_{\nu\lambda}+Bk_\nu k_\lambda)\epsilon_\sigma^{*\nu}\frac{-\Delta m^2 f_a+q^2 f_a^{-1}}{q^2-m_a^2+i\sqrt{q^2}\Gamma_a(q^2)}$$
$$+\frac{i}{\sqrt{4\omega E}}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)\left(-\frac{1}{f_a}\right)(Ag_{\nu\lambda}+Bk_\nu k_\lambda)\epsilon_\sigma^{*\nu}$$
$$=\frac{i}{\sqrt{4\omega E}}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)(Ag_{\nu\lambda}+Bk_\nu k_\lambda)\epsilon_\sigma^{*\nu}\frac{g^2 f_a m_\rho^2-if_a^{-1}\sqrt{q^2}\Gamma_a(q^2)}{q^2-m_a^2+i\sqrt{q^2}\Gamma_a(q^2)}. \tag{35}$$

It is necessary to point out that the Breit-Wigner formula of the axial-vector meson a_1 is new and is different from the one of the vector meson (see Sec. VII). This difference is caused by dynamical chiral symmetry breaking. It is also important to notice that the amplitude A, Eq. (27), derived in Ref. [12] strongly depends on the momentum. Because of the cancellation in Eq. (27), this dependence is significant.

The decay width of a_1 meson has been introduced. The a_1 dominance in $\tau\to\rho\pi\nu$ is revealed and the dominance is caused by the cancellation (34) which leads to the axial-vector current conservation. Using the vertex $\mathcal{L}^{\rho\pi\pi}$, Eq. (31), the matrix element is derived:

$$\langle\pi^+\pi^-\pi^-|\overline{\psi}\tau_+\gamma_\mu\gamma_5\psi|0\rangle=\frac{i}{\sqrt{8\omega_1\omega_2\omega_3}}\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{q^2}\right)\frac{g^2 f_a m_\rho^2-i\sqrt{q^2}f_a^{-1}\Gamma_a(q^2)}{q^2-m_a^2+i\sqrt{q^2}\Gamma_a(q^2)}\left\{\frac{f_{\rho\pi\pi}(k^2)}{k^2-m_\rho^2+ik\Gamma_\rho(k^2)}[A(k^2)(k_2-k_3)_\nu\right.$$
$$\left.+Bk_{1\nu}k_1\cdot(k_2-k_3)]+\frac{f_{\rho\pi\pi}(k'^2)}{k'^2-m_\rho^2+ik'\Gamma_\rho(k'^2)}[A(k'^2)(k_1-k_3)_\nu+Bk_{2\nu}k_2\cdot(k_1-k_3)]\right\}, \tag{36}$$

$$f_{\rho\pi\pi}(k^2)=\frac{2}{g}\left\{1+\frac{k^2}{2\pi^2 f_\pi^2}\left[\left(1-\frac{2c}{g}\right)^2-4\pi^2c^2\right]\right\},$$

$$\Gamma_\rho(k^2)=\frac{f_{\rho\pi\pi}^2}{48\pi}\frac{k^2}{m_\rho}\left(1-4\frac{m_\pi^2}{k^2}\right)^{3/2}, \tag{37}$$

where k_i $(i=1,2,3)$ are the momenta of π^-, π^-, and π^+, $k=k_2+k_3$, $k'=k_1+k_3$, $q=k_1+k_2+k_3$. $A(k^2)$ $[A(k'^2)]$ are obtained by taking $p^2=k^2$ $[k'^2]$ in Eq. (27) and Γ_a is defined in Eq. (39). The distribution of the decay width of $\tau^-\to\pi^-\pi^-\pi^+\nu$ is derived:

$$\frac{d\Gamma}{dq^2dk^2dk'^2}=\frac{G^2}{(2\pi)^5}\frac{\cos^2\theta_C}{3072m_\tau^3q^6}(m_\tau^2-q^2)^2(m_\tau^2+2q^2)\frac{g^4f_a^2m_\rho^4+q^2f_a^{-2}\Gamma_a^2(q^2)}{(q^2-m_a^2)^2+q^2\Gamma_a^2(q^2)}F(k^2,k'^2),$$

$$G(k^2,k'^2)=\left\{\frac{f_{\rho\pi\pi}(k^2)}{(k^2-m_\rho^2)^2+k^2\Gamma_\rho^2(k^2)}(k^2-m_\rho^2)[A(k^2)+k_1\cdot(k_2-k_3)B]+\frac{f_{\rho\pi\pi}(k'^2)}{(k'^2-m_\rho^2)^2+k'^2\Gamma_\rho^2(k'^2)}(k'^2-m_\rho^2)2A(k'^2)\right\}^2$$

$$+\left\{\frac{f_{\rho\pi\pi}(k^2)}{(k^2-m_\rho^2)^2+k^2\Gamma_\rho^2(k^2)}\sqrt{k^2}\Gamma_\rho(k^2)[A(k^2)+k_1\cdot(k_2-k_3)B]\right.$$
$$\left.+\frac{f_{\rho\pi\pi}(k'^2)}{(k'^2-m_\rho^2)^2+k'^2\Gamma_\rho^2(k'^2)}\sqrt{k'^2}\Gamma_\rho(k'^2)2A(k'^2)\right\}^2,$$

$$F(k^2,k'^2)=\tfrac{1}{2}\{G(k^2,k'^2)(q^2-k^2)^2+G(k'^2,k^2)(q^2-k'^2)^2\}. \tag{38}$$

Γ_a is derived from the vertices (26) and (31):

$$\Gamma_a(q^2)=\frac{1}{192(2\pi)^3 m_a q^4}\int dq_1^2 dq_2^2(q^2-q_1^2)^2\left\{\frac{f_{\rho\pi\pi}(q_1^2)}{(q_1^2-m_\rho^2)^2+q_1^2\Gamma_\rho(q_1^2)}(q_1^2-m_\rho^2)[A(q_1^2)+\tfrac{1}{2}(q_3^2-q_2^2)B]\right.$$
$$\left.+\frac{f_{\rho\pi\pi}(q_2^2)}{(q_2^2-m_\rho^2)^2+q_2^2\Gamma_\rho(q_2^2)}(q_2^2-m_\rho^2)2A(q_2^2)\right\}^2+\left\{\frac{f_{\rho\pi\pi}(q_1^2)}{(q_1^2-m_\rho^2)^2+q_1^2\Gamma_\rho(q_1^2)}\sqrt{q_1^2}\Gamma_\rho(q_1^2)[A(q_1^2)+\tfrac{1}{2}(q_3^2-q_2^2)B]\right.$$
$$\left.+\frac{f_{\rho\pi\pi}(q_2^2)}{(q_2^2-m_\rho^2)^2+q_2^2\Gamma_\rho(q_2^2)}\sqrt{q_2^2}\Gamma_\rho(q_2^2)2A(q_2^2)\right\}^2, \tag{39}$$

where $q_1^2=(q-k_1)^2$, $q_2^2=(q-k_2)^2$, $q_3^2=(q-k_3)^2$, and k_1, k_2, and k_3 are momentum of the three pions, respectively. There are two decay modes $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ which have equal branching ratios. The branching ratios are computed to be

$$B(\tau\rightarrow\pi^+\pi^-\pi^-\nu)=B(\tau\rightarrow\pi^-\pi^0\pi^0\nu)=6.3\%. \tag{40}$$

The comparison with experiments is presented in Table I. The distribution of $d\Gamma(\tau\rightarrow\pi^+\pi^-\pi^-\nu)/dq$ is shown in Fig. 1. From Fig. 1 the decay width is determined to be

$$\Gamma_a=386 \text{ MeV}. \tag{41}$$

The data are $\sim$400 MeV [27]. The comparison with experiments using the model [19] is presented in Table II. The starred results are taken from [19].

TABLE I. Branching ratios.

Experiment [24]	$B(2h^-h^+\nu)$%	$B(h^-2\pi^0\nu)$%
New W.A.	9.26 ± 0.26	9.21 ± 0.14
DELPHI(92-95)	$8.69\pm0.12\pm0.16$	$9.22\pm0.43\pm0.20$
ALEPH(89-93)	$9.46\pm0.10\pm0.11$	$9.32\pm0.13\pm0.10$
CELLO(90)		$9.1\pm1.3\pm0.9$
OPAL(91-94)	$9.83\pm0.10\pm0.24$	
L3(92)		$8.88\pm0.37\pm0.42$
CLEO(93)	8.7 ± 0.8	$8.96\pm0.16\pm0.449$
CLEO(95)	$9.47\pm0.07\pm0.20$	
CBALL(91) [44]		$5.7\pm0.5\pm1.4$
ARGUS(93)	$7.3\pm0.1\pm0.5$	
MAC(87)		$8.7\pm0.4\pm0.11$
BES [14]	$7.3\pm0.5(\pi^+\pi^-\pi^-)$	
Taula 2.4	7.0 ± 2.8	6.4 ± 2.8
This study	$6.3(\pi^+\pi^-\pi^-)$	$6.3(\pi^-\pi^0\pi^0)$

V. a_1 DOMINANCE IN $\tau\rightarrow f_1(1285)\pi\nu$

The f_1 meson is the chiral partner of the ω meson [12] and the mass formula of the f_1 meson is derived in Ref. [12]:

$$\left(1-\frac{1}{2\pi^2g^2}\right)m_{f_1}^2=6m^2+m_\omega^2, \quad m_{f_1}=1.21 \text{ GeV}. \tag{42}$$

The vertex of $f_1(1285)a_1\pi$ is presented in Ref. [12] (a factor of -4 has been lost),

$$\mathcal{L}^{f_1a_1\pi}=\frac{1}{\pi^2f_\pi}\frac{1}{g^2}\left(1-\frac{1}{2\pi^2g^2}\right)^{-1}\varepsilon^{\mu\nu\alpha\beta}f_\mu\partial_\nu\pi^i\partial_\alpha a_\beta^i. \tag{43}$$

The narrow width of the decay $f_1\rightarrow\rho\pi\pi$ is revealed from this vertex [12]. Using the substitution (6), the vertex $\mathcal{L}^{Wf_1\pi}$ is derived. The vertex $\mathcal{L}^{f_1a_1\pi}$ has abnormal parity; hence, it belongs to the WZW anomaly. Therefore, the WZW anomaly can be tested in τ mesonic decay. Only the

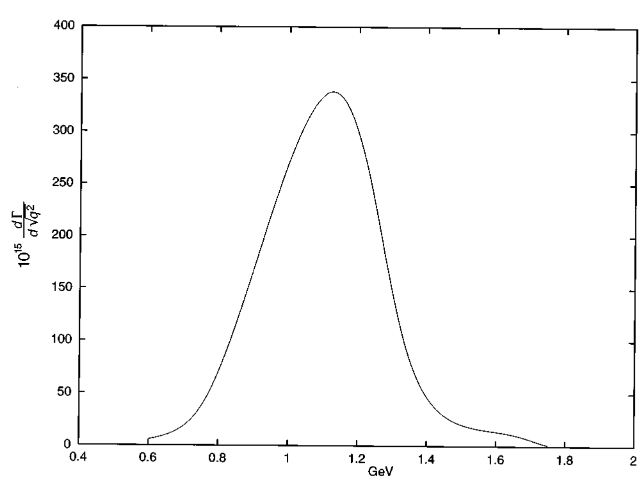

FIG. 1. Distribution of the decay rate of $\tau\rightarrow3\pi\nu$ vs the invariant mass of three pions.

TABLE II. Parameters of the a_1 meson.

Experiment	m_{a_1}(GeV)	Γ_{a_1}(GeV)
ARGUS [25]	1.211 ± 0.007	0.446 ± 0.021
DELCO*	1.180 ± 0.060	0.430 ± 0.190
Mark II*	1.250 ± 0.050	0.580 ± 0.100
ARGUS*	1.213 ± 0.011	0.434 ± 0.030
This study	1.20	0.386

axial-vector part of the weak interaction contributes to the decay $\tau\rightarrow f_1\pi\nu$. Using the Lagrangian $\mathcal{L}^A$, Eq. (10), it is obtained:

$$\langle f_1\pi|\overline{\psi}\tau_+\gamma_\mu\gamma_5\psi|0\rangle = -\frac{1}{\sqrt{4\omega E}}\frac{1}{\pi^2 f_\pi g^2}\left(1-\frac{1}{2\pi^2 g^2}\right)^{-1} \times\frac{g^2 f_a m_\rho^2 - iqf_a^{-1}\Gamma_a(q^2)}{q^2-m_a^2+iq\Gamma_a(q^2)}\varepsilon^{\mu\nu\alpha\beta}k_\nu q_\alpha\epsilon_\beta^{*\sigma}, \tag{44}$$

where k is the momentum of the pion and $q=p+k$; p is the momentum of the f_1 meson.

The decay width is derived:

$$\Gamma=\frac{G^2}{(2\pi)^3}\frac{\cos^2\theta_C}{128m_\tau^3}\int dq^2\frac{1}{q^4}(m_\tau^2-q^2)^2(m_\tau^2+2q^2) \times(q^2-m_f^2)^3\frac{f_a^4}{\pi^4 f_\pi^2}\frac{g^4 f_a^2 m_\rho^4+q^2 f_a^{-2}\Gamma_a^2(q^2)}{(q^2-m_a^2)^2+q^2\Gamma_a^2(q^2)}. \tag{45}$$

a_1 is dominant in this decay. The theoretical prediction of the branching ratio of this decay is

$$B(\tau\rightarrow f_1\pi\nu)=2.91\times10^{-4}. \tag{46}$$

The data are $(6.7\pm1.4\pm2.2)\times10^{-4}$ [28]. Two factors result in the small branching ratio. A small phase space is the first factor and the second factor is the anomalous coupling. The effective theory proposed in Ref. [12] is a theory at low energies; therefore, the derivative expansion is exploited. In this theory the anomalous couplings are at the fourth order in derivatives. Comparing with the couplings at the second order in derivatives, the anomalous couplings are weaker. This is the reason why the widths of ρ and a_1 are broader (the two vertices are at the second order in the derivative expansion) and ω and $f_1\rightarrow\rho\pi\pi$ are narrower.

The distribution of the invariant mass of $f_1\pi$ is shown in Fig. 2. The peak of the distribution results from both the effects of the threshold and the a_1 resonance.

The experimental measurement of this decay is a test of the Wess-Zumino-Witten anomaly and the mechanism proposed in this paper.

VI. $\tau\rightarrow K^*(892)K\nu$

The processes $\tau\rightarrow KK\pi\nu$ have been studied by many authors. The earlist study is done by using a chiral Lagrangian [3]. In Ref. [17] a chiral Lagrangian and three resonances

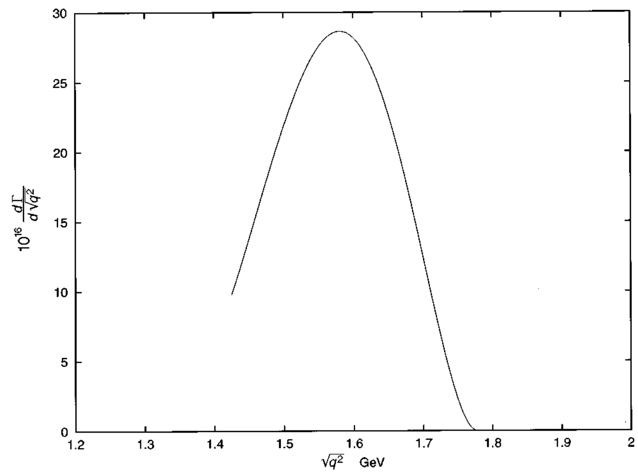

FIG. 2. Distribution of the decay rate of $\tau\rightarrow f\pi\nu$ vs the invariant mass of $f\pi$.

ρ, ρ', and ρ'' are used. In Ref. [4] the ρ meson field is introduced to a chiral Lagrangian. In Ref. [5] a comprehensive resonance model including vector and axial-vector resonances has been exploited. In this paper the process $\tau\rightarrow K^*K\nu$ is studied in terms of the same formulas (5) and (10) used to study $\tau\rightarrow3\pi\nu$ and $f_1\pi\nu$.

Both the vector and the axial-vector currents of the weak interactions contribute to the decay $\tau^-\rightarrow(K^*K)^-\nu$. The vector part comes from the anomaly and the vertex is presented in Ref. [12]:

$$\mathcal{L}^{K^*K\rho}=-\frac{N_C}{\pi^2 g^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta}d_{abi}K_\mu^a\partial_\nu\rho_\alpha^i\partial_\beta K^b. \tag{47}$$

$\mathcal{L}^{K^*KW}$ can be derived by using the substitution (4). There are three vertices in the axial-vector part: πK^*K, a_1K^*K, and WK^*K. As mentioned above, the latter can be derived by using the substitution (6) in the vertex a_1K^*K. The vertex $\mathcal{L}^{\pi K^*K}$ is given in Ref. [12]:

$$\begin{aligned}\mathcal{L}^{\pi K^*K}=if_{K^*K\pi}(q^2)\Bigg\{&-\frac{1}{\sqrt{2}}K_\mu^0(\pi^+\partial^\mu K^- - K^-\partial^\mu\pi^+)\\ &-\frac{1}{\sqrt{2}}K_\mu^+(\pi^-\partial^\mu\overline{K^0}-\overline{K^0}\partial^\mu\pi^-)\\ &+\tfrac{1}{2}K_\mu^0(\pi^0\partial^\mu\overline{K^0}-\overline{K^0}\partial^\mu\pi^0)\\ &-\tfrac{1}{2}K_\mu^+(\pi^0\partial^\mu K^- - K^-\partial^\mu\pi^0)\Bigg\}+\text{H.c.},\end{aligned} \tag{48}$$

where q^2 is the momentum squared of K^*. In the limit of $m_q=0$, $f_{K^*K\pi}(q^2)$ is the same as $f_{\rho\pi\pi}(q^2)$, Eq. (37). The vertex a_1K^*K is derived from Ref. [12]:

$$\begin{aligned}\mathcal{L}^{a_1K^*K}=&f_{abi}K_\mu^a K^b a_\nu^i\{A(q^2)_{K^*}g_{\mu\nu}+Bk_\mu k_\nu\}\\ &-f_{abi}DK_\mu^a\partial^\mu K^b\partial^\nu a_\nu^i,\end{aligned} \tag{49}$$

where $A(q^2)_{K*}$ is obtained by replacing m_ρ^2 by m_{K*}^2 in Eq. (27). In the limit of $m_q=0$, B and D are the same as Eqs. (27), (29), and (30). The vector matrix element is obtained from $\mathcal{L}^V$, Eq. (3), and the vertex (47):

$$\langle K^- K^{*0}|\bar{\psi}\tau_+\gamma_\mu\psi|0\rangle = -\frac{1}{\sqrt{4\omega E}}\frac{N_C}{\sqrt{2}\pi^2 g f_\pi}\frac{m_\rho^2 - i\sqrt{q^2}\Gamma_\rho(q^2)}{q^2-m_\rho^2+i\sqrt{q^2}\Gamma_\rho(q^2)} \times \varepsilon^{\mu\nu\alpha\beta}\epsilon_\nu^{*\sigma} q_\alpha k_\beta, \tag{50}$$

where k is the momentum of the kaon and $q=p+k$; p is the momentum of K^*. Because of $q^2>4m_K^2$, the decay mode of $\rho\to K\bar{K}$ is open and we have

$$\Gamma(\rho\to\pi\pi) = \frac{f_{\rho\pi\pi}^2(q^2)}{48\pi}\frac{q^2}{m_\rho}\left(1-\frac{4m_\pi^2}{q^2}\right)^{3/2},$$

$$\Gamma(\rho\to K\bar{K}) = \frac{f_{\rho\pi\pi}^2(q^2)}{96\pi}\frac{q^2}{m_\rho}\left(1-\frac{4m_K^2}{q^2}\right)^{3/2},$$

$$\Gamma_\rho(q^2) = \Gamma(\rho\to\pi\pi)+\Gamma(\rho\to K\bar{K}). \tag{51}$$

The axial-vector matrix element is derived by using $\mathcal{L}^{K^*K\pi}$, Eq. (48), $\mathcal{L}^{a_1 K^* K}$, Eq. (49), $\mathcal{L}^{WK^*K}$, and $\mathcal{L}^A$, Eq. (10):

$$\langle K^- K^{*0}|\bar{\psi}\tau_-\gamma_\mu\gamma_5\psi|0\rangle = -\frac{i}{\sqrt{4\omega E}}\frac{1}{\sqrt{2}}\frac{g^2 f_a m_\rho^2 - i\sqrt{q^2} f_a^{-1}\Gamma_a(q^2)}{q^2-m_a^2+\sqrt{q^2}\Gamma_a(q^2)}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)[A(q^2)_{K*}g_{\nu\lambda}+Bk_\nu k_\lambda - Dk^\nu p^\lambda]\epsilon_\sigma^{*\lambda}. \tag{52}$$

The decay width is derived:

$$\frac{d\Gamma}{dq^2}(\tau^-\to K^{*0}K^-\nu) = \frac{G^2\cos^2\theta_C}{64m_\tau^3 q^4}\frac{1}{(2\pi)^3}[(q^2+m_{K*}^2-m_K^2)^2-4q^2m_{K*}^2]^{1/2}(m_\tau^2-q^2)^2(m_\tau^2+2q^2) \times\left\{\frac{3}{\pi^4 g^2 f_\pi^2}\frac{m_\rho^4+q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2+q^2\Gamma_\rho^2(q^2)}[(p\cdot q)^2-q^2m_{K*}^2] + \frac{1}{2}\frac{g^4 f_a^2 m_\rho^4 + f_a^{-2}q^2\Gamma_a^2(q^2)}{(q^2-m_a^2)^2+q^2\Gamma_a^2(q^2)}\left[A^2(q^2)_{K*}-\frac{1}{3}\frac{(q\cdot k)^2}{q^2}2A(q_{K*}^2 B - m_{K*}^2 D^2)\right]\right\}. \tag{53}$$

The distribution of $d\Gamma/dq$ is shown in Fig. 3. There is a peak located at 1.51 GeV which is caused by both threshold and resonance effects. The branching ratio is computed to be

$$B(\tau\to K^{*0}K^-\nu) = 0.392\%.$$

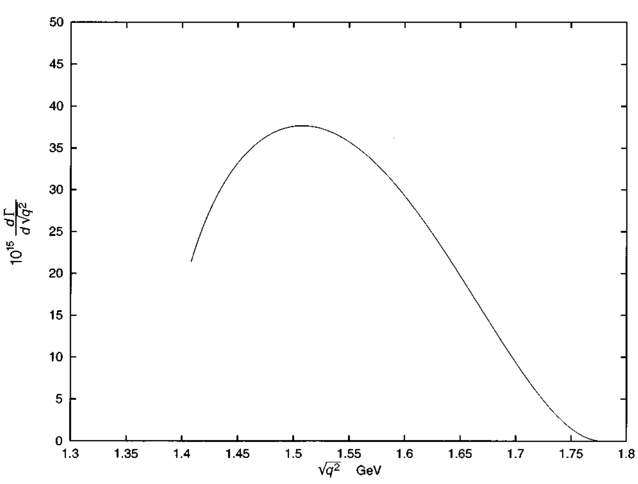

FIG. 3. Distribution of the decay rate of $\tau\to K^*K\nu$ vs the invariant mass of K^*K.

The calculation shows that the vector current is the dominant contributor and the contribution of the axial-vector current is only 7.5% of the decay rate. The data are CLEO [29], $(0.32\pm0.08\pm0.12)\%$, and ARGUS [30], $(0.20\pm0.05\pm0.04)\%$. The branching ratio of $\tau^-\to K^{*-}K^0\nu$ is the same as $\tau^-\to K^{*0}K^-\nu$.

VII. CVC AND $e^+e^-\to\pi\pi$

As discussed in Ref. [14], CVC works very well in both meson productions in e^+e^- annihilation and τ decays. As a test of the theory explored in this paper, the same theory [12] is used to study $e^+e^-\to\pi^+\pi^-$.

The expression of VMD [12] is

$$\frac{e}{2}g\{-\tfrac{1}{2}F^{\mu\nu}(\partial_\mu\rho_\nu^0-\partial_\nu\rho_\mu^0)+A^\mu j_\nu\}. \tag{54}$$

Using the substitution

$$\rho_\mu \to \frac{e}{2}gA_\mu,$$

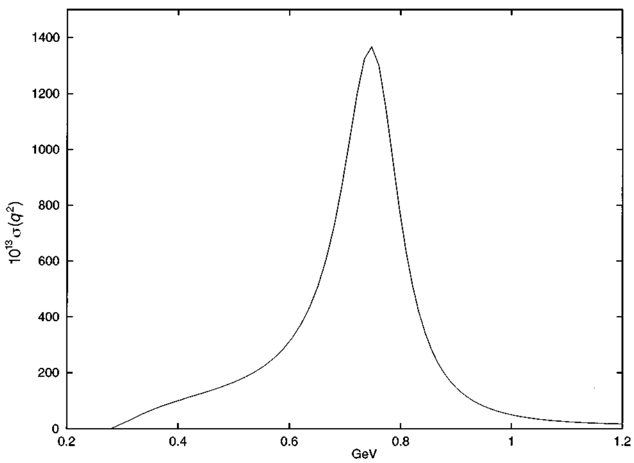

FIG. 4. Cross section of $e^+e^-\to\pi\pi$ vs the invariant mass of $\pi\pi$.

the current j_μ is obtained from $\mathcal{L}^{\rho\pi\pi}$, Eq. (31). There are two diagrams: The photon is coupled to $\pi\pi$ directly and the photon is via the ρ meson coupled to $\pi\pi$. The matrix element is derived:

$$\begin{aligned}&\langle\pi^+\pi^-|\overline{\psi}\tau_3\gamma_\mu\psi|0\rangle\\&=\frac{1}{\sqrt{4\omega_1\omega_2}}gf_{\rho\pi\pi}(q^2)\left\{1-\frac{q^2}{q^2-m_\rho^2+iq\Gamma_\rho(q^2)}\right\}\\&\quad\times(k_1-k_2)_\mu\\&=\frac{1}{\sqrt{4\omega_1\omega_2}}gf_{\rho\pi\pi}(q^2)\\&\quad\times\frac{-m_\rho^2+iq\Gamma_\rho(q^2)}{q^2-m_\rho^2+iq\Gamma_\rho(q^2)}(k_1-k_2)_\mu,\end{aligned}\tag{55}$$

where k_1 and k_2 are momenta of π^+ and π^-, respectively, and $q^2=(k_1+k_2)^2$. The cross section is found to be

$$\begin{aligned}\sigma=&\frac{\pi\alpha^2}{12}\frac{1}{q^2}\left(1-\frac{4m_\pi^2}{q^2}\right)^{3/2}g^2f^2_{\rho\pi\pi}(q^2)\\&\times\frac{m_\rho^4+q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2+q^2\Gamma_\rho^2(q^2)}.\end{aligned}\tag{56}$$

The numerical results are shown in Fig. 4. Theoretical results are in good agreement with the data [31]. Systematic study of meson production in e^+e^- collisions will be presented somewhere else.

The pion form factor is found from Eq. (56):

$$|F(q^2)|^2=\left(\frac{g}{2}f_{\rho\pi\pi}(q^2)\right)^2\frac{m_\rho^4+q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2+q^2\Gamma_\rho^2(q^2)}.$$

The new point in this study is that the coupling $f_{\rho\pi\pi}$, Eq. (37), is a function of q^2. As a matter of fact, $f_{\rho\pi\pi}(q^2)$ is the form factor of the vertex $\rho\pi\pi$. The chiral theory of mesons presented in Ref. [12] is a theory at low energies (the energy scale Λ is determined to be 1.6 GeV [12]) and covariant derivative expansion is exploited. In Eq. (37) part of the q^2 dependence of $f_{\rho\pi\pi}$, $(q^2/2\pi^2f_\pi^2)(1-2c/g)^2$, comes from the fourth order in derivatives and $(q^2/2\pi^2f_\pi^2)(-4\pi^2c^2)$ comes from

$$-\tfrac{1}{8}\mathrm{Tr}\rho^{\mu\nu}\rho_{\mu\nu},$$

where $\rho_{\mu\nu}$ is the strength of the non-Abelian ρ field [12]. The radius of the charged pion is derived from the pion form factor in the spacelike region of q^2:

$$\langle r^2\rangle_\pi=\frac{6}{m_\rho^2}+\frac{3}{\pi^2f_\pi^2}\left\{\left(1-\frac{2c}{g}\right)^2-4\pi^2c^2\right\}.$$

In Ref. [12] $g=0.35$ is chosen and the last two terms are canceled out. In this paper we choose $g=0.39$ to have better fits and

$$\langle r^2\rangle_\pi=(0.393+0.0549)\ \mathrm{fm}^2=0.447\ \mathrm{fm}^2.$$

The first number is from the ρ pole and the second comes from the form factor $f_{\rho\pi\pi}$ and it is 12.2% of the total value. The data [32] are (0.44 ± 0.01) fm^2.

It is necessary to point out that the new expression of the pion form factor still results in VMD and the $\rho\pi\pi$ coupling constant is substituted by the form factor of $\rho\pi\pi$. On the other hand, the $\rho\pi\pi$ form factor increases the value of the pion radius.

In the chiral limit, the form factors of the vertices $\rho K\overline{K}$, ρK^*K, $K^*K\pi$, and $K^*K\eta$ are the same. These form factors result in physical effects in corresponding τ decays.

VIII. $\tau\to\omega\pi\nu$

This process has been studied in Ref. [8] by using the abnormal vertex $\rho\omega\pi$. The effects of excited ρ mesons have been taken into account. In this paper we only take the contribution of the ρ meson. The $\omega\rho\pi$ vertex is presented in Ref. [12]:

$$\mathcal{L}^{\omega\rho\pi}=-\frac{N_C}{\pi^2g^2f_\pi}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\omega_\nu\rho^i_\alpha\partial_\beta\pi^i.$$

The vertices of $\pi^0\gamma\gamma$, $\omega\pi\gamma$, $\rho\pi\gamma$, and $\omega3\pi$ are via the VMD derived from this vertex and theoretical results agree with the data well. The coupling constant of this $\omega\rho\pi$ vertex is different from the one presented in Ref. [8]. The decay width of $\tau\to\omega\pi\nu$ is derived:

$$\begin{aligned}\Gamma=&\frac{G^2}{128m_\tau^3}\frac{\cos^2\theta_C}{(2\pi)^3}\int dq^2\frac{1}{q^4}(m_\tau^2-q^2)^2(m_\tau^2+2q^2)\\&\times(q^2-m_\omega^2)^3\frac{3}{\pi^4g^2f_\pi^2}\frac{m_\rho^4+q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2+q^2\Gamma_\rho^2(q^2)}.\end{aligned}\tag{57}$$

The numerical result of the branching ratio is

$$B=1.2\%,$$

and the experiment is $(1.6\pm0.5)\%$ [27]. This result is the same as the one obtained in Ref. [8] when only the ρ meson is taken. The distribution of the invariant mass of ω and π is shown in Fig. 5.

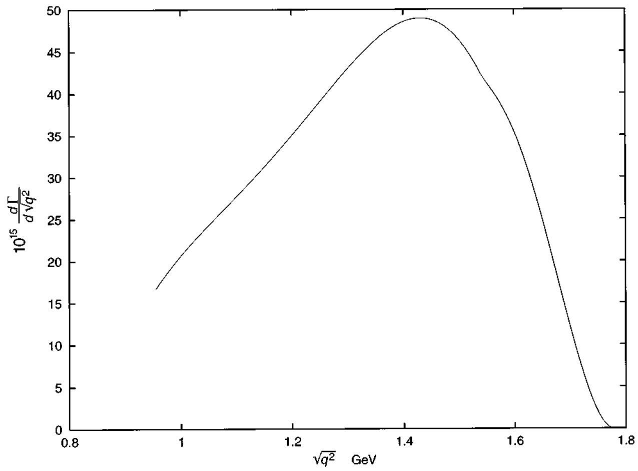

FIG. 5. Distribution of the decay rate of $\tau\to\omega\pi\nu$ vs the invariant mass of $\omega\pi$.

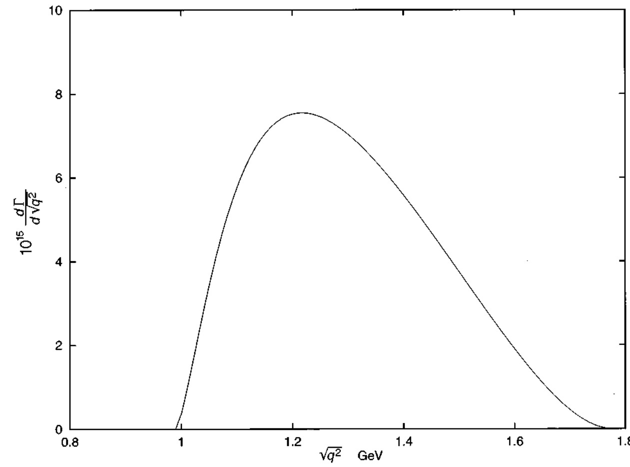

FIG. 6. Distribution of the decay rate of $\tau\to K\overline{K}\nu$ vs the invariant mass of KK.

IX. $\tau\to K\overline{K}\nu$

This process has been studied by several groups [33]. The a_1 field does not couple to KK. The vertex a_1KK in which the tensor $\varepsilon^{\mu\nu\alpha\beta}$ must be involved cannot be constructed. Therefore, only the vector current contributes to this decay. The vertex $\mathcal{L}^{vKK}$ has been used to calculate the electric form factors of charged kaons and neutral kaons and theoretical predictions are in good agreements with the data [12]. We use the same vertex (only the isovector part) to calculate the decay rate of $\tau\to KK\nu$. This is a test of CVC. The related vertex is presented in Ref. [12]:

$$\mathcal{L}^{\rho K\overline{K}}=\frac{i}{\sqrt{2}}f_{\rho KK}\{\rho^-_\mu(K^+\partial^\mu\overline{K^0}-\overline{K^0}\partial^\mu K^+)-\rho^+_\mu(K^-\partial^\mu K^0-K^0\partial^\mu K^-)\},\tag{58}$$

where $f_{\rho KK}$ is the same as the $f_{\rho\pi\pi}$, Eq. (37), in the limit of $m_q=0$. By using VMD, Eq. (3), and the vertex (58), the matrix element is derived:

$$\langle K^-K^0|\overline{\psi}\tau_+\gamma_\mu\psi|0\rangle=\frac{1}{\sqrt{4\omega_1\omega_2}}\frac{1}{\sqrt{2}}(k_1-k_2)_\mu g f_{\rho\pi\pi}(q^2)\times\frac{-m_\rho^2+iq\Gamma_\rho(q^2)}{q^2-m_\rho^2+iq\Gamma_\rho(q^2)},\tag{59}$$

where k_1 and k_2 are momenta of two kaons, respectively, and $q=k_1+k_2$. Using this matrix element, the decay width is obtained:

$$\frac{d\Gamma}{dq^2}(\tau\to K^0\overline{K^-}\nu)=\frac{G^2}{(2\pi)^3}\frac{\cos^2\theta_C}{384m_\tau^3}(m_\tau^2-q^2)^2(m_\tau^2+2q^2)\times\left(1-\frac{4m_K^2}{q^2}\right)^{3/2}g^2f^2_{\rho\pi\pi}(q^2)\frac{m_\rho^4+q^2\Gamma_\rho^2(q^2)}{(q^2-m_\rho^2)^2+q^2\Gamma_\rho^2(q^2)}.\tag{60}$$

The branching ratio is computed to be

$$B=0.27\%,$$

and the data are TPC/2γ [34], <0.26; ALEPH [35], $0.26\pm0.09\pm0.02$; CLEO [36], $0.151\pm0.021\pm0.022$. The distribution of the decay rate is shown in Fig. 6.

Because of the effects of the threshold and the ρ resonance, there is a peak in the distribution and it is positioned at 1.17 GeV.

X. FORM FACTORS OF $\pi^-\to e\gamma\nu$

As a test of $\mathcal{L}^{V,A}$, Eqs. (3) and (10), we study the vector and the axial-vector form factors of $\pi^-\to e\gamma\nu$. In $\pi^-\to\gamma e\nu$ there is inner bremsstrahlung from the lepton and meson and structure-dependent term [27]. The form factors of the structure-dependent term have been studied [27].

The structure-dependent form factors are defined as [27]

$$M^V_{\rm SD}+M^A_{\rm SD}=e\frac{G}{\sqrt{2}}\cos\theta_C m_\pi\epsilon^*_\nu\overline{\nu}\gamma_\mu(1-\gamma_5)e\times\{F^V\varepsilon^{\mu\nu\alpha\beta}p_{\pi\alpha}k_\beta+iF^A[g^{\mu\nu}k\cdot p_\pi-k^\mu p^\nu_\pi]+iRtg^{\mu\nu}\},\tag{61}$$

where k and p_π are momenta of the photon and pion, respectively, and $t=k^2$. It is known that F^V is via CVC determined by the amplitude of $\pi^0\to2\gamma$. In this theory the amplitude of $\pi^0\to2\gamma$ obtained by a triangle anomaly is obtained by combining the vertex $\mathcal{L}^{\omega\rho\pi}$ and VMD. Using L^V, Eq. (3), $\mathcal{L}^{\omega\rho\pi}$, and VMD, it is obtained:

$$F^V=\frac{m_\pi}{2\sqrt{2}\pi^2f_\pi}=0.0268.\tag{62}$$

The experiments [27] are 0.014 ± 0.009, $0.023^{+0.015}_{-0.013}$

The form factors F^A and R are determined by calculating the matrix element $\langle\gamma|\overline{\psi}\tau_-\gamma_\mu\gamma_5\psi|\pi^+\rangle$ which is via VMD found from $\langle\rho^0|\overline{\psi}\tau_-\gamma_\mu\gamma_5\psi|\pi^+\rangle$, Eq. (35). However, for pion weak radiative decay besides the axial-vector current conservation (in the limit $m_q=0$) the electric current is conserved too. In order to satisfy the electric current conservation, the divergence of the ρ field which is ignored in the vertex $\mathcal{L}^{a_1\rho\pi}$, Eq. (26), must be kept and the term derived from the effective Lagrangian of Ref. [12] is

$$-D\epsilon_{ijk}a^i_\mu\partial^\mu\pi^j\partial^\nu\rho^k_\nu, \tag{63}$$

where D is given in Eq. (30) Adding the term (63) to the matrix element (35) we obtain

$$\langle\gamma|\overline{\psi}\tau_-\gamma_\mu\gamma_5\psi|\pi^+\rangle=\frac{ie}{\sqrt{4\omega_\pi\omega_\gamma}}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)\times\{Ag_{\lambda\nu}+Bp_{\pi\nu}p_{\pi\lambda}+Dk_\nu k_\lambda\}\times\epsilon^{*\lambda}_\sigma\frac{1}{2}g^3f_a\frac{1}{q^2-m_a^2}\frac{m_\rho^2}{m_\rho^2-k^2}, \tag{64}$$

where $q=p_\pi-k$. Using the expressions of A, Eq. (27), B, Eq. (29), and D, Eq. (30), it is proved that

$$A+k\cdot p_\pi B+Dk^2=0. \tag{65}$$

Equation (65) guarantees electric current conservation. Ignoring q^2 and k^2, the two form factors are found:

$$F^A=\frac{1}{2\sqrt{2}\pi^2}\frac{m_\pi}{f_\pi}\frac{m_\rho^2}{m_a^2}\left(1-\frac{2c}{g}\right)\left(1-\frac{1}{2\pi^2g^2}\right)^{-1}=0.0102, \tag{66}$$

$$R=\frac{g^2}{2\sqrt{2}}\frac{m_\pi}{f_\pi}\frac{m_\rho^2}{m_a^2}\left\{\frac{2c}{g}+\frac{1}{\pi^2g^2}\left(1-\frac{2c}{g}\right)\right\}\left(1-\frac{1}{2\pi^2g^2}\right)^{-1}. \tag{67}$$

The experimental values [27] of F^A are 0.0106 ± 0.006, 0.0135 ± 0.0016, and 0.011 ± 0.003.

XI. EFFECTIVE LAGRANGIAN OF $\Delta s=1$ WEAK INTERACTIONS

It is natural to generalize the expressions of $\mathcal{L}^{V,A}$, Eqs. (3) and (10), to the case of three flavors. The vector part of the weak interaction, instead of the ρ meson in Eq. (3), the $K^*(892)$ meson, takes part in

$$\mathcal{L}^{Vs}=\frac{g_W}{4}\sin\theta_C\frac{1}{f_{K^*}}\{-\tfrac{1}{2}(\partial_\mu W^+_\nu-\partial_\nu W^+_\mu)(\partial^\mu K^{-\nu}-\partial^\nu K^{-\mu})+(\partial_\mu W^-_\nu-\partial_\nu W^-_\mu)(\partial^\mu K^{+\nu}-\partial^\nu K^{+\mu})+W^+_\mu j^{-\mu}+W^-_\mu j^{+\mu}\}, \tag{68}$$

where $j^\pm_\mu$ are obtained by substituting

$$K^\pm_\mu\to\frac{g_W}{4}\frac{1}{f_{K^*}}\sin\theta_C W^\pm_\mu$$

into the vertex in which K_μ fields are involved. In the chiral limit, f_{K^*} is determined to be g^{-1} [12]. This Lagrangian has been used to calculate the form factors of K_{l3} [12] and the results are in good agreement with the data.

For the axial-vector part $\mathcal{L}^A$ there are two 1^+ K mesons: $K_1(1400)$ and $K_1(1275)$. In Ref. [12] the chiral partner of the $K^*(892)$ meson, the K_1 meson, is coupled to

$$\overline{\psi}\lambda^a\gamma_\mu\gamma_5\psi. \tag{69}$$

The mass of this K_1 meson is derived as

$$\left(1-\frac{1}{2\pi^2g^2}\right)m^2_{K_1}=6m^2+m^2_{K^*},\quad m_{K_1}=1.32\ \text{GeV}. \tag{70}$$

Theoretical value of m_{K_1} is lower than the mass of the $K_1(1400)$ and greater than the $K_1(1270)$'s mass. The widths of three decay modes ($K_1\to K^*\pi$, $K\rho$, $K\omega$) are calculated [12]. It is found that the $K^*\pi$ channel is dominant; however, $B(K\rho)$ is about 11%. The data [27] show that the branching ratio of $K_1(1400)$ decaying into $K\rho$ is very small. Therefore, the meson coupled to the quark axial-vector current is not a pure $K_1(1400)$ state; instead, it is a mixture of the two K_1 mesons. This state is coupled to the quark axial-vector current, Eq. (69), and it is K_a:

$$K_a=\cos\theta K_1(1400)+\sin\theta K_1(1270),$$
$$K_b=-\sin\theta K_1(1400)+\cos\theta K_1(1270). \tag{71}$$

In this theory K_a is coupled to the quark axial-vector current and the amplitudes of $\tau\to K_a\nu$ are from the tree diagrams and at $O(N_C)$ [12]. The production of K_b in τ decay is through loop diagrams of mesons which is at order 1 in the large N_c expansion [12]. This theory predicts a small branching ratio for K_b production in τ decays.

In the limit of $m_q=0$, the currents $\overline{\psi}\lambda_a\gamma_\mu\psi$ and $\overline{\psi}\lambda_a\gamma_\mu\gamma_5\psi$ form an algebra of $SU(3)_L\times SU(3)_R$. In this theory K_a is taken as the chiral partner of the K^* meson. The axial-vector part of the weak interaction $\mathcal{L}^A$, Eq. (10), is generalized to the case of $\Delta s=1$:

$$\mathcal{L}^{As}=-\frac{g_W}{4}\frac{1}{f_a}\sin\theta_C\{-\tfrac{1}{2}(\partial_\mu W^\pm_\nu-\partial_\nu W^\pm_\mu)(\partial^\mu K^{\mp\nu}_a-\partial^\nu K^{\mp\mu}_a)+W^{\pm\mu}j^\mp_\mu\}-\frac{g_W}{4}\sin\theta_C\Delta m^2f_aW^\pm_\mu K^{\mp,\mu}_a-\tfrac{1}{4}\sin\theta_C f_KW^\pm_\mu\partial^\mu K^\mp, \tag{72}$$

where $j^\pm_\mu$ are obtained by substituting

$$K^\pm_{a\mu}\to-\frac{g_W}{4f_a}\sin\theta_C W^\pm_\mu$$

into the vertex in which K_a fields are involved. In the limit of $m_q=0$, f_a and Δm^2 are the same as Eqs. (23) and (25) and $f_K=f_\pi$. $\mathcal{L}^{As}$, Eq. (72), can be exploited to study τ mesonic decays. On the other hand, τ mesonic decays of $\Delta s=1$ provide a crucial test of $\mathcal{L}^{As}$.

XII. K_a DOMINANCE IN $\tau\to K\pi\pi\nu$ DECAY

The processes $\tau\to K\pi\pi\nu$ have been studied by many authors. In Ref. [3] a chiral Lagrangian of the pseudoscalars with the introduction of vector resonances (ρ and K^*) has been used to calculate the branching ratios of $\tau\to K\pi\pi$. In Ref. [4] a chiral Lagrangian of pseudoscalars and ρ mesons is exploited. In Ref. [37] the mixture of the two K_1 resonances is phenomenologically taken into account in studying the decay $\tau\to K_1(1400)[K_1(1270)]\nu$. In Ref. [5] meson vertices (independent of momentum) and normalized Breit-Wigner propergators of the resonances are exploited.

In this theory, like $\tau\to 3\pi\nu$, the contribution of the contact terms containing more than three mesons to the processes $\tau\to K\pi\pi\nu$ is too small and the processes are dominated by $\tau\to K^*\pi\nu$ and $K\rho\nu$.

A. $\tau\to K^*\pi\nu$

We study the decay $\tau\to K^*\pi\nu$ first. Both the vector and axial-vector currents contribute to the decay $\tau^-\to\overline{K^{*0}}\pi^-\nu$. The vertex $\mathcal{L}^{\pi K^*\overline{K^*}}$ contributes to the vector part and has abnormal parity. It is from an anomaly. This vertex is derived from

$$-i\frac{2m}{f_\pi}\pi^i\langle\overline{\psi}\tau_i\gamma_5\psi\rangle.$$

The method obtaining the vertex $\mathcal{L}^{\pi K^*\overline{K^*}}$ from this quantity is the same as the one used to derive the vertices of ηvv $(v=\rho,\omega,\phi)$ in Ref. [12]:

$$\mathcal{L}^{\pi K^*\overline{K^*}}=-\frac{N_C}{\sqrt{2}\pi^2g^2f_\pi}\varepsilon^{\mu\nu\alpha\beta}\Big\{\partial_\mu K_\nu^+\partial_\alpha\overline{K_\beta^0}\pi^- +\partial_\mu K_\nu^-\partial_\alpha K_\beta^0\pi^+ +\frac{1}{\sqrt{2}}\pi^0(\partial_\mu K_\nu^+\partial_\alpha K_\nu^- -\partial_\mu K_\nu^0\partial_\alpha\overline{K_\beta^0}). \tag{73}$$

The vertex $\mathcal{L}^{WK^{*0}\pi^-}$ is derived by using the substitution. Using $\mathcal{L}^{Vs}$, Eq. (68), and the vertex (73), the vector matrix element is obtained:

$$\langle\overline{K^{*}}^0\pi^-|\overline{\psi}\lambda_+\gamma_\mu\psi|0\rangle =\frac{-1}{\sqrt{4\omega E}}\frac{1}{\sqrt{2}}\frac{N_C}{\pi^2g^2f_\pi}\frac{m_{K^*}^2-i\sqrt{q^2}\Gamma_{K^*}(q^2)}{q^2-m_{K^*}^2+i\sqrt{q^2}\Gamma_{K^*}(q^2)} \times\varepsilon^{\mu\nu\alpha\beta}k_\nu p_\alpha\epsilon_\beta^{*\sigma}, \tag{74}$$

where p and k are momentum of K^* and pion, respectively, and $q=k+p$.

The axial-vector matrix element is obtained by using the vertices $K_aK^*\pi$, $KK^*\pi$ which are presented in Ref. [12]. In the chiral limit, the expression of the matrix element of the axial-vector current is similar to Eq. (35):

$$\langle\overline{K^{*}}^0\pi^-|\overline{\psi}\lambda_+\gamma_\mu\gamma_5\psi|0\rangle=\frac{i}{\sqrt{4\omega E}}\frac{1}{\sqrt{2}}\left(\frac{q_\mu q_\nu}{q^2}-g_{\mu\nu}\right)\epsilon_\sigma^{*\lambda}\Bigg\{\frac{g^2f_am_{K^*}^2-iqf_a^{-1}\Gamma_{K_1(1400)}(q^2)}{q^2-m_{K_1(1400)}^2+iq\Gamma_{K_1(1400)}(q^2)}\cos\theta[A_{K_1(1400)}(q^2)_{K^*}g^{\nu\lambda} +B_{K_1(1400)}k^\nu k^\lambda]+\frac{g^2f_am_{K^*}^2-iqf_a^{-1}\Gamma_{K_1(1270)}(q^2)}{q^2-m_{K_1(1270)}^2+iq\Gamma_{K_1(1270)}(q^2)}\sin\theta[A_{K_1(1270)}(q^2)_{K^*}g^{\nu\lambda}+B_{K_1(1270)}k^\nu k^\lambda]\Bigg\}. \tag{75}$$

Let us determine the amplitudes $A_{K_1(1400)}$, $B_{K_1(1400)}$, $A_{K_1(1270)}$, and $B_{K_1(1270)}$. Equation (71) is written as

$$K_1(1400)=\cos\theta K_a-\sin\theta K_b,$$

$$K_1(1270)=\sin\theta K_a+\cos\theta K_b. \tag{76}$$

The vertex of K_1VP is presented in Ref. [12]:

$$\mathcal{L}^{K_1VP}=f_{abc}\{AK_{1\mu}^aV^{b\mu}P^c-BK^{a\mu}V^{b\nu}\partial_\mu\partial_\nu P^c\}. \tag{77}$$

It is similar to Eq. (27), the amplitude $A_{K_a}^{K^*}$ being determined to be

$$A(q^2)_{K_a}^{K^*}=\frac{2}{f_\pi}gf_a\Bigg\{\frac{m_{K_a}^2}{g^2f_a^2}-m_{K^*}^2 +m_{K^*}^2\left[\frac{2c}{g}+\frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right] +q^2\left[\frac{1}{2\pi^2g^2}-\frac{2c}{g}-\frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right]\Bigg\}. \tag{78}$$

B_{K_a} is the same as Eq. (29). The amplitudes $A_{K_b}^{K^*}$ and $B_{K_b}^{K^*}$ are unknown and we take them as parameters. Both $K_1(1400)$ and $K_1(1270)$ decay to $K\rho$ and $K\omega$. Using the SU(3) coefficients, for both K_1, it is determined that

$$B(K\omega)=\tfrac{1}{3}B(K\rho).$$

This relation agrees with the data [27] reasonably well. For the $K\rho$ decay mode $A^{\rho}_{K_b}$ and $B^{\rho}_{K_b}$ are the other two parameters. In the decays of the two K_1 mesons the momentum of the pion or kaon is low; therefore, the decay widths are insensitive to the amplitude B. We take

$$B^{K^*}_{K_b}=B^{\rho}_{K_b}\equiv B_b\,.$$

The decay width of the K_1 meson is derived from Eq. (77):

$$\Gamma_{K_1}=\frac{k}{32\pi}\frac{1}{\sqrt{q^2}m_{K_1}}\left\{\left(3+\frac{k^2}{m_v^2}\right)A^2(q^2)-A(q^2)B(q^2+m_v^2)\frac{k^2}{m_v^2}+\frac{q^2}{m_v^2}k^4B^2\right\}, \tag{79}$$

where $q^2=m^2_{K_1}$, $v=K^*,\rho$, k is the momentum of the pion or kaon,

$$k=\left\{\frac{1}{4m^2_{K_1}}(m^2_{K_1}+m_v^2-m_P^2)^2-4m_v^2\right\}^{1/2},$$

and m_P is the mass of the pion or kaon.

We choose the parameters as

$$\theta=30°,\quad A_b^{K^*}=-4.5\ \text{GeV},$$
$$A_b^{\rho}=5.0\ \text{GeV},\quad B_b=0.8\ \text{GeV}^{-1}, \tag{80}$$

from which the decay widths are obtained:

$$\Gamma(K_1(1400)\to K^*\pi)=159\ \text{MeV},$$
$$\Gamma(K_1(1400)\to K\rho)=10.5\ \text{MeV},$$
$$\Gamma(K_1(1270)\to K^*\pi)=12.4\ \text{MeV},$$
$$\Gamma(K_1(1270)\to K\rho)=26.8\ \text{MeV}. \tag{81}$$

The value of θ is about the same as the one determined in Ref. [37]. The data [27] are $163.4(1\pm0.13)$ MeV, 5.22 ± 5.22 MeV, $14.4(1\pm0.27)$ MeV, and $37.8(1\pm0.28)$ MeV, respectively.

Using the two matrix elements (74) and (75), the distribution of the decay rate is derived:

$$\frac{d\Gamma}{dq^2}(\tau^-\to\overline{K^{*0}}\pi^-\nu)=\frac{G^2}{(2\pi)^3}\frac{\sin^2\theta_C}{128m_\tau^3q^4}(m_\tau^2-q^2)^2(m_\tau^2+2q^2)\{(q^2+m^2_{K^*}-m^2_\pi)^2-4q^2m^2_{K^*}\}^{1/2}$$
$$\times\left\{\frac{6}{\pi^4g^2f_\pi^2}\frac{m^4_{K^*}+q^2\Gamma^2_{K^*}(q^2)}{(q^2-m^2_{K^*})^2+q^2\Gamma^2_{K^*}(q^2)}[(p\cdot q)^2-q^2m^2_{K^*}]+|A|^2\left[1+\frac{1}{12m^2_{K^*}q^2}(q^2-m^2_{K^*})^2\right]\right.$$
$$\left.-(BA^*+B^*A)\frac{1}{24m^2_{K^*}q^2}(q^2+m^2_{K^*})(q^2-m^2_{K^*})^2+\frac{|B|^2}{48m^2_{K^*}q^2}(q^2-m^2_{K^*})^4\right\}, \tag{82}$$

where p is the momentum of K^*, q^2 is the invariant mass squared of $K^*\pi$, and

$$A=\frac{g^2f_am^2_{K^*}-i\sqrt{q^2}f_a^{-1}\Gamma_{K_1(1400)}}{q^2-m^2_{K_1(1400)}+i\sqrt{q^2}\Gamma_{K_1(1400)}}\cos\theta A^{K^*}_{K_1(1400)}+\frac{g^2f_am^2_{K^*}-i\sqrt{q^2}f_a^{-1}\Gamma_{K_1(1270)}}{q^2-m^2_{K_1(1270)}+i\sqrt{q^2}\Gamma_{K_1(1270)}}\sin\theta A^{K^*}_{K_1(1270)},$$

$$B=\frac{g^2f_am^2_{K^*}-i\sqrt{q^2}f_a^{-1}\Gamma_{K_1(1400)}}{q^2-m^2_{K_1(1400)}+i\sqrt{q^2}\Gamma_{K_1(1400)}}\cos\theta B^{K^*}_{K_1(1400)}+\frac{g^2f_am^2_{K^*}-i\sqrt{q^2}f_a^{-1}\Gamma_{K_1(1270)}}{q^2-m^2_{K_1(1270)}+i\sqrt{q^2}\Gamma_{K_1(1270)}}\sin\theta B^{K^*}_{K_1(1270)}, \tag{83}$$

where

$$A^{K^*}_{K_1(1400)}=\cos\theta A_a^{K^*}-\sin\theta A_b^{K^*},\quad B^{K^*}_{K_1(1400)}=\cos\theta B_a-\sin\theta B_b\,,$$
$$A^{K^*}_{K_1(1270)}=\sin\theta A_a^{K^*}+\cos\theta A_b^{K^*},\quad B^{K^*}_{K_1(1270)}=\sin\theta B_a+\cos\theta B_b\,. \tag{84}$$

In the range of q^2 the main decay channels of K^* are $K\pi$ and $K\eta$ [the vertex of $K^*K\eta$ is shown in Eq. (88)]. The decay width of K^* is derived:

$$\Gamma(q^2)_{K^*}=\frac{f^2_{\rho\pi\pi}(q^2)}{8\pi}\frac{k^3}{\sqrt{q^2}m_{K^*}}+\cos^2 20°\frac{f^2_{\rho\pi\pi}(q^2)}{8\pi}\frac{k'^3}{\sqrt{q^2}m_{K^*}},$$

$$k=\left\{\frac{1}{4q^2}(q^2+m_K^2-m_\pi^2)^2-m_K^2\right\}^{1/2},$$

$$k'=\left\{\frac{1}{4q^2}(q^2+m_K^2-m_\eta^2)^2-m_K^2\right\}^{1/2}. \quad (85)$$

In $\Gamma_{K_1}(q^2)$ the decay modes $K^*\pi$, $K\rho$, and $K\omega$ are included:

$$\Gamma(q^2)_{K_1}=\frac{k}{32\pi}\frac{1}{\sqrt{q^2}m_{K_1}}\left\{\left(3+\frac{k^2}{m_{K^*}^2}\right)A^2(q^2)_{K^*}-A(q^2)_{K^*}B(q^2+m_{K^*}^2)\frac{k^2}{m_{K^*}^2}+\frac{q^2}{m_{K^*}^2}k^4B^2\right\}+\frac{4}{3}\frac{k'}{32\pi}\frac{1}{\sqrt{q^2}m_{K_1}}$$

$$\times\left\{\left(3+\frac{k'^2}{m_\rho^2}\right)A^2(q^2)-A(q^2)B(q^2+m_\rho^2)\frac{k'^2}{m_\rho^2}+\frac{q^2}{m_\rho^2}k'^4B^2\right\},$$

$$k=\left\{\frac{1}{4q^2}(q^2+m_{K^*}^2-m_\pi^2)^2-m_{K^*}^2\right\}^{1/2},$$

$$k'=\left\{\frac{1}{4q^2}(q^2+m_K^2-m_\rho^2)^2-m_K^2\right\}^{1/2}. \quad (86)$$

For $K_1(1270)$ $\Gamma(K_1(1270)\to K_0^*(1430)\pi)=25.2$ MeV is included. The distribution is shown in Fig. 7 and the branching ratio is calculated:

$$B(\tau^-\to\overline{K^{*0}}\pi^-\nu)=0.23\%.$$

The contribution of the vector current is about 7.4%. Therefore, K_a is dominant in this decay. The data are $0.38\pm0.11\pm0.13\%$ (CLEO [38]), $0.25\pm0.10\pm0.05\%$ (ARGUS [39]).

There is another decay channel $\tau^-\to K^{*-}\pi^0\nu$ whose branching ratio is one-half of $B(\tau^-\to\overline{K^{*0}}\pi^-\nu)$. The total branching ratio is

$$B(\tau^-\to\overline{K}\pi\nu)=0.35\%.$$

The narrow peak in Fig. 7 is from $K_1(1270)$ and the wider peak comes from $K_1(1400)$. The width is about 230 MeV.

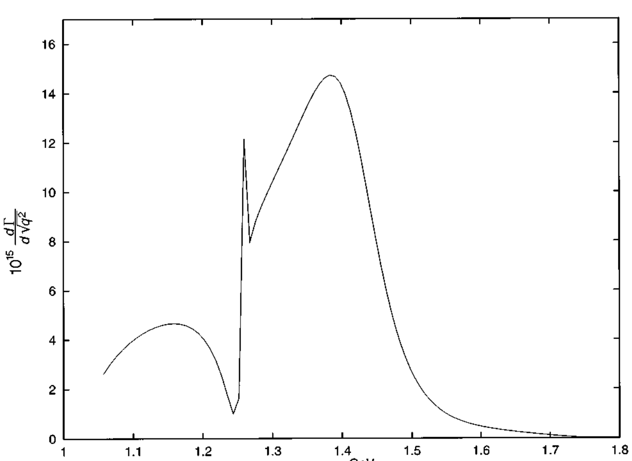

FIG. 7. Distribution of the decay rate of $\tau\to K^*\pi\nu$ vs the invariant mass of $K^*\pi$.

B. $\tau\to K\rho\nu$ and $K\omega\nu$

It is the same as $\tau\to K^*\pi\nu$; K_a dominates the decay $\tau\to K\rho\nu$. Both the vector and axial-vector currents contribute to this decay mode. The matrix element of the vector current, $\langle\overline{K^0}\rho^-|\bar{\psi}\lambda_+\gamma_\mu\psi|0\rangle$, is determined by the vertex $\mathcal{L}^{K^*K\rho}$, Eq. (47), and is the same as Eq. (50). The axial-vector matrix element $\langle\overline{K^0}\rho^-|\bar{\psi}\lambda_+\gamma_\mu\gamma_5\psi|0\rangle$ is obtained by substituting

$$K^*\to\rho,\quad K\to\pi$$

in Eq. (75). Using the same substitutions in Eq. (83), the distribution of the decay rate of $\tau\to K\rho\nu$ is found. The branching ratio of $\tau\to K\rho\nu$ (two modes $\overline{K^0}\rho^-$ and $K^-\rho^0$) is computed to be

$$B=0.75\times10^{-3}. \quad (87)$$

It is about 18% of $\tau\to K\pi\pi\nu$. The vector current makes an 8% contribution. The DELPHI Collaboration [40] has reported that $\tau\to K^*\pi\nu$ dominates the decay $\tau\to K\pi\pi\nu$ and $K\rho\nu$ decay mode have not been observed. The ALEPH Collaboration [41] has reported the $K^*\pi$ dominance and a branching ratio of $(30\pm11)\%$ for the $K\rho$ mode.

Because of the SU(3) coefficient, we expect

$$B(\tau\to K\omega\nu)=\tfrac{1}{3}B(\tau\to K\rho\nu).$$

The theoretical results are in reasonable agreement with the data. In this paper the spontaneous chiral symmetry-breaking effect (for the mass difference between K^* and K_a) is taken into account and the resonance formula is [Eq. (83)]

$$BW_{K_1}[s]\equiv\frac{-g^2f_a^2m_{K^*}^2+i\sqrt{q^2}\Gamma_{K_1}}{q^2-m_{K_1}^2+i\sqrt{q^2}\Gamma_{K_1}}.$$

Because the spontaneous chiral symmetry-breaking effect does not disappear in the limit of $q^2\to0$, we have a different

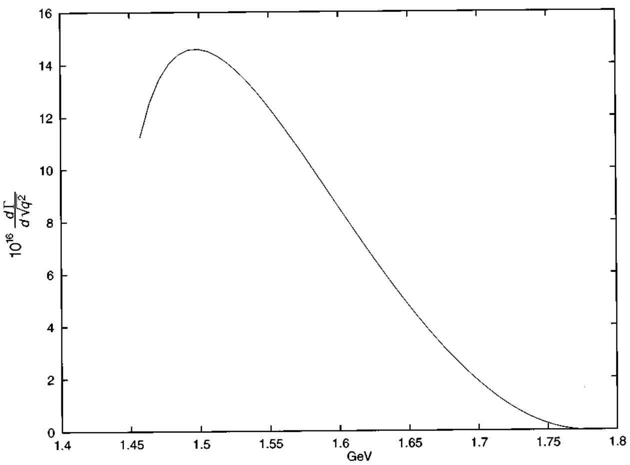

FIG. 8. Distribution of the decay rate of $\tau \to K^*\eta\nu$ vs the invariant mass of $K^*\eta$.

low energy limit which is $g^2 f_a^2 m_{K^*}^2/m_{K_1}^2$ On the other hand, in this theory the amplitude A strongly depends on q^2 and this dependence plays an important role in understanding the large branching ratio of the $K^*\pi$ mode and the smaller one for the $K\rho$ mode.

XIII. $\tau \to K^*\eta$

There are vector and axial-vector parts in this decay. The calculation of the decay rate is similar to the decay of $\tau \to K^*\pi\nu$. The vertices $\mathcal{L}^{K^*\overline{K^*}\eta}$ and $\mathcal{L}^{WK^*\eta}$ via the Lagrangian L^{Vs}, Eq. (68), contribute to the vector part and the vertices $\mathcal{L}^{K_1K^*\eta}$, $\mathcal{L}^{KK^*\eta}$, and $\mathcal{L}^{WK^*\eta}$ via L^{As} take the responsibility for the axial-vector part. The vertex $\mathcal{L}^{K^*\overline{K^*}\eta}$ comes from the anomaly. Using the same method deriving the vertices $\eta v v$ (in Ref. [12]), it is found that

$$\mathcal{L}^{K^*\overline{K^*}\eta} = -\frac{3a}{2\pi^2 g^2 f_\pi} d_{ab8}\varepsilon^{\mu\nu\alpha\beta}\eta\partial_\mu K_\nu^a \partial_\alpha K_\beta^b - \frac{3b}{2\pi^2 g^2 f_\pi}\varepsilon^{\mu\nu\alpha\beta}\eta\partial_\mu K_\nu^a\partial_\alpha K_\beta^a, \tag{88}$$

where a and b are the octet and singlet components of η, respectively, $a=\cos\theta$, $b=\sqrt{\frac{2}{3}}\cos\theta$, and $\theta=-20°$. Because of the cancellation between the two components, the vector matrix element is very small and can be ignored.

The vertices $\mathcal{L}^{K_1K^*\eta}$ and $\mathcal{L}^{KK^*\eta}$ contribute to the axial-vector matrix element and they are derived from the effective Lagrangian presented in Ref. [12]:

$$\mathcal{L}^{K_1K^*\eta} = a f_{ab8}\{A(q^2)_{K^*}K_\mu^a K^{b\nu}\eta - BK_\mu^a K_\nu^b \partial^{\mu\nu}\eta\}, \tag{89}$$

$$\mathcal{L}^{K^*K\eta} = a f_{K^*K\eta} f_{ab8} K_\mu^a (K^b\partial^\mu\eta - \eta\partial^\mu K^b), \tag{90}$$

where $f_{K^*K\eta}$ is the same as $f_{\rho\pi\pi}$, Eq. (37), in the limit of $m_q=0$. The decay width is similar to the one of $\tau \to K^*\pi\nu$:

$$\frac{d\Gamma}{dq^2}(\tau^- \to K^{*-}\eta\nu) = \frac{G^2}{(2\pi)^3}\cos^2 20° \frac{\sin^2\theta_C}{64 m_\tau^3 q^4}(m_\tau^2 - q^2)^2 (m_\tau^2 + 2q^2) \times\frac{3}{4}\left\{|A|^2\left[1+\frac{1}{12 m_{K^*}^2 q^2}(q^2 - m_{K^*}^2)^2\right] - (BA^* + B^*A)\frac{1}{24 m_{K^*}^2 q^2}(q^2 + m_{K^*}^2)(q^2 - m_{K^*}^2)^2 + \frac{|B|^2}{48 m_{K^*}^2 q^2}(q^2 - m_{K^*}^2)^4\right\}. \tag{91}$$

The distribution is shown in Fig. 8. The branching ratio is computed to be

$$B = 1.01\times 10^{-4}.$$

The axial-vector current is dominant.

XIV. $\tau \to K\eta\nu$

The decay $\tau \to \eta K\nu$ has been studied in terms of a chiral Lagrangian [3,7] and only the vector current contributes. The prediction is 1.2×10^{-4}. The experiments are CLEO [42], $(2.6\pm 0.5)\times 10^{-4}$; ALEPH [43], $(2.9^{+1.3}_{-1.2}\pm 0.7)\times 10^{-4}$.

In the effective chiral theory the vertex $K_1K\eta$ does not exist. The reason is that if it exists it has abnormal parity and comes from the anomaly in which there is an antisymmetric tensor. It is impossible to construct a vertex with an antisymmetric tensor by using K_1, K, and η fields. Therefore, only the vector current contributes to this process and K^* is dominant in this process. The vertex $\mathcal{L}^{K^*K\eta}$ is shown in Eq. (90). The decay width is found:

$$\frac{d\Gamma}{dq^2} = \frac{3}{4}\frac{G^2}{(2\pi)^3}\sin^2\theta_C \cos^2 20° \frac{1}{384 m_\tau^3}\frac{1}{q^2}(m_\tau^2 - q^2)^2 \times (m_\tau^2 + 2q^2)[(q^2 + m_\eta^2 - m_K^2)^2 - 4q^2 m_\eta^2]^{1/2} \times g^2 f_{\rho\pi\pi}^2(q^2)\frac{m_{K^*}^4 + q^2\Gamma_{K^*}^2(q^2)}{(q^2 - m_{K^*}^2)^2 + q^2\Gamma_{K^*}^2(q^2)}. \tag{92}$$

The branching ratio is computed to be

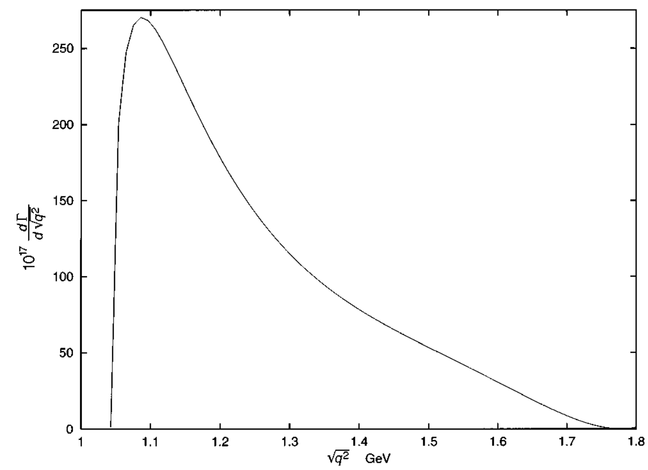

FIG. 9. Distribution of the decay rate of $\tau \rightarrow K\eta\nu$ vs the invariant mass of $K\eta$.

$$B(\tau^- \rightarrow \eta K^- \nu) = 2.22 \times 10^{-4}.$$

The distribution of the invariant mass of η and K is shown in Fig. 9. The figure indicates a peak at 1.086 GeV which is slightly above the threshold. The peak results from the effects of the threshold and the resonance.

The branching ratio of $\tau \rightarrow \eta' K\nu$ is 200 times smaller.

XV. FORM FACTORS OF $K^+ \rightarrow \gamma l \nu$

As a test of $\mathcal{L}^{Vs,As}$, Eqs. (68) and (72), we study the form factors of $K^- \rightarrow e\gamma\nu$. The form factors of $K^+ \rightarrow \gamma l\nu$ are calculated in the chiral limit. The vector form factor is determined by the vertex which comes from the anomaly [12]:

$$\mathcal{L}^{K^{*+}K^-\gamma} = -\frac{e}{2\pi^2 g f_\pi} \varepsilon^{\mu\nu\alpha\beta} K^+_\mu \partial_\beta K^- \partial_\nu A_\alpha. \quad (93)$$

The vector form factor is determined:

$$F^V = \frac{1}{2\sqrt{2}\pi^2} \frac{m_K}{f_\pi} = 0.095. \quad (94)$$

The vertices $\mathcal{L}^{K_1 K\gamma}$ can be found from Ref. [12]:

$$\mathcal{L}^{K_1 K\gamma} = \frac{i}{2} eg(Ag_{\nu\lambda} + Bp_\nu p_\lambda + Dk_\nu k_\lambda) K_1^{-\nu} K^+ A^\lambda. \quad (95)$$

$\mathcal{L}^{KK\gamma}$ is [12]

$$\mathcal{L}^{K^+K^-\gamma} = ie(K^+ \partial_\mu K^- - K^- \partial_\mu K^+) A^\mu. \quad (96)$$

The axial-vector form factors are derived:

$$F^A = \frac{1}{2\sqrt{2}\pi^2} \frac{m_K}{f_\pi} \frac{m^2_{K^*}}{m^2_{K_1}} \left(1 - \frac{2c}{g}\right) \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1} = 0.04,$$

$$m^2_{K_1} = (1.32\ \text{GeV})^2, \quad (97)$$

$$R = \frac{g^2}{2\sqrt{2}} \frac{m_K}{f_\pi} \frac{m^2_{K^*}}{m^2_{K_1}} \left\{ \frac{2c}{g} + \frac{1}{\pi^2 g^2} \left(1 - \frac{2c}{g}\right) \right\} \left(1 - \frac{1}{2\pi^2 g^2}\right)^{-1}$$

$$= 0.078. \quad (98)$$

We obtain $F^A + F^V = 0.135$, $F^A - F^V = -0.055$. The data [27] are $F^A + F^V = 0.147 \pm 0.011$, $0.150^{+0.018}_{-0.023}$, $F^A - F^V = < 0.49$. In the calculation of F_A the K_a is used, and we obtain $F_A = 0.032$. It is necessary to point out that the factor m_K in Eqs. (88), (91), and (92) comes from the definitions of the form factors.

XVI. CONCLUSIONS

The Lagrangian of the weak interaction of mesons consists of a vector part and an axial-vector part. In the chiral limit, VMD takes responsibility for the vector part. Based on chiral symmetry and spontaneous chiral symmetry breaking the Lagrangian of the axial-vector part of weak interactions of mesons is determined. The whole Lagrangian is derived from the effective chiral theory of mesons. All the vertices of mesons are obtained from the same theory. This theory provides a unified study for τ mesonic decays. The a_1 dominance in the matrix elements of the $\Delta s = 0$ axial-vector currents and K_a dominance in the ones of $\Delta s = 1$ axial-vector currents in τ decays are found. All theoretical studies are done in the limit of $m_q = 0$ and the results are in reasonable agreement with the data. There are many other τ mesonic decay modes that can be studied by this theory.

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[1] Y.S. Tsai, Phys. Rev. D **4**, 2821 (1971).
[2] R. Fischer, J. Wess, and F. Wagner, Z. Phys. C **3**, 313 (1980).
[3] G. Aubrecht II, N. Chahrouri, and K. Slanec, Phys. Rev. D **24**, 1318 (1981).
[4] E. Braaten, R.J. Oakes, and S.M. Tse, Int. J. Mod. Phys. A **5**, 2737 (1990).
[5] R. Decker, E. Mirkes, R. Sauer, and Z. Was, Z. Phys. C **58**, 445 (1993); M. Finkemeier and E. Mirkes, *ibid.* **69**, 243 (1996).
[6] G. Kramer and W.F. Palmer, Z. Phys. C **25**, 195 (1984); **39**, 423 (1988).
[7] A. Pich, Phys. Lett. B **196**, 561 (1987).
[8] R. Decker, Z. Phys. C **36**, 487 (1987).
[9] E. Braaten, R. Oakes, and S.Z. Tse, Phys. Rev. D **36**, 2187 (1987).
[10] R. Decker and E. Mirkes, Phys. Rev. D **47**, 4012 (1993).
[11] J.J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969).

[12] B.A. Li, Phys. Rev. D **52**, 5165 (1995); **52**, 5184 (1995).
[13] J. Wess and B. Zumino, Phys. Lett. B **223**, 95 (1971); E. Witten, Nucl. Phys. **37B**, 422 (1983).
[14] S.I. Eidelman and V.N. Ivanchenko, in *Tau 94*, Proceedings of the Third Workshop on Tau Lepton Physics, Montreux, Switzerland, 1994, edited by L. Rolandi [Nucl. Phys. B (Proc. Suppl.) **40,** 131 (1995)].
[15] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).
[16] J. Schwinger, Phys. Lett. **24B**, 473 (1967); J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); S. Weinberg, *ibid.* **166**, 1568 (1968); B. W. Lee and H. T. Nieh, *ibid.* **166**, 1507 (1968).
[17] J.J. Gomez-Cadennas, M.C. Gonzalez-Garcia, and A. Pich, Phys. Rev. D **42**, 3093 (1990).
[18] M.G. Bowler, Phys. Lett. B **182**, 400 (1986); N.A. Tornquist, Z. Phys. C **36**, 695 (1987); J.H. Kuhn and A. Santamaria, *ibid.* **48**, 445 (1990); M.K. Volkov, Yu.P. Ivanov, and A.A. Osipov, *ibid.* **49**, 563 (1991).
[19] N. Isgur, C. Mornongstar, and C. Reader, Phys. Rev. D **39**, 1357 (1989).
[20] CELLO Collaboration, H.J. Behrend *et al.*, Z. Phys. C **46**, 537 (1990).
[21] ALEPH Collaboration, D. Decamp *et al.*, Z. Phys. C **54**, 211 (1992).
[22] DELPHI Collaboration, P. Abreu *et al.*, Z. Phys. C **55**, 555 (1992).
[23] CLEO Collaboration, G. Crawford, in *Proceedings of the Second Workshop on Tau Lepton Physics*, Columbus, Ohio, 1992, edited by K.K. Gan (World Scientific, Singapore, 1992), p. 183.
[24] H. Evans, presented at the Fourth International Workshop on Tau Lepton Physics, Estes Park, Colorado, 1996 (unpublished).
[25] ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **58**, 61 (1993).
[26] BES Collaboration, High Energy Phys. Nucl. Phys. **19**, 385 (1994).
[27] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[28] CLEO Collaboration, T.E. Coan *et al.*, Report No. CLEO CONF 96-15, ICHEP96 PA01-085 (unpublished).
[29] CLEO Collaboration, M. Goldberg *et al.*, Phys. Lett. B **251**, 223 (1990).
[30] ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **68**, 215 (1995).
[31] See paper by M. Benayoun *et al.*, Z. Phys. C **58**, 31 (1993).
[32] E.B. Dally *et al.*, Phys. Rev. Lett. **48**, 375 (1982).
[33] S.I. Eidenlman and V.N. Ivanchenko, Phys. Lett. B **257**, 437 (1991); S. Nelson and A. Pich, *ibid.* **304**, 359 (1993).
[34] TPC/2γ Collaboration, H. Aihara *et al.*, Phys. Rev. Lett. **59**, 751 (1987).
[35] ALEPH Collaboration, D. Buskulic *et al.*, Z. Phys. C **70,** 579 (1996).
[36] CLEO Collaboration, T. Coan *et al.*, Phys. Rev. D **53**, 6037 (1996).
[37] H. Lipkin, Phys. Lett. B **303**, 119 (1993); M. Suzuki, Phys. Rev. D **47**, 1252 (1993).
[38] CLEO Collaboration, M. Goldberg *et al.*, Phys. Lett. B **251**, 223 (1992).
[39] ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **68**, 215 (1995).
[40] W. Hao Ph.D. thesis, NIKHEF-H, 1996.
[41] ALEPH Collaboration, M. Davier, presented at the Fourth International Workshop on Tau-Lepton Physics [24].
[42] CLEO Collaboration, J. Bartelt *et al.*, Phys. Rev. Lett. **76**, 4119 (1996).
[43] ALEPH Collaboration, D. Buskulic *et al.*, Report No. CERN-PDE/96-103 (unpublished).
[44] Crystalball Collaboration, D. Antreasyan *et al.*, Phys. Lett. B **259**, 216 (1991).