Going beyond the peaking approximation in the PQCD analysis of exclusive heavy meson pair production

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In the PQCD analyses of the exclusive production of higher generation hadrons, the quark distribution amplitude of the heavy quark system has often been approximated by a δ function from the nonrelativistic consideration. Going beyond the peaking approximation, the factorization of the covariant hard scattering amplitude from the nonperturbative quark distribution amplitude is no longer valid. We therefore use the light-cone time-ordered perturbation theory which is the step prior to the usual factorization formula and calculate the form factor of a pseudoscalar meson composed of a heavy quark and antiquark. However, we find that the numerical results for the cross section of exclusive heavy meson pair production in e^+e^- annihilation are not much different from those of the peaking approximation. [S0556-2821(97)00103-3]
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I. INTRODUCTION

It has been pointed out [1] that exclusive pair production of heavy hadrons, $|Q_1,Q_2\rangle$ and $|Q_1,Q_2,Q_3\rangle$, consisting of higher-generation quarks ($Q_i=t,b,c$) can be reliably predicted within the framework of perturbative QCD (PQCD). In this framework, the invariant amplitude $\mathcal{M}$ for exclusive processes factorizes into the convolution of the valence-quark distribution amplitudes $\phi(x_i,q^2)$ with the hard scattering amplitude T_H [2]:

$$\mathcal{M}=\int [dx_i]\int [dy_i]\phi(x_i,q^2)T_H(x_i,y_i,q^2)\phi(y_i,q^2), \tag{1}$$

where $[dx_i]=\delta(1-\Sigma_{k=1}^{n}x_k)\Pi_{k=1}^{n}dx_k$ and $n=2,3$ is the number of quarks in the valence Fock state. Since the collinear divergences are summed in $\phi(x_i,q^2)$, T_H can be systematically computed as a perturbation expansion in $\alpha_s(q^2)$. The distribution amplitude

$$\phi(x_i,q^2)=\int^{\vec{k}_\perp^2<|q^2|}[d^2\vec{k}_{\perp i}]\psi^{(q^2)}(x_i,k_{\perp i}), \tag{2}$$

where

$$[d^2\vec{k}_{\perp i}]=2(2\pi)^3\delta\left[\sum_{j=1}^{n}\vec{k}_{\perp j}\right]\prod_{i=1}^{n}\frac{d^2\vec{k}_{\perp i}}{2(2\pi)^3}$$

is computed from the valence wave function of the hadron at equal time $\tau=t+z/c$ on the light cone and gives the probability amplitude for the constituents with light-cone momentum fraction $x_i=(k_i^0+k_i^z)/\Sigma_{i=1}^{n}(k_i^0+k_i^z)$ to combine into the hadron with relative transverse momentum up to the scale q^2. Although there have been efforts to calculate the distribution amplitude using nonperturbative methods such as the QCD sum rule [3] and lattice calculation [4], it is not yet well known for systems made of light quarks (u,d,s). However, for heavy quark systems it can be essentially determined by nonrelativistic considerations; i.e., a δ function was used for the distribution amplitude of the heavy meson, $|Q_1,Q_2\rangle$:

$$\phi(x_i,q^2)=\frac{f_M}{2\sqrt{3}}\,\delta\left[x_1-\frac{m_1}{m_1+m_2}\right], \tag{3}$$

where f_M is the meson decay constant and m_1 and m_2 are the masses of the quarks. Most of the works including some recent calculations involving heavy quark systems employed a similar peaking approximation [5–8].

In the example of meson form factor calculations, the following procedure was taken. At leading order of α_s, the contribution to T_H is dominated by the single-gluon-exchange diagrams shown in Fig. 1. To calculate T_H, each hadron is replaced by its collinear on-shell constituents. It is common to assign the ith constituent's momentum k_i (l_i) by $k_i=x_iP$ ($l_i=y_iP'$), where x_i (y_i) represents the longitudinal momentum fractions of the total momentum P (P') of the hadron in the initial (final) state. However, it can be easily seen from the on-mass-shell conditions

$$m_i^2=k_i^2=x_i^2P^2=x_i^2M_H^2,$$
$$m_i^2=l_i^2=y_i^2P'^2=y_i^2M_H^2,$$

which lead to $x_i=y_i=m_i/M_H$. If one can neglect the masses m_i and M_H, then these constraints on the values of x_i and y_i may not play any role. On the other hand, if one cannot neglect the masses as in heavy hadron production processes, then these constraints restrict the choice of the quark distribution amplitude, and the only consistent quark distribution amplitude would be given by $\delta(x_i-m_i/M_H)$ [Eq. (3)] [9].

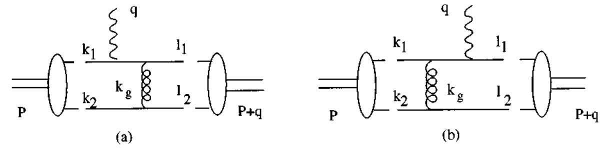

FIG. 1. Leading-order one-gluon-exchange diagram to T_H.

However, it may be important to investigate the case that $\phi(x)$ is not an exact δ function even though it is a highly peaked function at $x_i=m_i/\Sigma_i m_i$. The contributions from the end-point regions could be significant in exclusive form factor calculations. More importantly, as we illustrated in the previous paragraph, the factorized formula given by Eq. (1) is no longer valid when $\phi(x)$ is not an exact δ function. We thus use light-cone time-ordered perturbation theory which is the step prior to Eq. (1) in order to consider the case beyond the peaking approximation; e.g., the invariant amplitude $\mathcal{M}$ involving two mesons is given by

$$\mathcal{M}=\int dx\, dy\, d^2\vec{k}_\perp d^2\vec{l}_\perp \psi(x,\vec{k}_\perp)T(x,y,\vec{k}_\perp,\vec{l}_\perp,\vec{q}_\perp)\psi(y,\vec{l}_\perp), \tag{4}$$

where $\psi(x,\vec{k}_\perp)$ is the light-cone wave function of the two-body Fock state and $T(x,y,\vec{k}_\perp,\vec{l}_\perp,\vec{q}_\perp)$ is obtained by the two-body irreducible diagrams. The same step was taken in recent pion form factor calculations [10]. An analogue of Eq. (4) was also used in the recent analyses of B meson decays [11]. In this paper, using Eq. (4), we present the analysis of the pair production of heavy pseudoscalar mesons with a light-cone wave function that is not exactly a δ function. Then we compare our results with the previous peaking approximation results using Eq. (1). In Sec. II, the formulation used in this work is detailed and the results of the light-cone time-ordered diagram calculations are presented. The numerical results are presented in Sec. III and the summary and conclusions are given in Sec. IV.

II. FORMULATION

The electromagnetic interaction vertex of a pseudoscalar meson is determined by the Lorentz- and gauge-invariant form factor $S(q^2)$:

$$\Gamma_\mu(q^2)=(P_\mu+P'_\mu)S(q^2). \tag{5}$$

In terms of $S(q^2)$, the cross section of the pseudoscalar meson pair (MM) production in unpolarized e^+e^- annihilation is given by [1]

$$\frac{d\sigma}{d\Omega}(e^+e^-\rightarrow M\overline{M})=\frac{3\beta^3}{32\pi}\sigma_{e^+e^-\rightarrow\mu^+\mu^-}\sin^2\theta|S(q^2)|^2, \tag{6}$$

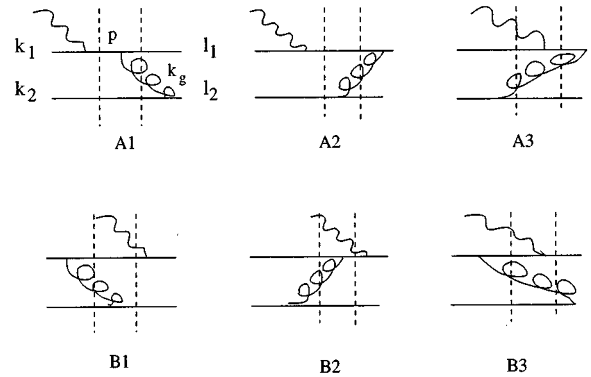

FIG. 2. Leading-order light-cone time-ordered diagrams for the T. In each diagram, the instantaneous diagrams for the intermediate quark and gluon are implicitly included by using the technique shown in Ref. [13].

where $\beta=\sqrt{1-4M_H^2/q^2}$ and M_H is the meson mass.

In order to calculate the form factor $S(q^2)$, we use the ordinary Drell-Yan frame (i.e., $q^+=0$) and set $\mathcal{M}=S(q^2)$ in Eq. (4) [12]. For the calculation of T in Eq. (4), one cannot assign $k_i=x_iP$ because $\Sigma_i k_i^-\neq P^-$. Instead, we determine k_i^- from the on-mass-shell condition $k_i^2=m_i^2$. In the leading order of the light-cone PQCD, T is given by six light-cone time-ordered diagrams shown in Fig. 2: i.e., $T=A_1+A_2+A_3+B_1+B_2+B_3$. In each diagram, the instantaneous diagrams for the intermediate quark and gluon are included using the technique shown in Ref. [13]. In the light-cone gauge $A^+=0$, the gluon propagator is given by

$$d_{\mu\nu}=-g_{\mu\nu}+\frac{(k_g)_\mu\eta_\nu+(k_g)_\nu\eta_\mu}{k_g^+}, \tag{7}$$

where $\eta^+=1$, $\eta^-=0$, and $\vec{\eta}_\perp=0$. In the leading twist, the light-cone gauge parts proportional to $1/k_g^+$ in Eq. (7) explicitly cancel out among six diagrams and the sum of six diagrams is identical to the usual covariant T_H. Beyond the leading twist, the cancellation between the light-cone gauge part ($1/k_g^+$ terms) and the higher Fock-state contribution has been discussed in Ref. [10]. With these considerations, we calculated all six diagrams and the results are summarized as follows:

$$A_1=\frac{\theta(y_2-x_2)N}{(y_2-x_2)\{M^2+q_\perp^2-[(k_\perp+q_\perp)^2+m_1^2]/x_1-(k_\perp^2+m_2^2)/x_2\}}\times\frac{1}{\{M^2+q_\perp^2-[(y_1q_\perp+l_\perp^2)^2+m_1^2]/y_1-(k_\perp^2+m_2^2)/x_2-(y_2q_\perp-l_\perp+k_\perp)^2/(y_2-x_2)\}}, \tag{8}$$

$$A_2=\frac{\theta(x_2-y_2)N}{(x_2-y_2)\{M^2+q_\perp^2-[(k_\perp+q_\perp)^2+m_1^2]/x_1-(k_\perp^2+m_2^2)/x_2\}}\times\frac{1}{\{M^2+q_\perp^2-[(k_\perp+q_\perp)^2+m_1^2]/x_1-(y_2q_\perp-l_\perp+k_\perp)^2/(x_2-y_2)-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2\}}, \tag{9}$$

$$A_3=\frac{\theta(x_2-y_2)N}{(x_2-y_2)\{M^2-(k_\perp^2+m_1^2)/x_1-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2-(y_2q_\perp-l_\perp+k_\perp)^2/(x_2-y_2)\}}$$
$$\times\frac{1}{\{M^2+q_\perp^2-[(k_\perp+q_\perp)^2+m_1^2]/x_1-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2-(y_2q_\perp-l_\perp+k_\perp)^2/(x_2-y_2)\}},\qquad(10)$$

$$B_1=\frac{\theta(x_2-y_2)N}{(x_2-y_2)\{M^2-(k_\perp^2+m_1^2)/x_1-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2-(y_2q_\perp-l_\perp+k_\perp)^2/(x_2-y_2)\}}$$
$$\times\frac{1}{\{M^2-[(y_2q_\perp-l_\perp)^2+m_1^2]/y_1-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2\}},\qquad(11)$$

$$B_2=\frac{\theta(y_2-x_2)N}{(y_2-x_2)\{M^2-[(y_2q_\perp-l_\perp)^2+m_1^2]/y_1-(k_\perp^2+m_2^2)/x_2-(y_2q_\perp-l_\perp+k_\perp)^2/(y_2-x_2)\}}$$
$$\times\frac{1}{\{M^2-[(y_2q_\perp-l_\perp)^2+m_1^2]/y_1-[(y_2q_\perp-l_\perp)^2+m_2^2]/y_2\}},\qquad(12)$$

$$B_3=\frac{\theta(y_2-x_2)N}{(y_2-x_2)\{M^2-[(y_2q_\perp-l_\perp)^2+m_1^2]/y_1-(k_\perp^2+m_2^2)/x_2-(y_2q_\perp-l_\perp+k_\perp)^2/(y_2-x_2)\}}$$
$$\times\frac{1}{\{M^2+q_\perp^2-[(y_1q_\perp+l_\perp)^2+m_1^2]/y_1-(k_\perp^2+m_2^2)/x_2-(y_2q_\perp-l_\perp+k_\perp)^2/(y_2-x_2)\}},\qquad(13)$$

where

$$N=\frac{8}{x_1x_2y_1y_2}[x_2^2y_1y_2q_\perp^2+y_1y_2k_\perp^2+x_1x_2l_\perp^2+2x_2y_1y_2q_\perp\cdot k_\perp+x_2(x_1y_1+x_2y_2)q_\perp\cdot l_\perp+(x_1y_1+x_2y_2)k_\perp\cdot l_\perp+x_1y_1m_2^2+x_2y_2m_1^2-(x_1-y_1)(x_2-y_2)m_1m_2].\qquad(14)$$

As we mentioned before, the sum of A_i and B_i ($i=1,2,3$) is same with the usual T_H given in the literature [13] if and only if all the masses (M,m_1,m_2) and transverse momenta ($k_\perp,l_\perp$) are neglected. Also, if we keep the masses as $m_i=x_iM$ and $M=m_1+m_2$, but neglect $k_\perp$ and $l_\perp$ (i.e., the peaking approximation), then the sum of Eqs. (8)–(14) is identical to our previous result shown in Ref. [1]. However, in this paper, we include all the mass terms and the $k_\perp$ and $l_\perp$ terms as shown in Eqs. (8)–(14) and use $\psi(x,\vec{k}_\perp)$, which leads to a highly peaked quark distribution amplitude, but not exactly a δ function. Then we compare the cross section for pair production of the heavy meson with our previous result [1] in the peaking approximation.

III. NUMERICAL RESULTS

For comparison with our previous result, we use the two-body light-cone wave function, frequently used in the literature [14],

$$\psi(x,\vec{k}_\perp)=N\ \exp\left[\left(M^2-\frac{\vec{k}_\perp^{\,2}+m_1^2}{x}-\frac{\vec{k}_\perp^{\,2}+m_2^2}{(1-x)}\right)\Big/(8\beta^2)\right],\qquad(15)$$

where $\beta^2=(M-m_1-m_2)^2$. If we neglect the binding energy, i.e., $\beta=0$, then Eq. (15) becomes the δ function peaking at $x=m_1/(m_1+m_2)$ and $\vec{k}_\perp=0$, which is ultimately equivalent with the peaking approximation given by Eq. (3). However, in this paper, we vary the value of β and investigate the difference of the result from our previous result of the peaking approximation.

The cross section for the pair production of the heavy meson $B_c(b\overline{c})$ with different M_H values (i.e., different binding energies) are predicted in Fig. 3 using our formula given by Eqs. (6) and (15). The results for the cross sections are given in units of R with the $\mu^+\mu^-$ rate as reference. As had been discussed in Ref. [1], the form factors for the heavy hadrons are normalized by the constraint that the Coulomb contribution to the form factor equals the total hadronic charge at $q^2=0$. Further, by the correspondence principles, the form factor should agree with the standard nonrelativistic calculation at small momentum transfer. In Ref. [1], all of these constraints are satisfied by introducing a parameter $\gamma=v m_\tau$, which sets the scale for capture into the wave function in relative transverse momentum. However, in our case, the higher-twist terms in the energy denominators of T play the role of this parameter and, in principle, we do not need any extra parameter to fix the normalization. Since we are interested in comparing with the δ-function result shown in Ref. [1], we set $\gamma=m_2$ for the $\beta=0$ limit and neglect the e_2 contribution [15]. Then, our result with $M_H=m_1+m_2$, i.e.,

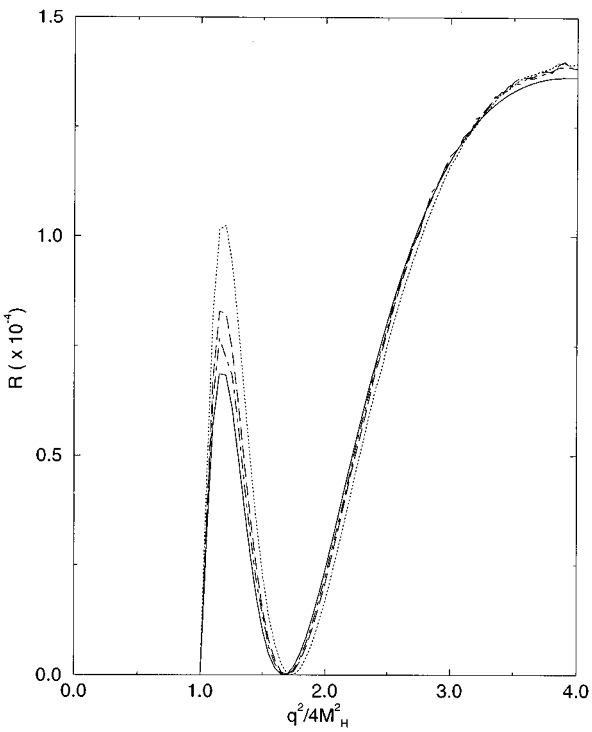

FIG. 3. Predictions of $R_{M_1M_2}=\sigma(e^+e^-\to M_1M_2)/\sigma(e^+e^-\to\mu^+\mu^-)$ for $M_1M_2=B_CB_C$. The normalization is fixed by taking $\gamma/m_2=1$. The values of β in MeV for the solid, dot-dashed, dashed, and dotted curves are 0, 30, 50, and 100, respectively.

$\beta=0$, in Fig. 3 reproduces the zero-binding result of Ref. [1]. As we can see in Fig. 3, for binding energy up to 100 MeV, our results are very close to the results from the peaking approximation. Even if we increase the binding energy, we did not find any large deviation from the peaking approximation and the qualitative feature remained the same.

IV. SUMMARY AND CONCLUSIONS

The peaking approximation has been used frequently in PQCD analyses involving heavy quark systems. We have investigated the validity of the peaking approximation for heavy quark systems by analyzing the pseudoscalar heavy meson pair production process in e^+e^- annihilation. We found that in the PQCD analysis, using the assignment of $k_i^\mu=x_iP^\mu$, the δ function is the only valid quark distribution amplitude to be consistent with the Lorentz and gauge invariance of the hard scattering amplitude. Thus, we used Eq. (4) instead of Eq. (1) as our starting point. We have computed all the light-cone time-ordered diagrams in the leading order of α_s, but including the higher-twist effects arising in the lowest Fock component of the hadron. The analytic results were summarized in Sec. II. However, the numerical results indicate that the peaking approximation may not be a bad approximation after all in the calculations involving heavy quark systems. Although we have focused on pseudoscalar heavy meson pair production processes, the same features should apply to other types of heavy mesons (vector, axial vector, etc.). We also expect that the general features of our discussion apply to other heavy hadron processes including heavy meson decay.

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[1] S. J. Brodsky and C.-R. Ji, Phys. Rev. Lett. **55**, 2257 (1985).

[2] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); Phys. Rev. Lett. **43**, 545 (1979).

[3] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B246**, 52 (1984); Phys. Rep. **112**, 173 (1984).

[4] See, for example, S. Gottlieb and A. S. Kronfeld, Phys. Rev. D **33**, 227 (1986).

[5] C.-R. Ji and A. F. Sill, Phys. Rev. D **34**, 3350 (1986).

[6] C. E. Carlson and J. Milana, Phys. Lett. B **301**, 237 (1993).

[7] F. Amiri and C.-R. Ji, Phys. Rev. D **38**, 776 (1988).

[8] M. S. Baek, S. Y. Choi, and H. S. Song, Phys. Rev. D **50**, 4363 (1994).

[9] Anselmino *et al.* have also considered mass corrections terms to the hard scattering amplitude in the processes of charmonium decays into light hadron pairs such as $\pi\pi$ or $p\bar{p}$: M. Anselmino, F. Caruso, and F. Murgia, Phys. Rev. D **42**, 3218 (1990); M. Anselmino, R. Cancelliere, and F. Murgia, *ibid.* **46**, 5049 (1992). They calculated the scattering amplitude by the assignment $k_i=x_iP$ and avoided the gauge invariance problem by postulating that the quarks masses are not fixed, but instead given by x_iM_H or y_iM_H. We do not adopt this scheme in this work of heavy meson pair production.

[10] C.-R. Ji, A. Pang, and A. Szczepaniak, Phys. Rev. D **52**, 4038 (1995).

[11] H.-n. Li, Phys. Lett. B **348**, 597 (1995); H.-n. Li and H.-L. Yu, *ibid.* **353**, 301 (1995).

[12] Complications that arise when the $q^+\neq 0$ frame is used for form factor analyses were pointed out in a few works: S. J. Brodsky and J. R. Hiller, Phys. Rev. D **46**, 2141 (1992); M. Sawicki, *ibid.* **46**, 474 (1992); M. Burkardt and A. Langnau, *ibid.* **44**, 3857 (1991).

[13] S. J. Brodsky and G. P. Lepage, Phys. Rev. Lett. **53**, 545 (1979); Phys. Lett. **87B**, 359 (1979); G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

[14] Z. Dziembowski, Phys. Rev. D **37**, 778 (1988).

[15] The normalization of $R_{M\bar{M}}$ in Fig. 1(a) of Ref. [1] is actually fixed by $\gamma=m_2$, instead of $\gamma=m_r$ as stated in Ref. [1]. In the case of $m_1\gg m_2$, the two different choices of γ values gives almost the same result. The choice of $\epsilon=\gamma$ in Eq. (11) of Ref. [1] is in fact valid only in equal mass case.