

New Goldstone multiplet for partially broken supersymmetry

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The partial spontaneous breaking of rigid $N=2$ supersymmetry implies the existence of a massless $N=1$ Goldstone multiplet. In this paper we show that the spin- $(1/2,1)$ Maxwell multiplet can play this role. We construct its full nonlinear transformation law and find the invariant Goldstone action. The spin-1 piece of the action turns out to be of Born-Infeld-type, and the full superfield action is duality invariant. This leads us to conclude that the Goldstone multiplet can be associated with a D-brane solution of superstring theory for $p=3$. In addition, we find that $N=1$ chirality is preserved in the presence of the Goldstone-Maxwell multiplet. This allows us to couple it to $N=1$ chiral and gauge field multiplets. We find that arbitrary Kähler and superpotentials are consistent with partially broken $N=2$ supersymmetry. [S0556-2821(97)00102-1]

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I. INTRODUCTION

The spontaneous breaking of rigid supersymmetry gives rise to a massless spin-1/2 Goldstone field $\psi_\alpha(x)$ [1]. When $N=2$ supersymmetry is broken to $N=1$, the Goldstone fermion belongs to a massless multiplet of the unbroken $N=1$ supersymmetry. One obvious Goldstone candidate is the $N=1$ chiral multiplet $(A+iB, \psi_\alpha, F+iG)$. In [2] we used this multiplet to construct a nonlinear realization of partially broken $N=2$ supersymmetry. We found that the complex spin-0 field $A+iB$ is the Goldstone boson associated with the broken central charge generator of $N=2$ supersymmetry; the complex auxiliary field $F+iG$ parametrizes the coset $SU(2)/U(1)$ of the automorphism group $SU(2)$. The superthreebrane of Liu, Hughes, and Polchinski [3] provides a different, but on-shell-equivalent representation of the same Goldstone multiplet.

A second candidate Goldstone multiplet is the $N=1$ vector, or Maxwell, multiplet (A_m, ψ_α, D) . In this case the superpartners of the spin-1/2 Goldstone field are an Abelian gauge field A_m and a real auxiliary field D . We will show that the Maxwell multiplet provides a second consistent Goldstone multiplet for partially broken $N=2$ supersymmetry. We will construct its invariant action and its couplings to $N=1$ matter fields.

Perhaps the most striking feature of the new Goldstone multiplet is its unification of a Goldstone and a gauge field. The theory of Goldstone fields is based on the formalism of nonlinear realizations, which is usually associated with finite-dimensional groups [4,5]. However, gauge fields can also be interpreted as Goldstone fields associated with infinite-dimensional symmetry groups [6]. This suggests that the full symmetry of the new multiplet is some infinite-dimensional extension of $N=2$ supersymmetry. As we shall see, the gauge field A_m has only nonminimal interactions; in other words, the field appears only via its field strength, so the gauge invariance is hidden. Hence we can use the original formalism of [4,5] to study the properties of the Goldstone-Maxwell multiplet.

This paper can be viewed as an outgrowth of an early attempt to partially break $N=2$ supersymmetry [7]. The

problem of ghost states is resolved by requiring the Goldstone multiplet to be an irreducible representation of $N=1$ supersymmetry. Recently, Antoniadis, Partouche, and Taylor [8] constructed a model with partially broken $N=2$ supersymmetry. In their model, the second supersymmetry is realized nonlinearly. However, their action involves an extra, massive, $N=1$ multiplet. Our approach is completely model independent; if the extra matter is integrated out, their action must reduce to ours.

Like the chiral Goldstone $N=1$ multiplet [2], the Goldstone-Maxwell multiplet has a superstring interpretation. It is related to the recently discovered Dirichlet p -branes [9]. These objects are solutions of the superstring equations of motion that can be viewed as dynamical membranes in $(p+1)$ world volume space. D-branes characteristically break half of the superstring supersymmetries and involve a $(p+1)$ -dimensional gauge field with the Born-Infeld action. Until now, only the bosonic parts of the D-brane actions have been constructed. We propose that the Goldstone-Maxwell action, after eliminating the auxiliary fields, is precisely the supersymmetric, gauge-fixed, D-brane action for $p=3$ (in a flat background): The gauge field A_m has a Born-Infeld action, and the full Goldstone-Maxwell action is duality invariant.

This paper is organized as follows. In Sec. II we review the formalism of nonlinear realizations. As we shall see, this technique has an ambiguity when applied to $N=2$ supersymmetry: dimensionless invariants can be used to modify the covariant derivatives and the covariant constraints. However, requiring consistency of the constraints fixes the ambiguity. In Sec. III we find a set of consistent constraints, to third order in the Goldstone fields. We then solve the constraints in terms of the ordinary $N=1$ Maxwell multiplet, and derive the broken supersymmetry transformations of the Goldstone-Maxwell multiplet to second order in fields. In Sec. IV we present the full nonlinear transformation law and derive the invariant action for the Goldstone-Maxwell multiplet. Surprisingly, we find that the gauge field is governed by the Born-Infeld action, and that the full action is invariant under a superfield duality transformation. In Sec. V we show that $N=1$ chirality is preserved in the presence of the Goldstone-Maxwell multiplet. This allows us to generalize the Kähler

potential to the case of partially broken $N=2$ supersymmetry. It also permits us to construct $N=2$ extensions of the general superpotential for chiral $N=1$ superfields, as well as the kinetic and Fayet-Iliopoulos terms for $N=1$ gauge superfields. Section VI contains concluding remarks.

II. NONLINEAR REALIZATIONS

In this section we review basics of nonlinear realizations as applied to $N=2$ supersymmetry. We begin with the $N=2$ supersymmetry algebra:

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a P_a, & \{S_\alpha, \bar{S}_{\dot{\alpha}}\} &= 2\sigma_{\alpha\dot{\alpha}}^a P_a, \\ \{Q_\alpha, S_\beta\} &= 0, & \{Q_\alpha, \bar{S}_{\dot{\alpha}}\} &= 0, \end{aligned} \quad (1)$$

where Q_α and S_α are the two supersymmetry generators and P_a is the four-dimensional momentum operator. In what follows, we take Q_α to be the unbroken $N=1$ supersymmetry generator, and S_α to be its broken counterpart.

Following the formalism of nonlinear realizations [4,5], we consider the coset space G/H , where G is the $N=2$ supersymmetry group and $H=SO(3,1)$ is its Lorentz subgroup. We parametrize the coset element Ω as

$$\Omega = \exp[i(x^a P_a + \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})] \exp[i(\psi^\alpha S_\alpha + \bar{\psi}_{\dot{\alpha}} \bar{S}^{\dot{\alpha}})]. \quad (2)$$

Here x, θ , and $\bar{\theta}$ are coordinates of $N=1$ superspace, while $\psi^\alpha = \psi^\alpha(x, \theta, \bar{\theta})$ and its conjugate $\bar{\psi}_{\dot{\alpha}} = \bar{\psi}_{\dot{\alpha}}(x, \theta, \bar{\theta})$ are Goldstone $N=1$ superfields of dimension $-1/2$. Note that these superfields are reducible; they contain spins up to $3/2$. In the next section we will reduce the representations by imposing $N=2$ covariant irreducibility constraints.

The group G acts on the coset space by left multiplication:

$$g\Omega = \Omega' h \quad (g \in G, h \in H). \quad (3)$$

In particular, under S supersymmetry, with $g = \exp[i(\eta S + \bar{\eta} \bar{S})]$, this implies

$$x'^a = x^a + i(\eta \sigma^a \bar{\psi} - \psi \sigma^a \bar{\eta}), \quad \theta' = \theta, \quad \bar{\theta}' = \bar{\theta}, \quad (4)$$

and

$$\begin{aligned} \psi'^\alpha(x', \theta', \bar{\theta}') &= \psi^\alpha(x, \theta, \bar{\theta}) + \eta^\alpha, \\ \bar{\psi}'_{\dot{\alpha}}(x', \theta', \bar{\theta}') &= \bar{\psi}_{\dot{\alpha}}(x, \theta, \bar{\theta}) + \bar{\eta}_{\dot{\alpha}}. \end{aligned} \quad (5)$$

The Cartan one-form $\Omega^{-1} d\Omega$,

$$\begin{aligned} \Omega^{-1} d\Omega &= i[\omega^\alpha(P) P_a + \omega^\alpha(Q) Q_\alpha + \bar{\omega}_{\dot{\alpha}}(\bar{Q}) \bar{Q}^{\dot{\alpha}} \\ &\quad + \omega^\alpha(S) S_\alpha + \bar{\omega}_{\dot{\alpha}}(\bar{S}) \bar{S}^{\dot{\alpha}}], \end{aligned} \quad (6)$$

defines covariant $N=1$ superspace coordinate differentials

$$\begin{aligned} \omega^\alpha(P) &= dx^a + i(d\theta \sigma^a \bar{\theta} + d\bar{\theta} \sigma^a \theta \\ &\quad + d\psi \sigma^a \bar{\psi} + d\bar{\psi} \sigma^a \psi), \\ \omega^\alpha(Q) &= d\theta^\alpha, \\ \bar{\omega}_{\dot{\alpha}}(\bar{Q}) &= d\bar{\theta}_{\dot{\alpha}}, \end{aligned} \quad (7)$$

and covariant Goldstone one-forms

$$\omega^\alpha(S) = d\psi^\alpha, \quad \bar{\omega}_{\dot{\alpha}}(\bar{S}) = d\bar{\psi}_{\dot{\alpha}}. \quad (8)$$

The supervielbein matrix E_M^A is found by expanding the one-forms $\omega^A \equiv (\omega^\alpha(P), \omega^\alpha(Q), \bar{\omega}_{\dot{\alpha}}(\bar{Q}))$ with respect to the $N=1$ superspace coordinate differentials $dX^M = (dx^m, d\theta^\mu, d\bar{\theta}_{\dot{\mu}})$,

$$\omega^A = dX^M E_M^A. \quad (9)$$

In a similar fashion, the covariant derivatives of the Goldstone superfield ψ^α are found by expanding $\omega^\alpha(S) = \omega^A \mathcal{D}_A \psi^\alpha$, which implies $\mathcal{D}_A \psi^\alpha = E_A^{-1M} \partial_M \psi^\alpha$. These covariant derivatives can be explicitly written as

$$\mathcal{D}_a = \omega_a^{-1m} \partial_m,$$

$$\begin{aligned} \mathcal{D}_\alpha &= D_\alpha - i(D_\alpha \psi \sigma^a \bar{\psi} + D_\alpha \bar{\psi} \sigma^a \psi) \omega_a^{-1m} \partial_m, \\ \bar{\mathcal{D}}_{\dot{\alpha}} &= \bar{D}_{\dot{\alpha}} - i(\bar{D}_{\dot{\alpha}} \psi \sigma^a \bar{\psi} + \bar{D}_{\dot{\alpha}} \bar{\psi} \sigma^a \psi) \omega_a^{-1m} \partial_m, \end{aligned} \quad (10)$$

where $\omega_m^a \equiv \delta_m^a + i(\partial_m \psi \sigma^a \bar{\psi} + \partial_m \bar{\psi} \sigma^a \psi)$ and $D_\alpha, \bar{D}_{\dot{\alpha}}$ are the ordinary flat $N=1$ superspace spinor derivatives.

$N=1$ matter superfields $\Phi(x, \theta, \bar{\theta})$ transform as follows under the full group G :

$$\Phi'(x', \theta', \bar{\theta}') = R(h) \Phi(x, \theta, \bar{\theta}), \quad (11)$$

where $R(h)$ is a matrix in a representation of the stability subgroup, $H=SO(3,1)$. Since there is no H connection in the right-hand side of Eq. (6), the covariant derivatives of the matter and Goldstone $N=1$ superfields are identical.

In what follows we will need the algebra of the covariant derivatives. This algebra can be worked out with the help of Eq. (10):

$$\begin{aligned} \{\mathcal{D}_\alpha, \mathcal{D}_\beta\} &= -2i(\mathcal{D}_\alpha \psi^\gamma \mathcal{D}_\beta \bar{\psi}^\gamma + \mathcal{D}_\beta \psi^\gamma \mathcal{D}_\alpha \bar{\psi}^\gamma) \sigma_{\gamma\gamma}^a \mathcal{D}_a, \\ \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} &= 2i\sigma_{\alpha\dot{\beta}}^a \mathcal{D}_a - 2i(\mathcal{D}_\alpha \psi^\gamma \bar{\mathcal{D}}_{\dot{\beta}} \bar{\psi}^\gamma \\ &\quad + \bar{\mathcal{D}}_{\dot{\beta}} \psi^\gamma \mathcal{D}_\alpha \bar{\psi}^\gamma) \sigma_{\gamma\gamma}^a \mathcal{D}_a, \\ [\mathcal{D}_a, \mathcal{D}_a] &= -2i(\mathcal{D}_\alpha \psi^\gamma \mathcal{D}_a \bar{\psi}^\gamma + \mathcal{D}_a \psi^\gamma \mathcal{D}_\alpha \bar{\psi}^\gamma) \sigma_{\gamma\gamma}^b \mathcal{D}_b. \end{aligned} \quad (12)$$

An important feature of this formalism is the existence of two dimensionless invariants, $\bar{\mathcal{D}}_{\dot{\alpha}} \psi_\alpha$ and $\mathcal{D}_\alpha \psi_\beta$ (together with their complex conjugates). These invariants render the standard formalism of nonlinear realizations somewhat ambiguous. For example, one can multiply the covariant derivative \mathcal{D}_α by one function of these invariants, or shift $\mathcal{D}_\alpha \psi_\beta$ by

another. This ambiguity will prove important in the next section; the reason for it and the way to overcome it will be discussed in the conclusions.

III. CONSISTENT COVARIANT CONSTRAINTS

The Goldstone superfield $\psi^\alpha(x, \theta, \bar{\theta})$ is a reducible representation of unbroken $N=1$ supersymmetry with highest spin $3/2$. It contains the spin- $(1, 1/2)$ Maxwell multiplet, but it also contains ghosts. The only way to eliminate the ghosts is to impose appropriate irreducibility constraints on the Goldstone superfield.

In this section we will find a set of consistent, $N=2$ covariant constraints which reduce ψ^α to the $N=1$ Maxwell multiplet. The elucidation of the proper constraints is complicated by the dimensionless invariants discussed in the previous section. Therefore we will adopt a perturbative approach and present a set of constraints that are consistent to the third order in the Goldstone fields.

As is well known (see, e.g., [10]), the Maxwell multiplet is described by a chiral $N=1$ field strength W_α of dimension $3/2$:

$$\bar{D}_{\dot{\alpha}} W_\alpha = 0. \quad (13)$$

The superfield W_α satisfies the irreducibility constraint

$$D^\alpha W_\alpha + \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} = 0. \quad (14)$$

The second constraint (14) must also satisfy a consistency condition: its left-hand side must vanish under D^2 (and \bar{D}^2).

The two constraints are solved by $W_\alpha = i\bar{D}^2 D_\alpha V$, where $V(x, \theta, \bar{\theta})$ is the Maxwell prepotential. The field V is defined modulo chiral gauge transformations, $\delta V = i(\Lambda - \bar{\Lambda})$, with $\bar{D}_{\dot{\alpha}} \Lambda = 0$.

To lowest order, we can identify

$$\psi_\alpha|_{\text{lin}} = \kappa^2 W_\alpha, \quad (15)$$

where the constant κ (of dimension -2) is the scale of S -supersymmetry breaking. In what follows, we set $\kappa = 1$.

Our aim is to generalize Eqs. (13) and (14) to obtain a set of constraints that are covariant under $N=2$ supersymmetry. The new constraints must be consistent and reduce to Eqs. (13) and (14) in the linearized approximation.

We begin by generalizing¹ Eq. (13):

$$\bar{D}_{\dot{\beta}} \psi_\alpha = 0. \quad (16)$$

Note that the right-hand side of this equation can, in principle, involve any power of the dimensionless invariants $D^\alpha \psi_\beta$ and $\bar{D}_{\dot{\beta}} \psi_\alpha$. However, it is easy to see that Eq. (16) is consistent as it stands. Indeed, Lorentz covariance implies that any terms on the right side (16) must be at least linear in $\bar{D}\psi$ and $D\bar{\psi}$. Hence the most general modification of Eq. (16) has the form $\bar{D}_{\dot{\alpha}} \psi_\alpha = M_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}} \bar{D}_{\dot{\beta}} \psi_\beta + N_{\alpha\dot{\alpha}}{}^{\beta\dot{\beta}} D_{\beta\dot{\beta}} \bar{\psi}_{\dot{\beta}}$, where the matrices M and N are at least linear in the Gold-

stone fields. This equation, together with its conjugate, imply Eq. (16). An important corollary of this result is the fact that $N=1$ chirality is preserved in the Goldstone background:

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0. \quad (17)$$

We will discuss the geometrical meaning of covariant chirality in Sec. V.

We now turn to Eq. (14). The simplest, most naive generalization, $\mathcal{D}^\alpha \psi_\alpha + \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = 0$, is not consistent at order $\mathcal{O}(\psi^3)$. Applying $\bar{\mathcal{D}}^2$ to the left-hand side gives

$$\begin{aligned} \bar{\mathcal{D}}^2(\mathcal{D}^\alpha \psi_\alpha + \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}) &= 4D\psi\partial^{\dot{\alpha}\alpha}\psi^\beta\partial_{\beta\dot{\alpha}}\psi_\alpha \\ &\quad + 8D^\alpha\psi^\beta\partial^{\dot{\alpha}\gamma}\psi_\gamma\partial_{\beta\dot{\alpha}}\psi_\alpha + \mathcal{O}(\psi^5). \end{aligned} \quad (18)$$

It is remarkable that there exists the following $N=2$ covariant generalization of the Maxwell constraints:

$$\begin{aligned} \mathcal{D}^\alpha \psi_\alpha + \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} - \frac{1}{2} \mathcal{D}^\alpha \psi^\beta \mathcal{D}_\beta \psi_\alpha \mathcal{D}^\gamma \psi_\gamma - \frac{1}{2} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} \bar{\mathcal{D}}_{\dot{\beta}} \bar{\psi}_{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\gamma}} \bar{\psi}^{\dot{\gamma}} \\ = \mathcal{O}(\psi^5). \end{aligned} \quad (19)$$

This constraint is consistent to order $\mathcal{O}(\psi^3)$, in the sense that $\bar{\mathcal{D}}^2$ [left-hand side (19)] = $\mathcal{O}(\psi^5)$.

The ambiguity of the standard nonlinear realization is completely fixed by the consistency requirements. In fact, higher-order terms can be added to the left-hand side of Eq. (19) to make it consistent to all orders. The structure of these higher-order corrections is likely to be related to hidden symmetries of the Goldstone-Maxwell multiplet.

The consistent covariant constraints (16) and (19) can be solved in terms of the $N=1$ Maxwell field strength W_α :

$$\psi_\alpha = W_\alpha + \frac{1}{4} \bar{\mathcal{D}}^2(\bar{W}^2) W_\alpha - i W^\beta \bar{W}^{\dot{\beta}} \partial_{\beta\dot{\beta}} W_\alpha + \mathcal{O}(W^5), \quad (20)$$

where $W^2 = W^\alpha W_\alpha$ and $\bar{W}^2 = \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$.

In what follows an important role is played by the nonlinear transformations of W_α under the second supersymmetry. To find them, let us first consider the form variation of ψ_α under S supersymmetry:

$$\begin{aligned} \delta^* \psi_\alpha &\equiv \psi'_\alpha(x, \theta, \bar{\theta}) - \psi_\alpha(x, \theta, \bar{\theta}) \\ &= \eta_\alpha - i(\eta^\beta \bar{\psi}^{\dot{\beta}} - \psi^\beta \bar{\eta}^{\dot{\beta}}) \partial_{\beta\dot{\beta}} \psi_\alpha. \end{aligned} \quad (21)$$

For the Maxwell field strength W_α , this implies

$$\delta^* W_\alpha = \eta_\alpha - \frac{1}{4} \bar{\mathcal{D}}^2(\bar{W}^2) \eta_\alpha - i \partial_{\alpha\dot{\alpha}}(W^2) \bar{\eta}^{\dot{\alpha}} + \mathcal{O}(W^4). \quad (22)$$

Note that this transformation preserves the defining linear constraints (13) and (14). The corresponding Maxwell prepotential transformation is

$$\delta^* V = \frac{i}{4} (\bar{\theta}^2 + \bar{W}^2) \theta \eta - \frac{i}{4} (\theta^2 + W^2) \bar{\theta} \bar{\eta} + \mathcal{O}(W^4). \quad (23)$$

¹Equation (16) was first discussed in similar context in [7].

The commutator of two such transformations reduces to an ordinary translation (plus a gauge transformation), as required by the algebra (1).

Using Eq. (22), we can find the $N=2$ invariant Goldstone-Maxwell action (to order W^6)

$$S_{\text{Goldst}} = \frac{1}{4} \int d^4x d^2\theta W^2 + \frac{1}{4} \int d^4x d^2\bar{\theta} \bar{W}^2 + \frac{1}{8} \int d^4x d^4\theta W^2 \bar{W}^2 + O(W^6). \quad (24)$$

The gauge field contribution to this action has the form

$$(S_{\text{Goldst}})|_{\text{gauge}} = \int d^4x \left[-\frac{1}{4} F_{mn} F^{mn} - \frac{1}{32} (F_{mn} F^{mn})^2 + \frac{1}{8} F_{mn} F^{nk} F_{kl} F^{lm} \right] + O(F^6). \quad (25)$$

The action (25) coincides with the expansion of the Born-Infeld action

$$S_{\text{BI}} = - \int d^4x \sqrt{-\det(\eta_{mn} + F_{mn})}. \quad (26)$$

In the next section we will see that this is not an accident; the full nonlinear action for the gauge field is precisely that of Born and Infeld [11].

IV. THE GOLDSTONE-MAXWELL MULTIPLER

A. The nonlinear transformation law

In this section we will extend the results of the previous section to all orders. Instead of generalizing the constraints (16) and (19), we will work directly with the $N=1$ Maxwell superfield W_α . We stress that all results of this section are nonperturbative.

We begin with the full nonlinear transformation law for W_α . To preserve the defining constraints (13) and (14) it must have the form

$$\delta^* W_\alpha = \eta_\alpha - \frac{1}{4} \bar{D}^2 \bar{X} \eta_\alpha - i \partial_{\alpha\dot{\alpha}} X \bar{\eta}^{\dot{\alpha}}, \quad (27)$$

where X is a chiral $N=1$ superfield which satisfies $\bar{D}_{\dot{\alpha}} X = 0$. The commutator of two such transformations obeys the $N=2$ algebra (1) if X transforms as

$$\delta^* X = 2 W^\alpha \eta_\alpha. \quad (28)$$

Note that the commutator of two such transformations gives the correct algebra.

The following recursive expression for X is chiral and has the required transformation properties:

$$X = \frac{W^2}{1 - (1/4) \bar{D}^2 \bar{X}}. \quad (29)$$

We will not derive this equation since it was guessed. However, once found, it can be justified by its consistency with Eqs. (27) and (28).²

Equation (29) can be used to expand X in powers of W^2 and its derivatives,

$$X = W^2 + \frac{1}{4} W^2 \bar{D}^2 (\bar{W}^2) + \frac{1}{16} W^2 [(\bar{D}^2 \bar{W}^2)^2 + \bar{D}^2 (\bar{W}^2 \bar{D}^2 \bar{W}^2)] + \dots \quad (30)$$

More importantly, it can also be used to find an explicit expression for X . To this end, we transform Eq. (29) as

$$X = W^2 + \frac{W^2}{4} \frac{\bar{D}^2 \bar{X}}{1 - \frac{1}{4} \bar{D}^2 \bar{X}} = W^2 + \frac{1}{4} \bar{D}^2 \left[\frac{W^2 \bar{W}^2}{[1 - (1/4) D^2 X][1 - (1/4) \bar{D}^2 \bar{X}]} \right]. \quad (31)$$

We note that the numerator in the square brackets involves the squares of the anticommuting spinor superfields W_α and $\bar{W}_{\dot{\alpha}}$. Since $W_\alpha W_\beta W_\gamma = 0$ and $\bar{W}_{\dot{\alpha}} \bar{W}_{\dot{\beta}} \bar{W}_{\dot{\gamma}} = 0$, the terms in the denominator which contain an undifferentiated W or \bar{W} must vanish. This implies that $D^2 X$ enters the denominator only in the ‘‘effective’’ form

$$(D^2 X)_{\text{eff}} = \frac{D^2 W^2}{1 - (1/4) (\bar{D}^2 \bar{X})_{\text{eff}}}. \quad (32)$$

This equation, together with its complex conjugate, gives rise to a quadratic equation for $(D^2 X)_{\text{eff}}$, with the solution³

$$(D^2 X)_{\text{eff}} = 2 + B - 2 \sqrt{1 - A + \frac{1}{4} B^2}, \quad (33)$$

where

$$A = \frac{1}{2} (D^2 W^2 + \bar{D}^2 \bar{W}^2),$$

$$B = \frac{1}{2} (D^2 W^2 - \bar{D}^2 \bar{W}^2). \quad (34)$$

Substituting this into Eq. (31), we find an explicit expression for X :

²Note that there can exist only one superfield X with the required properties: given two such superfields, X and X' , $X - X'$ is invariant under S supersymmetry. No such invariant of dimension 1 can be built from W_α , except for a constant. The constant part of X is fixed by requiring X vanish at $W_\alpha = 0$.

³The second solution does not vanish at $W=0$ and should be discarded.

$$X = W^2 + \frac{1}{2} \bar{D}^2 \left[\frac{W^2 \bar{W}^2}{1 - \frac{1}{2} A + \sqrt{1 - A + (1/4) B^2}} \right]. \quad (35)$$

B. The action

The superfield X plays a second important role: it is also a chiral density for the invariant Goldstone-Maxwell action. Indeed, the transformation property (28) implies that the chiral integral

$$\int d^4x d^2\theta X \quad (36)$$

is invariant under $N=2$ supersymmetry. The Q supersymmetry is manifest in Eq. (36), while the S invariance follows from the fact that $\int d^4x d^2\theta W^\alpha \eta_\alpha$ is a surface term. The Goldstone-Maxwell action is nothing but the real part of the invariant (36),⁴

$$\begin{aligned} S_{\text{GM}} &= \frac{1}{4} \int d^4x d^2\theta X + \frac{1}{4} \int d^4x d^2\bar{\theta} \bar{X} \\ &= \frac{1}{4} \int d^4x d^2\theta W^2 + \text{h.c.} \\ &\quad + \frac{1}{4} \int d^4x d^2\theta d^2\bar{\theta} \frac{W^2 \bar{W}^2}{1 - (1/2)A + \sqrt{1 - A + (1/4)B^2}}. \end{aligned} \quad (37)$$

By construction, this action is invariant under $N=2$ supersymmetry, where the second supersymmetry is given by Eqs. (27), (35), and (34). It is written in terms of the $N=1$ Maxwell multiplet field strength W_α and its derivatives.

Physically, the action (37) describes nonminimal couplings of massless spin-1/2 and spin-1 particles, with first- and second-order equations of motion, respectively. It does not involve any ghost states.

It is instructive to analyze the bosonic part of the action. To this end we set the fermionic field $W_\alpha|_{\theta=0} = 0$, and use the identities

$$\begin{aligned} D^\alpha W_\beta &= \frac{1}{4} (\sigma^{mn} F_{mn})_\beta{}^\alpha + \frac{i}{4} \delta_\beta^\alpha D, \\ D^2 W^2 &= -\frac{1}{2} F_{mn} F^{mn} - \frac{i}{2} F_{mn} \tilde{F}^{mn} + D^2, \end{aligned} \quad (38)$$

which hold at $W_\alpha|_{\theta=0} = 0$. Since Grassmann integration is equivalent to differentiation, Eqs. (37) and (38) imply that the real field D enters the bosonic action in a bilinear way. Therefore on shell, $D=0$, and the gauge field strength F_{mn} contains all the bosonic degrees of freedom.

The action for the gauge field F_{mn} can be written as

$$\begin{aligned} S_{\text{bosonic}} &= \int d^4x \left[1 - \left(1 + \frac{1}{2} F_{mn} F^{mn} + \frac{1}{8} (F_{mn} F^{mn})^2 \right. \right. \\ &\quad \left. \left. - \frac{1}{4} F_{mn} F^{nk} F_{kl} F^{lm} \right)^{1/2} \right]. \end{aligned} \quad (39)$$

Since in four dimensions

$$\begin{aligned} -\det(\eta_{mn} + F_{mn}) &= 1 + \frac{1}{2} F_{mn} F^{mn} + \frac{1}{8} (F_{mn} F^{mn})^2 \\ &\quad - \frac{1}{4} F_{mn} F^{nk} F_{kl} F^{lm}, \end{aligned} \quad (40)$$

this action coincides (up to additive constant) with the Born-Infeld action (26). It is remarkable that the Born-Infeld form of the gauge field action is dictated by the partially broken $N=2$ supersymmetry. Since the gauge field is a superpartner of the Goldstone fermion, this hints strongly that a Goldstone-type symmetry underlies the Born-Infeld action.

We should mention that the action (37) was first constructed in [12] as an $N=1$ generalization of the Born-Infeld action.⁵ As pointed out in [12], the $N=1$ supersymmetry is not sufficient to fix the action (37). Indeed, one can modify the $d^4\theta$ part of the $N=1$ Lagrangian by replacing $D^2 W^2 \rightarrow D^2 W^2 + a (D^\alpha W_\alpha)^2$, where a is any number. This clearly does not change the Born-Infeld form of the gauge field action. Note, however, that this modification is not consistent with the transformations (27) and (35). It is the second, nonlinear supersymmetry which unambiguously defines the form of the Goldstone-Maxwell action.

C. Duality

We now turn to the duality properties of the Goldstone-Maxwell action. Let us first recall that the Born-Infeld action possesses a certain duality invariance [13]. The duality stems from the fact that the action involves the field strength only; therefore one can relax the Bianchi identity $\epsilon^{mnlk} \partial_n F_{kl} = 0$ by including a Lagrange multiplier term

$$S_{\text{BI}}(F_{mn}) \rightarrow S_{\text{BI}}(F_{mn}) + \frac{1}{2} \int d^4x \tilde{A}_m \epsilon^{mnlk} \partial_n F_{kl}, \quad (41)$$

where \tilde{A}_m is the multiplier field. If one varies this action with respect to F_{mn} , and substitutes back the result, one recovers the Born-Infeld action for \tilde{A}_m itself.

The Goldstone-Matter action (37) enjoys a similar self-duality. This can be seen as follows. We first relax the Bianchi identity (14) by adding a superfield Lagrange multiplier term to Eq. (37):

$$\begin{aligned} S_{\text{GM}}(W) &\rightarrow S_{\text{GM}}(W) + \frac{i}{2} \int d^4x d^2\theta \tilde{W}^\alpha W_\alpha \\ &\quad - \frac{i}{2} \int d^4x d^2\bar{\theta} \bar{W}_\alpha \bar{W}^{\dot{\alpha}}. \end{aligned} \quad (42)$$

Here \tilde{W}^α is an $N=1$ Maxwell multiplet which serves as an $N=1$ Lagrange multiplier, and W_α is an arbitrary chiral $N=1$ superfield. Varying with respect to \tilde{W}^α reimposes the Maxwell constraint on W_α , while varying with respect to W_α gives rise to the Goldstone-Maxwell action for the field \tilde{W}^α .

To see how this works, let us first vary Eq. (42) with respect to W_α . This gives

⁴The imaginary part of Eq. (36) reduces to a surface term.

⁵We are grateful to E. Ivanov for introducing us to this paper.

$$W^\alpha = -i\widetilde{W}^\alpha \left(1 - \frac{1}{4}\overline{D}^2\overline{X}\right)t^{-1}, \quad (43)$$

where t satisfies the recursive equation

$$t = 1 - \frac{1}{4}\overline{D}^2(\widetilde{W}^2\overline{t}^{-1}). \quad (44)$$

We then substitute Eq. (43) back into Eq. (42). Let us focus on the part of the action which is an integral over the full $N=1$ superspace. The trick of Sec. IV can be used to write t and $\overline{D}^2\overline{X}$ in the ‘‘effective’’ form

$$t_{\text{eff}} = 1 - \frac{\overline{D}^2\widetilde{W}^2}{4\overline{t}_{\text{eff}}}, \quad \left(1 - \frac{1}{4}\overline{D}^2\overline{X}\right)_{\text{eff}} = \frac{t(\overline{t} - t + 1)}{\overline{t} + t - 1} \Big|_{\text{eff}}. \quad (45)$$

Solving Eq. (45) for t and substituting back into the action, we recover the Goldstone-Maxwell action (37) for the $N=1$ Maxwell superfield \widetilde{W}_α .

In this section we established that the Goldstone-Maxwell action possesses partially broken $N=2$ supersymmetry, that it is self-dual, and that its bosonic part reduces to the Born-Infeld action. These are exactly the properties that are expected from a supersymmetric D-brane action [9]. Thus we may conclude that Eq. (37) is, in fact, the gauge-fixed D-brane action in a flat background (after the auxiliary fields are eliminated).

V. $N=1$ MATTER AND THE GOLDSTONE-MAXWELL MULTIPLY

A. Chirality

The chirality constraint (16) and integrability of the covariant spinor derivatives (17) allow us to define $N=1$ chiral superfields in the Goldstone background:

$$\overline{D}_\alpha\Phi = 0. \quad (46)$$

To understand complex geometry behind this covariant chirality, and to explicitly solve the constraints (16) and (46), we consider another, complex parametrization Ω_L of the coset space G/H [see Eq. (2)]:

$$\Omega_L = \exp[i(x_L^a P_a + \theta^\alpha Q_\alpha + \psi^\alpha S_\alpha)] \exp[i(\overline{\theta}_\alpha \overline{Q}^\alpha + \overline{\psi}_\alpha \overline{S}^\alpha)], \quad (47)$$

where

$$x_L^a = x^a - i\theta\sigma^a\overline{\theta} - i\psi\sigma^a\overline{\psi}. \quad (48)$$

Since the generators $\overline{Q}^\alpha, \overline{S}^\alpha$ and the Lorentz generators form a (complexified) subalgebra \widetilde{H} of G , the coordinates $(x_L^a, \theta^\alpha, \psi^\alpha)$ of the coset G/\widetilde{H} form a closed subspace under $N=2$ supersymmetry:

$$x_L'^a = x_L^a - 2i\theta\sigma^a\overline{\epsilon} - 2i\psi\sigma^a\overline{\eta},$$

$$\theta'^\alpha = \theta^\alpha + \epsilon^\alpha, \quad (49)$$

and

$$\psi'^\alpha = \psi^\alpha + \eta^\alpha, \quad (50)$$

where ϵ^α and η^α are the first and second supersymmetry transformation parameters. This implies that we can choose a surface in this space in an $N=2$ covariant way:

$$\psi^\alpha = \psi^\alpha(x_L, \theta). \quad (51)$$

This equation is equivalent to the holomorphicity condition

$$\left(\frac{\partial}{\partial\theta^\alpha}\right)_L \psi^\alpha(x_L, \theta, \overline{\theta}) = 0. \quad (52)$$

In fact, using Eq. (52), one can show that the spinor covariant derivative \overline{D}^α becomes a partial derivative in terms of the complex coordinates $(x_L, \theta, \overline{\theta})$:

$$\overline{D}^\alpha = \left(\frac{\partial}{\partial\theta^\alpha}\right)_L. \quad (53)$$

Thus the holomorphicity condition (52) is equivalent to the $N=1$ chirality constraint (16), and the Goldstone-Maxwell superfield ψ^α is a chiral superfield. It is now obvious that the general solution to the covariant chirality constraint (46) is given by

$$\Phi = \Phi(x_L^a, \theta^\alpha). \quad (54)$$

B. Chiral superspace invariants

Having defined the chiral subspace (x_L^a, θ^α) , and chiral superfields covariant under $N=2$ supersymmetry, we are ready to construct the superspace invariants associated with chiral superfields. But first we need a chiral density whose transformation compensates for the chiral volume transformations

$$d^4x_L' d^2\theta' = (1 - 2i\partial_a^L \psi\sigma^a\overline{\eta}) d^4x_L d^2\theta. \quad (55)$$

One way to obtain such a density is to take the vielbein superdeterminant $E = \text{Ber}(E_M^A)$ and then change from the real coordinates $(x, \theta, \overline{\theta})$ to the complex coordinates $(x_L, \theta, \overline{\theta})$:

$$E_L \equiv E \text{Ber}\left(\frac{\partial(x_L, \theta, \overline{\theta})}{\partial(x, \theta, \overline{\theta})}\right). \quad (56)$$

The density E_L transforms correctly,

$$E_L' = (1 + 2i\partial_a^L \psi\sigma^a\overline{\eta}) E_L, \quad (57)$$

but it is not chiral: in the linearized approximation, $E_L = 1 + 2i\partial_a\psi\sigma^a\bar{\psi} + \mathcal{O}(\psi^4)$. The chirality can be restored with the help of the dimensionless invariants,

$$\begin{aligned}\hat{E}_L &= E_L \left(1 + \frac{1}{2} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}_{\dot{\beta}} \bar{\mathcal{D}}^{\dot{\alpha}} \bar{\psi}^{\dot{\beta}} + \mathcal{O}(\psi^4) \right) \\ &= 1 - \frac{1}{4} \bar{\mathcal{D}}^2(\bar{W}^2) + \mathcal{O}(\psi^4).\end{aligned}\quad (58)$$

The density \hat{E}_L transforms correctly, but is chiral [up to $\mathcal{O}(\psi^4)$ terms], as can be seen by its expansion in terms of W_{α} .

Having found the chiral density, we are ready to write the general $N=1$ superpotential in the Goldstone background. The coupling is just

$$S_{\text{superpot}} = \int d^4x_L d^2\theta \hat{E}_L P(\Phi), \quad (59)$$

where $P(\Phi)$ is an arbitrary holomorphic function of chiral superfields. This coupling is invariant under $N=2$ supersymmetry, up to $\mathcal{O}(\psi^4)$.

The discussion of the $N=1$ chiral matter interactions can be generalized to include gauge multiplets as well. The Maxwell gauge superfield is a real $N=1$ superfield, $\mathcal{A}(x, \theta, \bar{\theta})$, that is a scalar under $N=2$ supersymmetry:

$$\mathcal{A}'(x', \theta', \bar{\theta}') = \mathcal{A}(x, \theta, \bar{\theta}). \quad (60)$$

Under gauge symmetry, the superfield \mathcal{A} transforms as

$$\delta\mathcal{A} = i[\xi(x_L^a, \theta) - \bar{\xi}(\bar{x}_L^a, \bar{\theta})]. \quad (61)$$

where $\xi(x_L^a, \theta)$ is a (covariantly) chiral transformation parameter.

The kinetic term for \mathcal{A} can be written as an integral over chiral superspace. The first step is to construct the chiral gauge field strength, \mathcal{W}_{α} , in terms of the Maxwell superfield, \mathcal{A} , and the Goldstone superfield, W_{α} . The field \mathcal{W}_{α} must be a tensor under gauge symmetry as well as supersymmetry. It is

$$\begin{aligned}\mathcal{W}_{\alpha} &= i \left[\delta_{\alpha}^{\beta} + \frac{1}{8} \delta_{\alpha}^{\beta} \bar{\mathcal{D}}^2 \bar{W}^2 + \bar{\mathcal{D}}_{\dot{\beta}} (\mathcal{D}_{\alpha} W^{\beta} \bar{W}^{\dot{\beta}}) \right] [\bar{\mathcal{D}}^2 \mathcal{D}_{\beta} \\ &\quad + 4i(\mathcal{D}_{\beta\dot{\alpha}} W^{\gamma} \bar{\mathcal{D}}^{\dot{\alpha}} \bar{W}^{\dot{\gamma}}) \bar{\mathcal{D}}_{\dot{\gamma}} \mathcal{D}_{\beta}] \mathcal{A} + \mathcal{O}(W^4).\end{aligned}\quad (62)$$

Then the supersymmetric and gauge-invariant action is just

$$S_{\text{gauge}} = \frac{1}{4} \int d^4x d^2\theta \hat{E}_L \mathcal{W}^{\alpha} \mathcal{W}_{\alpha} + \text{H.c.}, \quad (63)$$

where \hat{E}_L is the chiral density defined above.

C. Full superspace invariants

The kinetic part of the chiral superfield action can also be written in the Goldstone background. In flat $N=1$ superspace, the kinetic action involves a Kähler potential, $K(\Phi, \bar{\Phi})$,

$$S_{\text{kin}}^{\text{flat}} = \int d^4x d^4\theta K(\Phi, \bar{\Phi}), \quad (64)$$

which is defined up to holomorphic Kähler transformations:

$$K' = K + \Lambda(\Phi) + \bar{\Lambda}(\bar{\Phi}). \quad (65)$$

To generalize Eqs. (64) and (65) in the Goldstone background, we need a real density \hat{E} with the property

$$\int d^4x d^4\theta \hat{E} f(x_L, \theta) = 0 \quad (66)$$

for an arbitrary chiral function f . Expanding the density $E = 1 + i\partial_a\psi\sigma^a\bar{\psi} + i\partial_{\dot{a}}\bar{\psi}\sigma^{\dot{a}}\psi + \mathcal{O}(\psi^4)$ and the function $f(x_L^a, \theta) = (1 - i\psi\sigma^b\bar{\psi}\partial_b)f(x^a - i\theta\sigma^a\bar{\theta}, \theta) + \mathcal{O}(\psi^4)$, we see that E does not satisfy Eq. (66). As above, we can use the dimensionless invariants to define a new density with the property (66):

$$\hat{E} = E(1 - \mathcal{D}\psi\bar{\mathcal{D}}\bar{\psi}) + \mathcal{O}(\psi^4). \quad (67)$$

The Kähler potential part of the matter action is simply

$$S_{\text{kin}} = \int d^4x d^4\theta \hat{E} K(\Phi, \bar{\Phi}), \quad (68)$$

and the Kähler potential enjoys the the invariance (65).

As discussed above, the matter couplings can be extended to include $N=1$ gauge multiplets as well. The kinetic term is easy to construct in the chiral subspace. Its associated Fayet-Iliopoulos term is given by

$$S_{\text{FI}} = \int d^4x d^4\theta \hat{E} \mathcal{A}, \quad (69)$$

which is invariant under gauge and $N=2$ supersymmetry transformations.

Thus we have seen that all the usual self-couplings of $N=1$ matter can be extended to the case of partially broken $N=2$ supersymmetry with the help of the Goldstone-Maxwell multiplet. A similar result holds for partially broken $N=2$ supersymmetry with a chiral Goldstone multiplet [2].

VI. CONCLUSIONS

In this paper we showed that there exists a new Goldstone multiplet for partially broken $N=2$ supersymmetry, the Goldstone-Maxwell multiplet. We found its exact nonlinear supersymmetry transformation and constructed the invariant Goldstone-Maxwell action. We also worked out the first perturbative terms of the $N=1$ matter couplings to the Goldstone-Maxwell multiplet. We found that the superspace description of the Goldstone-Maxwell multiplet requires two constraints, Eqs. (16) and (19). These constraints are presently on a different footing. The first, Eq. (16), is known in its full form; it has a clear geometrical interpretation in terms of $N=1$ chirality preservation. The second, Eq. (19), is only known in a perturbative expansion.

The derivation of the second constraint is obscured by two dimensionless invariants, $\mathcal{D}_{(\alpha}\psi_{\beta)}$ and $\mathcal{D}^{\alpha}\psi_{\alpha}$. These in-

variants can be identified (at $\theta=0$) with the gauge field strength, $F_{\alpha\beta}$, and the auxiliary field, D . It is instructive to compare this situation with that of the chiral Goldstone multiplet [2]. There *all* fields of the Goldstone multiplet were associated with symmetries, so each had a geometrical interpretation. For the case at hand, this suggests that we are missing the Goldstone-type symmetries associated with the gauge field strength and the auxiliary field of the Goldstone-Maxwell multiplet.

In fact, the D field of the Goldstone-Maxwell multiplet can be interpreted as the Goldstone field associated with the following subgroup of the $SU(2)$ automorphism group of the $N=2$ algebra:

$$\delta\theta^\alpha = i\lambda\psi^\alpha, \quad \delta\psi^\alpha = i\lambda\theta^\alpha. \quad (70)$$

Under such a transformation, the field D is shifted by the constant parameter λ . This $U(1)$ transformation is a symme-

try of the defining constraints (16) and (19). Note that the rest of the automorphism group $SU(2)$ explicitly breaks these constraints.

If we were to extend G in G/H by Eq. (70), we would eliminate the dimensionless invariant $\mathcal{D}^\alpha\psi_\alpha$. However, we would still have to contend with the dimensionless invariant associated with the gauge field strength, $\mathcal{D}_{(\alpha}\psi_{\beta)}$. This suggests that there exists an extension of $N=2$ supersymmetry which associates a Goldstone-like symmetry with this field strength.

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