

Intermediate scale as a source of lepton flavor violation in SUSY SO(10)

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In supersymmetric SO(10) grand unified models, we examine the lepton flavor violation process $\mu \rightarrow e \gamma$ from having the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry broken at an intermediate scale M_I below the SO(10) grand unification scale M_G . Even in the case that supersymmetry is broken by universal soft terms introduced at the scale M_G , we find significant rates for $\mu \rightarrow e \gamma$ with $M_I \sim 10^{12}$ GeV or less. These rates are further enhanced if the universal soft terms appear at a scale greater than M_G . [S0556-2821(96)50213-4]

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It has recently been pointed out [1–3] that significant lepton flavor violation can arise in supersymmetric (SUSY) grand unified theories. The origin of this flavor violation resides in the largeness of the top quark Yukawa coupling and the assumption that supersymmetry is broken by flavor uniform soft breaking terms communicated to the visible sector by gravity at a scale M_X . Assuming that M_X is the reduced Planck scale which is much greater than the grand unification scale M_G , renormalization effects cause the third generation multiplet of squarks and sleptons which belong to the same multiplet as the top in the grand unified theory (GUT) to become lighter than those of the first two generations. The slepton and the charged lepton mass matrices can no longer be simultaneously diagonalized thus inducing lepton flavor violation through a suppression of the Glashow-Iliopoulos-Maiani (GIM) mechanism in the slepton sector. This effect is more pronounced in SO(10) models than in SU(5) where the left-handed slepton mass matrices remain degenerate. The evolution of soft terms from M_X to M_G causes these flavor violations, which disappear when $M_X = M_G$. Here, we explore another class of theories which are SUSY SO(10) GUT's which break down to an intermediate gauge group G_I before being broken to the standard model (SM) gauge group at the scale M_I . In this class of theories, even if $M_X = M_G$, lepton flavor violation arises due to the effect of the third generation neutrino Yukawa coupling on the evolution of the soft leptonic terms from the grand unification scale to the intermediate scale. Depending on the location of the intermediate scale M_I and the size of the top Yukawa coupling at M_G , these rates can be within one order of magnitude of the current experimental limit. Our results will also indicate that if $M_X > M_G$ in SUSY SO(10) models with an intermediate scale, the predicted rates of lepton violating processes are further enhanced. We will concentrate on the decay $\mu \rightarrow e \gamma$ as an example since experimentally it is likely to be the most viable.

SO(10) [4] has many outstanding virtues that recommend it as a group for grand unification. Among them are (1) all fermions are in a single representation, (2) a possibility exists to understand the observed patterns of fermion masses and mixing [5] due in large part to useful SO(10) Clebsch factors, (3) small nonzero neutrino masses are generated

through the see-saw mechanism [6,7] and, (4) a scenario for baryogenesis is possible [8]. Further, SUSY is an attractive feature for GUT's because it provides a solution to the fine-tuning problem in the Higgs potential and, as a bonus, the third generation SM fermion masses are more easily accounted for [9]. In addition, SUSY SO(10) can accommodate a simple mechanism to solve the doublet-triplet splitting problem [10]. (SUSY) SO(10) also has the interesting feature that it allows an intermediate scale M_I before breaking to the standard model. In models where $M_I \sim 10^{10} - 10^{12}$ GeV, one can naturally get a neutrino mass in the interesting range of $\sim 3 - 10$ eV, which could serve as hot dark matter which may be needed to explain the observed large scale structure formation of the universe [11]. In models without an intermediate gauge symmetry, in principle one could produce a τ -neutrino Majorana mass that is much less than the GUT breaking scale, as for example via nonrenormalizable operators involving Higgs bosons in the SO(10) $\overline{16}$ representation or a small and carefully chosen Yukawa coupling to a $\overline{126}$ field. However, this would suffer from the further problem of abandoning $b - \tau$ Yukawa coupling unification except possibly in the case of high $\tan\beta$ [12]. As we have also verified, this problem is significantly reduced for the case of an intermediate scale [13,14]. The window $\sim 10^{10} - 10^{12}$ GeV is also of the right size for a hypothetical Peccei-Quinn (PQ) symmetry to be broken so as to solve the strong CP problem without creating phenomenological or cosmological problems [15]. Models which allow $M_I \sim 1$ TeV are also interesting since they would predict relatively light new gauge fields, as for example $SU(2)_R$ charged gauge bosons W_R . The thrust of this study will be on flavor nonconservation in the leptonic sector for $M_I < M_G$ and $M_X = M_G$ in SUSY SO(10). Towards this goal, we study one model where $M_I \sim 10^{11} - 10^{12}$ GeV range, one where M_I can be as low as the TeV range with left-right gauge symmetry being preserved in G_I , and one where M_I could have any value between the TeV scale and the GUT-breaking scale without violating any bounds on gauge couplings obtained from Z physics.

We will consider scenarios for which the intermediate gauge symmetry is $G_I \equiv SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$\times \text{SU}(3)_c$ although the concept that we illustrate in this paper is applicable to any GUT-breaking scenario for which an intermediate gauge symmetry exists under which right-handed neutrinos transform nontrivially. For the purpose of keeping the calculation as simple as possible, we will consider only the situation of a low value of $\tan\beta$. We will choose to use the arbitrary value $\tan\beta=2$ in our examples. (However, analogous to the dependence on $\tan\beta$ in the case of lepton flavor violation generated by physics above the GUT-breaking scale [3,16], using larger values of $\tan\beta$ tends to produce even greater values of lepton flavor violation.) $\tan\beta$ not being $\sim m_t/m_b$ requires two bidoublets, which transform as $(2,2,0,1)$ under the gauge symmetry G_I , to account for all the standard model fermion masses in a natural fashion. As a consequence the Yukawa couplings that give masses to the τ lepton and bottom quark are small enough so that terms of order λ_b^2 may be neglected in the renormalization group equations (RGE's). Below the scale M_I only the Yukawa coupling λ_t is relatively large, however, above M_I the top quark Yukawa coupling and the τ -neutrino Yukawa coupling are both large. In fact, at the GUT scale these two couplings are equal. The presence of this large τ -neutrino Yukawa coupling above the intermediate breaking scale causes the third generation slepton masses to be less than the other two generations, and hence the lepton flavor violation gets generated. It is also possible to produce lepton flavor violation in the minimal supersymmetric standard model (MSSM) with the introduction of an arbitrary intermediate scale $v_R \sim 10^{12}$ GeV for Majorana masses as in Ref. [17] and M_X again taken at the reduced Planck scale. We do not find this approach theoretically well motivated, and it also suffers from the technical shortcomings of the unification scale being taken at the reduced Planck scale while the MSSM gauge couplings apparently unify at a much lower scale M_G and the scale of v_R not being associated with any breaking of the gauge symmetry.

The superpotential terms which will be responsible for giving the SM fermion masses have the following form when G_I is the gauge symmetry:

$$W_Y = \lambda_{Q_u} Q_L \Phi_2 Q_R + \lambda_{L_\nu} L_L \Phi_2 L_R + \lambda_{Q_d} Q_L \Phi_1 Q_R + \lambda_{L_e} L_L \Phi_1 L_R, \quad (1)$$

where all group and generation indices have been suppressed, and $Q_{L,R}$ and $L_{L,R}$ represent the quark and lepton superfields which transform as doublets under $\text{SU}(2)_L$ or $\text{SU}(2)_R$ and Φ_1 and Φ_2 are the two bidoublets. We have assumed that Φ_2 contains the MSSM Higgs doublet which gives masses to the up quarks and Dirac masses for the neutrinos. Φ_1 contains the doublet which gives masses to the down quarks and the charged leptons. These Yukawa couplings run in G_I as

$$\mathcal{D} \ln \lambda_{Q_{uj}}^2 = - \sum_i c_i^{(\lambda_Q)} g_i^2 + (3+4\delta_{j3}) \lambda_{Q_i}^2 + \lambda_{\nu_\tau}^2, \quad (2)$$

$$\mathcal{D} \ln \lambda_{Q_{dj}}^2 = - \sum_i c_i^{(\lambda_Q)} g_i^2 + 4\delta_{j3} \lambda_{Q_i}^2, \quad (3)$$

$$\mathcal{D} \ln \lambda_{L_{vj}}^2 = - \sum_i c_i^{(\lambda_L)} g_i^2 + 3\lambda_{Q_i}^2 + (1+4\delta_{j3}) \lambda_{\nu_\tau}^2, \quad (4)$$

$$\mathcal{D} \ln \lambda_{L_{ej}}^2 = - \sum_i c_i^{(\lambda_L)} g_i^2 + 4\delta_{j3} \lambda_{\nu_\tau}^2, \quad (5)$$

where j refers to generation and i refers to the gauge group,

$$c^{(\lambda_Q)} = \left(3, 3, \frac{1}{6}, \frac{16}{3} \right), c^{(\lambda_L)} = \left(3, 3, \frac{3}{2}, 0 \right), \quad (6)$$

and we have used

$$\mathcal{D} \equiv \frac{16\pi^2}{2} \frac{d}{dt}, \quad (7)$$

where $t = \ln(\mu/\text{GeV})$ with μ being the scale.

Now we give the RGE's for the soft SUSY-breaking parameters which we need in the effective G_I theory. First of all, there are gaugino masses M_i corresponding to each g_i . Secondly, corresponding to each trilinear superpotential coupling λ_i there is a trilinear scalar term with the coupling $A_i \lambda_i$ at M_X . Finally there are soft scalar mass terms for each of the fields $Q_{L,R}$, $L_{L,R}$, and $\Phi_{1,2}$. The RGE's for these parameters are

$$\mathcal{D} M_i = b_i g_i^2 M_i, \quad (8)$$

$$\mathcal{D} A_{Q_{uj}} = \sum_i c_i^{(\lambda_Q)} g_i^2 M_i + (3+4\delta_{j3}) \lambda_{Q_i}^2 A_{Q_i} + \lambda_{\nu_\tau}^2 A_{\nu_\tau}, \quad (9)$$

$$\mathcal{D} A_{Q_{dj}} = \sum_i c_i^{(\lambda_Q)} g_i^2 M_i + 4\delta_{j3} \lambda_{Q_i}^2 A_{Q_i}, \quad (10)$$

$$\mathcal{D} A_{L_{vj}} = \sum_i c_i^{(\lambda_L)} g_i^2 M_i + 3\lambda_{Q_i}^2 A_{Q_i} + (1+4\delta_{j3}) \lambda_{\nu_\tau}^2 A_{\nu_\tau}, \quad (11)$$

$$\mathcal{D} A_{L_{ej}} = \sum_i c_i^{(\lambda_L)} g_i^2 M_i + 4\delta_{j3} \lambda_{\nu_\tau}^2 A_{\nu_\tau}, \quad (12)$$

$$\mathcal{D} M_{Q_{jL,R}}^2 = - \sum_i c_i^{(Q_{L,R})} g_i^2 M_i^2 + 2\lambda_{Q_i}^2 X_Q \delta_{j3}, \quad (13)$$

$$\mathcal{D} M_{L_{jL,R}}^2 = - \sum_i c_i^{(L_{L,R})} g_i^2 M_i^2 + 2\lambda_{\nu_\tau}^2 X_L \delta_{j3}, \quad (14)$$

$$\mathcal{D} M_{\Phi_1}^2 = - \sum_i c_i^{(\Phi)} g_i^2 M_i^2, \quad (15)$$

$$\mathcal{D} M_{\Phi_2}^2 = - \sum_i c_i^{(\Phi)} g_i^2 M_i^2 + 3\lambda_{Q_i}^2 X_Q + \lambda_{\nu_\tau}^2 X_L, \quad (16)$$

where

$$X_Q \equiv M_{Q_L}^2 + M_{Q_R}^2 + M_{\Phi_2}^2 + A_{Q_i}^2, \quad (17)$$

$$X_L \equiv M_{L_L}^2 + M_{L_R}^2 + M_{\Phi_2}^2 + A_{L_\tau}^2, \quad (18)$$

and

$$c^{(Q_L)} = \left(3, 0, \frac{1}{6}, \frac{16}{3}\right), c^{(Q_R)} = \left(0, 3, \frac{1}{6}, \frac{16}{3}\right), \quad (19)$$

$$c^{(L_L)} = \left(3, 0, \frac{3}{2}, 0\right), c^{(L_R)} = \left(0, 3, \frac{3}{2}, 0\right), \quad (20)$$

$$c^{(\Phi)} = (3, 3, 0, 0). \quad (21)$$

At the scale M_G , we assume a universal form to the soft SUSY-breaking parameters, i.e., all gaugino masses $M_i(M_G) = m_{1/2}$, all trilinear scalar couplings $A_i(M_G) = A_0$, and all soft scalar masses $m_i^2(M_G) = m_0^2$. We also assume $\lambda_{Q_{l,b}}(M_G) = \lambda_{L_{\nu,\tau}}(M_G)$ since quarks and leptons become unified in SO(10). At the scale M_I , we match the G_I effective theory parameters with the MSSM parameters in the usual fashion. We run all the RGE's according to MSSM [18–20] down to the top scale which we take to be 175 GeV. All RGE's are integrated numerically. We note that since the rank of the SM gauge group is one less than G_I , the soft scalar masses may receive D -term contributions proportional to an additional parameter at the intermediate scale, however, for simplicity we take this extra unknown parameter to be zero.

We examine three unification scenarios as our examples. Since in all of our examples no $\mathbf{126} + \overline{\mathbf{126}}$ representation fields are used to be compatible with superstring derived models, SO(10) singlets are used to give neutrino Majorana masses as explained in Ref. [21]. Also, in all the models we will discuss the value of the b -quark running mass m_b tends to be in the vicinity of 4.9 GeV, which is a little high, however, the threshold corrections to m_b can easily be of the order of 10% [22]. Also, whatever operators give masses to the other two generations of down quarks and charged leptons could be of the right size to fix this problem.

Scenario (a): This model is case V of Ref. [13]. In the effective theory above the scale M_I , the number n_H of bidoublet, (2,2,0,1), copies is 2 and number n_X of SU(2) $_R$ doublet, (1,2,1/2,1)+(1,2,-1/2,1) copies belonging to $\mathbf{16} + \overline{\mathbf{16}}$ representation of SO(10) is 4. The scalar components of these right-handed doublets develop the vacuum expectation value (VEV) which breaks G_I to the SM gauge group. In this scenario, we use $M_I \approx 10^{12}$ GeV and $M_G \approx 10^{15.6}$ GeV leading to $\alpha_s(M_Z) \approx 0.129$. Scenario (b): This is the model presented in Ref. [23]. It is the only example we use for which D parity is not broken at M_G and hence left-right parity ($g_L = g_R$) is preserved in G_I . In this model above M_I , $n_H = 2$, $n_X = 1$ along with one (2,1,-1/2,1)+(2,1,1/2,1) superfield belonging to $\mathbf{16} + \overline{\mathbf{16}}$ representation as demanded by D parity, and in addition to this minimal field content there exist two copies of (1,1,-1/3,3)+(1,1,1/3, $\bar{3}$) and (1,1,-1,1)+(1,1,1,1) from the $\mathbf{10}$ and $\mathbf{120}$ representations of SO(10), respectively. This particle content allows $M_I \sim 1$ TeV with $M_G \approx 10^{16}$ GeV. We use MSSM below the scale M_I for convenience although in the original work [23] the two Higgs doublet model (2HDM) has been used. The value of α_s with 2HDM below M_I is about 0.117 and with MSSM it becomes about 0.129. Our result however has very little sensitivity to the value of α_s . One might have expected this

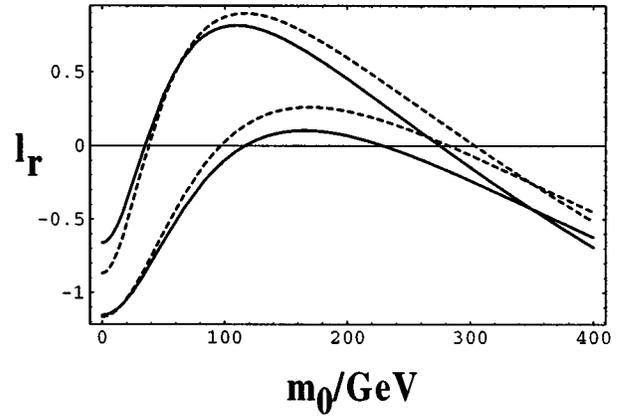


FIG. 1. $l_r \equiv \log_{10}(B/B_{\text{expt}})$ in scenario (a) is plotted as a function of the universal soft mass m_0 . The solid lines correspond to $\mu > 0$, while the dashed lines correspond to $\mu < 0$. The upper two lines in the vicinity of $m_0 = 150$ GeV are for $m_{1/2} = 120$ GeV, and the lower two lines are for $m_{1/2} = 200$ GeV. $\lambda_{Q_{l,G}} = 3.54$ for all the lines.

scenario to have greater lepton flavor violation than scenario (a) has since it has a lower intermediate scale, however this is not true due to the fact the generationally blind gaugino loop contribution to the slepton masses is greater in this scenario since α_G is greater.

Scenario (c): This is the model discussed in Ref. [24]. In this model M_G is predicted to be exactly the same as in the conventional SUSY SO(10) breaking with no intermediate scale and the scale M_I can have any value between the TeV and the GUT scales. In this model, $n_H = 1$ and $n_X = 3$. Since there is only one Higgs bidoublet, this model prefers large values of $\tan\beta$ with $\lambda_t = \lambda_b$ at M_I . Nevertheless, the introduction of nonrenormalizable operators can allow for small $\tan\beta$. Since this model has the unique property that M_I is arbitrary, we use this model as an example of how the $\mu \rightarrow e\gamma$ branching ratio changes as a function of M_I . The value of the α_s at the weak scale is about 0.122.

For all three scenarios, we run the gauge couplings at two loops although we neglect the small effect with low $\tan\beta$ of the Yukawa couplings on the gauge coupling running. The G_I gauge β functions for scenarios (a) and (c) are given in Ref. [24]. The G_I gauge β functions for scenarios (b) are given in Ref. [23].

The expression with which we calculate the width for $\mu \rightarrow e\gamma$ is given by Eqs. (29)–(31) in Ref. [2]. This expression [25] for the width has been determined from the mass interaction basis, which is an excellent approximation for the low $\tan\beta$ parameter space which we consider. The Cabibbo-Kobayashi-Maskawa (CKM) elements needed for the amplitude are calculated at the intermediate scale. We use the neutralino mass matrix as given by Eq. (44) of Ref. [19].

In Figs. 1–3, we show our results by plotting the function

$$l_r \equiv \log_{10} \left(\frac{B}{B_{\text{expt}}} \right), \quad (22)$$

where B is the predicted $\mu \rightarrow e\gamma$ branching ratio and $B_{\text{expt}} = 4.9 \times 10^{-11}$ being the experimental 90% confidence limit upper bound on the branching ratio. In all the figures,

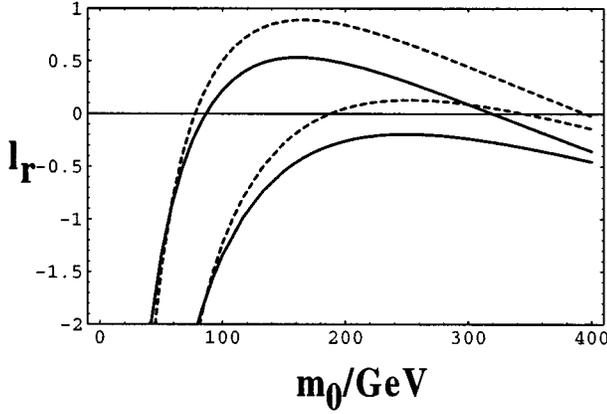


FIG. 2. l_r in scenario (b) is plotted as a function of the universal soft mass $m_{1/2}$. The solid lines correspond to $\mu > 0$ and the dashed lines correspond to $\mu < 0$. The upper two lines around $m_0 = 150$ GeV are for $m_{1/2} = 120$ GeV, and the lower two lines in that region are for $m_{1/2} = 200$ GeV. $\lambda_{Q_{tG}} = 3.54$.

we assume that the universal M_G scale trilinear soft scalar interaction coupling $A_0 = 0$. In any parameter space which we show, we have checked that the lightest slepton is not lighter than 43 GeV and the lightest neutralino is not lighter than 20 GeV, so as to be consistent with their experimental lower bounds of these masses. In general, we will find that $\mu < 0$ gives a greater branching ratio than $\mu > 0$, where μ is the bilinear MSSM Higgs superpotential coupling which we have calculated at the tree level [see, for example, Eq. (22) of Ref. [19]]. This is because with $A_0 = 0$, A_i is always negative at the weak scale, and the part proportional to $\mu \tan\beta + A_i$ in the $\mu \rightarrow e\gamma$ amplitude has the dominant contribution.

In Fig. 1, we show l_r as predicted by scenario (a) as a function of the universal soft mass m_0 for the cases of universal gaugino mass $m_{1/2} = 120$ GeV and $m_{1/2} = 200$ GeV. We have taken the M_G scale top quark Yukawa coupling $\lambda_{Q_{tG}} = 3.54$. The dashed lines correspond to $\mu < 0$ while the solid ones correspond to $\mu > 0$. The upper two lines in the vicinity of $m_0 = 150$ GeV correspond to $m_{1/2} = 120$ GeV, and the lower two lines in the same region correspond to $m_{1/2} = 200$ GeV. We find the size of lepton flavor violation through $\mu \rightarrow e\gamma$ predicted by these two choices of gaugino masses are fairly typical of those predicted by lighter $m_{1/2} < 200$ GeV gaugino masses over the given range of m_0 .

In Fig. 2, we show l_r as predicted by scenario (b) again as a function of the universal soft mass m_0 for the cases of universal gaugino mass $m_{1/2} = 120$ GeV and $m_{1/2} = 200$ GeV. The dashed lines correspond to $\mu < 0$ while the solid ones correspond to $\mu > 0$. The upper two lines around $m_0 = 150$ GeV correspond to $m_{1/2} = 120$ GeV, and the lower two lines in the same region correspond to $m_{1/2} = 200$ GeV. Once again, we have taken the M_G scale top quark Yukawa coupling $\lambda_{Q_{tG}} = 3.54$. In this example, we plot only those points where l_r is less than two orders of magnitude beneath the current experimental limit.

In Fig. 3, we show l_r in scenario (c) as a function of $\log_{10}(M_I/\text{GeV})$ for the cases $\lambda_{Q_{tG}} = 3.54$ and $\lambda_{Q_{tG}} = 1.38$. The dashed lines correspond to $\mu < 0$ while the solid ones

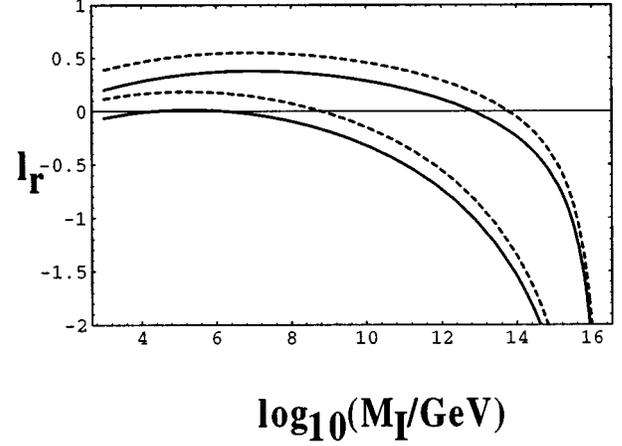


FIG. 3. l_r in scenario (c) is plotted as a function of $\log_{10}(M_I/\text{GeV})$. The solid lines correspond to $\mu > 0$; the dashed lines correspond to $\mu < 0$. The upper two lines around $M_I = 10^8$ GeV correspond to $\lambda_{Q_{tG}} = 3.54$, and the lower two lines in the same region correspond to $\lambda_{Q_{tG}} = 1.38$. $m_0 = m_{1/2} = 180$ GeV for all the lines.

correspond to $\mu > 0$. The values of m_0 and $m_{1/2}$ are both chosen to be 180 GeV for all the lines. The upper two lines around $M_I \sim 10^8$ GeV correspond to $\lambda_{Q_{tG}} = 3.54$, and the lower two lines in the same region correspond to $\lambda_{Q_{tG}} = 1.38$. Here we want to show the dependence of l_r on the intermediate scale. For values of M_I less than about 10^6 GeV, we see that the two cases of $\lambda_{Q_{tG}}$ predict relatively similar values of l_r . Notice that l_r has a maximum for $M_I \sim 10^7$ GeV, rather than l_r monotonically increasing as M_I is decreased. This is caused by the fact that the gaugino contribution to the scalar masses is increased with decreasing M_I and that $\lambda_{L_{\nu\tau}}$ is no longer of order $\lambda_{Q_{tG}}$ at the intermediate scale since it has a much lower fixed point than $\lambda_{Q_{tG}}$.

With M_X taken at the reduced Planck scale, as expected one finds an enhanced branching ratio for $\mu \rightarrow e\gamma$. It is almost impossible to find parameter space with smaller values of μ (i.e., $|\mu| < 500$ GeV). As a result the parameter space gets restricted. As an example, in the scenario (a), the l_r is 1.61 when $m_0 = 100$ GeV and $m_{1/2} = 400$ GeV with μ is around 800 GeV. A similar situation also happens in the cases of other two scenarios.

Between the GUT's and the intermediate scale, the large τ -neutrino Yukawa coupling affects the slepton sector only; the flavor violation in the quark sector does not get modified significantly by the inclusion of this intermediate scale. This implies that the parameter space analysis by the constraint from $b \rightarrow s\gamma$ with the universal soft terms at the GUT-breaking scale [26] or at the reduced Planck scale [27,16] still holds in this case unless light $SU(2)_R$ gauge fields exist [28].

In conclusion, we find that an intermediate gauge symmetry breaking is a significant source of lepton flavor violation in SUSY SO(10) models with GUT scale uniform soft SUSY-breaking terms. In fact, the present limit on $\mu \rightarrow e\gamma$ rate already puts some limits on the soft breaking parameters m_0 and $m_{1/2}$. The cause of the lepton flavor violation is simply that the τ -neutrino's Yukawa coupling is equal to that

of the top quark at the GUT scale and that the τ -neutrino's Yukawa coupling reduces the mass of the third generation sleptons relative to that of the first two generations via their renormalization group evolution from the GUT-breaking scale down to the intermediate gauge symmetry breaking scale. Hence the principle discussed in this paper is applicable to all SUSY grand unified models which have the $B-L$ gauge symmetry broken at an intermediate scale. Of course in such models as we have considered, if the scale at which the soft terms appear is higher than the GUT-breaking scale, for example at the reduced Planck scale, the predicted rate of lepton flavor violation gets enhanced as well. We shall present other consequences of an intermediate gauge symmetry, such as additional contributions to the electric dipole moments of the electron and neutron, in a more detailed publication. Although as we have discussed an inter-

mediate gauge symmetry breaking scale in a grand unified theory can be useful and offers interesting phenomenology, such models also have an additional scale of symmetry breaking to be explained. However, our results do not depend on the nature of the mechanism that determines this scale. We also note that if one desires a model with neutrino Majorana masses or PQ-symmetry breaking in the range of 10^{10} – 10^{12} GeV then an additional scale has already been introduced into the model, and perhaps the simplest explanation of which would be to associate it with the breaking of a gauge symmetry at that scale.

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