

New effective Feynman-like rules for the multi-Regge QCD asymptotics of inclusive multijet production

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New effective Feynman-like rules are defined for inclusive multijet cross sections in the multi-Regge regime. The solution of the Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation is used as a starting point. The resulting rules involve conformal weight and rapidity as a momentum and a coordinate, respectively, and are translation invariant in the coordinates. We use the effective rules to calculate ultrahigh-energy asymptotics of inclusive multijet production. The dependence on the parton densities occurs only in the overall normalization of the asymptotic cross sections. [S0556-2821(96)50113-X]

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The Balitskii-Fadin-Kuraev-Lipatov (BFKL) Pomeron [1,2] lies on the border of the known territory of perturbative QCD. It may give access to a new physics of parton liquids [3] and it has inspired unexpected theoretical developments [4], but it first requires experimental justification. A natural place to look for that is in inclusive jet production in high-energy hadron collisions [5–15]. The main problems are (i) to define selection criteria for the events that would ensure the applicability of the BFKL analysis [16] and (ii) to manage the complexity of the BFKL asymptotics for the cross sections. We deal here with the second problem. Namely, we give a new representation for the asymptotic cross sections by means of effective Feynman-like rules. They factorize the full expressions for the asymptotic cross sections into vertices and propagators of an effective theory. A crucial property of the presented rules is that they include propagators corresponding to the parton distributions. That allows one to treat systematically integrations over parameters of untagged most forward or backward jets that are indispensable to calculations of quantities measurable with the detectors of limited acceptance [14].

We illustrate the use of the effective rules in a calculation of the ultrahigh-energy asymptotic behavior of the cross sections. We begin by defining kinematic regime. We consider the inclusive differential cross section of N -jet production at a fixed scale of large (with respect to the hadronic scale) transverse momenta and very large (much larger than the transverse momenta) total energy of colliding hadrons. The invariant mass of any jet pair is also supposed to be much larger than the transverse momenta. This is known as the multi-Regge regime. As discussed in [14], the cross section in this regime is a sum of terms each of which is, crudely speaking, a product of BFKL Pomerons:

$$d\sigma_N^{\text{sum}} = d\sigma_N + \int \left(d\Omega_f \frac{d\sigma_{f+N}}{d\Omega_f} + \frac{d\sigma_{N+b}}{d\Omega_b} d\Omega_b \right) + \int d\Omega_f \frac{d\sigma_{f+N+b}}{d\Omega_f d\Omega_b} \Omega_b, \quad (1)$$

where $d\sigma_N^{\text{sum}}$ is the differential cross section on the phase space of N massless partons (jets for us are descendants of massless partons), $d\Omega_{f(b)}$ is the differential volume of the most forward (backward) jet phase space, and, for example, $d\sigma_{f+N}$ is the cross section on the phase space of N tagged jets plus the untagged most forward jet of an event. The first term of the sum comes from events with the most forward and backward jets both among the tagged jets (and it is absent in the case of $N=1$).

We will describe simple rules to write down $d\sigma_N$ of Eq. (1). To this end consider the simplest representative case of $N=3$. $d\sigma_3$ integrated over the phase space of the most forward jet is (see Eq. (9) of Ref. [14])

$$\begin{aligned} \int d\Omega_f \frac{d\sigma_{f+2}}{d\Omega_f} &= d\Omega_2 \frac{\alpha_S C_A}{k_{2\perp}^2} \frac{\alpha_S C_A}{k_{3\perp}^2} \frac{2\alpha_S C_A}{\pi^2} \int_{x_2}^1 dx_1 F_A(x_1, \mu_1^2) \\ &\times \int_{\mu_1}^{x_1 \sqrt{s}} \frac{d^2 k_{1\perp}}{k_{1\perp}^2} \int d^2 q_{\perp} f^{\text{BFKL}}(k_{1\perp}, q_{\perp}, y_1(x_1, k_{1\perp})) \\ &\times f^{\text{BFKL}}(q_{\perp} + k_{2\perp}, k_{3\perp}, y) F_B(x_3, \mu_2^2), \end{aligned} \quad (2)$$

where $d\Omega_2 = dx_2 dx_3 d^2 k_{2\perp} d^2 k_{3\perp}$.

Let us explain the notations and meaning of Eq. (2). The differential cross section of three jets integrated over the phase space of the most forward jet is on the left-hand side (lhs) of the equation. The same integration in the right-hand side (rhs) is over the longitudinal (x_1) and transverse ($k_{1\perp}$) momenta of the most forward jet. Longitudinal momenta are normalized to the half of the total energy and thus x_1 is the fraction of the hadron momentum carried by the parton that scattered to produce the most forward jet. As such it enters also as a variable of the effective parton distribution function

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F_A [17] of the hadron A . Another variable on which $F_{A,B}$ depend is the factorization scale $\mu_{1,2}$ (one can take $\mu_{1,2} = \mu \sim \min\{k_{i\perp}\}$). $d\Omega_2$ is the differential volume of the phase space of the two jets (all jets but the most forward of the set $f+2$) while x_i and $k_{i\perp}$ are their longitudinal and transverse momenta, respectively. The most remarkable objects in the rhs are f^{BFKL} . They describe correlations between transverse momenta of t -channel Reggeized gluons [1] emitted from a pair of tagged jets nearest in rapidity space, and depend on the rapidity intervals

$$y_1(x_1, k_{1\perp}) = \ln(x_1 k_{1\perp} / x_2 k_{2\perp}), \quad y = \ln(x_2 x_3 s / k_{2\perp} k_{3\perp}) \quad (3)$$

spanned by the jet pairs (in the above formulas it is supposed that the middle jet 2 is in the forward direction; s is the squared total energy of the collision). The superscript BFKL is to recall that f^{BFKL} is the solution of the Balitskii-Fadin-Kuraev-Lipatov equation [1]. The f^{BFKL} depending on y_1 is called in Ref. [14] the adjacent (to the hadron A) Pomeron, while that depending on y is the inner Pomeron (it is developed between the tagged jets). The solution for the BFKL equation has the integral representation [1]

$$f^{\text{BFKL}}(k_{1\perp}, k_{2\perp}, y) = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \chi_{n,\nu}(k_{1\perp}) e^{y\omega(n,\nu)} \chi_{n,\nu}^*(k_{2\perp}), \quad (4)$$

where the asterisk means complex conjugation,

$$\chi_{n,\nu}(k_{\perp}) = (k_{\perp}^2)^{-1/2+i\nu} e^{in\varphi/2\pi} \quad (5)$$

are Lipatov's eigenfunctions, and

$$\omega(n,\nu) = \frac{2\alpha_S C_A}{\pi} \left[\psi(1) - \text{Re}\psi\left(\frac{|n|+1}{2} + i\nu\right) \right]$$

are Lipatov's eigenvalues. Here ψ is the logarithmic derivative of the Euler Γ function. The summation in Eq. (4) runs over conformal spin indices n and the integration is over conformal dimension $d=1-2i\nu$. Combinations $h=1+n/2-i\nu$, $\bar{h}=1-n/2-i\nu$ are known as conformal weights.

Here we should comment on the present status of our basic formulas in Eqs. (1) and (2). They are similar to one of the formulas of the naive parton model prior to the proofs of the QCD factorization theorems (see for review Ref. [18]). The recent phenomenological estimations of the applicability of the formulas such as Eqs. (1) and (2) see, e.g., Ref. [19], and the attempt to prove the relevant factorization in Ref. [20].

From now on we start to restructure $d\sigma_{f+2}$ from Eq. (2). First, one can integrate out the transverse momentum q_{\perp} of the t -channel Reggeized gluon. To this end, the following formula may be used:

$$\int \frac{d^2 q_{\perp}}{q_{\perp}^2} \chi_{n,\nu}^*(q_{\perp}) \chi_{m,\lambda}(q_{\perp} + k_{\perp}) = \pi \chi_{n,\nu}^*(k_{\perp}) \chi_{m,\lambda}(k_{\perp}) \times \frac{i^{|m-n|-|m|+|n|} \Gamma\left(\frac{|m-n|}{2} + 1 - i(\lambda - \nu)\right) \Gamma\left(\frac{|m|+1}{2} + i\lambda\right) \Gamma\left(\frac{|n|+1}{2} - i\nu\right)}{\frac{|m-n|}{2} - i(\lambda - \nu + i\epsilon) \Gamma\left(\frac{|m-n|}{2} + 1 + i(\lambda - \nu)\right) \Gamma\left(\frac{|m|+1}{2} - i\lambda\right) \Gamma\left(\frac{|n|+1}{2} + i\nu\right)}, \quad (6)$$

where $i\epsilon$ takes care of the singularity at $m-n=\lambda-\nu=0$. The result of this transverse momentum integration is an integral representation for $d\sigma_{f+2}$. Next, we rewrite it in new variables for jet momenta and momenta of incoming hadrons A and B . To parametrize the light-cone components of hadron momenta $p_A^+, p_B^-, s = p_A^+ p_B^-$ we take

$$x_0^+ = \ln(p_A^+ / \mu), \quad x_4^- = -\ln(p_B^- / \mu), \quad (7)$$

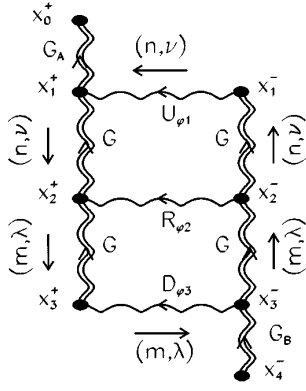
and to parametrize jet four-momenta k_i , $i=1,2,3$,

$$x_i^+ = \ln(k_i^+ / \mu), \quad x_i^- = -\ln(k_i^- / \mu), \quad (8)$$

where $k_{i\pm} = k_{i0} \pm k_{i3}$ are the light-cone components of k_i ($i=1$ corresponds to the most forward jet above). A virtue of

these variables is that the cross section is invariant under translations $x_i^+ \rightarrow x_i^+ + a$, $x_i^- \rightarrow x_i^- + a$:

$$\frac{s d\sigma_3}{\pi^4 \prod_{i=1}^3 dx_i^+ dx_i^- \frac{d\varphi_i}{2\pi}} = \left(\frac{\alpha_S C_A}{2\pi^2}\right)^3 \sum_n \sum_m \int d\nu \int d\lambda [G_A(x_0^+ - x_1^+; \mu) \times G(x_1^+ - x_2^+; -n, -\nu) G(x_2^+ - x_3^+; -m, -\lambda)] \times [U_{\varphi_1}(x_1^+ - x_1^-; n, \nu) R_{\varphi_2}(m-n, \lambda - \nu) \times D_{\varphi_3}(x_3^+ - x_3^-; -m, -\lambda)] G(x_1^- - x_2^-; n, \nu) \times G(x_2^- - x_3^-; m, \lambda) G_B(x_3^- - x_4^-; \mu)], \quad (9)$$

FIG. 1. Diagrammatic representation of 3-jet cross section σ_3 .

where $d\sigma_3$ stands for $d\sigma_{f+2}$ of Eq. (2), φ_i is the azimuthal angle of the i th jet, and an explicit form of the ‘‘propagators’’ $G_{A,B}$, G , U , R , and D is

$$G_{A,B}(x; \mu) = \theta(x) F_{A,B}(e^{-x}, \mu^2),$$

$$G(x; n, \nu) = \theta(x) \exp \left[-ix \left(\nu + i \frac{1 + \omega(n, \nu)}{2} \right) \right],$$

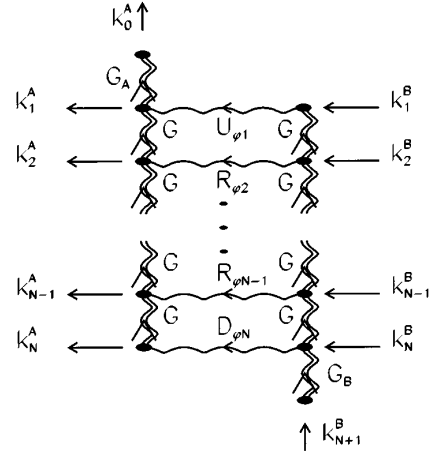
$$U_\varphi(x; n, \nu) = \theta(x) i^{|n|} e^{in\varphi} \frac{\Gamma \left(\frac{|n|+1}{2} - i\nu \right)}{\Gamma \left(\frac{|n|+1}{2} + i\nu \right)},$$

$$R_\varphi(n, \nu) = \frac{i^{|n|} e^{in\varphi}}{\frac{|n|}{2} - i(\nu + i\epsilon)} \frac{\Gamma \left(\frac{|n|}{2} + 1 - i\nu \right)}{\Gamma \left(\frac{|n|}{2} + 1 + i\nu \right)},$$

$$D_\varphi(x; n, \nu) = (-1)^{|n|} U_\varphi(x; n, \nu). \quad (10)$$

Note the role of θ functions from Eq. (10) in Eq. (9): they provide the right ordering of the components of the hadron and jet momenta ($x_{A,i}^+$ and $x_{B,i}^-$ decrease with an increment in i , and transverse momenta of the most forward or backward jets are larger than the factorization scale μ); the same ordering is seen in the limits of integration over x_1 and $k_{1\perp}$ from Eq. (2).

We now consider the rhs of Eq. (9) as corresponding to a graph of Fig. 1. Namely, the first factor $(\alpha_S C_A / 2\pi^2)^3$ may be redistributed among the vertices of the graph; summations over n, m and integrations over ν, λ correspond to an integration over loop momenta. Each momentum has a discrete (n or m) and a continuous (ν or λ) component; the first expression in square brackets corresponds to the left-hand side vertical line of the graph and the last to the right one. Factors U and D of the middle square bracket correspond to the up and down border rungs of the ladder graph respectively and R to the middle rung. Note also that the lines of the graph are oriented and the sign of ‘‘momentum’’ variables of the propagators depend on the direction of the momentum flow.

FIG. 2. Graph corresponding to the N -jet cross section σ_N .

The next step is to note that one obtains a more symmetric representation for the Feynman-like rhs of Eq. (9) if one replaces loop momentum integrations by equivalent integrations over additional x and φ variables per vertex. To this end, one multiplies the propagators of Eq. (10) by exponentials of products of the additional variables and momenta in such a way that the additional integrations provide momentum conservation at the vertices. The momentum integrations may then be performed independently for each ‘‘propagator’’; this will define the propagators in the ‘‘coordinate’’ representation. In this way one arrives at diagrams whose vertices are parametrized by two x variables and an azimuthal angle. One may equally look at the resulting Feynman rules in the momentum representation. Each momentum will consist of a discrete variable and two continuous variables.

We now describe the Feynman-like rules in the momentum representation for the graph of Fig. 2 and then define $d\sigma_N$ in terms of the analytic expression corresponding to the graph. Each vertex of the graph of Fig. 2 gives a factor $\sqrt{\alpha_S C_A / 2\pi^2}$. Each momentum comprises two continuous and one discrete variables [for example, $k^{A,i} = (k_1^{A,i}, k_2^{A,i}, n^{A,i})$]. Momenta flowing along the arrows are calculated with momentum conservation at the vertices as linear combinations of the external and the loop momenta. There are lines of six types: G_A , G_B , G , U , D , and R . Each line gives the following factor depending on its momentum and, for the ladder rungs, on the azimuthal angles φ_i of the corresponding jets:

$$G_{A,B}(k) = g_{A,B}(k_1), \quad (11)$$

$$G(k) = \frac{1}{2\pi i} \frac{1}{k_2 - k_1 + i \frac{1 + \omega(n, k_2)}{2} - i\epsilon}, \quad (12)$$

$$U_\varphi(k) = \frac{1}{2\pi i} \frac{-1}{k_1 + i\epsilon} e^{in\varphi + iu(n, k_2)}, \quad (13)$$

$$D_\varphi(k) = \frac{1}{2\pi i} \frac{-1}{k_1 + i\epsilon} e^{in\varphi + id(n, k_2)}, \quad (14)$$

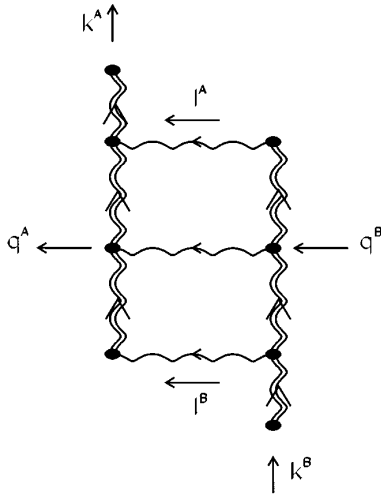


FIG. 3. The graph giving leading contribution to the asymptotic inclusive single-jet production cross section.

$$R_\varphi(k) = \delta(k_1) \frac{1}{|n|/2 - i(k_2 + i\epsilon)} e^{in\varphi + ir(n, k_2)}, \quad (15)$$

where

$$g_{A,B}(k) = \int \frac{dx}{2\pi} e^{ikx} \theta(x) F_{A,B}(e^{-x}, \mu^2), \quad (16)$$

$$iu(n, k) = i \frac{\pi}{2} |n| + \ln \frac{\Gamma\left(\frac{|n|+1}{2} - ik\right)}{\Gamma\left(\frac{|n|+1}{2} + ik\right)}, \quad (17)$$

$$id(n, k) = -i \frac{\pi}{2} |n| + \ln \frac{\Gamma\left(\frac{|n|+1}{2} - ik\right)}{\Gamma\left(\frac{|n|+1}{2} + ik\right)}, \quad (18)$$

$$ir(n, k) = i \frac{\pi}{2} |n| + \ln \frac{\Gamma\left(\frac{|n|}{2} + 1 - ik\right)}{\Gamma\left(\frac{|n|}{2} + 1 + ik\right)}. \quad (19)$$

The product is integrated over the loop momenta with the measure

$$\sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dl_1 \int_{-\infty}^{\infty} dl_2. \quad (20)$$

The result of the integration is multiplied by

$$\delta\left(\sum_{i=0}^N k_1^{A,i} - \sum_{i=0}^N k_1^{B,i}\right). \quad (21)$$

The final result is a function that we will denote as

$$I_N(k^{A,0}, \dots, k^{A,N}; k^{B,1}, \dots, k^{B,N+1}; \varphi_1, \dots, \varphi_N) \\ = I_N(k^A; k^B; \varphi). \quad (22)$$

This completes the description of the Feynman rules.

The cross section in terms of $I_N(k^A, k^B, \varphi)$ is

$$\frac{sd\sigma_N}{\pi^4 \prod_{i=1}^N dx_i^+ dx_i^- \frac{d\varphi_i}{2\pi}} = \int \left(\prod_{i=0}^N dk_1^{A,i} e^{-ik_1^{A,i} x_i^+} \right) \\ \times \left(\prod_{i=1}^{N+1} dk_1^{B,i} e^{ik_1^{B,i} x_i^-} \right) I_N^0(k^A; k^B; \varphi), \quad (23)$$

where I_N^0 is I_N at $k_2^{A,i} = k_2^{B,i} = n^{A,i} = n^{B,i} = 0$ and the energy variable of the collision s is connected with x_0^+, x_{N+1}^- by $s = \mu^2 e^{x_0^+ - x_{N+1}^-}$ [see Eq. (7)].

To illustrate the use of the above rules let us calculate the asymptotic inclusive single-jet cross section at high energy. The leading contribution comes from events with the untagged most forward and most backward jets:

$$\frac{sd\sigma_1^{\text{sum}}}{\pi^4 dx^+ dx^- \frac{d\varphi}{2\pi}} \approx \int dx_f^+ dx_f^- \frac{d\varphi_f}{2\pi} \\ \times \frac{sd\sigma_3}{\pi^4 dx_f^+ dx_f^- \frac{d\varphi_f}{2\pi} dx^+ dx^- \frac{d\varphi}{2\pi} dx_b^+ dx_b^- \frac{d\varphi_b}{2\pi}} \\ \times dx_b^+ dx_b^- \frac{d\varphi_b}{2\pi}. \quad (24)$$

Then the use of the above rules gives (see Fig. 3)

$$\frac{sd\sigma_1^{\text{sum}}}{\pi^4 dx^+ dx^- \frac{d\varphi}{2\pi}} \approx \left(\frac{\alpha_S C_A}{2\pi^2} \right)^3 \frac{i}{(2\pi)^6} \int dk^A dq^A e^{-ik^A x_A^+ - iq^A x^+} dk^B dq^B e^{ik^B x_B^- + iq^B x^-} d^2 l^A d^2 l^B \\ \times \left[g_A(k^A) \frac{1}{\left(l_1^A - l_2^A - k^A + i \frac{1 + \omega(0, l_2^A)}{2} - i\epsilon \right) \left(l_2^A - l_1^A + i \frac{1 + \omega(0, l_2^A)}{2} - i\epsilon \right)} \right] [A \rightarrow B] \\ \times \frac{\exp[iu(0, l_2^A) + id(0, l_2^B) + ir(0, -l_2^A - l_2^B)]}{(l_1^A + i\epsilon)(l_1^B + i\epsilon)(l_2^A + l_2^B - i\epsilon)} \delta(k^A + q^A - l_1^A - l_1^B) \delta(k^A + q^A - k^B - q^B), \quad (25)$$

where the second expression in square brackets is obtained from the first one by the substitution $A \rightarrow B$ and we took into account the fact that $\omega(0, k)$ is an even function. As $s = \mu^2 e^{x_A^+ - x_B^-}$, we are interested in the limit $x_A^+ \rightarrow \infty$, $x_B^- \rightarrow -\infty$. To calculate it, we first integrate out q^A, q^B by means of the δ functions, then take the residues at $k^A = l_1^A - l_2^A + i[1 + \omega(0, l_2^A)/2]$, $k^B = l_1^B - l_2^B + i[1 + \omega(0, l_2^B)/2]$ (only these poles contribute to the asymptotic limit), then at $l_1^A = l_2^A + i[1 + \omega(0, l_2^A)/2]$, $l_1^B = l_2^B + i[1 + \omega(0, l_2^B)/2]$, and finally take the remaining integrations over l_2^A, l_2^B in the saddle-point approximation (the saddle point is $l_2^A = l_2^B = 0$). The net result is

$$\frac{sd\sigma_1^{\text{sum}}}{\pi^4 dx^+ dx^- d\varphi/2\pi} \approx \left(\frac{\alpha_S C_A}{2\pi^2} \right)^3 \frac{e^{\alpha_P(x_A^+ - x_B^-)} M_A(\alpha_P, \mu^2) M_B(\alpha_P, \mu^2)}{(2\pi\alpha_P)^2 \sqrt{14\alpha_S C_A \zeta(3)(x_A^+ - x_B^-)}}, \quad (26)$$

where

$$\alpha_P = 1 + 4(\alpha_S C_A / \pi) \ln 2 \quad (27)$$

is the BFKL Pomeron intercept [2] and

$$M_{A,B}(\alpha_P, \mu^2) = \int_0^1 dx x^{\alpha_P - 1} F_{A,B}(x, \mu^2) \quad (28)$$

are moments of the parton distribution functions.

Note that the asymptotic cross section is independent of the jet parameters and depends on the parton distribution functions only by an overall normalization factor. One may assess the usefulness of the above representation of the cross

sections trying to reproduce without it the asymptotic of Eq. (26) by integration over parameters of most forward or backward jets of the corresponding cross sections from Refs. [5,9] where the single-jet production was considered under fixed parameters of most forward or backward jets. This integration changes the dependence of the asymptotic cross section of Refs. [5,9] on the parameters of the tagged jet.

The moments of Eq. (28) will enter also the asymptotics of the inclusive multijet cross section. This may be obtained along the same lines as the single-jet asymptotic limit of Eq. (26). We will present this elsewhere.

The use of the above rules for the inclusive dijet production reproduces the results of Ref. [14]. In particular, one may obtain a diagrammatic representation for the BFKL structure functions of Ref. [14].

We would like to stress that presented effective Feynman-like rules for description of inclusive cross sections are complementary to the effective field theory of interacting Reggeized and physical gluons (Ref. [21]) that describes ‘‘exclusive’’ processes. However, exact relation of our effective rules and effective theory of Ref. [21] requires further study.

To sum up, we introduced new effective Feynman-like rules for inclusive multijet cross sections in the multi-Regge regime, and used them to calculate an ultrahigh-energy asymptotic limit of single-jet production.

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- [1] L.N. Lipatov, *Yad. Fiz.* **23**, 642 (1976) [*Sov. J. Nucl. Phys.* **23**, 338 (1976)]; E.A. Kuraev, L.N. Lipatov, and V.S. Fadin, *Zh. Eksp. Teor. Fiz.* **71**, 840 (1976) [*Sov. Phys. JETP* **44**, 443 (1976)]; **72**, 377 (1977) [**45**, 199 (1977)]; Ya.Ya. Balitskii and L.N. Lipatov, *Yad. Fiz.* **28**, 1597 (1978) [*Sov. J. Nucl. Phys.* **28**, 822 (1978)].
- [2] L.N. Lipatov, *Zh. Eksp. Teor. Fiz.* **90**, 1536 (1986) [*Sov. Phys. JETP* **63**, 904 (1986)].
- [3] E. Laenen and E. Levin, *Ann. Rev. Nucl. Part. Sci.* **44**, 199 (1994).
- [4] G.P. Korchemsky, *Nucl. Phys.* **B443**, 255 (1995).
- [5] M.G. Ryskin, *Yad. Fiz.* **32**, 259 (1980) [*Sov. J. Nucl. Phys.* **32**, 133 (1980)].
- [6] A.H. Mueller and H. Navelet, *Nucl. Phys.* **B282**, 727 (1987).
- [7] A.H. Mueller and W.-K. Tang, *Phys. Lett. B* **284**, 123 (1992).
- [8] J. Kwiecinski, A.D. Martin, and P.J. Sutton, *Phys. Lett. B* **287**, 254 (1992); *Phys. Rev. D* **46**, 921 (1992).
- [9] V. Del Duca, M.E. Peskin, and W.-K. Tang, *Phys. Lett. B* **306**, 151 (1993).
- [10] V. Del Duca and C.R. Schmidt, *Phys. Rev. D* **49**, 4510 (1994); **51**, 2150 (1995).
- [11] W.J. Stirling, *Nucl. Phys.* **B423**, 56 (1994).
- [12] N.N. Nikolaev and B.G. Zakharov, *Phys. Lett. B* **332**, 177 (1994).
- [13] R. Ragazzon and D. Treleani, *Z. Phys. C* **65**, 305 (1995); R. Ragazzon and D. Treleani, *Phys. Rev. D* **53**, 55 (1996).
- [14] V.T. Kim and G.B. Pivovarov, *Phys. Rev. D* **53**, 6 (1996).
- [15] V.T. Kim and G.B. Pivovarov, Report No. INR-0904/95, hep-ph/9510247, 1995, to appear in *Proceedings of the Workshop on Particle Theory and Phenomenology*, IITAP, Ames, 1995.
- [16] S. Ahn, V.T. Kim, B.B. Levchenko, and G.B. Pivovarov (work in progress).
- [17] B.L. Combridge and C.J. Maxwell, *Nucl. Phys.* **B239**, 429 (1984).
- [18] J.C. Collins, D.E. Soper, and G. Sterman, in *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller (World Scientific, Singapore, 1989), pp. 1–91.
- [19] J.R. Cudell, A. Donnachie, and P.V. Landshoff, Report No. ULG-PNT-96-1-JRC, hep-ph/9602284, 1996 (unpublished).
- [20] I. Balitsky, *Nucl. Phys.* **B463**, 99 (1996).
- [21] L.N. Lipatov, *Nucl. Phys.* **B452**, 369 (1995).