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D-brane recoil and infrared divergences in string theory

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It is shown that there are logarithmic operators in D-brane backgrounds that lead to infrared divergences in open-string loop amplitudes. These divergences can be canceled by changing the closed-string background by operators that correspond to the D-brane moving with constant velocity after some instant in time, since it is precisely such operators that give rise to the appropriate ultraviolet divergences in the closed-string channel. [S0556-2821(96)50418-2]

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Duality symmetries of various types have been of great interest in recent times. Such symmetries may lead to insights into nonperturbative aspects of string theory, perhaps leading to contact with observed physics. This hope rests to some extent on the celebrated work of Seiberg and Witten [1] on supersymmetric Yang-Mills theories. These symmetries predict the existence of solitonic objects in certain vacua that are dual to ''fundamental'' objects in the dual vacua. These solitonic objects play vital roles in a complete understanding of low-energy physics, as in the work of Strominger [2]—it is, therefore, important to understand their dynamical properties.

Polchinski [3] showed that Dirichlet boundary conditions in open-string theories, previously studied for possible relevance to hadronic applications of string physics, can be interpreted as an operational definition of solitons that carry Ramond-Ramond charge in closed-string theories. These boundary conditions amount to an amazingly simple exact description of solitons (called D-branes) which are quite complicated from the spacetime perspective, and are therefore a good laboratory for the study of dynamical properties of solitons in string theory [4].

In the present work, we will use a variant of the Fischler-Susskind [5] mechanism to treat the problem of D-brane recoil. The problem of soliton recoil in field theory is already somewhat nontrivial, since one has to isolate the contributions of the Goldstone modes that arise from broken symmetries, such as translation invariance. In string theory, it is not immediately clear how one could isolate such modes in a consistent manner, especially from an exact description of the string theory soliton [4]. The problem of soliton recoil in string theory in a general case has been studied by Fischler, Paban, and Rozali [6], and in following work by Kogan and Mavromatos [7]. Our method will be much closer to Ref. [5], and our conclusions will differ from Refs. [6,7]. We will, however, confirm a ''postulate'' of Kogan and Mavromatos concerning logarithmic operators [8] in soliton backgrounds in string theory.

All the interesting elements of the problem are already evident in the case of a 0-brane, which is just a particle from a spacetime perspective. We will even restrict ourselves to the case of the bosonic string, since no further conceptual light is shed on this problem by considering the supersymmetric strings. Of course, these solitonic objects are only stable in supersymmetric theories, so an extension of our calculation to such cases is definitely of interest.

The bosonic static 0-brane is described by imposing Dirichlet boundary conditions on the spatial coordinates of the string, while keeping Neumann boundary conditions on the timelike coordinate. Thus

$$X^i(\mathrm{boundary})=0, \quad \partial_n X^0(\mathrm{boundary})=0,$$

where ∂_n is the derivative normal to the boundary of the string world sheet, and describes a 0-brane located at $x^i=0$. Such a configuration obviously breaks translation invariance in the spatial directions, and there are vertex operators $V^i \equiv \oint \partial_n X^i$ that translate the 0-brane that correspond to Goldstone modes. These vertex operators, however, correspond to translations of the entire world line of the 0-brane, and cannot be used directly to describe recoil. There are infrared divergences in annulus amplitudes (which correspond to an open-string loop correction to the disk amplitude), and we shall see that they come from operators that are rather closely related to these vertex operators.

Recall that the Fischler-Susskind mechanism [5] cancels infrared closed-string divergences due to massless dilatons at the one-loop level with a cosmological constant on the sphere. We are interested in an infrared divergence in an open-string channel at the annulus level, which we aim to cancel with an ultraviolet divergence in a closed-string channel at the disk level.

To this end, we first calculate the annulus amplitude describing the scattering of one closed-string tachyon off a 0-brane. This calculation can be done in a variety of ways. The simplest is perhaps the operator formulation, in which case we need to calculate $\mathrm{tr}V(k_1)\Delta V(k_2)\Delta$, with $\Delta^{-1}\equiv L_0-1$, and $V(k_i)$ are the closed-string vertex operators, integrated across the propagating open string. This calculation can also be formulated in the closed-string channel as $\langle B|\Delta V(k_1)\Delta V(k_2)\Delta|B\rangle$, where $|B\rangle$ is a state in the closed-string Fock space that imposes the appropriate boundary conditions on the end of the closed-string world sheet. In either case, these calculations are uninteresting in themselves, as far as the oscillator parts of the contractions are concerned—they give the standard η function form of the

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determinant of the Laplacian on the annulus. What is interesting, is the zero-mode trace, which we discuss in detail. Writing

$$\Delta \equiv \int_0^1 dx\ x^{L_0-2}, \quad L_0 = 2p^2 + N,$$

the zero-mode trace for a 0-brane is

$$\int \frac{dq^0}{2\pi} \langle q^0 | \exp(-ik_1^0 x^0) x_1^{-2(p^0)^2} \exp(-ik_2^0 x^0) x_2^{-2(p^0)^2} | q^0 \rangle.$$

For our purposes, the important point is the dependence on x_i, since we are interested in canceling divergences that arise from $x_1 \to 0$, with x_2 held fixed (and vice versa). This is the limit when the annulus amplitude degenerates into a disk amplitude with two open-string insertions on the boundary. The zero-mode trace gives in this limit

$$\delta(k_1^0 + k_2^0) \sqrt{\frac{1}{\ln(x_1)}} f(x_2, k_2^0).$$

This is the most important point of our calculation—the 0-brane background implies that the zero-mode trace has changed from the standard open string, which has 26 zero-mode integrals, giving a factor of $\ln(x_1)^{-13}$. Since $x_1 < 1$, there is an assumed analytic continuation, or an $i\epsilon$ prescription in performing the Gaussian integral. We shall find that precisely the same analytic continuation is needed for the ultraviolet divergence we will find below, so the manner in which one chooses to define the Gaussian integral is irrelevant, provided it is chosen consistently.

Now, in the complete amplitude, we have, in the limit $x_1 \to 0$ (neglecting divergences due to the pathologies of the bosonic open string that have no dependence on the momenta of the closed-string vertex operators, and hence no bearing on the problem of recoil),

$$g_{\rm st} \int_{x_1 \approx 0} \frac{dx_1}{x_1 \sqrt{8\pi \ \ln(x_1)}} A_{\rm disk}(k_1, k_2),$$

where

$$A_{\rm disk} = \langle V(k_1) V(k_2) V^i V^i \rangle.$$

We have found a rather peculiar feature, the divergence in the integral over x_1 is proportional to $\sqrt{|\ln(\epsilon)|}$, where $-\ln(\epsilon)$ is the large-time infrared cutoff. This must come from open-string states that have a two-point function of the form $\langle \phi(x) \phi(0) \rangle = \ln(x)/x^2$. The operators V^i are garden-variety conformal fields, not capable of such behavior. The appearance of logarithmic operators in string backgrounds corresponding to solitons was postulated by Kogan and Mavromatos [7]. We have therefore confirmed their conjecture. We will explicitly find these operators, which have a simple geometric interpretation, in the following.

Getting back to the matter at hand, we wish to cancel this divergence with a change in the closed-string background. We are looking for a closed-string vertex operator that will lead to an ultraviolet divergence in a closed-string channel, which would, therefore, be equivalent to an infrared divergence in an open-string channel. One might naively think that one would need *two* open-string insertions, since the coefficient of the divergence is $A_{\rm disk}$, but here we need to recall that the operators V^i are very special operators, since they produce infinitesimal motions of the entire 0-brane world line. The effect of inserting any number of these operators can be directly shown to be the same as multiplying the amplitude with factors of the total momentum carried by the external vertex operators. (We ignore terms involving contractions between two insertions of V^i because such terms have no external momentum dependence.) We therefore consider a closed-string vertex operator of the form

$$V_{\rm recoil} \equiv \alpha^i \int d^2 z\, \partial_\alpha (f(X^0) \partial^\alpha X^i),$$

with f to be determined. By construction, such an operator gives a contribution only from boundary terms on the world sheet (which is the disk or the upper half plane), but it must be regulated near the boundary because of the expectation value of $f(X^0)$. In fact, since the tangential derivative of X^i at the boundary vanishes, and energy is conserved at the order to which we are working, an insertion of $V_{\rm recoil}$ is the same as an insertion of $\alpha^i V^i$, but multiplied by this divergent expectation value.

Consider $f(X^0 - c) \equiv \int (dq/2\pi) \exp[iq(X^0 - c)] g(q^2)$. [We assume that $f(X^0)$ has been normal ordered on the sphere.] We have, when $f(X^0 - c)(z)$ is close to the boundary of the upper half plane,

$$\langle f(X^0 - c)(i\epsilon/2) \rangle = \int \frac{dq}{2\pi} g(q^2) \exp(-iqc)(\epsilon)^{-q^2}.$$

When $g(q^2) = 1/q^2$, this gives the dependence on ϵ that we need to cancel the divergence coming from the annulus. Note that any c dependence is only in the nonsingular terms. It is, however, considerably more illuminating to write

$$f(X^0 - c) = (X^0 - c) \Theta(X^0 - c),$$

where Θ is the step function. Thus, we have derived exactly what we would naively have predicted: The deformation of the D-brane background is precisely such that the D-brane starts moving at some time with constant velocity. What is this velocity? By comparing the annulus divergence with the disk divergence due to $V_{\rm recoil}$, we find

$$\alpha^i = 8\pi \sqrt{2} g_{\rm st} (k_1 + k_2)^i.$$

Recall that $\alpha^i X^0$ is the position of the soliton, and the mass of the 0-brane is expected $\propto 1/g_{\rm st}$, so this is exactly what we expect in soliton recoil. One could treat this as a leading-order determination of the mass of the 0-brane.

On simple kinematic grounds, the momentum change of the D-brane is of order 1, but the energy change is $O(g)$. Energy conservation is related to the appearance of c, which is arbitrary at the order to which we are calculating in this Rapid Communication. As explained in slightly different language by Fischler, Paban, and Rozali [6], the parameter c characterizes *different conformal field theories*. For calculations at $O(g^0)$, i.e., at the annulus level, any sum over c such that the total weight is 1 gives the same answer, e.g., one could sum over c as $\lim_{S \uparrow \infty} S^{-1} \int_{-S/2}^{S/2} dc$. At the next order [9], we expect that the measure for the sum over c will be specified by the cancellation of divergences that lead to energy conservation. It is important to note that there is always just one sum over c, for any number of insertions of V_{recoil}. These calculations are somewhat involved technically since one must evaluate two-loop open-string amplitudes, and will be given elsewhere [9].

In summary, we have arrived at a pleasing picture of D-brane recoil: We have found logarithmic operators in the annulus amplitude, as conjectured in Ref. [7]. We have canceled divergences in the disk amplitude due to insertions of V_{recoil} against divergences in the annulus amplitude due to the logarithmic operators, in an ultraviolet↔infrared reversal of the Fischler-Susskind mechanism. The form of V_{recoil} we found has a simple and manifestly correct physical meaning. The next step is to extend these computations to the physical case of the supersymmetric strings, and to compactifications that give rise to other finite mass D-branes. At higher orders, one can also derive energy conservation, though that likely involves including other operators, such as $\oint \partial_\tau X^0$, along with corrections to V_{recoil} [9]. Presumably, higher-order corrections will smooth out the abrupt change in the soliton trajectory we have found at the leading order. Carrying out the same calculation for p-branes in flat space gives no divergence for $p>1$, and only a $\ln \ln \epsilon$ divergence for 1-branes, presumably related to the properties of massless scalar fields on the world sheet of the 1-brane.

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[1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994).
[2] A. Strominger, Nucl. Phys. **B451**, 96 (1995).
[3] J. Polchinski, Phys. Rev. Lett. **75**, 4724 (1995); for earlier references, see J. Polchinski, S. Chaudhuri, and C. V. Johnson, ''Notes on D-branes,'' ITP Santa Barbara Report No. NSF-ITP-96-003, hep-th/9602052 (unpublished).
[4] C. Callan and A. Felce, ''Soliton mass in string theory,'' report (unpublished).
[5] W. Fischler and L. Susskind, Phys. Lett. B **171**, 383 (1986).
[6] W. Fischler, S. Paban, and M. Rozali, Phys. Lett. B **352**, 298 (1995).
[7] I. Kogan and N. Mavromatos, Phys. Lett. B **375**, 111 (1996).
[8] V. Gurarie, Nucl. Phys. **B410**, 535 (1993).
[9] V. Periwal and Ø. Tafjord (work in progress).