

Causality and the Doppler peaks

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A considerable experimental effort is underway to detect the ‘‘Doppler peaks’’ in the angular power spectrum of the cosmic microwave anisotropy. These peaks offer unique information about structure formation in the universe. One key issue is whether structure could have formed by the action of causal physics within the standard hot big bang, or whether a prior period of inflation was required. Recently there has been some discussion of whether causal sources could reproduce the pattern of Doppler peaks produced by the standard adiabatic theory. This paper gives a rigorous definition of causality, and a causal decomposition of a general source. I present an example of a very simple causal source that mimics the standard adiabatic theory, accurately reproducing the behavior of the local intrinsic temperature perturbations. [S0556-2821(96)50318-8]

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Existing theories of cosmic structure formation are of two types. In the first, the hot big bang is assumed to have started out smooth. Structure then forms as the result of a symmetry-breaking phase transition and phase ordering. In the second, an epoch of inflation *prior* to the hot big bang is invoked. Whilst both mechanisms are causal, causality imposes a much stronger constraint in the former case (Fig. 1), because the initial conditions for the perturbation variables are established on a Cauchy surface Σ *within* the hot big bang (Fig. 1). The causal nature of the Einstein-matter field equations then implies the vanishing of all correlations between all local perturbation variables at spacetime points whose backward light cones fail to intersect on Σ . In the inflationary case, by construction the relevant surface Σ lies so far before $\tau=0$ that there is no useful constraint.

Could observations distinguish the causally constrained theories from inflationary ones? The cosmic microwave anisotropy is the best hope of a direct probe, giving us a picture of the universe on the surface of last scattering. This surface cuts through many regions which that ‘‘causally disconnected’’ (quotes indicate a standard hot big bang definition) at that time. If the only contributions to the microwave anisotropy were local effects, like temperature and velocity perturbations in the photon-baryon fluid, one could check whether ‘‘superhorizon’’ perturbations were present by measuring the autocorrelation function of the anisotropy map. If this was consistent with zero beyond some angular scale (twice that subtended by the ‘‘causal horizon’’ at last scattering, of order 2° with standard recombination), one could conclude that the perturbations were indeed causally constrained.

The complication that spoils this test is that a significant component of the microwave anisotropy is generated after last scattering, by the integrated effect of time dependent gravitational potentials along photon paths. This is after all how cosmic defects produce a scale invariant spectrum of microwave anisotropies on very large angular scales [consis-

tent with the Cosmic Background Explorer (COBE) results] even though these theories *are* causally constrained.

Nevertheless, the local contributions to the microwave anisotropy do have a signature distinguishing them from the foreground due to the integrated effect. This is the presence of ‘‘Doppler’’ peaks in the angular power spectrum, caused by phase-coherent oscillations in the photon-baryon fluid prior to recombination. The location of these peaks is mainly determined by the temperature perturbations in the photon-baryon fluid, a completely local effect. This Rapid Communication will address the question of whether the peak locations can be used as a discriminator between inflationary and noninflationary theories of structure formation.

Crittenden and I suggested a connection between causality and peak location [1] following an analysis of the cosmic texture theory, in which the Doppler peaks are phase-shifted relative to those in standard inflation, the biggest peak occurring at higher multipole l (smaller angular scales) than in the standard inflationary theory. Albrecht, Magueijo, and collaborators [2] raised the important issue of decoherence, and gave a detailed discussion of the behavior of the Doppler peaks for different models of causal sources, in particular

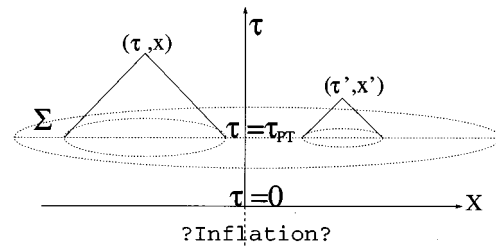


FIG. 1. Causal constraint on theories of structure formation where the standard hot big bang starts out homogeneous. The vertical axis shows conformal time τ , with $\tau=0$ corresponding to the initial singularity (in the absence of inflation). Correlations between any local variables at any two spacetime points vanish if their backward light cones fail to intersect on Σ , the spacelike hypersurface $\tau = \tau_{PT}$ just prior to the phase transition. In inflationary theories, there is no singularity at $\tau=0$, instead there is a preceding epoch of inflation during which longer range correlations are established.

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those motivated by the study of Robinson and Wandelt [3]. Most recently Hu and White [4] have addressed these issues, claiming that the pattern of Doppler peaks predicted by the simplest inflation model is “essentially unique and its confirmation would have deep implications for the causal structure of the early universe.” In this Rapid Communication I develop a formalism for dealing with decoherent but causal sources. I exhibit a causal source that closely mimics the inflationary pattern of temperature perturbations in the photon-baryon fluid, and the corresponding contribution to the microwave anisotropy.

Structure formation within the standard hot big bang requires the presence of a source term in the Einstein equations in addition to the usual metric and matter variables. Cosmic string and texture each provide an example of such a source. The perturbations are most simply dealt with in the fluid approximation, which is reasonable for our purposes. In the synchronous gauge, the relevant equations read [5]

$$\ddot{\delta}_C + \frac{\dot{a}}{a} \dot{\delta}_C = 4\pi G \sum_N (1 + 3c_N^2) \rho_N a^2 \delta_N + \mathcal{S}, \quad (1)$$

$$\ddot{\delta}_R + \frac{\dot{a}}{a} (1 - 3c_S^2) \dot{\delta}_R = c_S^2 \nabla^2 \delta_R + \frac{4}{3} \ddot{\delta}_C + \frac{4}{3} \frac{\dot{a}}{a} (1 - 3c_S^2) \dot{\delta}_C. \quad (2)$$

Overdots denote derivatives with respect to τ , $a(\tau)$ is the scale factor, δ_C and δ_R are the contrast in dark matter and radiation densities, and c_S is the speed of sound. The sum over N includes dark matter, the photon-baryon fluid and neutrinos. I assume the “canonical” parameter values $\Omega_{\text{CDM}} = 0.95$, $\Omega_B = 0.05$, and $h = 0.5$. The fluctuating part of the external source is taken to have stress energy tensor $\Theta_{\mu\nu}$, and $\mathcal{S} = 4\pi G (\Theta_{00} + \Theta_{ii})$. The initial conditions to be used with Eqs. (1) and (2) are the vanishing of the pseudoenergy $\tau_{00} = \Theta_{00} + \sum_N \rho_N a^2 \delta_N + (\dot{a}/a) \dot{\delta}_C / (4\pi G)$, and adiabaticity, $\delta_R = \delta_\nu = \frac{4}{3} \delta_C$. This corresponds to starting with a universe in which the energy density and space curvature are uniform.

It is a good approximation to treat the radiation as a perfect fluid, obeying Eqs. (1) and (2) from initial conditions set up deep in the radiation era, up to recombination. The intrinsic temperature perturbation from a Fourier mode \mathbf{k} is then given by $(\delta T/T)_i(\mathbf{k}) = \frac{1}{4} \delta_R(\mathbf{k})$, and its contribution to the angular power spectrum of anisotropies is given by $C_l \propto \int k^2 dk (\delta T/T)_i^2(k) j_l(k\tau_0)^2$ with τ_0 the conformal time today [6]. This dominates the anisotropy pattern on small angular scales.

Equations (1) and (2) are linear, and it follows that all correlations between local observables are completely determined by the unequal time correlation functions of the source stress energy tensor. In particular, for \mathcal{S} the causality constraint reads

$$\xi(r, \tau, \tau') = \langle \mathcal{S}(r, \tau) \mathcal{S}(0, \tau') \rangle = 0, \quad r > \tau + \tau'. \quad (3)$$

The sharp edge on ξ leads to oscillations in its three dimensional Fourier transform $\tilde{\xi}(k, \tau, \tau')$ at large k . Integration by parts produces

$$\tilde{\xi}(k, \tau, \tau') \sim \left[-\frac{R}{k^2} \cos kr + \frac{R_{,r}}{k^3} \sin kr + \dots \right]_{\tau}^{\tau + \tau'}, \quad (4)$$

where $R(r) = r\xi(r)$, and $R_{,r} = dR/dr$. (If $\xi \sim r^{-2}$ at small r , as it does for strings, then $\tilde{\xi}$ has an additional k^{-1} term). The leading term is not necessarily oscillatory, but there *must* be oscillatory subleading terms. Most of the *Ansätze* for $\tilde{\xi}$ in the literature do not have this feature and are therefore manifestly acausal. They may still be useful as approximations, but it is desirable to develop a formalism in which causality is rigorously built in.

The discussion simplifies if we assume scaling [7]. Then dimensional analysis implies that

$$\tilde{\xi} = \langle S(\mathbf{k}, \tau) S^*(\mathbf{k}, \tau') \rangle = \tau^{-(1/2)} \tau'^{-(1/2)} X(k\tau, k\tau'). \quad (5)$$

Regarded as a matrix with indices τ , and τ' , X is real and symmetric and can therefore be represented as

$$X(k\tau, k\tau') = \sum_{\alpha} P_{\alpha} f_{\alpha}(k\tau) f_{\alpha}(k\tau'), \quad (6)$$

with $f_{\alpha}(k\tau)$ a set of orthonormalised eigenfunctions of $X(k\tau, k\tau')$ regarded as an integral operator, with corresponding eigenvalues P_{α} . In the terminology of [1], $f_{\alpha}(k\tau)$ is a “master” function. As in quantum mechanics, we have a pure, “coherent” state if P_{α} is nonzero for only a single value of α , otherwise we have a mixed, “incoherent” state. Equation (6) shows that a general source may be represented as an incoherent sum of coherent sources. The P_{α} ’s must be positive for all α because they are the expectation value of a quantity squared.

This representation is useful because the contribution of each individual term in the α sum is straightforwardly calculable, by using the source $\tau^{-(1/2)} f_{\alpha}(k\tau)$ in the linearized Einstein equations. A bonus is that the assumption of scaling allows one to infer the correct initial conditions for the perturbations. For small k , Eq. (6) assures us that, if $\tilde{\xi}(0)$ exists, $f(k\tau)$ must tend to a (possibly zero) constant. Then energy conservation, $\dot{\Theta}_{00} + (\dot{a}/a)(\Theta_{00} + \Theta_{ii}) \approx 0$ for $k\tau \ll 1$, and the assumption of scaling (by dimensions, $\Theta_{00} \propto \tau^{-(1/2)}$) allow one to unambiguously determine the contribution to Θ_{00} appropriate to each $f_{\alpha}(k\tau)$.

Now let us return to the causality constraint (3). Consider a single term in the sum over α in Eq. (6). The contribution it makes to $\langle S(r, \tau) S(0, \tau') \rangle$ is proportional to the convolution of $f_{\alpha}(r, \tau)$ with $f_{\alpha}(r, \tau')$. If the $f_{\alpha}(r, \tau)$ have compact support, a simple argument [8] shows that $f_{\alpha}(r, \tau) = 0$ for all $r > \tau$. This gives a nice geometrical picture of how the causality constraint works. For each α the master function $f_{\alpha}(r, \tau)$ is the profile of a ball of radius τ , and the convolution of $f_{\alpha}(r, \tau)$ with $f_{\alpha}(r, \tau')$ clearly vanishes if the separation of the ball centers is greater than $\tau + \tau'$.

Determining the form of the f_{α} and P_{α} would be very interesting in any particular causal scenario. Here however, I want to see whether anything useful can be learnt by considering all possible f_{α} ’s and P_{α} ’s. The power spectrum in the general case is just a sum of the power spectra for different such f ’s with positive coefficients, so if for example we can show that the C_l for every f has positive slope for $l < l_{\text{max}}$, it

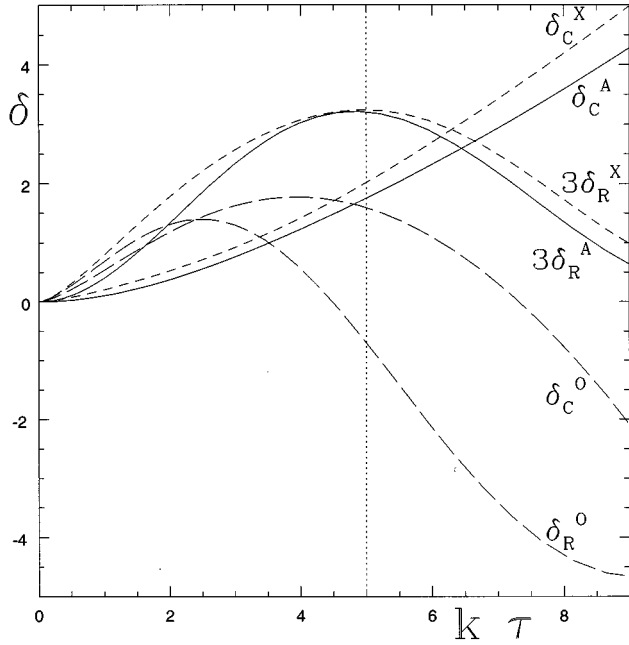


FIG. 2. The evolution of perturbation modes for radiation δ_R and dark matter δ_C as a function of conformal time τ , for fixed k . The value of k chosen has $k\tau_{\text{rec}}=5$, and in the inflationary theory the corresponding mode has its first maximum at recombination. The superscript A denotes the standard inflationary theory, X shows the extreme causal model which mimics the case A , and O shows the opposite extreme causal model which mimics instead the texture model.

follows that the total power spectrum will too. In this way we can set a lower limit on the location of the first Doppler peak.

A basis for all functions $f(r)$ is provided by the family $r^2 f(r, \tau) = \delta(r - A\tau)$, with $0 < A < 1$. In Fourier space we have $f(k, \tau) = \sin Ak\tau / (Ak\tau)$. In one extreme, with only short-range correlations, $f(k, \tau)$ is nearly constant. In the other, it has its first zero at $k\tau = \pi$. The equal time correlation functions corresponding to this family of master functions are smooth functions: one finds $\xi(r, \tau, \tau) \sim 1/(r\tau^3)$ for $r < 2A\tau$, $\xi(r) = 0$ for $r > 2A\tau$. If the master functions $f(r)$ do not change sign for all $r < \tau$ (which is unlikely), then any $f(r)$ can be represented as a sum of the above basis functions with positive coefficients.

We now proceed to solve Eqs. (1) and (2) for this family of source functions, with $S = f_k(\tau)/\tau^{1/2}$. Each Fourier mode of δ_R starts out small and grows (like $\tau^{3/2}$). After horizon crossing it oscillates as an acoustic wave. At the instant of last scattering, all modes are caught at a particular phase of their oscillations, and those that are at maximum amplitude produce the Doppler peaks in C_l . Figure 2 shows the time evolution of a single Fourier mode of δ_R and δ_C in the two extreme cases ($A=0$, denoted O , and $A=1$, denoted X), and in the standard adiabatic theory with no source. The approximate scale invariance means that the same graph very roughly represents $\delta_R(k, \tau_{\text{rec}})$ as a function of k at recombination. One can translate $k\tau_{\text{rec}}$ into multipole moment l by the approximate relation $l \sim k\tau_0 \sim 50k\tau_{\text{rec}}$. Peaks in $\delta_R(k, \tau)^2$ are, through the integral given above, translated into peaks in C_l .

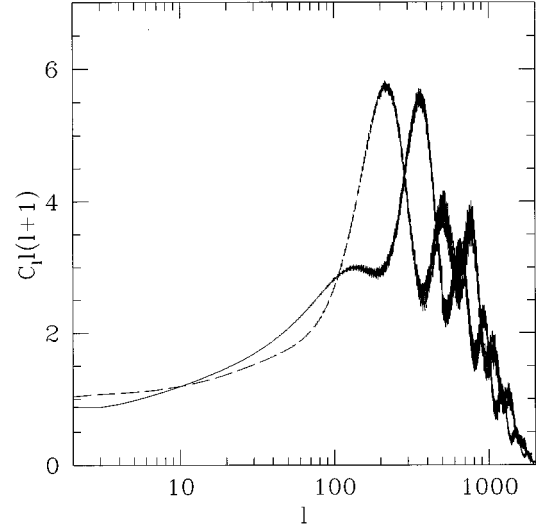


FIG. 3. The anisotropy power spectra for the standard inflationary theory (dashed line) and the texture theory (solid line). The family of sources studied here produces peak locations that approximately (at the 10% level) interpolate between these two cases. As the parameter A is dialed from 1 to 0, the position of the first peak moves from $l \sim 240$ to $l \sim 120$, and decreases in amplitude. The higher peaks shift down in l by a similar amount, with the second peak growing to become the highest peak. Note that the texture curve shown here includes the vector and tensor contributions, which help to emphasise the “hidden” peak at $l \sim 120$.

In the causal theories, δ_C is forced to start out growing with sign *opposite* to the source S , because the total pseudoenergy τ_{00} must initially be zero. As time goes on, S starts to drive δ_C . If S always has the same sign, as in the case $A \ll 1$, δ_C changes sign as it becomes driven by S . The forcing term for the radiation, $\ddot{\delta}_C$, then changes sign around $k\tau = 1$, so while δ_R initially grows with the opposite sign to S , it is later driven to the same sign as S . Because the sign change in $\ddot{\delta}_C$ occurs early (at $k\tau \sim 1$) the first oscillation in the radiation has small amplitude. Because C_l is really an integral over k , as mentioned above, the first Doppler peak, at $l \sim 120$, is smeared by the contribution of higher k , and may be effectively hidden. The main peak is that due to the next oscillation, the one that is really driven by S . This one occurs at $l \sim 380$, compared to the main peak in the standard adiabatic case at $l \sim 220$. Inside the horizon, the radiation oscillates sinusoidally, and higher peaks occur at shifts $\Delta l \approx 280n$, $n = 1, 2, 3, \dots$ to the right. It is interesting that this case ($A \ll 1$) reproduces the main features of the texture models presented in [1] and [9].

Next, consider the case where there *is* a sign change in S around horizon crossing. As before δ_C and δ_R start out with the opposite sign to S , because of compensation. But here, if S changes sign early enough, δ_C *does not* have to change sign. The radiation forcing term $\ddot{\delta}_C$ is always positive, and the first peak in δ_R is not small. As can be seen in Fig. 2, the extreme case $A=1$ mimics the standard adiabatic model rather closely. I have computed the power spectrum $\delta_R(k, \tau)^2$ at recombination in the $A=1$ theory, for $\Omega_B = 0.05, 0.1$, and 0.2 (and $\Omega_{\text{CDM}} = 1 - \Omega_B$). In all cases the result is similar to the analogous standard adiabatic theory,

both in the peak location and in the pattern of peak heights.

It may be useful to visualize this in terms of the C_l spectra. Figure 3 contrasts the standard adiabatic C_l 's with those of the texture model [1,9]. The simple family of causal theories I have just discussed roughly speaking interpolates between these two curves. At $A=1$ the first peak is close to that of the adiabatic theory, and as one decreases A , the peaks move to lower l . The first peak decreases in amplitude, for the reason discussed above, and moves leftward from $l\sim 220$ to $l\sim 120$. The second increases in amplitude, and moves from $l\sim 500$ to $l\sim 380$.

The calculations reported here include only the δ_R contribution to C_l , whereas the theoretical curves shown include all contributions, including the radiation velocity terms and the Sachs-Wolfe integral. In the inflationary case, the δ_R term alone gives a good overall representation of the C_l spectrum for $l>100$, and particularly the peak locations. It is therefore reasonable to expect that, at least in a similar situation where the Sachs-Wolfe integral and radiation velocity terms are subdominant, the complete spectrum of C_l 's will be similar. Computation of the Sachs-Wolfe integral, however, requires specifying the full stress tensor $\Theta_{\mu\nu}$ of the source, not just the combination \mathcal{S} that was sufficient here. In a companion paper [10], I extend the definition of the source stress tensor to one in which the Sachs-Wolfe integral is small, and show that with this extended definition of the source, the qualitative conclusions reached here hold for the complete C_l spectrum.

Continuing with the discussion of the δ_R contribution alone, what general conclusions can be drawn? Any master function f can be represented by a sum of the basis above:

since δ_R follows the same $\tau^{3/2}$ evolution up to $k\tau\sim 2.5$ for all of them, it follows that this will be true in the general case. Translated into C_l 's, this means that they cannot have a peak below $l\sim 120$. This is the real, and perhaps disappointing, causality constraint on scaling causal sources. Can we push the first peak to higher l than in the standard theory? Within the family considered, the limit for the first peak is close to the adiabatic position $l\sim 240$. It follows that this is an upper limit in any theory where the master functions $f(r/\tau)$ are strictly non-negative. However, if the $f(r/\tau)$ do change sign, the first peak may be pushed to much higher l . For example adding the negative of the ‘‘O’’ case to the ‘‘X’’ case in Fig. 2 pushes the first Doppler peak to $l\sim 400$. Similar examples produce first Doppler peaks at even higher l .

In conclusion, I have proposed a new formalism within which causal sources can be studied. As a first application, I have exhibited a very simple family of strictly causal sources that produce local contributions to the anisotropy that approximately interpolate between the standard adiabatic prediction and the texture prediction given in [1] and [9]. It follows that the causality constraint cannot on its own be used to distinguish between inflationary and noninflationary theories, and extra details are needed. It will be very interesting to determine the master functions f_α and coefficients P_α for specific scenarios.

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- [1] R. Crittenden and N. Turok, *Phys. Rev. Lett.* **75**, 2642 (1995).
 [2] A. Albrecht, D. Coulson, P. Ferreira, and J. Magueijo, *Phys. Rev. Lett.* **76**, 1413 (1996); J. Magueijo, A. Albrecht, D. Coulson, and P. Ferreira, *ibid.* **76**, 2617 (1996).
 [3] J. Robinson and B. Wandelt, *Phys. Rev. D* **53**, 618 (1996).
 [4] W. Hu and M. White, IAS Reports No. astro-ph/9602019 and astro-ph/9602020, 1996 (unpublished).
 [5] See, e.g., S. Veeraraghavan and A. Stebbins, *Astrophys. J.* **365**, 37 (1990).
 [6] For a review see G. Efstathiou, in *Physics of the Early Universe*, edited by A. Heavens, A. Davies, and J. Peacock (Scottish Universities Summer School Publications, Edinburgh, 1990).
 [7] The matter radiation transition does cause a departure from exact scaling, introducing the additional scale τ_{eq} . This can be incorporated into the formalism but the effect is likely to have little impact on the location of the main ‘‘Doppler’’ peaks and

I shall ignore it.

- [8] It is enough to consider the equal time correlator ($\tau=\tau'$) for this argument. Assume $f(r,\tau)=0$ beyond some scale $r=L$. Consider the convolution of f with itself, with argument $2L-\epsilon$. Only the outermost nonzero shell of f overlaps, and the contribution is proportional to $f(L-\epsilon)^2$. Since the convolution must be zero, it follows that $f(L-\epsilon)^2$ must be zero. Reapplying the argument for all L down to $L=\tau$, we see that $f(r,\tau)=0$ for all $r>\tau$. Because the full correlator (6) is a sum of such convolutions, with non-negative coefficients, the argument shows that *all* the $f_\alpha(r,\tau)$ vanish for $r>\tau$.
 [9] R. Durrer, A. Gangui, and M. Sakellariadou, *Phys. Rev. Lett.* **76**, 579 (1996).
 [10] N. Turok, DAMTP Report No. 96/69, astro-ph 9607109, 1996 (unpublished).
 [11] W. Hu, D. N. Spergel, and M. White, IAS Report No. 1996, astro-ph 9605193, 1996 (unpublished).