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$\mathrm{U}_A(1)$ symmetry restoration in QCD with N_f flavors

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Recently, there have been reports that the chirally restored phase of QCD is effectively symmetric under $\mathrm{U}(N_f)\times \mathrm{U}(N_f)$ rather than $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$. We supplement their argument by including the contributions from topologically nontrivial gauge-field configurations and discuss how the conclusions are modified. General statements are made concerning the particle spectrum of QCD with light N_f flavors in the high-temperature chirally restored phase. [S0556-2821(96)50215-8]

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In relation to the exciting possibilities of probing the QCD phase transition in relativistic heavy-ion collisions, there have been an increasing number of studies on the nature and physical consequence of chiral symmetry restoration in QCD [1]. Of particular interest is the possible changes in meson properties such as masses, decay width, or the spectral density in general as they might be observed directly through particle multiplicity or dilepton spectrum [2].

Recently, there have been reports that the chirally restored phase of QCD is effectively symmetric under $\mathrm{U}(N_f)\times\mathrm{U}(N_f)$ rather than $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$ [3,4]. In particular, Cohen [4] has shown, from a general consideration of the QCD partition function, that the two-point function of the spin-zero multiplet of $\mathrm{U}(N_f)\times\mathrm{U}(N_f)$, namely, π,σ,δ,η', etc., becomes degenerate if the spontaneously broken chiral symmetry is restored at high temperature. However, it should be noted that the argument in Ref. [4] is not complete in the sense that it does not take into account contributions from topologically nontrivial gauge-field configurations. In this Rapid Communication, we supplement the work in [4] by including the contributions from the nontrivial topological sectors and give general statements on the nature of chiral symmetry restoration of QCD with light N_f flavors.

Let us start with the Euclidean partition function of QCD:

$$Z[J]=\int D[A]D[\Psi\overline{\Psi}]\exp[-S_{\mathrm{QCD}}-\overline{\Psi}J\Psi], \qquad (1)$$

where $S_{\mathrm{QCD}}=\frac{1}{4}F^2+\overline{\Psi}\not{D}\Psi$. J could represent an external source or the mass matrix. Here we will neglect the gauge-fixing and ghost terms that will not be relevant for our discussion. Consider integrating out the quark fields,

$$Z[J]=\int D[A]e^{-S_{\mathrm{YM}}}\mathrm{Det}[\not{D}+J], \qquad (2)$$

where $S_{\mathrm{YM}}\equiv(1/4)F^2$. The integration over gauge fields A can have topologically nontrivial configurations with the topological charge $\nu=(g^2/32\pi^2)\int d^4xF\tilde{F}\neq 0$:

$$Z[J]=\sum_{\nu} Z[J]_{\nu}. \qquad (3)$$

The configuration with nonzero ν is always accompanied by $n_+(n_-)$ number of right-handed (left-handed) fermion zero mode satisfying $\not{D}\psi_0=0$. Furthermore, the index theorem tells that $\nu=n_+-n_-$. Our arguments below are not limited to specific gauge configurations such as the Belavin-Polyakov-Schwartz-Tyupkin (BPST) instanton [5]. Topological gauge configurations ($\nu\neq 0$) have zero measure in the chiral limit ($J=0$) as can be seen from the fermion determinant in Eq. (2). However, in the presence of an external source or finite quark mass, these zero-mode configurations can have nontrivial contributions.

In the presence of one topological charge, the zero-mode contribution can be separated out from the fermion determinant, and the $\nu=1$ sector of the partition function becomes [6]

$$Z[J]_{\nu=1}=\int D[A]_{\nu=1}e^{-S_{\mathrm{YM}}}\mathrm{Det}'[\not{D}+J] \times\mathrm{det}_{st}\left(\int d^4x\overline{\psi_0}(x)J_{st}\psi_0(x)\right). \qquad (4)$$

Here, det_{sf} stands for the determinant in the flavor space that is denoted by the index $s,f=1,\ldots,N_f$. The rest of the fermion determinant is denoted by Det', which now does not have a zero-mode contribution. ψ_0 corresponds to the zero-mode solution in the presence of a nontrivial gauge field configuration of $\nu=1$. The explicit form of the functional integral for the case of the BPST instanton solution has been worked out by 't Hooft [7]. A similar study for the periodic instanton relevant to the finite temperature system is given in [8].

Given the tools, we will now study n-point functions at arbitrary temperature, paying special attention to the contributions from the nontrivial topological sector and its relation to chiral symmetry restoration.

For simplicity, let us choose $N_f=2$. The case with more flavors can be easily generalized, although they have important differences as we will show later. We will also restrict our formulas to include only $\nu=0,\pm 1$ to clarify the role played by the nontrivial topological sector.

To begin with, the formula for the quark condensate reads

$$\langle\overline{q}q\rangle=\langle\overline{u}u+\overline{d}d\rangle=\frac{-1}{Z[J=m]}\left(\frac{\delta Z}{\delta J_{uu}}\bigg|_{J=m}+(u\to d)\right)$$
$$=\frac{-1}{Z}\int D[A]_{\nu=0}e^{-S_{\mathrm{YM}}}\mathrm{Det}'[\not{D}+m]\mathrm{tr}[S(0,0)]$$
$$+\frac{-1}{Z}\int D[A]_{\nu=\pm 1}e^{-S^{\mathrm{YM}}}\mathrm{Det}'[\not{D}+m]$$
$$\times\left[\overline{\psi_0}(0)\psi_0(0)\int d^4y\,\overline{\psi_0}(y)m_d\psi_0(y)+(u\to d)\right], \tag{5}$$

where $S(x,y)\equiv\langle x|(\not{D}+m)^{-1}|y\rangle$.

One finds that the first term in Eq. (5) gives the Casher-Banks formula [9]

$$\langle\overline{q}q\rangle=-\pi\langle\rho(\lambda=0)\rangle, \tag{6}$$

if one uses the identity

$$\mathrm{tr}[S(0,0)]=\int d\lambda\,\rho(\lambda)\frac{2m}{\lambda^2+m^2}\xrightarrow{m\to 0}\pi\rho(\lambda=0), \tag{7}$$

with $\rho(\lambda)$ being the density of fermion states with eigenvalue λ ($\not{D}\Psi=i\lambda\Psi$) in the presence of the gauge field A_μ.

The second term of Eq. (5) is from the $\nu=\pm 1$ sector and gives a correction of $O(m)$ to $\langle\overline{q}q\rangle$. Terms with higher ν have higher powers of m. For general N_f, the topological correction starts at $O(m^{N_f-1})$. Thus, in the chiral limit, all contributions from the $\nu\neq 0$ sector vanish and the Casher-Banks formula becomes exact. Specifically, the Casher-Banks formula links chiral symmetry breaking to the state density of fermions with nearly zero virtuality ($\lambda\sim 0$) coming only from the $\nu=0$ gauge configurations. However, the fact that the exact zero modes in $\nu\neq 0$ sector cannot contribute to the formula in the chiral limit does not imply that the topological configurations are irrelevant to the chiral symmetry breaking. In the $\nu=0$ sector, there exists many instanton + anti-instanton pairs that could cause nearly zero modes: this can be explicitly seen in the chiral symmetry breaking in the phenomenological instanton-liquid model [10].

One should note here that the weighting function $e^{-S_{\mathrm{YM}}}\mathrm{Det}'[\not{D}+m]$ is positive semidefinite. Hence, Eq. (5) implies that the restoration of chiral symmetry at high temperature (i.e., vanishing of $\langle\overline{q}q\rangle$) is possible only when $\rho(\lambda=0)$ vanishes before averaging over the gauge configurations [4].

A similar analysis with Eq. (5) can be made for two-point functions and higher. Let us concentrate on the difference between $\sigma-\sigma$ and $\delta-\delta$ two-point functions:

$$\langle\overline{q(x)}q(x),\overline{q(0)}q(0)\rangle-\langle\overline{q(x)}\tau^3q(x),\overline{q(0)}\tau^3q(0)\rangle$$
$$=\frac{1}{Z}\int D[A]_{\nu=0}e^{-S_{\mathrm{YM}}}\mathrm{Det}'[\not{D}+m]\mathrm{tr}[S(x,x)]\mathrm{tr}[S(0,0)]$$
$$+\frac{1}{Z}\int D[A]_{\nu=\pm 1}e^{-S_{\mathrm{YM}}}\mathrm{Det}'[\not{D}+m]$$
$$\times 4\overline{\psi_0}(x)\psi_0(x)\overline{\psi_0}(0)\psi_0(0)+O(m). \tag{8}$$

Again, the first term comes from the $\nu=0$ sector and the second from the $\nu=\pm 1$. In this case, the contribution from the $\nu=\pm 1$ part remains even in the chiral limit. At high temperature when chiral symmetry gets restored, $S(x,x)\propto\rho(\lambda=0)\sim O(m)$. (A more detailed proof of this is given in Ref. [4].) From this, Cohen concludes that, when chiral symmetry gets restored, the above difference vanishes and the σ and δ become degenerate. However, as can be clearly seen in Eq. (8), the nontrivial topological configurations do not vanish and the difference remains. This result is quite general. Consider taking the difference between two point functions such as $\langle J_1(x),J_1(0)\rangle-\langle J_2(x),J_2(0)\rangle$. Whenever we need a chiral rotation and a $\mathrm{U}(1)_A$ rotation to go from the spin-zero current J_1 to J_2, nontrivial topological configurations contribute even in the chiral limit, and the difference does not vanish even when chiral symmetry gets restored at high temperature.

It should be noted, however, that for general N_f, effects of $\nu\neq 0$ sector would be of $O(m^{N_f-2})$ and vanish in the chiral limit for $N_f\geqslant 3$. For QCD in the real world, it is proportional to m_s($\sim$150 MeV) and the effect may not be negligible.

When the current J_1 and J_2 have spin-1 or 2, nontrivial topological contributions vanish due to the chirality of the fermion zero mode irrespective of N_f [11].

For general n-point functions ($n\geqslant 3$) or their differences, there again is going to be a nontrivial contribution from the $\nu\neq 0$ sector. Generalization to higher N_f in this case is also easy. One immediately finds that the topologically nontrivial configurations start to contribute from n-point functions where $n=N_f$.

Let us now summarize our main conclusions.

(1) $\nu\neq 0$ sectors do not contribute to two-point functions for $N_f\geqslant 3$ in the $\mathrm{SU}(N_f)$ symmetric chiral limit. In this case, the physical spectrum will effectively be multiplets of $\mathrm{U}(N_f)\times\mathrm{U}(N_f)$, as was claimed in [4].

(2) If $N_f=2$, $\nu\neq 0$ sectors do contribute differently to the two-point functions of π,δ,σ,η even in the chiral limit and even when chiral symmetry is restored at high temperature. In this case, the argument in [4] does not apply.

(3) In the real world, SU(3) is not exact and $m_s\sim 150$ MeV. In this case, $\nu\neq 0$ sectors do contribute with possible non-negligible effects even at high temperature.

Quantitative estimates of the magnitude of $\nu\neq 0$ sectors in cases 2 and 3 at high temperature depend on the density of topological configurations at high temperature. Although such density could be highly suppressed and $\mathrm{U}_A(1)$ symmetry could be *effectively* restored at high temperature, our arguments in this letter show that there is always a finite amount of $U_A(1)$ symmetry breaking in the particle spectrum even when chiral symmetry restoration occurs at high temperature.

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