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QCD inequalities, the high temperature phase of QCD, and $\mathrm{U}(1)_A$ symmetry

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Inequalities for QCD functional integrals are used to establish that if the QCD functional integral does not get contributions from a set of zero measure, then the high-temperature chirally restored phase of QCD is effectively symmetric under $\mathrm{U}(N_f)\times\mathrm{U}(N_f)$ rather than $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$. If this assumption is correct, there are no effects due to anomalous breaking of $\mathrm{U}(1)_A$ on correlation functions in this phase. [S0556-2821(96)50115-3]

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One of the most important features of QCD is that it has an approximate $\mathrm{SU}(N_f)\times\mathrm{SU}(N_f)$ chiral symmetry that is spontaneously broken. For the purposes of this Rapid Communication, it will be assumed that the symmetry is exact or, more precisely, that corrections due to finite quark masses are small and can be handled via chiral perturbation theory. If one were to study QCD above some critical temperature this symmetry would be restored. The nature of this restored phase is of more than purely theoretical interest since ultrarelativistic heavy-ion collisions are expected to lead to thermalized regions of space with a temperature above T_c. For simplicity in the present discussion it will be assumed that $N_f=2$. The question of how the result depends on the number of light flavors is interesting and subtle [1,2].

This Rapid Communication addresses the question of what can be learned about the chirally restored phase directly from QCD via purely analytic means. If one makes certain technical assumptions about the functional integral, one can deduce nontrivial—and rather surprising—things about the nature of this phase by exploiting QCD inequality techniques similar to those used by Wiengarten, Vafa, and Witten [3] in studies of QCD at $T=0$.

In particular it will be shown that if a certain set of zero measure does not afflict the functional integral then, above T_c, the phase is effectively symmetric under $\mathrm{U}(2)\times\mathrm{U}(2)$ in the sense that operators can be classified into multiplets associated with representations of $\mathrm{U}(2)\times\mathrm{U}(2)$ and that correlation functions of operators in a given multiplet are identical. This means, for example, that the two-point correlation function in the π channel is degenerate with the correlation function in the η' channel. This is surprising since the $\mathrm{U}(2)\times\mathrm{U}(2)$ symmetry of the QCD Lagrangian is broken by the $\mathrm{U}(1)_A$ anomaly to $\mathrm{SU}(2)\times\mathrm{SU}(2)$. Moreover, the $\mathrm{U}(1)_A$ anomaly is at the operator level and thus the anomaly exists independent of temperature. As stressed by 't Hooft [4], the anomaly may be thought of as providing a mechanism for explicit (as opposed to spontaneous) symmetry breaking. Thus, it seems *a priori* implausible that restoration of chiral $\mathrm{SU}(2)\times\mathrm{SU}(2)$ should imply invariance under $\mathrm{U}(2)\times\mathrm{U}(2)$. Indeed, in a classic early review of the subject of instantons and the $\mathrm{U}(1)_A$ problem, Coleman [5] asserts precisely the viewpoint that the $\mathrm{U}(1)_A$ symmetry remains broken above the restoration temperature with the symmetry breaking decreasing at high temperatures as a power of $1/T$. Moreover Meggiolaro [6] has recently constructed a model motivated by lattice calculations of the topological susceptibility [7] in which $\mathrm{U}(1)_A$ remains broken above the chiral restoration temperature.

The idea that the chirally restored phase is symmetric under $\mathrm{U}(2)\times\mathrm{U}(2)$ rather than $\mathrm{SU}(2)\times\mathrm{SU}(2)$ is not new. Shuryak previously raised this possibility [8]. The present approach is novel, however, in that it derives this result formally from the QCD functional integral. Before discussing the present derivation, it is useful to review Shuryak's arguments. One argument is based on lattice calculations of screening masses that purport to show that above T_c, the π and σ screening masses are degenerate (within numerical noise) [9]. The calculated ''σ'' correlation functions only included the quark-line connected part. However, this quark-line connected part is the entire correlator in the scalar-isovector (δ) channel. Thus, the lattice calculations indicate a degeneracy between the π and δ screening masses. The π and the δ do not belong to the same $SU(2)\times SU(2)$ multiplet; they do, however, belong to the same $U(2)\times U(2)$ multiplet.

Shuryak also argues for $U(2)\times U(2)$ restoration from the instanton liquid model [10,11]. Recall that the solution of the $U(1)_A$ problem requires both the $U(1)_A$ anomaly and the contribution of nontrivial topological configurations [5]; topology is necessary for the anomalous violation of $U(1)_A$ symmetry to have physical manifestations. In the instanton liquid model [10,11] instantons provide the only source for these configurations. As has long been known, a finite density of instantons leads to chiral symmetry breaking [12]. In the instanton liquid model, instantons also provide the only source of $SU(2)\times SU(2)$ chiral symmetry breaking. Thus, in this model, the same mechanism is responsible both for $SU(2)\times SU(2)$ chiral symmetry breaking and for allowing the $U(1)_A$ symmetry breaking due to the anomaly to have physical consequences. In such a model, if one were in a phase in which $SU(2)\times SU(2)$ chiral symmetry were unbroken, it would follow that instanton effects are turned off. This, in turn, suggests that the anomaly will not have physical effects in any of the correlation functions: all observables will behave as though the phase is $U(2)\times U(2)$ symmetric. The mechanism responsible for this in the model is believed to be the condensation of instantons and anti-instantons into topologically neutral ''molecules'' [13].

The present analysis is based on properties of the QCD functional integral. The essential physics is best understood from the quark propagator in a given gluon background field. When the propagator is written in terms of a spectral representation, all $U(1)_A$-violating effects come from eigenmodes in the neighborhood of zero virtuality; i.e., $\lambda=0$ modes, where the Dirac eigenequation is $\not{D}\psi_j=i\lambda_j\psi_j$. While it is not immediately obvious how to establish in general that all $U(1)_A$-violating amplitudes come from the region of $\lambda=0$, it is easy to establish for given $U(1)_A$-violating amplitudes by studying the spectral representaton of the propagator in the context of the functional integral. Moreover, it is easy to prove that the density of states of the Euclidean Dirac operator, $\not{D}$ at $\lambda=0$, is zero for *any* gauge configurations with boundary conditions consistent with a temperature greater than the T_c (excluding, perhaps, a set of zero measure). Thus, one can show that these $U(1)_A$ violating amplitudes vanish.

The fact that above T_c all gauge configurations yield a vanishing density of states at zero virtuality can be seen quite transparently. The chiral condensate $\langle\overline{q}q\rangle$ is related to the density of states at zero averaged over gluon field configurations [14]: $\langle\overline{q}q\rangle=-\pi\langle\rho_A(\lambda=0)\rangle$ where ρ_A is the density of states in a given background gluon field configuration and angular brackets indicate averaging over the gluon field configurations weighted by $e^{-S_{\rm YM}}{\rm Det}[\not{D}-m]$. This applies to the finite-temperature case provided the average over gluons only includes configurations periodic in Euclidean time with a periodicity of $\beta=1/T$ and the fermion determinant is evaluated for antiperiodic configurations. Above T_c, $\langle\langle\overline{q}q\rangle\rangle_T=0$, implying that $\langle\langle\rho_A(0)\rangle\rangle_T=0$ (where the double angular brackets indicate a thermal average). However, $\rho_A(\lambda)$ is a density; accordingly it is positive semidefinite (i.e., $\geqslant 0$): $\rho_A(\lambda)\geqslant 0$. Moreover the weighting function $e^{-S_{\rm YM}}\,{\rm Det}[\not{D}-m]$ is also positive semidefinite [3]. An averaged quantity that is never negative cannot have an average of zero unless the quantity is zero for all configurations (except, perhaps, a set of measure zero): $\rho_A(0)=0$ for configurations consistent with the boundary conditions for $T>T_c$.

Before discussing how this works out in specific cases, it is worth stressing the generality of the result. It depends only on the fact that $\rho_A(0)$ goes to zero above the phase transition for all configurations and that $U(1)_A$ violating amplitudes come from modes in the neighborhood of $\lambda=0$. It does not depend on the detailed mechanism that generates a nonzero $\rho_A(0)$ below T_c.

To make the discussion concrete, consider the two-point correlation function of scalar and pseudoscalar quark bilinears. There are four distinct operators: pseudoscalar-isovector, $i\overline{q}\gamma_5\tau q$, (the π channel); scalar-isoscalar, $\overline{q}q$ (σ); pseudoscalar-isoscalar, $i\overline{q}\gamma_5 q$, (η'); and scalar-isovector, $\overline{q}\tau q$ (δ). These bilinears are denoted as J_π, J_σ, $J_{\eta'}$, and J_δ. The σ and π form a distinct $SU(2)\times SU(2)$ multiplet from the δ and η'. Under $U(2)\times U(2)$, however, they are all part of a single multiplet.

The thermal two-point correlation function $\Pi(\mathbf{x})$ of two equal-time quark bilinear operators at fixed temperature, T, is defined by

$$\Pi_J(\mathbf{x})\equiv\langle\langle J(\mathbf{x})J(\mathbf{0})\rangle\rangle_T-\langle\langle J(\mathbf{x})\rangle\rangle_T\langle\langle J(\mathbf{0})\rangle\rangle_T, \tag{1}$$

where the double angular brackets indicate thermal average and $J(\mathbf{x})=\overline{q(x)}\Gamma q(x)$ and Γ is a matrix in Dirac and flavor space. One can write this as a Euclidean functional integral:

$$\begin{aligned}\Pi_J(\mathbf{x})=&-\frac{1}{Z}\int_T D[A]e^{-S_{\rm YM}}{\rm Det}[\not{D}-m_q]\\&\times[{\rm tr}[S_A(\mathbf{x},\mathbf{0})\Gamma S_A(\mathbf{x},\mathbf{0})\Gamma]\\&-{\rm tr}[S_A(\mathbf{x},\mathbf{x})\Gamma]\,{\rm tr}[S_A(\mathbf{0},\mathbf{0})\Gamma]],\end{aligned} \tag{2}$$

where the subscript T indicates the finite T boundary conditions (periodic in A); Z is the partition function; $S_{\rm YM}$ is the Euclidean action of the Yang-Mills field; the Det indicates a functional determinant with the fermion modes satisfying antiperiodic boundary conditions; $S_A(\mathbf{x},\mathbf{y})$ is the Euclidean space quark propagator in the presence of a background gauge field A; and the traces are over color, flavor, and Dirac spaces.

A finite quark mass m_q is included in the previous expression—it will be sent to zero only at the end of the calculation. For technical reasons it is simpler to work in a box of finite volume V (which makes all of the modes discrete) and to let the volume of the box go to infinity at the end of the problem. The ordering of these two limits is critical. One must take the $V\rightarrow\infty$ limit before taking the chiral limit [15].

There are two distinct contributions to this functional integral: a term with a single trace and a term with two traces. They are the quark-line connected and quark-line disconnected pieces, respectively. If the up and down quark masses are equal (as is assumed here), $[S_A(\mathbf{x},\mathbf{0}),\tau]=0$, and the connected piece of an isoscalar correlator (e.g., the σ channel) is identical to the connected piece of an isovector correlator with the same spatial quantum numbers (e.g., the δ channel).

The difference between the σ and δ correlation functions, $\Pi_\sigma(x)-\Pi_\delta(x)$, is $U(1)_A$ violating. As noted above, in the functional integral this difference comes entirely from the quark-line disconnected piece:

$$\Pi_\sigma(\mathbf{x})-\Pi_\delta(\mathbf{x})=\frac{1}{Z}\int_T D[A]e^{-S_{\mathrm{YM}}}\mathrm{Det}[\not{D}-m_q] \times \mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]\ \mathrm{tr}[S_A(\mathbf{0},\mathbf{0})]. \tag{3}$$

If it can be shown that above the $SU(2)\times SU(2)$ chiral restoration temperature

$$\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]=O(m_q) \tag{4}$$

for *all* gauge configurations consistent with the boundary conditions, then it follows that $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]\mathrm{tr}[S_A(\mathbf{0},\mathbf{0})]=O(m_q^2)$ for all gauge configurations and thus the weighted average over gauge configurations will also be $O(m_q^2)$ from which Eq. (3) implies

$$\Pi_\sigma(\mathbf{x})-\Pi_\delta(\mathbf{x})=O(m_q^2). \tag{5}$$

This in turn implies that in the chiral limit of $m_q\to 0$ $\Pi_\sigma(\mathbf{x})-\Pi_\delta(\mathbf{x})\to 0$. That is, this $U(1)_A$-violating matrix element vanishes.

If the validity of Eq. (4) is established, then one has proven that this $U(1)_A$ violating amplitude vanishes. To begin use a spectral representation for S_A

$$S_A(\mathbf{x},\mathbf{y})=\sum_j \frac{\psi_j(\mathbf{x})\psi_j^\dagger(\mathbf{y})}{i\lambda_j-m_q}, \tag{6}$$

where the modes are eigenmodes of the Dirac operator. From the fact that $\{\gamma_5,\not{D}\}=0$, it follows that if ψ_j is an eigenmode with eigenvalue $i\lambda_j$, then $\gamma_5\psi_j$ is an eigenmode with eigenvalue $-i\lambda_j$. This in turn implies

$$\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]=\sum_j \frac{-m_q\psi_j^\dagger(\mathbf{x})\psi_j(\mathbf{x})}{\lambda_j^2+m_q^2}. \tag{7}$$

It is apparent from Eq. (7) that, as advertised, in the limit of $m_q\to 0$, contributions to $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]$ come entirely from modes near $\lambda=0$. More significantly, given the standard convention that the quark mass is positive, then $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]\leqslant 0$ for any gauge configuration.

Above T_c, we know that chiral condensate $\langle\langle\overline{q}q\rangle\rangle_T$ vanishes, or to be more precise is order m_q and vanishes when the chiral limit is taken. The chiral condensate can be written as a functional integral

$$N_f\langle\langle\overline{q}q(\mathbf{x})\rangle\rangle_T=\frac{1}{Z}\int_T D[A]e^{-S_{\mathrm{YM}}}\ \mathrm{Det}[\not{D}-m_q]\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})] =-O(m_q). \tag{8}$$

At this stage, it is worth recalling that $e^{-S_{\mathrm{YM}}}\ \mathrm{Det}[\not{D}-m_q]$ is positive semidefinite for all gauge configurations while $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]$ is negative semidefinite. Thus, the integrand in Eq. (8) is negative semidefinite. This means that there can be no cancellations in the integral—the only way that the integral can be $O(m_q)$ is if the contributions from all gauge configurations are $O(m_q)$ (except perhaps from a fraction of configurations that goes to zero in the chiral limit). Thus Eq. (4) has been shown to be true for all gauge configurations contributing to the functional integral except for contributions that become a set of measure zero in the chiral limit. Assuming this set of measure zero can be safely ignored in the evaluation of Eq. (3)—an issue that will be discussed at the end of this letter—one concludes that since Eq. (4) is true so is Eq. (5); thus in the chiral limit of $m_q\to 0$ these σ and δ correlators are identical.

Having established this, it is immediately obvious that $\Pi_\pi(\mathbf{x})$, $\Pi_{\eta'}(\mathbf{x})$, $\Pi_\sigma(\mathbf{x})$, and $\Pi_\delta(\mathbf{x})$ must all be identical above T_c in the $m_q\to 0$ limit. $SU(2)\times SU(2)$ chiral restoration implies that $\Pi_\pi(\mathbf{x})=\Pi_\sigma(\mathbf{x})$ and $\Pi_{\eta'}(\mathbf{x})=\Pi_\delta(\mathbf{x})$, while Eq. (5) implies that $\Pi_\delta(\mathbf{x})=\Pi_\sigma(\mathbf{x})$ — all members of the $U(2)\times U(2)$ multiplet are identical. Although from this argument it is clear that $\Pi_{\eta'}(\mathbf{x})=\Pi_\pi(\mathbf{x})$, it is useful to demonstrate this directly from functional integral inequalities as it demonstrates a technique that is useful for studying other multiplets.

The functional integral for this difference can be written as

$$\Pi_\pi(\mathbf{x})-\Pi_{\eta'}(\mathbf{x})=\frac{1}{Z}\int_T D[A]e^{-S_{\mathrm{YM}}}\ \mathrm{Det}[\not{D}-m_q] \times \mathrm{tr}[S_A(\mathbf{x},\mathbf{x})\gamma_5]\mathrm{tr}[S_A(\mathbf{0},\mathbf{0})\gamma_5]. \tag{9}$$

The first step in proving that $\Pi_\pi(\mathbf{x})-\Pi_{\eta'}(\mathbf{x})$ goes to zero above T_c is to show that

$$|\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})\gamma_5]|\leqslant|\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]| \tag{10}$$

for any gauge configurations. This is easily established using the spectral decomposition of the propagator and the fact that $\psi_j^\dagger(\mathbf{x})(1+\gamma_5)^2\psi_j(\mathbf{x})\geqslant 0$ for any ψ_j. By comparing Eq. (3) with Eq. (9) and using Eq. (10) and the fact that $e^{-S_{\mathrm{YM}}}\mathrm{Det}[\not{D}-m_q]$ is positive semidefinite, one sees that $|\Pi_\pi(\mathbf{x})-\Pi_{\eta'}(\mathbf{x})|\leqslant|\Pi_\sigma(\mathbf{x})-\Pi_\delta(\mathbf{x})|$. Since the right-hand side of this inequality goes to zero, the left-hand side does as well, and thus the degeneracy of the π and η' channels above T_c has been demonstrated directly from the QCD functional integrals.

The technique used to establish that π and η' correlation functions are identical above the phase transition by showing that the absolute value of their difference is less than or equal to $|\Pi_\sigma-\Pi_\delta|$, can be immediately generalized for other channels. In this way, one can show that vector and pseudovector, isovector and isoscalar (i.e., the ω, ρ, f_1, and

a_1) correlation functions are all identical above T_c. Again this is an identification of U(2)×U(2) symmetry since only the ρ and a_1 are connected by SU(2)×SU(2) chiral symmetry. The same method allows one to show that the tensor and pseudotensor, isoscalar and isovector correlators are identical above T_c.

There is a loophole in the demonstration of the U(2)×U(2) nature of the chirally restored phase given above. In particular, it was assumed that contributions to the functional integral in Eq. (8), which were a set of measure zero in the $m_q \rightarrow 0$ limit, do not contribute to the functional integral in Eq. (3). By inspection, it is clear that so long as $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})]$ is finite for all gauge configurations [after a gauge invariant and SU(2)×SU(2) chiral invariant ultraviolet regularization], then the set of measure zero cannot effect the functional integral in Eq. (3). To discuss a set of measure zero with infinite contributions it is sensible to first introduce an infrared cutoff regulator, ϵ where $\epsilon \rightarrow 0$ corresponds to the infinite volume and $m_q \rightarrow 0$ limits with $V m_q^3 \rightarrow \infty$. The loophole in the general argument is that there could be configurations for which $\mathrm{tr}[S_A(\mathbf{x},\mathbf{x})] \sim \epsilon^{-1/2}$, which have a weight proportional to ϵ. In such a case $\langle\langle \overline{q}q(\mathbf{x})\rangle\rangle_T \sim \epsilon^{1/2}$, which vanishes in the $\epsilon \rightarrow 0$ limit while $\{\Pi_\sigma(\mathbf{x}) - \Pi_\delta(\mathbf{x})\} \sim O(1)$. Thus, there is apparently the possibility that the chiral condensate vanishes as the regulator goes to zero while the $U(1)_A$ violating amplitude $\{\Pi_\sigma(\mathbf{x}) - \Pi_\delta(\mathbf{x})\}$ does not. It has been suggested recently [1,2] that spatially isolated regions with a topological winding number of ± 1 have a probability that goes as $m_q^{N_f}$ (where N_f is the number of light flavors) . Such a configuration gives rise to an isolated zero mode that causes S_A to go as $1/m_q$. Such a situation with $N_f = 2$ satisfies the loophole condition with m_q^2 playing the role of ϵ.

In summary, neglecting the loophole discussed above, it has been shown directly from the QCD function integral that above T_c the correlation functions for quark bilinears in a given U(2)×U(2) multiplet are identical. This indicates that the phase is invariant under U(2)×U(2) rather than SU(2)×SU(2). The anomalous $U(1)_A$ breaking does not split the U(2)×U(2) multiplets because the effects of the anomaly occur entirely through the quark-line disconnected parts of correlation functions and, in the $m_q \rightarrow 0$ limit, the quark-line disconnected parts contribute only due to the the modes near $\lambda = 0$. Above T_c the density of states at $\lambda = 0$ goes to zero and the anomaly ceases to play a role. On the other hand if high-temperature QCD does in fact have $U_A(1)$ violations then one can conclude that it must occur due to the loophole. This in turn allows one to constrain the possible forms of the spectral density at high temperatures.

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