Asymptotic scaling in the two-dimensional SU(3) σ model at correlation length 4×10^5

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We carry out a high-precision simulation of the two-dimensional SU(3) principal chiral model at correlation lengths ξ up to $\approx 4\times10^5$, using a multigrid Monte Carlo (MGMC) algorithm. We extrapolate the finite-volume Monte Carlo data to infinite volume using finite-size-scaling theory, and we discuss carefully the systematic and statistical errors in this extrapolation. We then compare the extrapolated data to the renormalization-group predictions. For $\xi \gtrsim 10^3$ we observe good asymptotic scaling in the bare coupling; at $\xi\approx 4\times10^5$ the nonperturbative constant is within 2–3 % of its predicted limiting value. [S0556-2821(96)50114-1]

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A key tenet of modern elementary-particle physics is the asymptotic freedom of four-dimensional non-Abelian gauge theories [1]. However, the nonperturbative validity of asymptotic freedom has been questioned [2], and numerical studies of lattice gauge theory have thus far failed to detect asymptotic scaling in the bare coupling [3]. Even in the simpler case of two-dimensional nonlinear σ models [4], numerical simulations at correlation lengths $\xi\sim 10$–100 have often shown discrepancies of order 10–50% from asymptotic scaling. In a recent paper [5] we employed a finite-size-scaling extrapolation method [6–9] to carry simulations in the O(3) σ model to correlation lengths $\xi\approx10^5$; the discrepancy from asymptotic scaling decreased from $\approx 25\%$ to $\approx 4\%$. In the present Rapid Communication we apply a similar technique to the SU(3) principal chiral model, reaching correlation lengths $\xi\approx 4\times10^5$ with errors $\lesssim 2\%$. For $\xi\gtrsim10^3$ we observe good asymptotic scaling in the bare parameter β; moreover, at $\xi\approx4\times10^5$ the nonperturbative ratio $\xi_{\rm observed}/\xi_{\rm theor,\,3\text{-loop}}$ is within 2–3 % of the predicted limiting value.

We study the lattice σ model taking values in the group SU(N), with nearest-neighbor action $\mathcal{H}(U) = -\beta\Sigma\ \mathrm{Re\ tr}(U_x^\dagger U_y)$. Perturbative renormalization-group computations predict that the infinite-volume correlation lengths $\xi^{(\exp)}$ and $\xi^{(2)}$ [10] behave as

$$\xi^{\#}(\beta) = C_{\xi^{\#}}\, e^{4\pi\beta/N}\left(\frac{4\pi\beta}{N}\right)^{-1/2}\left[1+\frac{a_1}{\beta}+\frac{a_2}{\beta^2}+\cdots\right] \quad (1)$$

as $\beta\to\infty$. Three-loop perturbation theory yields [12]

$$a_1 = -0.121019N + 0.725848N^{-1} - 1.178097N^{-3}. \quad (2)$$

The nonperturbative constant $C_{\xi^{(\exp)}}$ has been computed using the thermodynamic Bethe ansatz [13]:

$$C_{\xi^{(\exp)}} = \frac{\sqrt{e}}{16\sqrt{\pi}}\,\frac{\pi/N}{\sin(\pi/N)}\,\exp\left(-\pi\frac{N^2-2}{2N^2}\right). \quad (3)$$

The nonperturbative constant $C_{\xi^{(2)}}$ is unknown, but Monte Carlo studies indicate that $C_{\xi^{(2)}}/C_{\xi^{(\exp)}}$ lies between ≈ 0.985 and 1 for all $N\geqslant 2$ [14]; for $N=3$ it is 0.987 ± 0.002 [12]. Monte Carlo studies [16–18,12] of the SU(3) model up to $\xi\approx35$ have failed to observe asymptotic scaling (1); the discrepancy from Eqs. (1)–(3) is of the order of 10–20 %.

Our extrapolation method [8] is based on the finite-size-scaling (FSS) ansatz

$$\frac{\mathcal{O}(\beta,sL)}{\mathcal{O}(\beta,L)} = F_{\mathcal{O}}[\xi(\beta,L)/L;s] + O(\xi^{-\omega},L^{-\omega}), \quad (4)$$

where $\mathcal{O}$ is any long-distance observable, s is a fixed scale factor (here $s=2$), L is the linear lattice size, $F_{\mathcal{O}}$ is a universal function, and ω is a correction-to-scaling exponent. We make Monte Carlo runs at numerous pairs (β,L) and (β,sL); we then plot $\mathcal{O}(\beta,sL)/\mathcal{O}(\beta,L)$ versus $\xi(\beta,L)/L$, using those points satisfying both $\xi(\beta,L)\geqslant$ some value $\xi_{\min}$ and $L\geqslant$ some value $L_{\min}$. If all these points fall with good accuracy on a single curve, we choose a smooth fitting function $F_{\mathcal{O}}$. Then, using the functions F_ξ and $F_{\mathcal{O}}$, we extrapolate the pair $(\xi,\mathcal{O})$ successively from $L\to sL\to s^2L \to\cdots\to\infty$. See [8] for how to calculate statistical error bars on the extrapolated values.

We have chosen to use functions $F_{\mathcal{O}}$ of the form

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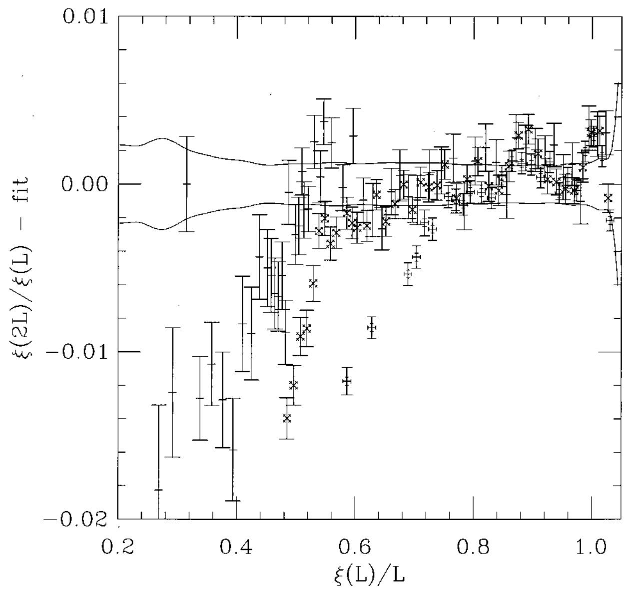

FIG. 1. Deviation of points from fit to F_ξ with $s=2$, $n=13$, $x_{\rm min}=(\infty,\infty,\infty,0.14,0)$. Symbols indicate $L=8$(┼), 16(⋇), 32(+). Error bars are one standard deviation. Curves near zero indicate statistical error bars ($\pm$ one standard deviation) on the function $F_\xi(x)$.

$$F_{\mathcal{O}}(x)=1+a_1e^{-1/x}+a_2e^{-2/x}+\cdots+a_ne^{-n/x}. \qquad (5)$$

We increase n until the χ^2 of the fit becomes essentially constant; the resulting χ^2 value provides a check on the systematic errors arising from corrections to scaling and/or from inadequacies of the form (5). The discrepancies between the

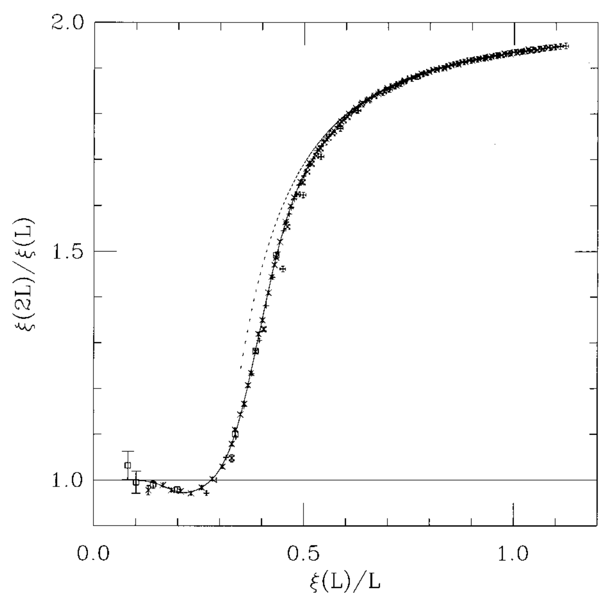

FIG. 2. $\xi(\beta,2L)/\xi(\beta,L)$ versus $\xi(\beta,L)/L$. Symbols indicate $L=8$(┼), 16(⋇), 32 (+), 64 (×), 128 (□). Error bars are one standard deviation. Solid curve is a thirteenth-order fit in Eq. (5), with $x_{\rm min}=(\infty,0.90,0.65,0.14,0)$ for $L=(8,16,32,64,128)$. Dashed curve is the perturbative prediction (6).

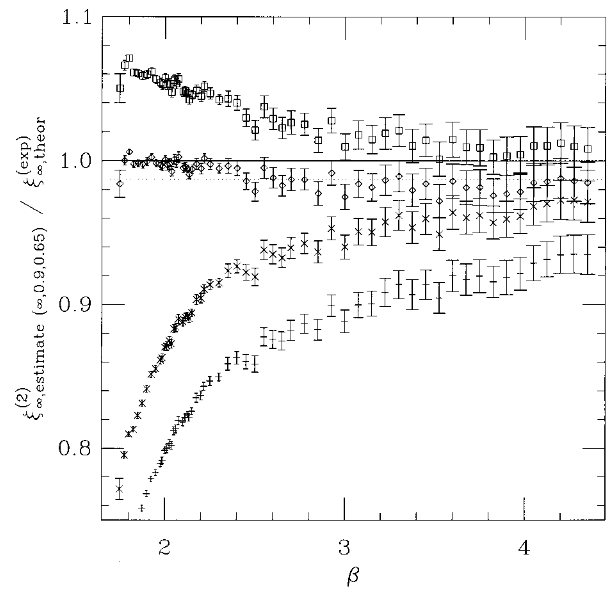

FIG. 3. $\xi^{(2)}_{\infty,\rm estimate(\infty,0.90,0.65)}/\xi^{(\rm exp)}_{\infty,\rm theor}$ versus β. Error bars are one standard deviation (statistical error only). There are four versions of $\xi^{(\rm exp)}_{\infty,\rm theor}$: standard perturbation theory in $1/\beta$ gives points + (2-loop) and × (3-loop); "improved" perturbation theory in $1-E$ gives points □ (2-loop) and ◇ (3-loop). Dotted line is the Monte Carlo prediction $C_{\xi^{(2)}}/C_{\xi^{(\rm exp)}}=0.987\pm0.002$ [12].

extrapolated values from different L at the same β can also be subjected to a χ^2 test. Further details on the method can be found in [8,5].

We simulated the two-dimensional SU(3) σ model using an XY-embedding multigrid Monte Carlo (MGMC) algorithm [19]. We ran on lattices $L=8$, 16, 32, 64, 128, 256 at 184 different pairs (β,L) in the range $1.65\leqslant\beta\leqslant4.35$ (corresponding to $5\lesssim\xi_\infty\lesssim4\times10^5$). Each run was between 4×10^5 and 5×10^6 iterations, and the total CPU time was one year on a Cray C-90 [20]. The raw data will appear in [21].

Our FSS data cover the range $0.08\lesssim x\equiv\xi(L)/L\lesssim1.12$, and we found tentatively that for $\mathcal{O}=\xi$ a thirteenth-order fit (5) is indicated (see Table I). There are significant corrections to scaling in the regions $x\lesssim0.84$ (respectively, 0.64, 0.52, 0.14) when $L=8$ (respectively, 16, 32, 64): see the deviations plotted in Fig. 1. We therefore investigated systematically the χ^2 of the fits, allowing different cuts in x for different values of L: see again Table I. A reasonable χ^2 is obtained when $n\geqslant13$ and $x_{\rm min}\geqslant(0.80,0.70,0.60,0.14,0)$ for $L=(8,16,32,64,128)$. Our preferred fit is $n=13$ and $x_{\rm min}=(\infty,0.90,0.65,0.14,0)$: see Fig. 2, where we compare also with the perturbative prediction

$$F_\xi(x;s)=s\left[1-\frac{aw_0\ln s}{2}x^{-2}-a^2\left(\frac{w_1\ln s}{2}+\frac{w_0^2\ln^2 s}{8}\right)x^{-4}+O(x^{-6})\right] \qquad (6)$$

valid for $x\gg1$, where $a=2N/(N^2-1)$, $w_0=N/(8\pi)$ and $w_1=N^2/(128\pi^2)$.

TABLE I. Degrees of freedom (DF), χ^2, χ^2/DF and confidence level for the nth-order fit (5) of $\xi(\beta,2L)/\xi(\beta,L)$ versus $\xi(\beta,L)/L$. The indicated $x_{\rm min}$ values apply to $L=8$, 16, 32, respectively; we always take $x_{\rm min}=0.14$, 0 for $L=64$, 128. Our preferred fit is shown in *italics*; other good fits are shown in **boldface**; bad fits are shown in roman.

$x_{\rm min}$	$n=11$	$n=12$	$n=13$	$n=14$	$n=15$
(0.50,0.40,0)	180 718.80	179 626.60	178 560.20	177 558.60	176 558.30
	3.99 0.0%	3.50 0.0%	3.15 0.0%	3.16 0.0%	3.17 0.0%
(∞,0.40,0)	154 673.80	153 566.30	152 533.00	151 532.10	150 531.80
	4.38 0.0%	3.70 0.0%	3.51 0.0%	3.52 0.0%	3.55 0.0%
(∞,∞,0)	108 236.00	107 172.40	106 154.80	105 154.70	104 153.40
	2.19 0.0%	1.61 0.0%	1.46 0.1%	1.47 0.1%	1.48 0.1%
(0.70,0.55,0.45)	162 288.30	161 219.20	160 183.00	159 182.50	158 182.30
	1.78 0.0%	1.36 0.2%	1.14 10.3%	1.15 9.8%	1.15 9.0%
(0.75,0.60,0.50)	150 222.40	149 172.20	148 129.90	147 129.80	146 129.80
	1.48 0.0%	1.16 9.4%	0.88 85.6%	0.88 84.3%	0.89 82.9%
(0.80,0.70,0.60)	129 173.90	128 135.00	**127 96.30**	**126 96.28**	**125 94.31**
	1.35 0.5%	1.05 32.0%	**0.76 98.1%**	**0.76 97.7%**	**0.75 98.1%**
(0.95,0.85,0.60)	111 150.30	110 107.20	**109 77.62**	**108 77.62**	**107 75.67**
	1.35 0.8%	0.97 55.8%	**0.71 99.0%**	**0.72 98.8%**	**0.71 99.1%**
(1.00,0.90,0.60)	105 139.20	104 100.90	**103 70.74**	**102 70.73**	**101 67.50**
	1.33 1.4%	0.97 56.7%	**0.69 99.4%**	**0.69 99.2%**	**0.67 99.6%**
(∞,0.90,0.65)	92 130.00	91 77.01	*90 60.85*	**89 58.66**	**88 58.31**
	1.41 0.6%	0.85 85.2%	*0.68 99.2%*	**0.66 99.5%**	**0.66 99.4%**
(∞,∞,0.65)	78 96.09	77 56.51	**76 49.55**	**75 46.63**	**74 45.94**
	1.23 8.1%	0.73 96.2%	**0.65 99.2%**	**0.62 99.6%**	**0.62 99.6%**
(∞,∞,∞)	52 55.85	**51 25.23**	**50 25.17**	**49 24.11**	**48 24.10**
	1.07 33.2%	**0.49 99.9%**	**0.50 99.9%**	**0.49 99.9%**	**0.50 99.8%**

The extrapolated values $\xi_\infty^{(2)}$ from different lattice sizes at the same β are consistent within statistical errors: only one of the 58 β values has a χ^2 too large at the 5% level, and summing all β values we have $\chi^2=64.28$ (103 DF, level = 99.9%).

In Table II we show the extrapolated values $\xi_\infty^{(2)}$ from our preferred fit and some alternative fits. The deviations between the different fits (if larger than the statistical errors) can serve as a rough estimate of the remaining systematic errors due to corrections to scaling. The statistical errors in our preferred fit are of the order of 0.5% (respectively, 0.9%, 1.1%, 1.3%, 1.5%) at $\xi_\infty \approx 10^2$ (respectively, 10^3, 10^4, 10^5,

TABLE II. Estimated correlation lengths $\xi_\infty^{(2)}$ as a function of β, from various extrapolations. Error bar is one standard deviation (statistical errors only). All extrapolations use $s=2$ and $n=13$. The indicated $x_{\rm min}$ values apply to $L=8$, 16, 32, respectively; we always take $x_{\rm min}=0.14$, 0 for $L=64$, 128. Our preferred fit is shown in *italic*; other good fits are shown in **boldface**; bad fits are shown in roman.

$x_{\rm min}$	$\beta=1.80$	$\beta=2.00$	$\beta=2.20$	$\beta=2.40$	$\beta=2.60$	$\beta=2.85$	$\beta=3.00$	$\beta=3.15$
(0.70,0.55,0.45)	10.455 (0.022)	24.903 (0.066)	57.13 (0.17)	129.68 (0.41)	290.5 (1.0)	794.9 (3.2)	1460 (6)	2687 (11)
(0.75,0.60,0.50)	10.454 (0.022)	24.886 (0.071)	57.50 (0.18)	130.83 (0.43)	293.0 (1.1)	801.7 (3.4)	1473 (6)	2709 (12)
(0.80,0.70,0.60)	**10.450 (0.021)**	**24.875 (0.073)**	**57.41 (0.22)**	**130.93 (0.64)**	**293.6 (1.6)**	**805.9 (5.0)**	**1482 (9)**	**2727 (17)**
(0.95,0.85,0.60)	**10.451 (0.021)**	**24.870 (0.071)**	**57.40 (0.21)**	**130.93 (0.63)**	**293.7 (1.6)**	**806.6 (6.1)**	**1483 (12)**	**2749 (25)**
(1.00,0.90,0.60)	**10.450 (0.022)**	**24.872 (0.069)**	**57.40 (0.21)**	**130.94 (0.63)**	**293.6 (1.6)**	**806.8 (5.9)**	**1484 (12)**	**2749 (25)**
(∞,0.90,0.65)	*10.446 (0.022)*	*24.859 (0.072)*	*57.40 (0.21)*	*131.00 (0.66)*	*295.2 (2.1)*	*809.6 (6.7)*	*1489 (13)*	*2761 (27)*
(∞,∞,0.65)	**10.447 (0.022)**	**24.863 (0.074)**	**57.40 (0.22)**	**131.01 (0.66)**	**295.0 (2.1)**	**809.7 (6.9)**	**1487 (14)**	**2759 (28)**
(∞,∞,∞)	**10.454 (0.022)**	**24.881 (0.074)**	**57.39 (0.22)**	**130.78 (0.66)**	**295.6 (2.3)**	**812.7 (9.8)**	**1482 (22)**	**2777 (49)**

$x_{\rm min}$	$\beta=3.30$	$\beta=3.45$	$\beta=3.60$	$\beta=3.75$	$\beta=3.90$	$\beta=4.05$	$\beta=4.20$	$\beta=4.35$
(0.70,0.55,0.45)	4957 (23)	9117 (46)	16780 (92)	30959 (182)	56766 (362)	105205 (707)	196197 (1396)	360864 (2792)
(0.75,0.60,0.50)	4995 (24)	9199 (47)	16938 (93)	31258 (185)	57265 (366)	106093 (736)	197949 (1419)	363905 (2880)
(0.80,0.70,0.60)	**5032 (32)**	**9268 (62)**	**17066 (118)**	**31492 (239)**	**57687 (456)**	**106807 (878)**	**199117 (1690)**	**366159 (3309)**
(0.95,0.85,0.60)	**5109 (49)**	**9411 (92)**	**17359 (178)**	**32059 (346)**	**58748 (650)**	**108781 (1237)**	**202868 (2360)**	**372553 (4392)**
(1.0,0.90,0.60)	**5110 (51)**	**9365 (99)**	**17299 (196)**	**31816 (372)**	**58308 (702)**	**107789 (1312)**	**200994 (2493)**	**369579 (4697)**
(∞,0.90,0.65)	*5132 (55)*	*9407 (105)*	*17377 (208)*	*31908 (398)*	*58594 (766)*	*108952 (1452)*	*201796 (2817)*	*371706 (5457)*
(∞,∞,0.65)	**5125 (55)**	**9391 (110)**	**17389 (229)**	**32008 (463)**	**58804 (941)**	**109440 (1886)**	**204587 (3779)**	**376704 (7722)**
(∞,∞,∞)	**5063 (102)**	**9295 (217)**	**16991 (447)**	**30912 (903)**	**55976 (1828)**	**104740 (3678)**	**192664 (7358)**	**359299 (14787)**

4×10^5), and the systematic errors are of the same order or smaller. The statistical errors at different β are strongly positively correlated.

In Fig. 3 (points + and ×) we plot $\xi^{(2)}_{\infty,\mathrm{estimate}(\infty,0.90,0.65)}$ divided by the two-loop and three-loop predictions (1)–(3) for $\xi^{(\exp)}$. The discrepancy from three-loop asymptotic scaling, which is $\approx 13\%$ at $\beta=2.0$ ($\xi_\infty\approx 25$), decreases to 2–3 % at $\beta=4.35$ ($\xi_\infty\approx 3.7\times10^5$). For $\beta\gtrsim 2.2$ ($\xi_\infty\gtrsim 60$) our data are consistent with convergence to a limiting value $C_{\xi^{(2)}}/C_{\xi^{(\exp)}}\approx 0.99$–1 with the expected $1/\beta^2$ corrections.

We can also try an ''improved expansion parameter'' [22,12] based on the energy $E=N^{-1}\langle \mathrm{Re}\,\mathrm{tr}(U_0^\dagger U_1)\rangle$. First we invert the perturbative expansion [12]

$$E(\beta)=1-\frac{N^2-1}{4N\beta}\left[1+\frac{N^2-2}{16N\beta}+\frac{0.0756-0.0634N^2+0.01743N^4}{N^2\beta^2}+O(1/\beta^3)\right] \quad (7)$$

and substitute into Eq. (1); this gives a prediction for ξ as a function of $1-E$. For E we use the value measured on the largest lattice (which is usually $L=128$); the statistical errors and finite-size corrections on E are less than 5×10^{-4}, and they induce an error less than 0.85% on the predicted ξ_∞ (less than 0.55% for $\beta\geqslant 2.2$). The corresponding observed/predicted ratios are also shown in Fig. 3 (points □ and ◇). The ''improved'' three-loop prediction is extremely flat, and again indicates a limiting value ≈ 0.99.

Further discussion of the conceptual basis of our analysis can be found in [5]. Details of this work, including an analysis of the susceptibility χ, will appear elsewhere [21].

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