

Color-octet J/ψ production in the Υ decay

Kingman Cheung*

Center for Particle Physics, University of Texas at Austin, Austin, Texas 78712

Wai-Yee Keung†

Physics Department, University of Illinois at Chicago, Illinois 60607-7059
and Fermilab, P.O. Box 500, Batavia, Illinois 60510

Tzu Chiang Yuan‡

Davis Institute for High Energy Physics, University of California at Davis, Davis, California 95616

(Received 6 March 1996)

The direct production rate of ψ in the Υ decay is shown to be dominated by the process $\Upsilon \rightarrow ggg^*$ followed by $g^* \rightarrow \psi$ via the color-octet mechanism proposed recently to explain the anomalous prompt charmonium production at the Fermilab Tevatron. We show that this plausibly dominant process has a branching ratio compatible with the experimental data. Further experimental study in this channel is important to test the significance of the color-octet component of the $c\bar{c}$ pair inside the ψ system. [S0556-2821(96)02113-3]

PACS number(s): 13.25.Gv, 12.38.Bx, 14.40.Gx

I. INTRODUCTION

The most appealing explanation of the excessive production rates of prompt ψ , ψ' , and χ_{cJ} observed at the Fermilab Tevatron [1,2] is given by the combination of the ideas of gluon fragmentation into quarkonium [3] and the color-octet mechanism [4], in which a gluon fragments into a color-octet 3S_1 $c\bar{c}$ pair which subsequently evolves nonperturbatively into the physical charmonium states by QCD dynamics. While the nonperturbative parameters associated with the color-octet mechanism must be extracted phenomenologically from the rates of prompt charmonium production, the prediction of the shape of the transverse momentum spectrum agrees well with the data [4–7]. A comprehensive review of these two theoretical developments and their implications at the Fermilab Tevatron and the CERN e^+e^- collider (LEP) can be found in Ref. [8]. If this mechanism is correct, it may give rise to many testable predictions for charmonium production in Z^0 decay [9], low energy e^+e^- annihilation [10], photoproduction at fixed target and experiments at the DESY ep collider HERA [11], hadroproduction at fixed target experiments [12] and at the CERN Large Hadron Collider (LHC) [13], and B -meson decays [14]. Double prompt quarkonium production from the color-octet mechanism has also been studied at the Fermilab Tevatron [15]. In this paper, we show that the color-octet mechanism can also provide the dominant contribution to ψ production in Υ decay.

The available experimental data on charmonium production in Υ decay are listed as

$$B(\Upsilon \rightarrow \psi + X) \begin{cases} = (1.1 \pm 0.4) \times 10^{-3} & \text{CLEO [16],} \\ < 1.7 \times 10^{-3} & \text{Crystal Ball [17],} \\ < 0.68 \times 10^{-3} & \text{ARGUS [18],} \end{cases} \quad (1)$$

in which the CLEO and the Crystal Ball results show a slight inconsistency. But it is interesting to note that these experiments can reach the branching fraction for $\Upsilon \rightarrow \psi + X$ at the level of 10^{-4} – 10^{-3} . Trottier [19] studied the indirect ψ production in Υ decay via the production of intermediate physical χ_c states, which decay radiatively into ψ . However, this indirect production of ψ contributes to a branching ratio less than 10^{-4} in Υ decay. Since the experimental branching fraction is already at the level of 10^{-4} – 10^{-3} , we definitely need a new production mechanism to explain the data. We also hope that the slight discrepancy in the above experiments can be resolved in the near future.

Conventional wisdom tells us that hadronically Υ decays predominantly through $b\bar{b}$ annihilation into three gluons. The rich gluon content in the final state makes it rather easy for the gluon to split into a $c\bar{c}$ pair in the color-octet 3S_1 configuration. If ψ can be formed from this color-octet configuration at a significant level as predicted by Braaten and Fleming [4] at the Fermilab Tevatron [1,2], charmonium states should be abundantly produced in Υ decay. A previous theoretical study [20] of the process $\Upsilon \rightarrow \psi + X$ was based on the color-evaporation model [21], with which the color-octet mechanism shares some common spirit, but the model fails to be systematic. Another qualitative estimate for $\Upsilon \rightarrow \psi + X$ can be found in Ref. [22].

In Sec. II, we will briefly review the description of the inclusive decay and production of quarkonium based on the nonrelativistic QCD (NRQCD) factorization formalism given by Bodwin, Braaten, and Lepage [23]. In Sec. III, we will discuss in detail several new color-octet processes relevant to ψ production in Υ decay allowed by the general factorization formula. In Sec. IV, we compare the production rates of different processes and discuss the energy spectrum of ψ in Υ decay.

II. NRQCD FACTORIZATION FORMALISM

The factorization formalism [23] for the inclusive decay and production of heavy quarkonium allows us to probe the

*Electronic address (internet): cheung@utpapa.ph.utexas.edu

†Electronic address (internet): keung@fnalv.fnal.gov

‡Electronic address (internet): yuanc@ucdhep.ucdavis.edu

complete quarkonium Fock space in a systematic and consistent manner based upon NRQCD. It can be straightforwardly applied to the case of inclusive charmonium production from bottomonium decay [19]. For the case of $Y \rightarrow \psi + X$, we have the factorization formula

$$d\Gamma(Y \rightarrow \psi + X) = \sum_{m,n} d\hat{\Gamma}_{mn} \langle Y | O_m | Y \rangle \langle O_n^\psi \rangle, \quad (2)$$

where $d\hat{\Gamma}_{mn}$ are the short-distance factors for a $b\bar{b}$ pair in the state m to decay into a $c\bar{c}$ pair in the state n plus anything, where m, n denote collectively the color, total spin, and orbital angular momentum of the heavy quark pairs. $d\hat{\Gamma}_{mn}$ can be calculated in perturbation theory as a series expansion in $\alpha_s(m_c)$ and/or $\alpha_s(m_b)$. Contributions to $d\hat{\Gamma}_{mn}$ that are sensitive to the quarkonium scales ($m_b v_b$ or $m_c v_c$ or smaller, where v_b and v_c are the relative velocities of the heavy quarks inside the bound states) and to Λ_{QCD} can be absorbed into the NRQCD matrix elements $\langle Y | O_m | Y \rangle$ and $\langle O_n^\psi \rangle$. We use the notation $\langle O_n^\psi \rangle$ to denote the vacuum expectation value $\langle 0 | O_n^\psi | 0 \rangle$ of the operator O_n^ψ . The nonperturbative factor $\langle Y | O_m | Y \rangle$ is proportional to the probability for the $b\bar{b}$ pair to be in the state m inside the physical bound state Y , while $\langle O_n^\psi \rangle$ is proportional to the probability for a point-like $c\bar{c}$ pair in the state n to form the bound state ψ . The relative importances of the various terms in the above double factorization formula (2) can be determined by the order of v_b or v_c in the NRQCD matrix elements and the order of α_s in the short-distance factors $d\hat{\Gamma}_{mn}$.

In the color-singlet model [24], the NRQCD matrix elements involved in the process $Y \rightarrow \psi + X$ are $\langle Y | O_1(^3S_1) | Y \rangle$ and $\langle O_1^\psi(^3S_1) \rangle$. According to the velocity scaling rules [23], they are scaled as $m_b^3 v_b^3$ and $m_c^3 v_c^3$, respectively, and can be related to the quarkonium wave functions as follows:

$$\langle Y | O_1(^3S_1) | Y \rangle \approx \frac{N_c}{2\pi} |R_Y(0)|^2, \quad (3)$$

$$\langle O_1^\psi(^3S_1) \rangle \approx 3 \frac{N_c}{2\pi} |R_\psi(0)|^2, \quad (4)$$

where N_c denotes the number of colors. Therefore, these color-singlet matrix elements can be determined from the leptonic widths of the Y and ψ . The short-distance factor in the color-singlet model for this direct process includes $b\bar{b}(\bar{^3S_1}, 1) \rightarrow c\bar{c}(\bar{^3S_1}, 1)gg$ and $b\bar{b}(\bar{^3S_1}, 1) \rightarrow c\bar{c}(\bar{^3S_1}, 1)gggg$. The two possible color configurations of the heavy quark pair are denoted by 1 for singlet and 8 for octet. The leading order Feynman diagrams for these processes are of order α_s^6 . Because of such a high order in the strong coupling constant, it is unlikely that these color-singlet processes can be the dominant production mechanism.

The first example of the double factorization formula such as (2) is the indirect ψ production in the decay $Y \rightarrow \chi_{cJ} + X$ with $\chi_{cJ} \rightarrow \psi\gamma$ considered by Trottier [19]. The short-distance factor $\hat{\Gamma}(b\bar{b}(\bar{^3S_1}, 1) \rightarrow c\bar{c}(\bar{^3P_J}, 1) + ggg)$ is of order α_s^5 , which is enhanced by a factor of $1/\bar{\alpha}_s$ compared with the

direct color-singlet processes mentioned in the previous paragraph. However, the infrared divergence in the leading order calculation of the short-distance factor indicates that the results are sensitive to the scale $m_c v_c$ or smaller. Therefore, in addition to the color-singlet matrix element $\langle O_1^{\chi_{cJ}}(\bar{^3P_J}) \rangle$ (scales such as $m_c^5 v_c^5$), one also needs to include the color-octet matrix element $\langle O_8^{\chi_{cJ}}(\bar{^3S_1}) \rangle$ (scales such as $m_c^3 v_c^5$) to absorb the infrared divergence. The short-distance process associated with the color-octet matrix element is $b\bar{b}(\bar{^3S_1}, 1) \rightarrow c\bar{c}(\bar{^3S_1}, 8) + gg$, which is of order α_s^4 . In this case, the introduction of the color-octet matrix element is required by perturbative consistency, since the infrared divergence would otherwise spoil the one-term factorization formula.

In the next section, we will consider the direct and indirect ψ productions in Y decay with short-distance factors of order α_s^3 and α_s^4 . These are possible only if higher Fock states of the ψ or Y are considered. We will also consider processes with short-distance factors of order $\alpha\alpha_s^3$, $\alpha^2\alpha_s$, and $\alpha^2\alpha_s^2$ that are suppressed by electromagnetic coupling but may or may not require higher Fock states of the quarkonia. In the following, we will first consider the produced $c\bar{c}$ pair in the color-octet 1S_0 , 3S_1 , or 3P_J configuration, which subsequently evolves into physical ψ described by the matrix elements $\langle O_8^\psi(^1S_0) \rangle$, $\langle O_8^\psi(^3S_1) \rangle$, or $\langle O_8^\psi(^3P_J) \rangle$, respectively. These color-octet matrix elements are suppressed by v_c^4 relative to the color-singlet matrix element $\langle O_1^\psi(^3S_1) \rangle$. The matrix element $\langle O_8^\psi(^3S_1) \rangle$ has been extracted from the Collider Detector at Fermilab (CDF) data [4–6], while two different combinations of the other color-octet matrix elements have been extracted from the CDF data [6] and from the photoproduction data by Amundson *et al.* [11].

Though of much smaller effects, we also consider the contributions by the higher Fock state of the color-octet $b\bar{b}$ pair inside the Y associated with the matrix element $\langle Y | O_8(^3S_1) | Y \rangle$, whose value has not yet been determined. An order of magnitude of this matrix element can, in principle, be estimated by considering the ratio

$$\frac{\langle Y | O_8(^3S_1) | Y \rangle \langle O_1^\psi(^3S_1) \rangle}{\langle Y | O_1(^3S_1) | Y \rangle \langle O_8^\psi(^3S_1) \rangle} \sim \left(\frac{v_b^2}{v_c^2} \right)^2, \quad (5)$$

which implies that its value should be highly suppressed. The ratio in Eq. (5) tells us that processes associated with a color-octet $c\bar{c}$ pair inside the produced ψ and a color-singlet $b\bar{b}$ pair inside the decaying Y are much more important than those with a color-octet $b\bar{b}$ pair inside the Y and a color-singlet $c\bar{c}$ pair inside the ψ . Using the value $\langle O_8^\psi(^3S_1) \rangle \approx 0.014 \text{ GeV}^3$, [4–6] $\langle O_1^\psi(^3S_1) \rangle \approx 3 \langle \psi | O_1(^3S_1) | \psi \rangle \approx 0.73 \text{ GeV}^3$ from the leptonic width of ψ , $\langle Y | O_1(^3S_1) | Y \rangle \approx 2.3 \text{ GeV}^3$ from the leptonic width of Y , and $v_c^2 \approx 0.3$ and $v_b^2 \approx 0.08$, we obtain $\langle Y | O_8(^3S_1) | Y \rangle \approx 3 \times 10^{-3} \text{ GeV}^3$. However, such a large value for this matrix element would substantially increase the hadronic width of Y , which would diminish the leptonic branching ratio to an unacceptable level. Obviously, this matrix element enters into the hadronic width of the Y via the short-distance process $b\bar{b}(\bar{^3S_1}, 8) \rightarrow g^* \rightarrow q\bar{q}$. In order not to

spoil the experimental value for the leptonic branching ratio and the total hadronic width of Υ , it is necessary to put a bound on the value of the matrix element $\langle Y | O_8(^3S_1) | Y \rangle$. The hadronic width of the Υ has the factored form

$$\begin{aligned} \Gamma(\Upsilon \rightarrow \text{light hadrons}) &= \left(\hat{\Gamma}(b\bar{b}(^3S_1, \underline{1}) \rightarrow ggg) \right. \\ &+ \sum_{q=u,d,s,c} \hat{\Gamma}(b\bar{b}(^3S_1, \underline{1}) \rightarrow q\bar{q}) \left. \langle Y | O_1(^3S_1) | Y \rangle \right. \\ &+ \sum_{q=u,d,s,c} \hat{\Gamma}(b\bar{b}(^3S_1, \underline{8}) \rightarrow q\bar{q}) \langle Y | O_8(^3S_1) | Y \rangle \\ &+ \dots, \end{aligned} \quad (6)$$

with the following short-distance factors calculated to leading order in α and α_s :

$$\hat{\Gamma}(b\bar{b}(^3S_1, \underline{1}) \rightarrow ggg) = \frac{20\alpha_s^3}{243m_b^2}(\pi^2 - 9), \quad (7)$$

$$\hat{\Gamma}(b\bar{b}(^3S_1, \underline{1}) \rightarrow f\bar{f}) = \frac{2\pi N_c Q_b^2 Q_f^2 \alpha^2}{3m_b^2}, \quad (8)$$

and

$$\hat{\Gamma}(b\bar{b}(^3S_1, \underline{8}) \rightarrow q\bar{q}) = \frac{\pi\alpha_s^2}{3m_b^2}. \quad (9)$$

In Eq. (8), N_c is 1 and 3 for f equals charged lepton l and light quark q , respectively; Q_f is the electric charge of the fermion f in unit of the positron charge; and we have set m_q and m_l to zeros in Eqs. (8) and (9) for simplicity. Using the following expression for muonic branching ratio

$$\begin{aligned} B(\Upsilon \rightarrow \mu^+ \mu^-) &= \frac{\Gamma(\Upsilon \rightarrow \mu^+ \mu^-)}{\Gamma(\Upsilon \rightarrow \text{light hadrons}) + \sum_{\ell} \Gamma(\Upsilon \rightarrow \ell^+ \ell^-)}, \end{aligned} \quad (10)$$

together with the experimental value for $B(\Upsilon \rightarrow \mu^+ \mu^-) = 0.0248 \pm 0.0007$ [25], we can obtain the bound on $\langle Y | O_8(^3S_1) | Y \rangle$

$$\langle Y | O_8(^3S_1) | Y \rangle \approx \left(1.9^{+5.1}_{-4.6} \right) \times 10^{-4} \text{ GeV}^3, \quad (11)$$

where we have allowed a 2σ variation on $B(\Upsilon \rightarrow \mu^+ \mu^-)$. Alternatively, we can obtain another bound by using the total width of Υ , but the result is not as good as the one given by Eq. (11). Therefore, in the rest of the paper we will use the value $\langle Y | O_8(^3S_1) | Y \rangle \sim 5 \times 10^{-4} \text{ GeV}^3$. With this value for $\langle Y | O_8(^3S_1) | Y \rangle$, we obtain the ratio

TABLE I. Input parameters used in our calculations.

NRQCD matrix elements	Value
$\langle O_1(^3S_1) \rangle \approx 3 \langle \psi O_1(^3S_1) \psi \rangle$	0.73 GeV^3
$\langle O_1'(^3S_1) \rangle \approx 3 \langle \psi' O_1(^3S_1) \psi' \rangle$	0.42 GeV^3
$ R'_{\chi_c}(0) ^2 \approx \frac{2\pi}{9} \langle O_1^{\chi_{c0}}(^3P_0) \rangle$	0.075 GeV^5 [27]
$\langle Y O_1(^3S_1) Y \rangle$	2.3 GeV^3 [Eq. (18)]
$\langle O_8(^3S_1) \rangle$	0.014 GeV^3 [4–6]
$\langle O_8'(^3S_1) \rangle$	0.0042 GeV^3 [4–6]
$\langle O_8'(^1S_0) \rangle \approx \langle O_8'(^3P_0) \rangle / m_c^2$	10^{-2} GeV^3 [6]
$\langle O_8^{\chi_{c0}}(^3S_1) \rangle$	0.0076 GeV^3 [5,8]
$\langle Y O_8(^3S_1) Y \rangle$	$5 \times 10^{-4} \text{ GeV}^3$ [Eq. (11)]
Other parameters	Value
m_c	1.5 GeV
m_b	4.9 GeV
$\alpha_s(2m_c)$	0.253
$\alpha_s(2m_b)$	0.179

$$\frac{\hat{\Gamma}(b\bar{b}(^3S_1, \underline{8}) \rightarrow g^* \rightarrow q\bar{q}) \langle Y | O_8(^3S_1) | Y \rangle}{\hat{\Gamma}(b\bar{b}(^3S_1, \underline{1}) \rightarrow ggg) \langle Y | O_1(^3S_1) | Y \rangle} \sim 0.017,$$

which is now sufficiently small posing no threat to the experimental value of the leptonic width in the Υ decay. The input parameters used in our later numerical calculations are summarized in Table I for convenience. Since all our new calculations are at tree level only, we, to be consistent, extract the color-singlet matrix elements from the leptonic widths of ψ, ψ' , and Υ using tree-level formulas.

III. COLOR-OCTET PROCESSES

We shall first consider two processes with a color-octet $c\bar{c}$ pair inside the produced ψ and a color-singlet $b\bar{b}$ pair in the decaying Υ .

$$\text{A. } b\bar{b}(^3S_1, \underline{1}) \rightarrow \gamma^* \rightarrow c\bar{c}(^2S+1L_J, \underline{8}) + g$$

The leading order diagrams for the process $b\bar{b}(^3S_1, \underline{1}) \rightarrow c\bar{c}(^2S+1L_J, \underline{8}) + g$ is of order $\alpha^2\alpha_s$, one of which is shown in Fig. 1(a). This is similar to the process $Z \rightarrow c\bar{c}(^2S+1L_J, \underline{8}) + g$, which is negligible because the short-distance factor is suppressed by powers of m_c^2/M_Z^2 from the quark propagator [9]. But in the present case it is only suppressed by powers of m_c^2/m_b^2 . We will restrict ourselves to the case of $L=0$ and 1 only. Although the contributions from higher values of L can be included easily, they are further suppressed by powers of v_c^2 . The inclusive production rate from these processes can be written as

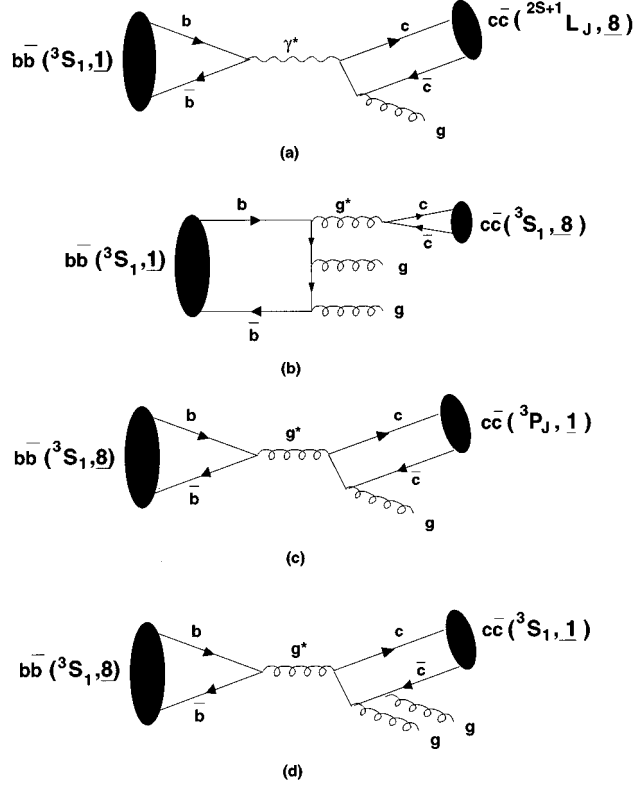


FIG. 1. Some of the contributing Feynman diagrams for the short-distance processes: (a) $b\bar{b}({}^3S_1, \underline{1}) \rightarrow \gamma^* \rightarrow c\bar{c}({}^{2S+1}L_J, \underline{8})g$; (b) $b\bar{b}({}^3S_1, \underline{1}) \rightarrow g^*gg \rightarrow c\bar{c}({}^3S_1, \underline{8})gg$; (c) $b\bar{b}({}^3S_1, \underline{8}) \rightarrow g^* \rightarrow c\bar{c}({}^3P_J, \underline{1})g$; and (d) $b\bar{b}({}^3S_1, \underline{8}) \rightarrow g^* \rightarrow c\bar{c}({}^3S_1, \underline{1})gg$.

$$\Gamma_{1a}(Y \rightarrow \psi + X) = \langle Y | O_1({}^3S_1) | Y \rangle \left[\langle O_8^\psi({}^1S_0) \rangle \hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^1S_0, \underline{8}) + g) + \sum_{J=0,1,2} \langle O_8^\psi({}^3P_J) \rangle \hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3P_J, \underline{8}) + g) \right]. \quad (12)$$

The short-distance factors are calculated to leading order and are given by

$$\hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^1S_0, \underline{8}) + g) = \frac{4\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{3} \frac{1}{m_b^4 m_c} (1 - \xi), \quad (13)$$

$$\hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3P_0, \underline{8}) + g) = \frac{4\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{9} \frac{1}{m_b^4 m_c^3} \frac{(1 - 3\xi)^2}{1 - \xi}, \quad (14)$$

$$\hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3P_1, \underline{8}) + g) = \frac{8\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{9} \frac{1}{m_b^4 m_c^3} \frac{1 + \xi}{1 - \xi}, \quad (15)$$

$$\hat{\Gamma}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3P_2, \underline{8}) + g) = \frac{8\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{45} \frac{1}{m_b^4 m_c^3} \frac{1 + 3\xi + 6\xi^2}{1 - \xi}, \quad (16)$$

where $\xi = M_\psi^2 / M_Y^2 \approx m_c^2 / m_b^2$. We have also used the nonrelativistic approximation for the mass of the quarkonium: $M_Y \approx 2m_b$ and $M_\psi \approx 2m_c$. We note that $b\bar{b}({}^3S_1, \underline{1}) \rightarrow \gamma^* \rightarrow c\bar{c}({}^3S_1, \underline{8}) + g$ vanishes. Using the heavy quark spin symmetry relation $\langle O_8^\psi({}^3P_J) \rangle \approx (2J+1) \langle O_8^\psi({}^3P_0) \rangle$ [23], the total width from these processes can be simplified as

$$\Gamma_{1a}(\Upsilon \rightarrow \psi + X) = \frac{4\pi^2 Q_c^2 Q_b^2 \alpha^2 \alpha_s}{3} \frac{\langle \Upsilon | O_1(^3S_1) | \Upsilon \rangle}{m_b^4 m_c} \left\{ \langle O_8^\psi(^1S_0) \rangle (1 - \xi) \right. \\ \left. + \frac{\langle O_8^\psi(^3P_0) \rangle}{3m_c^2} \left[\frac{(1 - 3\xi)^2}{1 - \xi} + \frac{6(1 + \xi)}{1 - \xi} + \frac{2(1 + 3\xi + 6\xi^2)}{1 - \xi} \right] \right\}, \quad (17)$$

which can be normalized by the leptonic width of Υ :

$$\Gamma(\Upsilon \rightarrow e^+ e^-) = \frac{2\pi Q_b^2 \alpha^2}{3} \frac{\langle \Upsilon | O_1(^3S_1) | \Upsilon \rangle}{m_b^2}. \quad (18)$$

Using $m_c = 1.5$ GeV, $m_b = 4.9$ GeV, $\alpha_s(2m_b) = 0.179$, $\langle O_8^\psi(^1S_0) \rangle \approx \langle O_8^\psi(^3P_0) \rangle / m_c^2 \approx 10^{-2}$ GeV³ [6], and $B(\Upsilon \rightarrow e^+ e^-) \approx 0.0252$ [25], the contribution from the above color-octet processes to the inclusive branching ratio $B(\Upsilon \rightarrow \psi + X)$ is only 1.6×10^{-5} , which is almost two orders of magnitude below the CLEO data (1).

B. $b\bar{b}({}^3S_1, \underline{1}) \rightarrow ggg^* \rightarrow c\bar{c}({}^3S_1, \underline{8}) + gg$

Figure 1(b) shows one of the six Feynman diagrams for the $b\bar{b}({}^3S_1, \underline{1})$ pair annihilating into three gluons with one of

the gluons converting into the $c\bar{c}({}^3S_1, \underline{8})$ pair. This process is of order α_s^4 and its calculation is very much similar to the process $\Upsilon \rightarrow gg \gamma^* \rightarrow gg l\bar{l}$ [26]. Introducing the scaling variables

$$x_v = \frac{E_\psi}{m_b}, \quad x_1 = \frac{E_{g_1}}{m_b}, \quad x_2 = \frac{E_{g_2}}{m_b}, \quad (19)$$

such that $x_v + x_1 + x_2 = 2$, where E_i stands for the energy of the particle i . In the rest frame of Υ , the differential decay width is given by

$$\frac{d\Gamma_{1b}}{dx_v dx_1}(\Upsilon \rightarrow \psi + X) = \frac{d\hat{\Gamma}}{dx_v dx_1}[b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3S_1, \underline{8}) + gg] \langle \Upsilon | O_1(^3S_1) | \Upsilon \rangle \langle O_8^\psi(^3S_1) \rangle, \quad (20)$$

with

$$\frac{d\hat{\Gamma}}{dx_v dx_1}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3S_1, \underline{8}) + gg) \\ = \frac{5\pi\alpha_s^4}{486m_c^3 m_b^2} \frac{1}{(x_v - 2\xi)^2 x_1^2 (2 - x_v - x_1)^2} \{ 2\xi^4 + 2\xi^3(6 - 4x_v + 2x_1 - x_v x_1 - x_1^2) \\ + 2\xi^2[11 - 16x_v + 6x_v^2 - (8 - 2x_v - x_v^2)x_1 + (4 + x_v)x_1^2] + \xi[4(1 - x_v)(4 - 5x_v + 2x_v^2) \\ - (32 - 44x_v + 14x_v^2)x_1 + (20 - 18x_v + x_v^2)x_1^2 - 2(2 - x_v)x_1^3 + x_1^4] + 2[2 - 6x_v + 7x_v^2 - 4x_v^3 + x_v^4 \\ - (6 - 13x_v + 9x_v^2 - 2x_v^3)x_1 + (7 - 9x_v + 3x_v^2)x_1^2 - 2(2 - x_v)x_1^3 + x_1^4] \}, \quad (21)$$

where the ranges of integration for x_v and x_1 are

$$2\sqrt{\xi} \leq x_v \leq 1 + \xi, \quad (22)$$

$$\frac{1}{2}(2 - x_v - \sqrt{x_v^2 - 4\xi}) \leq x_1 \leq \frac{1}{2}(2 - x_v + \sqrt{x_v^2 - 4\xi}). \quad (23)$$

One can integrate over x_1 to obtain the energy distribution of ψ :

$$\frac{d\hat{\Gamma}}{dx_v}(b\bar{b}({}^3S_1, \underline{1}) \rightarrow c\bar{c}({}^3S_1, \underline{8}) + gg) \\ = \frac{5\pi\alpha_s^4}{486m_c^3 m_b^2} \frac{1}{(2 - x_v)^3 (x_v - 2\xi)^2} \left[4(8 + 8\xi - 14\xi^2 - 2\xi^3 - 12x_v - 4\xi x_v + 10\xi^2 x_v + 5x_v^2 \right. \\ \left. - \xi x_v^2 - \xi^2 x_v^2)(1 + \xi - x_v) \ln \left(\frac{2 - x_v - \sqrt{x_v^2 - 4\xi}}{2 - x_v + \sqrt{x_v^2 - 4\xi}} \right) + (2 - x_v) \sqrt{x_v^2 - 4\xi} (16 + 28\xi + 20\xi^2 + 4\xi^3 - 24x_v - 36\xi x_v \right. \\ \left. - 12\xi^2 x_v + 14x_v^2 + 13\xi x_v^2 - 4x_v^3) \right]. \quad (24)$$

Numerically integrating x_v , we obtain the partial width

$$\Gamma_{1b}(Y \rightarrow \psi + X) = \frac{5\pi\alpha_s^4}{486m_c^3m_b^2} \langle Y | O_1(^3S_1) | Y \rangle \langle O_8^\psi(^3S_1) \rangle \times (0.571). \quad (25)$$

We note that the color-octet piece for the process $Y \rightarrow \chi_{cJ} + X$ considered by Trottier [19] can be obtained from the above Eq. (25) by simply replacing the matrix element $\langle O_8^\psi(^3S_1) \rangle$ with $\langle O_8^{\chi_{cJ}}(^3S_1) \rangle$. However, the expressions for the energy distributions (21) and (24) were not given explicitly in Ref. [19]. The prediction of the partial width is sensitive to the values of the two NRQCD matrix elements, the running coupling constant α_s , and the heavy quark masses. For convenience, we can normalize this partial width to the three-gluon width $\Gamma(Y \rightarrow ggg)$ given by Eq. (7) times the matrix element $\langle Y | O_1(^3S_1) | Y \rangle$:

$$\frac{\Gamma_{1b}(Y \rightarrow \psi + X)}{\Gamma(Y \rightarrow ggg)} = \frac{\pi\alpha_s}{8} \frac{\langle O_8^\psi(^3S_1) \rangle}{m_c^3} \frac{0.571}{\pi^2 - 9}. \quad (26)$$

It is insightful to take the scaling limit of $m_b \rightarrow \infty$ with $x_v = E_\psi/m_b$ held fixed in Eq. (24). In this scaling limit, one can deduce

$$\frac{\Gamma_{1b}(Y \rightarrow \psi + X)}{\Gamma(Y \rightarrow ggg)} \approx \frac{\pi\alpha_s}{8} \frac{\langle O_8^\psi(^3S_1) \rangle}{m_c^3}. \quad (27)$$

The right-hand side of Eq. (27) can be recognized to be three times the gluon fragmentation probability $P_{g \rightarrow \psi}$ in the color-octet mechanism obtained by Braaten and Fleming [4]. Thus, the above scaling limit corresponds to the fragmentation approximation. Comparing the above limit (27) with the exact result (26), we see that fragmentation is not a good approximation. However, this limit suggests that the scale to evaluate the α_s in Eq. (26) should be $2m_c$ instead of $2m_b$. With $\alpha_s(2m_c) = 0.253$, $\langle O_8^\psi(^3S_1) \rangle = 0.014 \text{ GeV}^3$ [4,6], and assuming $B(Y \rightarrow ggg) \approx B(Y \rightarrow \text{light hadrons}) = 0.92$ [25], we obtain

$$\frac{\Gamma_{1b}(Y \rightarrow \psi + X)}{\Gamma_{\text{total}}(Y)} = \frac{\Gamma_{1b}(Y \rightarrow \psi + X)}{\Gamma(Y \rightarrow ggg)} \times B(Y \rightarrow ggg) \approx 2.5 \times 10^{-4}. \quad (28)$$

This prediction is smaller than the CLEO data by merely a factor of 4, and is consistent with the bounds from Crystal Ball and ARGUS.

The color-octet process studied in this subsection applies to the case of ψ' as well, simply by replacing the matrix element $\langle O_8^\psi(^3S_1) \rangle$ with $\langle O_8^{\psi'}(^3S_1) \rangle$, whose value has also been extracted from the CDF data to be 0.0042 GeV^3 [4–7]. With $B(\psi' \rightarrow \psi + X) \approx 57\%$ [25], we obtain a branching ratio of 4.3×10^{-5} for the inclusive ψ production in the Y decay that comes indirectly from ψ' .

One can also consider the processes $Y \rightarrow \gamma g g^*$ followed by $g^* \rightarrow \psi(\psi')$ via the color-octet mechanism, and $Y \rightarrow g g \gamma^*$ followed by $\gamma^* \rightarrow \psi(\psi')$ in the color-singlet model. Up to overall normalization, the energy spectra of the ψ for these two processes are predicted to be the same as in Eq. (24). However, their partial widths are suppressed

by factors of $8\alpha/(15\alpha_s) \sim 0.02$ and $32\alpha^2 \langle O_1^\psi(^3S_1) \rangle / [45\alpha_s^2 \langle O_8^\psi(^3S_1) \rangle] \sim 0.06$, respectively, compared with the width of Eq. (25). Thus, they contribute a branching fraction about 2×10^{-5} in the inclusive decay $Y \rightarrow \psi + X$. The indirect contribution from the ψ' from these two processes is about 6×10^{-6} .

C. $b\bar{b}(^3S_1, 8) \rightarrow g^* \rightarrow c\bar{c}(^3P_J, 1) + g$

We now turn to the case where the $b\bar{b}$ pair inside the Y is in a color-octet 3S_1 state. The first process we consider is the production of χ_{cJ} from Y decay. The leading order Feynman diagram is depicted in Fig. 1(c). The factorization formula for the decay rate can be written as

$$\Gamma_{1c}(Y \rightarrow \chi_{cJ} + X) = \hat{\Gamma}(b\bar{b}(^3S_1, 8) \rightarrow c\bar{c}(^3P_J, 1) + g) \times \langle Y | O_8(^3S_1) | Y \rangle \langle O_1^{\chi_{cJ}}(^3P_J) \rangle. \quad (29)$$

We note that up to coupling constants, color factors, and NRQCD matrix elements, these processes are similar to the ones $b\bar{b}(^3S_1, 1) \rightarrow \gamma^* \rightarrow c\bar{c}(^3P_J, 8) + g$ considered in Sec. III A. The short-distance factors can be extracted easily from the previous calculations:

$$\hat{\Gamma}(b\bar{b}(^3S_1, 8) \rightarrow c\bar{c}(^3P_0, 1) + g) = \frac{\pi^2\alpha_s^3}{81} \frac{1}{m_b^4m_c^3} \frac{(1-3\xi)^2}{1-\xi}, \quad (30)$$

$$\hat{\Gamma}(b\bar{b}(^3S_1, 8) \rightarrow c\bar{c}(^3P_1, 1) + g) = \frac{2\pi^2\alpha_s^3}{81} \frac{1}{m_b^4m_c^3} \frac{1+\xi}{1-\xi}, \quad (31)$$

$$\begin{aligned} \hat{\Gamma}(b\bar{b}(^3S_1, 8) \rightarrow c\bar{c}(^3P_2, 1) + g) \\ = \frac{2\pi^2\alpha_s^3}{415} \frac{1}{m_b^4m_c^3} \frac{1+3\xi+6\xi^2}{1-\xi}. \end{aligned} \quad (32)$$

The matrix element $\langle O_1^{\chi_{cJ}}(^3P_J) \rangle$ is related to the wave function according to [23]

$$|R'_{\chi_c}(0)|^2 \approx \frac{2\pi}{9} \frac{\langle O_1^{\chi_{cJ}}(^3P_J) \rangle}{2J+1}. \quad (33)$$

Using the value of the matrix element $\langle Y | O_8(^3S_1) | Y \rangle \approx 5 \times 10^{-4} \text{ GeV}^3$ estimated in the last section $\alpha_s(2m_b) = 0.179$, $|R'_{\chi_c}(0)|^2 = 0.075 \text{ GeV}^5$ from the potential model calculation [27], and the branching ratios $B(\chi_{c1} \rightarrow \psi + \gamma) = 0.273$ and $B(\chi_{c2} \rightarrow \psi + \gamma) = 0.135$ (χ_{c0} has a negligible branching ratio into ψ) [25], we obtain the width $\Gamma_{1c}(Y \rightarrow \psi + X) \approx 0.05 \text{ eV}$ and thus a branching ratio of 9×10^{-7} . Therefore, these indirect contributions are negligible when compared with the indirect mechanism considered earlier by Trottier [19].

D. $b\bar{b}(\bar{^3S_1}, 8) \rightarrow g^* \rightarrow c\bar{c}(\bar{^3S_1}, 1) + gg$

One of the six leading Feynman diagrams for the short-distance process $b\bar{b}(\bar{^3S_1}, 8) \rightarrow g^* \rightarrow c\bar{c}(\bar{^3S_1}, 1) + gg$ is shown in Fig. 1(d). The corresponding differential width can be written as the factored form

$$\frac{d\Gamma_{1d}}{dx_v dx_1}(\Upsilon \rightarrow \psi + X) = \frac{d\hat{\Gamma}}{dx_v dx_1}[b\bar{b}(\bar{^3S_1}, 8) \rightarrow c\bar{c}(\bar{^3S_1}, 1)gg]\langle Y | O_8(\bar{^3S_1}) | Y \rangle \langle O_1^\psi(\bar{^3S_1}) \rangle \quad (34)$$

with the short-distance factor given by

$$\begin{aligned} \frac{d\hat{\Gamma}}{dx_v dx_1}(b\bar{b}(\bar{^3S_1}, 8) \rightarrow c\bar{c}(\bar{^3S_1}, 1)gg) \\ = \frac{5\pi\alpha_s^4}{486m_b^4 m_c} \frac{1}{(2-x_v)^2(1-\xi-x_1)^2(1+\xi-x_v-x_1)^2} \{ \xi^4 + 2\xi^3(8-5x_v+x_v^2-2x_1+x_vx_1 \\ + x_1^2) + \xi^2[28-46x_v+21x_v^2-4x_v^3-(28-26x_v+6x_v^2)x_1 + (14-6x_v)x_1^2] + 2\xi[6-17x_v+16x_v^2-6x_v^3 \\ + x_v^4 - (12-22x_v+12x_v^2-2x_v^3)x_1 + (10-12x_v+3x_v^2)x_1^2 - 2(2-x_v)x_1^3 + x_1^4] \\ - (1-x_1)(1-x_v-x_1)(1-x_v+2x_1-x_vx_1-x_1^2) \}, \end{aligned} \quad (35)$$

where the allowable ranges of x_v and x_1 are given in Eqs. (22) and (23). Integrating over x_1 , we obtain the energy distribution for ψ :

$$\begin{aligned} \frac{d\hat{\Gamma}}{dx_v}(b\bar{b}(\bar{^3S_1}, 8) \rightarrow c\bar{c}(\bar{^3S_1}, 1) + gg) \\ = \frac{5\pi\alpha_s^4}{486} \frac{1}{m_b^4 m_c} \frac{1}{(2-x_v)^2(x_v-2\xi)^3} \left\{ (4+20\xi+28\xi^2+16\xi^3-12x_v-36\xi x_v-24\xi^2 x_v+13x_v^2 \right. \\ + 14\xi x_v^2-4x_v^3)(x_v-2\xi)\sqrt{x_v^2-4\xi} - 4\ln\left(\frac{2\xi-x_v+\sqrt{x_v^2-4\xi}}{2\xi-x_v-\sqrt{x_v^2-4\xi}}\right)(2\xi+16\xi^2+6\xi^3-16\xi^4-8\xi^5-12\xi x_v \\ \left. - 20\xi^2 x_v+24\xi^3 x_v+x_v^2+20\xi^4 x_v+12\xi x_v^2-8\xi^2 x_v^2-17\xi^3 x_v^2-x_v^3-\xi x_v^3+5\xi^2 x_v^3) \right\}. \end{aligned} \quad (36)$$

The partial width is obtained numerically:

$$\begin{aligned} \Gamma_{1d}(\Upsilon \rightarrow \psi + X) = \frac{5\pi\alpha_s^4}{486} \frac{\langle Y | O_8(\bar{^3S_1}) | Y \rangle}{m_b^4} \frac{\langle O_1^\psi(\bar{^3S_1}) \rangle}{m_c} \\ \times (1.230). \end{aligned} \quad (37)$$

With the previous inputs, $\alpha_s(2m_b) = 0.179$, and $\langle O_1^\psi(\bar{^3S_1}) \rangle \approx 0.73 \text{ GeV}^3$ obtained from the leptonic width of ψ , we obtain a width of 0.02 eV only, which gives a branching ratio of 3×10^{-7} .

E. Other color-octet processes

When Υ annihilates into a $q\bar{q}$ pair via the s -channel photon γ^* , a bremsstrahlung virtual gluon emitted from the light quark line can become an octet $c\bar{c}(\bar{^3S_1}, 8)$, which then turns into ψ . The factored form of the rate for this process is

$$\begin{aligned} \Gamma_{1e}(\Upsilon \rightarrow \psi + X) = \hat{\Gamma}(b\bar{b}(\bar{^3S_1}, 1) \rightarrow c\bar{c}(\bar{^3S_1}, 8) + q\bar{q}) \\ \times \langle Y | O_1(\bar{^3S_1}) | Y \rangle \langle O_8^\psi(\bar{^3S_1}) \rangle. \end{aligned} \quad (38)$$

This process is of order $\alpha^2\alpha_s^2$, which is similar to a potentially dominant color-octet process in the prompt ψ production in Z^0 decay [9]. We can easily translate the formula from Ref. [9] to the present case. The partial width is given by

$$\begin{aligned} \frac{\Gamma_{1e}(\Upsilon \rightarrow \gamma^* \rightarrow q\bar{q}\psi)}{\Gamma(\Upsilon \rightarrow \gamma^* \rightarrow q\bar{q})} = \frac{\alpha_s^2(2m_c)}{36} \frac{\langle O_8^\psi(\bar{^3S_1}) \rangle}{m_c^3} \left\{ 5(1-\xi^2) \right. \\ - 2\xi \ln \xi + \left[2 \text{Li}_2\left(\frac{\xi}{1+\xi}\right) \right. \\ - 2 \text{Li}_2\left(\frac{1}{1+\xi}\right) - 2\ln(1+\xi)\ln \xi \\ \left. \left. + 3\ln \xi + \ln^2 \xi \right] (1+\xi)^2 \right\}. \end{aligned} \quad (39)$$

Here, $\text{Li}_2(x) = -\int_0^x (dt/t) \ln(1-t)$ is the Spence function and $\xi = (m_c/m_b)^2$. $\Gamma(\Upsilon \rightarrow q\bar{q})$ is given by Eq. (8) times the matrix element $\langle Y | O_1(\bar{^3S_1}) | Y \rangle$. Using $m_b = 4.9 \text{ GeV}$ and $m_c = 1.5 \text{ GeV}$, $\xi \approx 0.0937$ and the curly bracket in (39) is

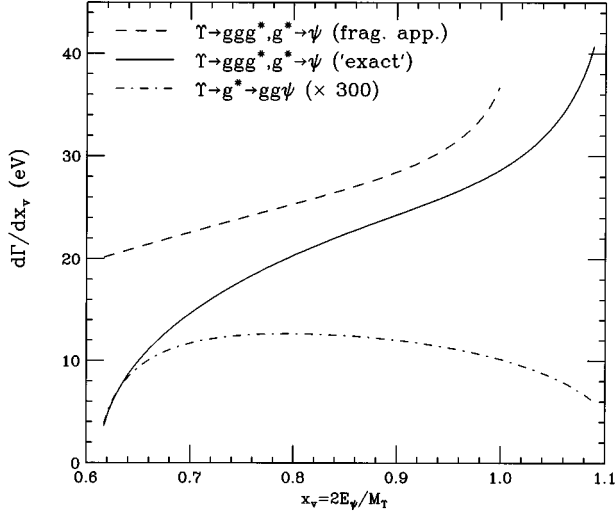


FIG. 2. The inclusive energy spectra $d\Gamma/dx_\psi$ of ψ in the decay $Y \rightarrow \psi + X$ with the following color-octet processes: $b\bar{b}(\bar{^3S_1}, 1) \rightarrow g^* g g \rightarrow c\bar{c}(\bar{^3S_1}, 8) g g$ (solid) as predicted by Eq. (24) compared with the fragmentation approximation (dashed), and $b\bar{b}(\bar{^3S_1}, 8) \rightarrow g^* \rightarrow c\bar{c}(\bar{^3S_1}, 1) g g$ as predicted by Eq. (36) multiplied by 300 (dashed-dotted).

about 1.1, as compared to the much larger value of 27.9 in the corresponding case in Z^0 decay [9] where $\xi = m_\psi^2/M_Z^2 \approx 1.1 \times 10^{-3}$. Therefore, the double logarithmic terms in Eq. (39) do not provide sufficient enhancement in our present case. Numerically, the ratio in Eq. (39) is only 8.3×10^{-6} , and so this process can be safely ignored.

In addition to the above processes, one can also consider a color-octet 3S_1 $b\bar{b}$ pair annihilating into a light quark pair via an s -channel gluon, followed by an off-shell photon bremsstrahlung off either the b quark line or the light quark line and eventually turns into the ψ . This process also involves the matrix element $\langle Y | O_8(^3S_1) | Y \rangle$ and, therefore, will be suppressed. One can also consider both the $b\bar{b}$ and $c\bar{c}$ pairs inside the quarkonia in the color-octet configuration. Processes associated with such configuration are further suppressed by powers of v_c^2 and v_b^2 compared to those we have studied in this paper. We ignore them in this work.

IV. DISCUSSIONS AND CONCLUSIONS

The energy distribution of charmonium in the Y decay can provide an interesting test for the NRQCD factorization formalism discussed in Sec. II. The energy distributions for the processes of Figs. 1(a) and 1(c) are just a delta function with the peak at one half of the Y mass, while the energy distributions for the processes of Figs. 1(b) and 1(d) are given by Eqs. (24) and (36), respectively, and are shown in Fig. 2. The solid curve in Fig. 2 is the energy spectrum of ψ for the dominant process $b\bar{b}(\bar{^3S_1}, 1) \rightarrow g^* g g \rightarrow c\bar{c}(\bar{^3S_1}, 8) g g$; it is monotonically increasing as

the ψ energy increases and eventually cut off by the kinematic limit. In the fragmentation approximation ($m_b \rightarrow \infty$ with E_ψ/m_b held fixed), the energy distribution of ψ for this subprocess is the same as that of the fragmenting gluon, up to an overall constant. The result using the fragmentation approximation is shown as the dashed curve in Fig. 2 for comparison. The fragmentation approximation has overestimated the exact result for all energies of the ψ . On the other hand, CLEO [16] obtained a relatively flat momentum spectrum. However, the rising spectrum shown by various mechanisms such as final state interactions of the soft gluons [28] and by relativistic corrections of the bound state [29], just to name a few possibilities. This situation is very similar to the photon spectrum in the decay $Y \rightarrow \gamma g g$ [28–30]. The dashed-dotted curve in Fig. 2 is the energy distribution for the suppressed channel $b\bar{b}(\bar{^3S_1}, 8) \rightarrow g^* \rightarrow c\bar{c}(\bar{^3S_1}, 1) g g$ and it has a much flatter shape.

Among the several processes that we have considered in this paper, those two with the color-octet $b\bar{b}$ pair inside the decaying Y are more than two orders of magnitude below the CLEO data and therefore negligible, while the other with the color-octet $c\bar{c}$ pair inside the produced ψ are relatively more important. The largest contribution is being the process shown in Fig. 1(b), which has a branching ratio of about 2.5×10^{-4} . The next largest contribution is the indirect process considered by Trottier [19]. From Eq. (21) of Ref. [19] and using the heavy quark spin symmetry relation $\langle O_8^{X_{cJ}}(^3S_1) \rangle \approx (2J+1) \langle O_8^{X_{c0}}(^3S_1) \rangle$ [23] and the value of the matrix element $\langle O_8^{X_{c0}}(^3S_1) \rangle = 0.0076 \text{ GeV}^3$ obtained by fitting the Fermilab Tevatron data [5,8], we obtain $\Sigma_J B_{\text{Trottier}}(Y \rightarrow X_{cJ} + X; X_{cJ} \rightarrow \psi \gamma) \approx 5.7 \times 10^{-5}$. Processes that are comparable to this one are (1) the process shown in Fig. 1(a) which has a branching ratio of 1.6×10^{-5} , (2) the processes $Y \rightarrow g g \gamma^* \rightarrow \psi + X$ and $Y \rightarrow \gamma g g^* \rightarrow \psi + X$ via the color-octet mechanism which have a combined branching ratio of about 2×10^{-5} , and finally, (3) the indirect contribution from ψ' having a branching ratio of about 5×10^{-5} . Therefore, adding up the contributions from all these processes, we obtain a branching ratio $B(Y \rightarrow \psi + X) \approx 4 \times 10^{-4}$, which is within 2σ of the CLEO data. Given these theoretical results, it would be very interesting to have more precise measurements of the inclusive rates and energy spectra of charmonium from the Y decay. This would provide a crucial test of the NRQCD factorization formalism applied simultaneously to both the bottomonium and charmonium systems.

ACKNOWLEDGMENTS

This work was supported in part by the United States Department of Energy under Grant Nos. DE-FG02-84ER40173, DE-FG03-93ER40757, and DE-FG03-91ER40674.

- [1] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **69**, 3704 (1992); **71**, 2537 (1993); **75**, 1451 (1995).
- [2] CDF Collaboration, F. Abe *et al.*, presented at the XVII International Symposium on Lepton-Photon Interactions, Beijing,

China, 1995 (unpublished); K. Byrum, in *Proceedings of the XXVII International Conference on High Energy Physics*, Glasgow, Scotland, 1994, edited by P. J. Bussey and I.G. Knowles (Institute of Physics, London, 1995); T. Daniels, in

- The Albuquerque Meeting*, Proceedings of the Eighth Meeting of the Division of Particles and Fields of the APS, Albuquerque, New Mexico, 1994, edited by S. Seidel (World Scientific, Singapore, 1995), Report No. hep-ex/9412013 (unpublished); A. Sansoni, Fermilab Report No. FERMILAB-CONF-95/263-E, to appear in the Proceedings of the Sixth International Symposium on Heavy Flavor Physics, Pisa, 1995 (unpublished).
- [3] E. Braaten and T. C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993); Phys. Rev. D **52**, 6627 (1995).
 - [4] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995).
 - [5] M. Cacciari, M. Greco, M.L. Mangano, and A. Petrelli, Phys. Lett. B **356**, 553 (1995).
 - [6] P. Cho and A. K. Leibovich, Phys. Rev. D **52** 150 (1996); Report No. CALT-68-2026, hep-ph/9511315 (unpublished).
 - [7] M. Cacciari and M. Greco, Phys. Rev. Lett. **73**, 1586 (1994); E. Braaten, M. A. Doncheski, S. Fleming, and M. L. Mangano, Phys. Lett. B **333**, 548 (1994); D. P. Roy and K. Sridhar, *ibid.* **339**, 141 (1994).
 - [8] E. Braaten, S. Fleming, and T.C. Yuan, Ann. Rev. Nucl. Part. Sci. (to be published).
 - [9] K. Cheung, W.-Y. Keung, and T. C. Yuan, Phys. Rev. Lett. **76**, 877 (1996); P. Cho, Phys. Lett. B **368**, 171 (1996).
 - [10] E. Braaten and Y.-Q. Chen, Phys. Rev. Lett. **76**, 730 (1996).
 - [11] J. Amundson, S. Fleming, and I. Maksymyk, Austin Report No. UTTG-10-95/MADPH-95-914, hep-ph/9601298 (unpublished); M. Cacciari and M. Krämer, DESY Report No. DESY 96-005, hep-ph/9601276 (unpublished); J. P. Ma, Nucl. Phys. B **460**, 109 (1996); P. Ko, J. Lee, and H. S. Song, Seoul Report No. SNUTP 95-116, hep-ph/9602223 (unpublished); R. Godbole, D.P. Roy, and K. Sridhar, Tata Institute Report No. TIFR/TH/95-57, hep-ph/9511433 (unpublished).
 - [12] W. K. Tang and M. Vanttinen, Phys. Rev. D **53**, 4851 (1996); S. Gupta and K. Sridhar, Tata Institute Report No. TIFR/TH/96-04, hep-ph/9601349 (unpublished).
 - [13] K. Sridhar, Tata Institute Report No. TIFR/TH/96-07, hep-ph/9602329 (unpublished).
 - [14] P. Ko, J. Lee, and H.S. Song, Phys. Rev. D **53**, 1409 (1996).
 - [15] V. Barger, R. J. N. Phillips, and S. Fleming, Madison Report No. MADPH-95-911, hep-ph/9510457 (unpublished).
 - [16] CLEO Collaboration, R. Fulton *et al.*, Phys. Lett. B **224**, 445 (1989).
 - [17] Crystal Ball Collaboration, W. Maschmann *et al.*, Z. Phys. C **46**, 555 (1990).
 - [18] ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **55**, 25 (1992).
 - [19] H. Trottier, Phys. Lett. B **320**, 145 (1994).
 - [20] H. Fritzsch and K.-H. Streng, Phys. Lett. **77B**, 299 (1978).
 - [21] H. Fritzsch, Phys. Lett. **67B**, 217 (1977).
 - [22] I.I.Y. Bigi and S. Nussinov, Phys. Lett. **82B**, 281 (1979).
 - [23] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. D **51**, 1125 (1995).
 - [24] For a review of color-singlet model, see, for example, G.A. Schuler, CERN Report No. CERN-TH.7170/94, hep-ph/9403387, Phys. Rep. (to be published).
 - [25] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
 - [26] J.P. Leveille and D. Scott, Phys. Lett. **95B**, 96 (1980).
 - [27] E.J. Eichten and C. Quigg, Phys. Rev. D **52**, 1726 (1995).
 - [28] R.D. Field, Phys. Lett. **164B**, 160 (1985).
 - [29] W.Y. Keung and I.J. Muzinich, Phys. Rev. D **27**, 1518 (1983).
 - [30] Crystal Ball Collaboration, A. Bizzeti *et al.*, Phys. Lett. B **267**, 286 (1991).