Nonleptonic two-body decays of D mesons in broken SU(3)

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Decays of the D mesons to two pseudoscalars, to two vectors, and to pseudoscalar plus vector are discussed in the context of broken flavor SU(3). A few assumptions are used to reduce the number of parameters. Amplitudes are fit to the available data, and predictions of branching ratios for unmeasured modes are made. [S0556-2821(96)05513-0]

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INTRODUCTION

Many data are available on the hadronic two-body decays of charmed mesons. Theoretical models that attempt to systematize the decay patterns have been available for many years. These models usually make dynamical assumptions in order to reduce the number of amplitudes that contribute to a particular decay. For example, the large N_c approximation [1,2], or the heavy-quark effective theory [3]. It is not clear *a priori* how well such approximations should work and hence how seriously to take a conflict between a prediction and a measured value. Another approach is to assume that the matrix elements factorize [4]. This model is quite successful in describing observed modes, but again, it is difficult to know whether a discrepancy is due to an incorrect measurement or the failure of the assumption. A more general approach based on a diagrammatic classification [5], with different assumptions, also exists. In many cases attempts are made to obtain predictions of unmeasured modes from these models.

SU(3) is badly broken in these decays, so models based on exact symmetry [6] are not useful. An analysis of the SU(3) breaking was begun in [7]. However, an attempt at a complete parameterization has been conspicuously missing, due to the large number of reduced matrix elements involved. We set out to remedy this omission. This work gives a full parameterization of the decays of the D mesons into final states of two pseudoscalars (PP), two vectors (VV), and a pseudoscalar plus a vector (PV), including SU(3) breaking. We assume that isospin is a good symmetry; the relations predicted between decay modes that follow from isospin are therefore respected by our fit. The elements of this parameterization—the particle representations, the weak Hamiltonian, the breaking operator, and the reduced matrix elements—are discussed in the following sections. We make only very few assumptions to limit the number of parameters. We fit the parameters to the available data of two-body decays and predict many unmeasured modes. Because a few of the parameters are not constrained, we indicate which branching fractions are needed to predict the rest of certain classes of modes. We comment on the case of $D_s \to \eta' \rho^+$, where the model is barely consistent with data.

I. PARTICLE STATES IN FLAVOR SU(3)

In a model based on flavor SU(3), the particles are denoted by their SU(3) representations. The fundamental representation is the triplet (**3**) of quarks u, d, and s. The three D mesons $\{D^0, D^+, D_s^+\}$ form an antitriplet ($\overline{\mathbf{3}}$) representation. The pseudoscalars $\{\pi^+, \pi^0, \pi^-, K^+, K^0, K^-, \overline{K}^0, \eta_8\}$ form an octet (**8**) representation, as do the vectors $\{\rho^+, \rho^0, \rho^-, K^{*+}, K^{*0}, K^{*-}, \overline{K}^{*0}, \omega_8\}$. The η_1 and ω_1 are each singlets. The physical η, η', ϕ, and ω are linear combinations of them, with mixing angles $-17.3°$ [8] and 39° [9] for η-η' and ϕ-ω, respectively.[1]

II. THE WEAK HAMILTONIAN

The decays of the D mesons are mediated by the weak Hamiltonian. Ignoring QCD corrections, the Hamiltonian in terms of the quark fields is

$$
\begin{aligned}
H_{\text{weak}} = {} & \frac{G_F}{\sqrt{2}} \cos^2\theta_C \overline{u}\gamma^\mu(1-\gamma_5)d\overline{s}\gamma_\mu(1-\gamma_5)c \\
& + \frac{G_F}{\sqrt{2}} \cos\theta_C \sin\theta_C \overline{u}\gamma^\mu(1-\gamma_5)s\overline{s}\gamma_\mu(1-\gamma_5)c \\
& - \frac{G_F}{\sqrt{2}} \cos\theta_C \sin\theta_C \overline{u}\gamma^\mu(1-\gamma_5)d\overline{d}\gamma_\mu(1-\gamma_5)c \\
& - \frac{G_F}{\sqrt{2}} \sin^2\theta_C \overline{u}\gamma^\mu(1-\gamma_5)s\overline{d}\gamma_\mu(1-\gamma_5)c. \qquad (1)
\end{aligned}
$$

Note that the operators $\overline{q}$ create quarks and so transform as a triplet, while q transforms as the antitriplet. Using the Clebsch-Gordan coefficients for the expansion of the product $\mathbf{3}\times\overline{\mathbf{3}}\times\mathbf{3}$, we can classify the operators according to irreducible representations of SU(3) as

$$
(\overline{u}d)(\overline{s}c) = -(1/\sqrt{2})\overline{6}(-\tfrac{2}{3},1,1) - (1/\sqrt{2})\mathbf{15}(-\tfrac{2}{3},1,1), \qquad (2)
$$

$$
(\overline{u}s) = (\overline{d}c) = (1/\sqrt{2})\overline{6}(\tfrac{4}{3},0,0) + (1/\sqrt{2})\mathbf{15}(\tfrac{4}{3},1,0),
$$

$$
\begin{aligned}
(\overline{u}d)(\overline{d}c) = {} & (1/\sqrt{8})\mathbf{3}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2}) + \tfrac{1}{2}\mathbf{3}'(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2}) - \tfrac{1}{2}\,\overline{\mathbf{6}}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2}) \\
& - (1/\sqrt{3})\mathbf{15}(\tfrac{1}{3},\tfrac{3}{2},\tfrac{1}{2}) - (1/\sqrt{24})\mathbf{15}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2}),
\end{aligned}
$$

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[1] K^* denotes $K^*(892)$; η' denotes $\eta'(958)$.

$$(\overline{u}s)(\overline{s}c)=(1/\sqrt{8})\mathbf{3}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})+\tfrac{1}{2}\mathbf{3}'(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})+\tfrac{1}{2}\;\overline{\mathbf{6}}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})+\sqrt{3/8}\;\mathbf{15}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2}),$$

where $(\overline{q}q')$ denotes $\overline{q}\gamma^{\mu}(1-\gamma_5)q'$. The numbers in parentheses are hypercharge, total isospin, and third component of isospin of the particular members of the SU(3) representations. The weak Hamiltonian can now be written in terms of the representations $\mathbf{3}$, $\mathbf{3}'$, $\overline{\mathbf{6}}$, and $\mathbf{15}$ as

$$\begin{aligned}H_{\text{weak}}=&G_F\sin^2\theta_C[-\tfrac{1}{2}\;\overline{\mathbf{6}}(\tfrac{4}{3},0,0)-\tfrac{1}{2}\;\mathbf{15}(\tfrac{4}{3},1,0)]\\&+G_F\cos^2\theta_C[-\tfrac{1}{2}\;\overline{\mathbf{6}}(-\tfrac{2}{3},1,1)-\tfrac{1}{2}\;\mathbf{15}(-\tfrac{2}{3},1,1)]\\&+G_F\cos\theta_C\sin\theta_C\cdot[(1/\sqrt{2})\overline{\mathbf{6}}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})\\&+(1/\sqrt{6})\mathbf{15}(\tfrac{1}{3},\tfrac{3}{2},\tfrac{1}{2})+(1/\sqrt{3})\mathbf{15}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})].\end{aligned}\tag{3}$$

Note that the $\mathbf{3}$ and $\mathbf{3}'$ representations do not appear in the uncorrected H_{weak} [10]. Because the QCD corrections are multiplicative and do not mix the SU(3) representations, the $\mathbf{3}$ and $\mathbf{3}'$ will also not appear in $H_{\text{weak}}(m_c)$.

Since the decays of the D mesons occur at the scale of the c-quark mass, we must allow the QCD evolution of the various operators from the W-mass scale, where Eq. (1) is valid, to the c-mass scale. The operators represented by the $\mathbf{15}$ are symmetric under quark interchange, and those represented by the $\overline{\mathbf{6}}$ are antisymmetric. The QCD renormalization of operators with these symmetry properties has been calculated [11]. We find that

$$\begin{aligned}\mathbf{15}\to\mathbf{15}\times\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{a_5^+}\times\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{a_4^+},\\ \overline{\mathbf{6}}\to\overline{\mathbf{6}}\times\left[\frac{\alpha_s(M_W)}{\alpha_s(m_b)}\right]^{a_5^-}\times\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{a_4^-},\end{aligned}\tag{4}$$

where

$$\begin{aligned}a_{N_f}^+=\frac{6}{33-2N_f},\\ a_{N_f}^-=\frac{-12}{33-2N_f},\end{aligned}\tag{5}$$

in the regime where there are N_f flavor degrees of freedom. Taking into account the change in the number of active flavors as the b-quark threshold is crossed, and using $\alpha(M_z)=0.119$, we obtain

$$\begin{aligned}\mathbf{15}&\to0.81\;\;\mathbf{15},\\ \overline{\mathbf{6}}&\to1.5\;\;\overline{\mathbf{6}}.\end{aligned}\tag{6}$$

With Eq. (3) as the boundary condition, we have

$$\begin{aligned}H_{\text{weak}}(m_c)=&\frac{G_F}{2}\sin^2\theta_C[-0.81\;\;\mathbf{15}(\tfrac{4}{3},1,0)-1.5\;\;\overline{\mathbf{6}}(\tfrac{4}{3},0,0)]+\frac{G_F}{2}\cos^2\theta_C[-0.81\;\;\mathbf{15}(-\tfrac{2}{3},1,1)-1.5\;\;\overline{\mathbf{6}}(-\tfrac{2}{3},1,1)]\\&+\frac{G_F}{2}\cos\theta_C\sin\theta_C[0.81\times(2/\sqrt{3})\mathbf{15}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})+0.81\times\sqrt{2/3}\mathbf{15}(\tfrac{1}{3},\tfrac{3}{2},\tfrac{1}{2})+1.5\times\sqrt{2}\overline{\mathbf{6}}(\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2})].\end{aligned}\tag{7}$$

Note that the QCD corrections do not introduce any new phases into the process. We can absorb these corrections into the reduced matrix elements (discussed below), and therefore they do not affect our analysis. In the absence of an independent determination of the matrix elements (e.g., from lattice gauge theory), the values of the QCD corrections are irrelevant.

III. PARAMETRIZATION

A. SU(3) breaking

For a complete parametrization of any process in flavor SU(3), we must include explicit breaking. Since we know that the source of flavor SU(3) breaking among the pions and kaons is the difference between the quark masses, we do this with an operator M which transforms as an $\mathbf{8}$. Although the quark mass difference is insufficient to explain the large SU(3) breaking that will be found, an octet is the simplest nontrivial operator that can be used.

We can express M as

$$M=\alpha\lambda_3+\beta\lambda_8,\tag{8}$$

where λ_i are the usual Gell-Mann matrices. The term in α represents breaking of the isospin SU(2) subgroup. This breaking, proportional to the difference between up and down quark masses, is expected to be very small and we neglect it in the following. The constant β can be absorbed into the reduced matrix elements. Hence M can be reduced to

$$M=\lambda_8.\tag{9}$$

B. Reduced matrix elements

Now consider the most general parametrization of the decays in the context of the flavor SU(3) symmetry. For each possible contraction of the representations into an SU(3) singlet there must be one parameter, i.e., one reduced matrix element. Each reduced matrix element is complex. The representations involved are those in Sec. I: D ($\overline{\mathbf{3}}$), H ($\overline{\mathbf{6}}\oplus\mathbf{15}$), and two of P and V (each $\mathbf{1}$ or $\mathbf{8}$). In addition, we must include all possible ways of involving the symmetry-breaking parameter M. We assume that the breaking is linear in M. We have chosen to contract D with H, then contract the products (PP, PV, VV) (and then possibly with M), and finally contract the two parts into the singlet. Our labels for the reduced matrix elements reflects this. For example, the matrix element denoted $[(DH_{15})_8((PP)_1M)_8]$ is obtained by contracting D and the **15** component of H into an octet, contracting PP into a singlet which combines with M to become another octet, and contracting the two resulting octets into the singlet.

Unfortunately, the above parametrization involves far more parameters than there exist data. Therefore we make two important assumptions. First, we assume that we can separate the spin and flavor dynamics of the processes, i.e., that the relative strengths of the reduced matrix elements are the same in the PP, PV, (VP), and VV cases. This implies that only forty-eight reduced SU(3) matrix elements are needed. They are labeled with S and O for the singlet and octet representations, rather than with PP, PV, or VV. They are listed in the Appendix. In order to distinguish the spin states we introduce two parameters, called A_{PV} and A_{VV} ($A_{PP}\equiv 1$). Second, we assume that the phase of each reduced matrix element is given solely by the representation of the product particles (before M is included). Bose symmetry for PP and VV and an appropriate phase rotation of the particle fields reduces the list of phases to $(\eta_1\eta_1)_1$, $(\eta_1\omega_1)_1$, $(\omega_1\omega_1)_1$, $(P\eta_1)_8$, $(P\omega_1)_8$, $(V\eta_1)_8$, $(V\omega_1)_8$, $(PP)_1$, $(PP)_{27}$, $(PV)_1$, $(PV)_{8'}$, $(PV)_{10}$, $(PV)_{\overline{10}}$, $(PV)_{27}$, $(VV)_1$, and $(VV)_{27}$. One should note that we cannot determine the relative phases between PP, PV, and VV. To the extent that all phases are introduced by final-state interactions, one can read off the relative phases of the product representations in Table VII. The complete list of parameters appears in Tables VI and VII.

The amplitude for each decay mode can be expressed as a sum over the reduced matrix elements with the appropriate Clebsch-Gordan coefficients:

$$A(D_j \to X_i) = \sum_k C_{ijk} R_k S_i . \tag{10}$$

Here R_k are the reduced SU(3) matrix elements and S_i are the parameters that we call $A_{PP}\equiv 1$, A_{PV}, and A_{VV}. The SU(3) Clebsch-Gordan factors C_i were calculated by computer. Many of the routines used are described in [12].

C. Linear combinations of reduced matrix elements

There are 45 measured values for the two-body decay modes and an additional 13 modes where upper limits exist.[2] It would appear that there are still more parameters than data, and therefore the model lacks predictability. However, there are only forty linearly independent combinations of the SU(3) reduced matrix elements that contribute to the possible decay modes of the D mesons. With the assumption of the last section concerning the phases of the reduced matrix elements, the linear combinations fall into these classes:

$$\begin{array}{ll}
\text{involving } (SS)_1: & L_1, \\
\text{involving } (SO)_8: & L_2, \ldots, L_8, \\
\text{involving } (OO)_1: & L_9, \\
\text{involving } (OO)_8: & L_{10}, \ldots, L_{16}, \\
\text{involving } (OO)_{8'}: & L_{17}, \ldots, L_{23}, \\
\text{involving } (OO)_{10}: & L_{24}, \ldots, L_{28}, \\
\text{involving } (OO)_{\overline{10}}: & L_{29}, \ldots, L_{33}, \\
\text{involving } (OO)_{27}: & L_{34}, \ldots, L_{40}
\end{array}.$$

We write them each as a sum over the reduced matrix elements, viz.,

$$L_n = \sum_i C'_{in} R_i , \tag{11}$$

and normalize them for convenience by setting

$$\sum_i C'^2_{in} = 1. \tag{12}$$

Now Eq. (10) is replaced by

$$A(D_j \to X_i) = \sum_n C''_{ijn} L_n S_i . \tag{13}$$

The L_n replace the reduced matrix elements in our parameterization of the amplitudes. The forty linearly independent combinations contain matrix elements including those that involve the breaking operator M. It is not possible to divide the linear combinations into a set that contains only matrix elements without M and a set containing only matrix elements with M. Of the forty combinations, three are not constrained by the available data. We call them L_1, L_2, and L_3. They are discussed below. A list of the independent L_n and the expressions for the amplitudes $A(D_i \to X)$ are given in the Appendix.

The replacement of the set of reduced matrix elements by the set of linear combinations that contribute to the possible decay modes reduces the number of parameters by eight. The total number is now 53. These parameters are fit to the data; the individual reduced matrix elements are no longer considered. The values of the linear combinations for the best fit are in Table VI. The signs have been absorbed into the C''_{ijn}. The units are fixed by Eq. (12) and by the units of the amplitudes, as given in Sec. IV.

The unconstrained combination L_1 contributes to the modes $D^0 \to \eta\eta$, $\eta\eta'$, $\eta'\eta'$, $\eta\phi$, $\eta\omega$, $\eta'\phi$, $\eta'\omega$, $\phi\phi$, $\phi\omega$, and $\omega\omega$. Because these modes are unobserved, the phases of $(\eta_1\omega_1)_1$, and $(\omega_1\omega_1)_1$ are also unconstrained. The remaining unconstrained linear combinations are L_2 and L_3. They contribute to the above modes, and also to modes of the types $D^0 \to \eta K^0$ and $D_s \to \eta K^+$. By "type" we mean a class of modes that contain mesons of the same flavors and charges. Thus the type $D_s \to \eta K^+$ contains the modes $D_s \to \eta K^+$, $\eta' K^+$, ηK^{*+}, $\eta' K^{*+}$, ϕK^+, ωK^+, ϕK^{*+}, ωK^{*+} and no others. With the exception of the limit on the branching ratio for $D_s \to \phi K^+$, there are no data for these modes. We still have some freedom in the definition of L_2 and L_3 that allows modes of the type $D_0 \to \eta K^0$ to depend on only one of them

[2] The data are from the Particle Data Group [9], together with [14] for the mode $D^+ \to K^{*0}\pi^+$.

TABLE I. Modes with positive experimental values. Branching ratios (BR) from data and from the fit are given.

Mode	Data BR	Fit BR
$D^0 \to K^- \pi^+$	0.0401 ± 0.0014	0.0400 ± 0.0014
$D^0 \to K^- K^+$	0.00454 ± 0.00029	0.00453 ± 0.00030
$D^0 \to \overline{K^0} \pi^0$	0.0205 ± 0.0026	0.0208 ± 0.0022
$D^0 \to \overline{K^0} K^0$	0.0011 ± 0.0004	0.00103 ± 0.00043
$D^0 \to \pi^- \pi^+$	0.00159 ± 0.00012	0.00159 ± 0.00012
$D^0 \to \pi^- K^+$	0.00031 ± 0.00014	$0.00031^{+0.00018}_{-0.00014}$
$D^0 \to \pi^0 \pi^0$	0.00088 ± 0.00023	0.00087 ± 0.00025
$D^0 \to \eta \overline{K^0}$	0.0068 ± 0.0011	0.0069 ± 0.0011
$D^0 \to \eta' \overline{K^0}$	0.0166 ± 0.0029	0.0168 ± 0.0028
$D^0 \to K^{*-} \rho^+$	0.059 ± 0.024	0.063 ± 0.016
$D^0 \to \overline{K^{*0}} \rho^0$	0.016 ± 0.004	0.0164 ± 0.0038
$D^0 \to \overline{K^{*0}} K^{*0}$	0.0029 ± 0.0015	$0.0029^{+0.0019}_{-0.0014}$
$D^0 \to \omega \overline{K^{*0}}$	0.011 ± 0.005	0.0099 ± 0.0044
$D^0 \to \phi \rho^0$	0.0019 ± 0.0005	0.00192 ± 0.00045
$D^0 \to K^- \rho^+$	0.104 ± 0.013	0.102 ± 0.013
$D^0 \to K^- K^{*+}$	0.0034 ± 0.0008	0.00323 ± 0.00080
$D^0 \to \overline{K^0} \rho^0$	0.0110 ± 0.0018	0.0110 ± 0.0017
$D^0 \to K^{*-} \pi^+$	0.049 ± 0.006	0.0495 ± 0.0058
$D^0 \to K^{*-} K^+$	0.0018 ± 0.0010	0.00209 ± 0.00087
$D^0 \to \overline{K^{*0}} \pi^0$	0.030 ± 0.004	0.0301 ± 0.0039
$D^0 \to \phi \overline{K^0}$	0.0083 ± 0.0012	0.0081 ± 0.0012
$D^0 \to \omega \overline{K^0}$	0.020 ± 0.004	0.0195 ± 0.0043
$D^0 \to \eta \overline{K^{*0}}$	0.019 ± 0.005	0.0204 ± 0.0049
$D^+ \to \overline{K^0} \pi^+$	0.0274 ± 0.0029	0.0262 ± 0.0028
$D^+ \to \overline{K^0} K^+$	0.0078 ± 0.0017	0.0086 ± 0.0016
$D^+ \to \pi^0 \pi^+$	0.0025 ± 0.0007	0.00257 ± 0.00067
$D^+ \to \eta \pi^+$	0.0075 ± 0.0025	0.0068 ± 0.0021
$D^+ \to \overline{K^{*0}} \rho^+$	0.021 ± 0.014	0.0398 ± 0.0092
$D^+ \to \overline{K^{*0}} K^{*+}$	0.026 ± 0.011	$0.0090^{+0.0054}_{-0.0041}$
$D^+ \to \overline{K^0} \rho^+$	0.066 ± 0.025	0.071 ± 0.018
$D^+ \to \pi^+ K^{*0}$	0.00046 ± 0.00015	0.00046 ± 0.00014
$D^+ \to \overline{K^{*0}} \pi^+$	0.022 ± 0.004	0.0217 ± 0.0041
$D^+ \to \overline{K^{*0}} K^+$	0.0051 ± 0.0010	0.00463 ± 0.00097
$D^+ \to \phi \pi^+$	0.0067 ± 0.0008	0.00674 ± 0.00078
$D^+ \to \phi K^+$	0.00039 ± 0.00020	$0.00039^{+0.00027}_{-0.00020}$
$D_s \to \overline{K^0} K^+$	0.035 ± 0.007	0.0319 ± 0.0059
$D_s \to \eta \pi^+$	0.019 ± 0.004	0.0204 ± 0.0039
$D_s \to \eta' \pi^+$	0.047 ± 0.014	0.054 ± 0.012
$D_s \to \overline{K^{*0}} K^{*+}$	0.056 ± 0.021	0.055 ± 0.018
$D_s \to \phi \rho^+$	0.065 ± 0.017	0.056 ± 0.014
$D_s \to \overline{K^0} K^{*+}$	0.042 ± 0.010	0.043 ± 0.011
$D_s \to \overline{K^{*0}} K^+$	0.033 ± 0.005	0.0328 ± 0.0053
$D_s \to \phi \pi^+$	0.035 ± 0.004	0.0349 ± 0.0040
$D_s + \eta \rho^+$	0.100 ± 0.022	0.100 ± 0.019

(choose L_2). This will allow us to estimate one of their branching fractions and thereby make some predictions of the other modes of this type.

IV. DATA AND FITTING THEREOF

The data used to determine the parameters are listed in Tables I–V. These are the modes for which there exist either experimental values or experimental limits. In the VV modes, S and D waves are possible. Data exist from E691 [15] for the modes $D_0 \to K^{*0} \rho_0$ and $D^+ \to K^{*0} \rho^+$. These are consistent with the S and D waves both having significant amplitudes and are inconsistent with either being zero. The ratios of S- and D-wave amplitudes from these two modes are taken as additional data, and the overall ratio of S- to D-wave amplitudes for the VV modes is allowed to vary in the fit. Its value is determined by the two modes mentioned above, and depends very little on the other data.

TABLE II. D^0 modes with predicted branching ratios. Experimental limits are given when available. All limits are at 90% confidence.

Mode	Data BR	Predicted BR	Predicted limit
$D^0 \to \pi^0 K^0$		$0.00017^{+0.00011}_{-0.00008}$	
$D^0 \to \phi \overline{K^{*0}}$		$0.00108^{+0.00073}_{-0.00054}$	
$D^0 \to \rho^0 K^{*0}$		$0.00038^{+0.00031}_{-0.00022}$	
$D^0 \to \pi^0 \rho^0$		$0.00014^{+0.00018}_{-0.00011}$	
$D^0 \to \pi^- \rho^+$		$0.093^{+0.133}_{-0.075}$	
$D^0 \to \rho^- \pi^+$		$0.094^{+0.136}_{-0.076}$	
$D^0 \to \eta \pi^0$		$0.0060^{+0.0092}_{-0.0050}$	
$D^0 \to \eta \rho^0$		$0.025^{+0.041}_{-0.021}$	<0.092
$D^0 \to K^{*-} K^{*+}$		$0.0024^{+0.0041}_{-0.0021}$	<0.0092
$D^0 \to \rho^0 K^0$		$0.0024^{+0.0041}_{-0.0021}$	<0.0091
$D^0 \to \eta' \overline{K^{*0}}$	<0.0011	$0.00018^{+0.00032}_{-0.00016}$	<0.00070
$D^0 \to \rho^- K^{*+}$		$0.00022^{+0.00038}_{-0.00020}$	<0.00085
$D^0 \to \rho^- K^+$		$0.0020^{+0.0035}_{-0.0018}$	<0.0078
$D^0 \to \phi \pi^0$		$0.024^{+0.049}_{-0.022}$	
$D^0 \to \pi^- K^{*+}$		$0.0019^{+0.0037}_{-0.0018}$	<0.0080
$D^0 \to \pi^0 K^{*0}$		$0.0020^{+0.0041}_{-0.0019}$	<0.0087
$D^0 \to \overline{K^0} K^{*0}$	<0.0008		<0.00052
$D^0 \to \omega \pi^0$			<0.086
$D^0 \to \eta' \rho^0$			<0.011
$D^0 \to \omega \rho^0$			<0.084
$D^0 \to \rho^- \rho^+$			<0.015
$D^0 \to \overline{K^{*0}} K^0$	<0.0015		<0.00061
$D^0 \to \eta' \pi^0$			<0.057
$D^0 \to \rho^0 \rho^0$			<0.0065

For each mode we remove the phase space and Cabibbo factors and reduce the branching ratio to a decay amplitude in arbitrary units. Because the vector particles have substantial widths, the phase space for modes involving a vector is integrated over the relativistic Breit-Wigner resonance for that resonance. The effect of this is important for those modes where the sum of the particle masses is within a few widths of the D mass. The modes $D^0 \to \phi K^{*0}$, $\phi \overline{K^{*0}}$, and $D^+ \to \phi K^{*+}$ would be forbidden if the widths were set to zero. Each amplitude is now expressed as a sum of Clebsch-Gordan coefficients times the parameters that represent the

TABLE III. D^+ modes with predicted branching ratios. Experimental limits are given when available. All limits are at 90% confidence.

Mode	Data BR	Predicted BR	Predicted limit
$D^+\to\rho^0\rho^+$		0.0066 ± 0.0023	
$D^+\to\eta K^+$		$0.0032^{+0.0030}_{-0.0020}$	
$D^+\to\pi^+K^0$		$0.017^{+0.018}_{-0.011}$	
$D^+\to\pi^0K^+$		$0.0086^{+0.0089}_{-0.0057}$	
$D^+\to\pi^0\rho^+$		$0.0034^{+0.0036}_{-0.0023}$	
$D^+\to\phi K^{*+}$		$0.00031^{+0.00035}_{-0.00022}$	
$D^+\to\rho^+K^{*0}$		$0.025^{+0.031}_{-0.018}$	
$D^+\to\rho^0K^{*+}$		$0.0095^{+0.0118}_{-0.0071}$	
$D^+\to\rho^+K^0$		$0.0087^{+0.0119}_{-0.0068}$	
$D^+\to\omega K^{*+}$		$0.0022^{+0.0031}_{-0.0018}$	
$D^+\to\omega\pi^+$	<0.007	$0.0024^{+0.0036}_{-0.0020}$	
$D^+\to\pi^0K^{*+}$		$0.0103^{+0.0162}_{-0.0087}$	
$D^+\to\omega\rho^+$		$0.0026^{+0.0049}_{-0.0023}$	<0.011
$D^+\to\overline{K^0}K^{*+}$		$0.0012^{+0.0026}_{-0.0011}$	<0.0054
$D^+\to\eta\rho^+$	<0.012	$0.0012^{+0.0022}_{-0.0011}$	<0.0048
$D^+\to\eta'K^+$		$0.0016^{+0.0041}_{-0.0015}$	<0.0082
$D^+\to\rho^0K^+$		$0.0018^{+0.0042}_{-0.0017}$	<0.0086
$D^+\to\eta'\pi^+$	<0.009	$0.00094^{+0.00237}_{-0.00092}$	<0.0048
$D^+\to\phi\rho^+$	<0.015		<0.0074
$D^+\to\omega K^+$			<0.0012
$D^+\to\eta'\rho^+$	<0.015		<0.00071
$D^+\to\rho^0\pi^+$	<0.0014		<0.00091
$D^+\to\eta'K^{*+}$			<0.000082
$D^+\to\eta K^{*+}$			<0.0022

reduced matrix elements, and finally as a sum over the linearly independent combinations of reduced matrix elements.

The parameters were fit to the data amplitudes with MINUIT, release 93.11 [13]. The total χ^2 was found to be 30.9 for seven degrees of freedom, indicating that the overall fit was poor. However, more than half of the χ^2 arose from only one mode. The mode in question is $D_s\to\eta'\rho^+$. The experimental value for the branching ratio $D_s\to\eta'\rho^+$ cannot be accommodated in our scheme. It is measured [16] to be larger than that for $D_s\to\eta\rho^+$, an *a priori* surprising result. We note that the angular distribution of the decay pions is barely consistent with that expected. A confirmation of this experimental value would be very significant as all other models [4] also predict a ratio of $B(D_s\to\eta'\rho^+)/B(D_s\to\eta\rho^+)$ of less than 1.

We decided to reject the experimental value for the branching fraction of $D_s\to\eta'\rho^+$. The result is a better fit, from which the branching ratios are reported in the tables. The total χ_2 is now 11.6 for six degrees of freedom. The best-fit values of the parameters are given in Tables VI and VII. The units correspond to $|A(D^0\to\pi^+K^-)|=1.15$.

TABLE IV. D_s modes with predicted branching ratios. Experimental limits are given when available. All limits are at 90% confidence.

Mode	Data BR	Predicted BR	Predicted limit
$D_s\to\pi^0K^+$		$0.0059^{+0.0048}_{-0.0034}$	
$D_s\to\pi^+K^{*0}$		$0.038^{+0.047}_{-0.028}$	
$D_s\to\eta'\rho^+$	0.120 ± 0.030[a]	$0.015^{+0.019}_{-0.011}$	
$D_s\to\pi^0K^{*+}$		$0.077^{+0.096}_{-0.058}$	
$D_s\to\rho^0K^{*+}$		$0.0126^{+0.0164}_{-0.0096}$	
$D_s\to\rho^+K^0$		$0.031^{+0.043}_{-0.024}$	
$D_s\to\rho^0K^+$		$0.049^{+0.071}_{-0.040}$	
$D_s\to\omega\rho^+$		$0.012^{+0.030}_{-0.012}$	<0.061
$D_s\to\pi^+K^0$	<0.007		<0.0015
$D_s\to K^0K^{*+}$			<0.00039
$D_s\to K^0K^+$			<0.00046
$D_s\to K^{*0}K^{*+}$			<0.00057
$D_s\to\rho^+K^{*0}$			<0.0080
$D_s\to K^{*0}K^+$			<0.00025
$D_s\to\omega\pi^+$	<0.017		<0.0090
$D_s\to\pi^0\rho^+$			<0.064
$D_s\to\rho^0\pi^+$	<0.0028		Not significant
$D_s\to\pi^+\pi^0$		$\equiv0$	
$D_s\to\rho^+\rho^0$		$\equiv0$	

[a]See text.

V. PREDICTIONS

A. Predictions from the fit parameters

From the fit values of the parameters the branching ratios of decay modes were calculated. We emphasize that our model of SU(3) breaking is such that modes whose branch-

TABLE V. Modes based on estimates. The only available experimental limit is shown. Values marked with asterisks are inputs.

Mode	Data BR	Fit BR (scheme A)	Fit BR (scheme B)	Fit BR (scheme C)
$D^0\to\eta K^0$		0.000054*	0.000054*	0.00035
$D^0\to\eta'K^0$		0.00046	0.00046	0.00085
$D^0\to\phi K^{*0}$		0.000019	0.000019	0.000016
$D^0\to\omega K^{*0}$		0.00027	0.00027	0.0012
$D^0\to\phi K^0$		0.00054	0.00054	0.000066*
$D^0\to\omega K^0$		0.000096	0.000096	0.0014
$D^0\to\eta K^{*0}$		0.00048	0.00048	0.00094
$D^0\to\eta'K^{*0}$		0.0000083	0.0000083	0.0000024
$D_s\to\eta K^+$		0.0027*	0.00041	0.0031
$D_s\to\eta'K^+$		0.017	0.052	0.015
$D_s\to\phi K^{*+}$		0.011	0.024	0.0095
$D_s\to\omega K^{*+}$		0.0057	0.028	0.0046
$D_s\to\phi K^+$	<0.0025	0.00051	0.0033*	0.00037*
$D_s\to\omega K^+$		0.0064	0.019	0.0055
$D_s\to\eta K^{*+}$		0.00083	0.00015	0.00094
$D_s\to\eta'K^{*+}$		0.00090	0.0028	0.00077

TABLE VI. Linear combinations and their fit values.

Linear combination	Amplitudes involved	Fit value
L_1	$(SS)_1$	
L_2		
L_3		
L_4		2.97 ± 0.66
L_5	$(SO)_8$	6 ± 26
L_6		8 ± 17
L_7		16 ± 18
L_8		6 ± 13
L_9	$(OO)_1$	2.4 ± 2.7
L_{10}		5.20 ± 0.12
L_{11}		1.7 ± 7.3
L_{12}		12.4 ± 2.3
L_{13}	$(OO)_8$	34 ± 12
L_{14}		9.5 ± 5.5
L_{15}		45 ± 21
L_{16}		52 ± 21
L_{17}		10.8 ± 4.1
L_{18}		55 ± 37
L_{19}		44 ± 38
L_{20}	$(OO)_{8'}$	45 ± 44
L_{21}		34 ± 25
L_{22}		42 ± 46
L_{23}		11 ± 47
L_{24}		4.7 ± 5.9
L_{25}		59 ± 44
L_{26}	$(OO)_{10}$	107 ± 63
L_{27}		39 ± 21
L_{28}		43 ± 37
L_{29}		21.1 ± 7.1
L_{30}		71 ± 67
L_{31}	$(OO)_{\overline{10}}$	97 ± 51
L_{32}		135 ± 74
L_{33}		115 ± 61
L_{34}		0.4 ± 2.5
L_{35}		26.2 ± 7.3
L_{36}		2.5 ± 2.9
L_{37}	$(OO)_{27}$	21.8 ± 4.6
L_{38}		9.2 ± 6.2
L_{39}		8.0 ± 8.6
L_{40}		19 ± 11

ing ratios are related by isospin satisfy those relations. In Table I are presented the modes for which there exist experimental values. Our calculated branching ratios are consistent with the data, with the exception of $D^+\to K^{*0}K^{*+}$ and $D_s\to\eta'\rho^+$. For the former the fit prefers a branching ratio that is three standard deviations below the reported experimental value. The latter was removed before the fit (see Sec. IV) because its experimental value was questioned. For this mode we predict a branching ratio of $(1.5^{+1.9}_{-1.1})\%$, well below the reported experimental value [16]. Tables II–IV contain modes for which there is no experimental information or for which there is an experimental limit. We have attempted to predict the branching ratio of each mode from the fit. However, in some cases the uncertainties are so large that we are able only to provide (90% confidence level) limits on the branching ratios. Notice that in all cases in which there are experimental limits, our predicted branching ratio or predicted limit is in the allowed region. We are unable to say anything about the mode $D_s\to\rho^0\pi^+$, because the uncertainty on its prediction is greater than the experimental limit.

TABLE VII. The remaining parameters and their fit values.

Parameter	Fit value
A_{PV}	0.91 ± 0.32
A_{VV}	1.40 ± 0.16
D/S wave	2.79 ± 0.41
Phase of $\eta_1\eta_1$	
Phase of $P\eta_1$	1.64 ± 0.45
Phase of PP_1	0.7 ± 1.2
Phase of PP_{27}	1.49 ± 0.16
Phase of $\omega_1\omega_1$	
Phase of $V\omega_1$	1.28 ± 0.52
Phase of VV_1	-3.0 ± 3.2
Phase of VV_{27}	0.53 ± 0.77
Phase of $\eta_1\omega_1$	
Phase of $P\omega_1$	0.01 ± 0.35
Phase of $V\eta_1$	-2.6 ± 2.5
Phase of PV_1	0.28 ± 1.5
Phase of $PV_{8'}$	-1.47 ± 0.38
Phase of PV_{10}	-2.49 ± 0.30
Phase of $PV_{\overline{10}}$	2.61 ± 0.23
Phase of PV_{27}	-2.86 ± 0.52

There are two modes, $D_s\to\pi^+\pi^0$ and $D_s\to\rho^+\rho^0$, which are forbidden in a model without isospin breaking. They are predicted to be identically zero. The modes that are kinematically forbidden are $D^0\to\eta'\eta'$, $\eta'\phi$, and $\phi\phi$. The modes involving the linear combinations L_1, L_2, and L_3 are discussed below. Any PP, PV, or VV mode not appearing in the tables is higher order in the weak coupling G_F.

B. Unconstrained linear combinations

There remain three linearly independent combinations of the reduced matrix elements that are not constrained by the data. The combination L_1 contributes only to modes of the type $D^0\to\eta\eta$. L_2 contributes to the types $D^0\to\eta\eta$ and $D^0\to\eta K^0$. L_3 contributes to these modes, and to modes of the type $D_s\to\eta K^+$.

The first unconstrained linear combination L_1 contributes only to amplitudes involving $(SS)_1$. These amplitudes, it is worth noting, are due entirely to SU(3) breaking. However, when we include the phases, we must make four estimates in order to obtain two predictions of modes of the type $D^0\to\eta\eta$. This would be an unproductive endeavor, and so we forego it.

In order to predict the modes of the types $D^0\to\eta K^0$ and $D_s\to\eta K^+$, we need two new inputs. In order to show the variability of the resulting predictions, we try three different sets of inputs. Scheme A is motivated by the recent CLEO measurement of the doubly suppressed mode $D^0 \rightarrow \pi^- K^+$ [17], in which this mode is found to have a branching ratio of about three times that expected from the corresponding unsuppressed mode, $D^0 \rightarrow \pi^+ K^-$. For this scheme, the two inputs are

$$\begin{aligned} B(D^0 - \eta K^0) &= 3\tan^4\theta_C B(D^0 \rightarrow \eta \overline{K^0}), \\ B(D_s \rightarrow \eta K^+) &= 3\tan^2\theta_C B(D_s \rightarrow \eta\pi^+). \end{aligned} \tag{14}$$

The linear combinations L_2 and L_3 are then constrained and the remaining branching ratios in the column for scheme A in Table V are found. The predictions for scheme B are based on the estimates

$$\begin{aligned} B(D^0 \rightarrow \eta K^0) &= 3\tan^4\theta_C B(D^0 \rightarrow \eta \overline{K^0}), \\ B(D_s \rightarrow \phi K^+) &= 3\tan^2\theta_C B(D_s \rightarrow \phi\pi^+). \end{aligned} \tag{15}$$

A third scheme (C) is considered also. It is based on these estimates:

$$\begin{aligned} B(D^0 \rightarrow \phi K^0) &= 3\tan^4\theta_C B(D^0 \rightarrow \phi \overline{K^0}), \\ B(D_s \rightarrow \phi K^+) &= \tfrac{1}{3}\tan^2\theta_C B(D_s \rightarrow \phi\pi^+). \end{aligned} \tag{16}$$

The resulting predictions are again in Table V. The spread in these values provides an indication of the expected ranges for these quantities.

One should note that arbitrary choices of the above modes may fail to give an acceptable fit, given the constraints from measured modes. For example, an apparently reasonable choice would have been

$$\begin{aligned} B(D^0 \rightarrow \eta K^0) &= \tan^4\theta_C B(D^0 \rightarrow \eta \overline{K^0}), \\ B(D_s \rightarrow \eta K^+) &= \tan^2\theta_C B(D_s \rightarrow \eta\pi^+). \end{aligned} \tag{17}$$

A consistent fit cannot be obtained to implement this. The parameters L_2 and L_3 could not be given values to accommodate $B(D^0 \rightarrow \eta K^0) < 0.0052\%$.

C. Modes involving axial vectors

There are a few modes involving axial vectors that have been observed or for which there are experimental limits. However, those that involve $K(1270)$ and $K(1400)$ are mixtures with the 1^{+-} octet, which we can call B since it includes the $b_1(1235)$. Therefore, in order to include these modes in our framework, we require two new parameters, A_{PA} and A_{PB}. In addition, we must also accommodate the mixing between $f_1^{(1)}$ and $f_1^{(8)}$ to become $f_1(1285)$ and $f_1(1510)$, as well as the new phases that are introduced. There are too few experimental observations of the PA and PB modes to make this endeavor fruitful. For that reason, they are not included here.

VI. COMMENTS ON MODELS

It is clear from the data alone that significant SU(3) breaking is necessary in any successful model of D decays. For example, $B(D^0 \rightarrow K^+K^-) = B(D^0 \rightarrow \pi^+\pi^-)$ in exact SU(3), yet they are in reality quite different. Models based on exact SU(3) [3,6,10,18] (or even on nonet symmetry [19]) are thus not admitted by the data.

Models of D decays based on heavy-quark effective theory (e.g., [3]) have as yet not developed to the point at which individual nonleptonic decays can be calculated. The question of whether heavy quark effective theory (HQET) is applicable to the c quark is still unsettled. The HQET is based on an expansion in the parameter

$$\frac{\Lambda_{\mathrm{QCD}}}{m_c} \simeq 0.2 \tag{18}$$

and assumes that it is small. Certainly this would be a good assumption in the case of the b quark, but perhaps not so here. Until we are able to calculate branching fractions in HQET, we must reserve judgement on its applicability to the D mesons.

Diagrammatical methods to the problem of D decays present us with a complementary approach to the one adopted in this work. The parameters in the SU(3) framework represent sums of diagrams in the diagrammatical approach. A very general diagrammatical calculation of branching fractions appears in [5]. Two shortcomings of their work lie in final-state interactions and in the inclusion of SU(3) breaking. The phases of the final-state interactions are added to the model, and are external to its central theme, and therefore appear as an *ad hoc* mechanism to force a fit. SU(3) breaking is added to the calculation as an additive correction to the diagrams in which it is believed to be important. However, there is also hidden breaking in the addition of phases in the final-state interactions. The result is a model in which the size and source of SU(3) breaking is not easily discerned. It is difficult to draw any conclusions from the application of such a model.

The factorization method is a special case of the diagrammatical approach. In it certain diagrams are considered unimportant (i.e., the annihilation diagrams). However, [4] find that these diagrams must be again included, as well as final-state transitions and intermediate resonances. The result is an eclectic model with little elegance. We are unable, because of the *ad hoc* features, to comment on the reliability and predictability of this model.

A description of nonleptonic D decays in a large-N_c (number of colors) expansion [2] is an elegant one with few parameters. In it, the source of SU(3) breaking is introduced by including nearby resonances. It is also a subset of the diagrammatical approach and neglects some diagrams based on their suppression by $1/N_c$. One may argue that these diagrams are larger than thought, and cite the fit of [5] as evidence of this. Nevertheless, [2] obtain excellent agreement with the data, with the exception of some modes involving η and η'. In this model, SU(3) breaking is introduced only through the inclusion of resonances in one class of diagram. They obtain, in agreement with our work, large breaking.

CONCLUSIONS

There now exist enough data to constrain all but three combinations of the reduced matrix elements of the broken

SU(3) model of the decays of D mesons with the two assumptions discussed in Sec. III B. We have assumed that the breaking of SU(3) is such that isospin remains a good symmetry; data indicate that this is the case. We have used these data to fix our parameters. Using the experimental information on 57 modes we are able to predict branching ratios or upper limits for an additional 53 modes. Only two measured modes are not easily accommodated in the fit. The measurement of a few additional modes involving η, η', ϕ, ω would enable another dozen or so modes to be predicted.

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APPENDIX: EXPLICIT EXPRESSIONS FOR DECAY AMPLITUDES

An assumption of Sec. III B is that we can introduce the parameters $A_{PP}\equiv 1$, A_{PV}, and A_{VV}, so that the reduced matrix element involving pseudoscalar-pseudoscalar decay modes is distinguished from the pseudoscalar-vector and vector-vector cases by these multiplicative factors. Below is a list of the resulting 48 reduced matrix elements. They are complex. Here O represents an octet of mesons, and S a singlet under SU(3). Without the assumption of Sec. III B there would be duplicate lists for PP, PV, VP, and VV decay modes. Only in the SO cases would the PV and VP matrix elements distinct. Subscripts denote irreducible SU(3) representations. Parentheses show the multiplication of representations:

$$\mathcal{M}_1=[(DH_{\bar{6}})_8(SSM)_8],\quad \mathcal{M}_2=[(DH_{15})_8(SSM)_8],$$

$$\mathcal{M}_3=[(DH_{\bar{6}})_8(SO)_8],\quad \mathcal{M}_4=[(DH_{15})_8(SO)_8],$$

$$\mathcal{M}_5=[(DH_{\bar{6}})_8(SOM)_8],\quad \mathcal{M}_6=[(DH_{\bar{6}})_8(SOM)_{8'}],$$

$$\mathcal{M}_7=[(DH_{\bar{6}})_{\overline{10}}(SOM)_{\overline{10}}],\quad \mathcal{M}_8=[(DH_{15})_8(SOM)_8],$$

$$\mathcal{M}_9=[(DH_{15})_8(SOM)_{8'}],\quad \mathcal{M}_{10}=[(DH_{15})_{\overline{10}}(SOM)_{\overline{10}}],$$

$$\mathcal{M}_{11}=[(DH_{15})_{27}(SOM)_{27}],\quad \mathcal{M}_{12}=[(DH_{\bar{6}})_8(OO)_8],$$

$$\mathcal{M}_{13}=[(DH_{\bar{6}})_8(OO)_{8'}],\quad \mathcal{M}_{14}=[(DH_{\bar{6}})_{\overline{10}}(OO)_{10}],$$

$$\mathcal{M}_{15}=[(DH_{15})_8(OO)_8],\quad \mathcal{M}_{16}=[(DH_{15})_8(OO)_{8'}],$$

$$\mathcal{M}_{17}=[(DH_{15})_{10}(OO)_{\overline{10}}],\quad \mathcal{M}_{18}=[(DH_{15})_{27}(OO)_{27}],$$

$$\mathcal{M}_{19}=[(DH_{\bar{6}})_8((OO)_1M)_8],\quad \mathcal{M}_{20}=[(DH_{\bar{6}})_8((OO)_8M)_8],$$

$$\mathcal{M}_{21}=[(DH_{\bar{6}})_8((OO)_8M)_{8'}],\quad \mathcal{M}_{22}=[(DH_{\bar{6}})_{\overline{10}}((OO)_8M)_{\overline{10}}],$$

$$\mathcal{M}_{23}=[(DH_{\bar{6}})_8((OO)_{8'}M)_8],\quad \mathcal{M}_{24}=[(DH_{\bar{6}})_8((OO)_{8'}M)_{8'}],$$

$$\mathcal{M}_{25}=[(DH)_{\overline{10}}((OO)_{8'}M)_{10}],\quad \mathcal{M}_{26}=[(DH_{\bar{6}})_8((OO)_{\overline{10}}M)_8],$$

$$\mathcal{M}_{27}=[(DH_{\bar{6}})_{\overline{10}}((OO)_{10}M)_{10}],\quad \mathcal{M}_{28}=[(DH_{\bar{6}})_8((OO)_{\overline{10}}M)_8],$$

$$\mathcal{M}_{29}=[(DH_6)_8((OO)_{27}M)_8],\quad \mathcal{M}_{30}=[(DH_{\bar{6}})_{\overline{10}}((OO)_{27}M)_{10}],$$

$$\mathcal{M}_{31}=[(DH_{15})_8((OO)_1M)_8],\quad \mathcal{M}_{32}=[(DH_{15})_8((OO)_8M)_8],$$

$$\mathcal{M}_{33}=[(DH_{15})_8((OO)_8M)_{8'}],\quad \mathcal{M}_{34}=[(DH_{15})_{10}((OO)_8M)_{\overline{10}}],$$

$$\mathcal{M}_{35}=[(DH_{15})_{27}((OO)_8M)_{27}],\quad \mathcal{M}_{36}=[(DH_{15})_8((OO)_{8'}M)_8],$$

$$\mathcal{M}_{37}=[(DH_{15})_8((OO)_{8'}M)_{8'}],\quad \mathcal{M}_{38}=[(DH_{15})_{10}((OO)_{8'}M)_{\overline{10}}],$$

$$\mathcal{M}_{39}=[(DH_{15})_{27}((OO)_{8'}M)_{27}],\quad \mathcal{M}_{40}=[(DH_{15})_8((OO)_{10}M)_8],$$

$$\mathcal{M}_{41}=[(DH_{15})_{27}((OO)_{10}M)_{27}],\quad \mathcal{M}_{42}=[(DH_{15})_8((OO)_{\overline{10}}M)_8],$$

$$\mathcal{M}_{43}=[(DH_{15})_{10}((OO)_{\overline{10}}M)_{\overline{10}}],\quad \mathcal{M}_{44}=[(DH_{15})_{27}((OO)_{\overline{10}}M)_{27}],$$

$$\mathcal{M}_{45}=[(DH_{15})_{8}((OO)_{27}M)_{8}],\quad \mathcal{M}_{46}=[(DH_{15})_{10}((OO)_{27}M)_{\overline{10}}],$$

$$\mathcal{M}_{47}=[(DH_{15})_{27}((OO)_{27}M)_{27}],\quad \mathcal{M}_{48}=[(DH_{15})_{27}((OO)_{27}M)_{27'}].$$

In the notation of [6], the three PP reduced matrix elements not involving SU(3) breaking are

$$S=A_{PP}\mathcal{M}_{12}^{*},\quad E=A_{PP}\mathcal{M}_{15}^{*},\quad T=A_{PP}\mathcal{M}_{18}^{*}.$$

The formulation of [6] does not involve SU(3) breaking. We explicitly add the SU(3)-breaking matrix elements in this work.

Under the assumptions of Sec. III B, the reduced matrix elements can be replaced by forty linearly independent combinations. The phases are removed from these combinations. Below is our choice of linearly independent combinations. The coefficients are combinations of Clebsch factors that make the L_n linearly independent:

$$L_1=0.845\,15\mathcal{M}_1+0.534\,52\mathcal{M}_2,$$

$$L_2=0.719\,20\mathcal{M}_3+0.454\,86\mathcal{M}_4+0.160\,82\mathcal{M}_5-0.359\,60\mathcal{M}_6+0.101\,71\mathcal{M}_8-0.227\,43\mathcal{M}_9-0.227\,43\mathcal{M}_{10}-0.083\,05\mathcal{M}_{11},$$

$$L_3=0.213\,26\mathcal{M}_3+0.134\,88\mathcal{M}_4-0.505\,47\mathcal{M}_5+0.305\,67\mathcal{M}_6-0.521\,52\mathcal{M}_7-0.319\,69\mathcal{M}_8+0.193\,32\mathcal{M}_9-0.328\,20\mathcal{M}_{10}+0.261\,02\mathcal{M}_{11},$$

$$L_4=0.426\,89\mathcal{M}_3-0.765\,07\mathcal{M}_4-0.096\,96\mathcal{M}_5-0.070\,03\mathcal{M}_6-0.181\,41\mathcal{M}_7+0.401\,57\mathcal{M}_8-0.044\,29\mathcal{M}_9+0.033\,07\mathcal{M}_{10}+0.144\,56\mathcal{M}_{11},$$

$$L_5=0.384\,66\mathcal{M}_3-0.178\,64\mathcal{M}_4+0.228\,42\mathcal{M}_5+0.604\,46\mathcal{M}_6+0.419\,79\mathcal{M}_7-0.433\,01\mathcal{M}_8-0.188\,77\mathcal{M}_9+0.050\,97\mathcal{M}_{10}+0.024\,82\mathcal{M}_{11},$$

$$L_6=-0.083\,26\mathcal{M}_3+0.079\,53\mathcal{M}_4+0.575\,53\mathcal{M}_5+0.045\,67\mathcal{M}_6-0.521\,51\mathcal{M}_7-0.074\,32\mathcal{M}_8-0.253\,76\mathcal{M}_9+0.273\,36\mathcal{M}_{10}+0.486\,63\mathcal{M}_{11},$$

$$L_7=0.205\,37\mathcal{M}_3+0.291\,44\mathcal{M}_4+0.141\,69\mathcal{M}_5+0.406\,65\mathcal{M}_6+0.006\,83\mathcal{M}_7+0.483\,32\mathcal{M}_8+0.604\,49\mathcal{M}_9+0.297\,10\mathcal{M}_{10}+0.011\,23\mathcal{M}_{11},$$

$$L_8=-0.035\,51\mathcal{M}_3+0.092\,63\mathcal{M}_4-0.156\,81\mathcal{M}_5-0.173\,10\mathcal{M}_6+0.496\,85\mathcal{M}_7+0.092\,27\mathcal{M}_8+0.071\,58\mathcal{M}_9-0.092\,63\mathcal{M}_{10}+0.816\,41\mathcal{M}_{11},$$

$$L_9=0.845\,15\mathcal{M}_{19}+0.534\,52\mathcal{M}_{31},$$

$$L_{10}=0.719\,20\mathcal{M}_{12}+0.454\,86\mathcal{M}_{15}+0.160\,82\mathcal{M}_{20}-0.359\,60\mathcal{M}_{21}+0.101\,71\mathcal{M}_{32}-0.227\,43\mathcal{M}_{33}-0.227\,43\mathcal{M}_{34}-0.083\,05\mathcal{M}_{35},$$

$$L_{11}=0.339\,37\mathcal{M}_{12}+0.214\,63\mathcal{M}_{15}-0.262\,68\mathcal{M}_{20}+0.587\,37\mathcal{M}_{21}-0.478\,80\mathcal{M}_{22}-0.166\,13\mathcal{M}_{32}+0.371\,48\mathcal{M}_{33}-0.107\,32\mathcal{M}_{34}+0.135\,65\mathcal{M}_{35},$$

$$L_{12}=-0.152\,73\mathcal{M}_{12}-0.096\,60\mathcal{M}_{15}-0.546\,43\mathcal{M}_{20}-0.388\,19\mathcal{M}_{21}-0.215\,33\mathcal{M}_{22}-0.345\,59\mathcal{M}_{32}-0.245\,51\mathcal{M}_{33}-0.460\,84\mathcal{M}_{34}+0.282\,17\mathcal{M}_{35},$$

$$L_{13}=0.152\,27\mathcal{M}_{12}+0.096\,31\mathcal{M}_{15}-0.286\,02\mathcal{M}_{20}+0.232\,22\mathcal{M}_{21}+0.851\,11\mathcal{M}_{22}-0.180\,89\mathcal{M}_{32}+0.146\,87\mathcal{M}_{33}-0.176\,96\mathcal{M}_{34}+0.147\,70\mathcal{M}_{35},$$

$$L_{14}=0.480\,23\mathcal{M}_{12}-0.756\,68\mathcal{M}_{15}-0.152\,19\mathcal{M}_{20}-0.001\,19\mathcal{M}_{21}+0.377\,97\mathcal{M}_{32}-0.000\,75\mathcal{M}_{33}+0.005\,25\mathcal{M}_{34}+0.175\,39\mathcal{M}_{35},$$

$$L_{15}=0.176\,31\mathcal{M}_{12}-0.266\,72\mathcal{M}_{15}+0.359\,33\mathcal{M}_{20}+0.332\,40\mathcal{M}_{21}-0.606\,92\mathcal{M}_{32}-0.537\,61\mathcal{M}_{33}+0.024\,08\mathcal{M}_{34}-0.014\,50\mathcal{M}_{35},$$

$$L_{16}=-0.051\,62\mathcal{M}_{12}+0.164\,03\mathcal{M}_{15}+0.302\,17\mathcal{M}_{20}-0.031\,89\mathcal{M}_{21}+0.087\,30\mathcal{M}_{32}-0.031\,98\mathcal{M}_{33}+0.164\,83\mathcal{M}_{34}+0.917\,77\mathcal{M}_{35},$$

$$L_{17}=0.719\,19\mathcal{M}_{13}+0.454\,86\mathcal{M}_{16}+0.160\,82\mathcal{M}_{23}-0.359\,60\mathcal{M}_{24}+0.101\,71\mathcal{M}_{36}-0.227\,43\mathcal{M}_{37}-0.227\,43\mathcal{M}_{38}-0.083\,05\mathcal{M}_{39},$$

$$L_{18}=0.124\,50\mathcal{M}_{13}+0.078\,74\mathcal{M}_{16}+0.566\,06\mathcal{M}_{23}+0.338\,91\mathcal{M}_{24}+0.253\,72\mathcal{M}_{25}+0.358\,01\mathcal{M}_{36}+0.214\,35\mathcal{M}_{37}+0.468\,07\mathcal{M}_{38}-0.292\,31\mathcal{M}_{39},$$

$$L_{19}=-0.350\,71\mathcal{M}_{13}-0.221\,80\mathcal{M}_{16}+0.217\,17\mathcal{M}_{23}-0.617\,11\mathcal{M}_{24}+0.459\,61\mathcal{M}_{25}+0.137\,35\mathcal{M}_{36}-0.390\,30\mathcal{M}_{37}+0.069\,31\mathcal{M}_{38}-0.112\,14\mathcal{M}_{39},$$

$$L_{20}=0.152\,27\mathcal{M}_{13}+0.096\,31\mathcal{M}_{16}-0.286\,02\mathcal{M}_{23}+0.232\,22\mathcal{M}_{24}+0.851\,11\mathcal{M}_{25}-0.180\,89\mathcal{M}_{36}+0.146\,87\mathcal{M}_{37}-0.176\,96\mathcal{M}_{38}+0.147\,70\mathcal{M}_{39},$$

$$L_{21}=-0.480\,23\mathcal{M}_{13}+0.756\,68\mathcal{M}_{16}+0.152\,19\mathcal{M}_{23}+0.001\,19\mathcal{M}_{24}-0.377\,97\mathcal{M}_{36}+0.000\,75\mathcal{M}_{37}-0.005\,25\mathcal{M}_{38}-0.175\,39\mathcal{M}_{39},$$

$$L_{22}=0.176\,31\mathcal{M}_{13}-0.266\,73\mathcal{M}_{16}+0.359\,33\mathcal{M}_{23}+0.332\,40\mathcal{M}_{24}-0.606\,92\mathcal{M}_{36}-0.537\,61\mathcal{M}_{37}+0.024\,09\mathcal{M}_{38}-0.014\,50\mathcal{M}_{39},$$

$$L_{23}=0.051\,62\mathcal{M}_{13}-0.164\,03\mathcal{M}_{16}-0.302\,17\mathcal{M}_{23}+0.031\,89\mathcal{M}_{24}-0.087\,31\mathcal{M}_{36}+0.031\,99\mathcal{M}_{37}-0.164\,82\mathcal{M}_{38}-0.917\,77\mathcal{M}_{39},$$

$$L_{24}=-0.866\,03\mathcal{M}_{14}+0.306\,19\mathcal{M}_{27}+0.395\,29\mathcal{M}_{41},$$

$$L_{25}=0.411\,00\mathcal{M}_{14}+0.464\,99\mathcal{M}_{26}+0.319\,68\mathcal{M}_{27}+0.294\,09\mathcal{M}_{40}+0.652\,83\mathcal{M}_{41},$$

$$L_{26}=-0.284\,75\mathcal{M}_{14}+0.671\,16\mathcal{M}_{26}-0.469\,81\mathcal{M}_{27}+0.424\,48\mathcal{M}_{40}-0.259\,94\mathcal{M}_{41},$$

$$L_{27}=-0.218\,22\mathcal{M}_{26}-0.763\,76\mathcal{M}_{27}-0.138\,01\mathcal{M}_{40}+0.591\,61\mathcal{M}_{41},$$

$$L_{28}=0.534\,52\mathcal{M}_{26}-0.845\,15\mathcal{M}_{40},$$

$$L_{29}=-0.738\,55\mathcal{M}_{17}-0.522\,23\mathcal{M}_{28}-0.330\,29\mathcal{M}_{42}-0.261\,12\mathcal{M}_{43}+0.067\,42\mathcal{M}_{44},$$

$$L_{30}=-0.558\,29\mathcal{M}_{17}+0.328\,98\mathcal{M}_{28}+0.208\,06\mathcal{M}_{42}+0.526\,36\mathcal{M}_{43}-0.509\,65\mathcal{M}_{44},$$

$$L_{31}=-0.124\,84\mathcal{M}_{17}+0.382\,52\mathcal{M}_{28}+0.241\,93\mathcal{M}_{42}-0.809\,17\mathcal{M}_{43}-0.353\,28\mathcal{M}_{44},$$

$$L_{32}=0.113\,96\mathcal{M}_{17}-0.644\,66\mathcal{M}_{28}+0.713\,51\mathcal{M}_{42}-0.249\,67\mathcal{M}_{44},$$

$$L_{33}=0.338\,06\mathcal{M}_{17}-0.239\,05\mathcal{M}_{28}-0.529\,15\mathcal{M}_{42}-0.740\,66\mathcal{M}_{44},$$

$$L_{34}=0.771\,84\mathcal{M}_{18}-0.272\,89\mathcal{M}_{29}+0.482\,40\mathcal{M}_{30}-0.172\,59\mathcal{M}_{45}+0.096\,48\mathcal{M}_{46}-0.046\,13\mathcal{M}_{47}-0.236\,33\mathcal{M}_{48},$$

$$L_{35}=0.463\,81\mathcal{M}_{18}-0.233\,17\mathcal{M}_{29}-0.590\,77\mathcal{M}_{30}+0.349\,24\mathcal{M}_{45}-0.431\,27\mathcal{M}_{46}+0.252\,97\mathcal{M}_{47}+0.097\,67\mathcal{M}_{48},$$

$$L_{36}=0.188\,97\mathcal{M}_{18}+0.706\,43\mathcal{M}_{29}+0.302\,74\mathcal{M}_{30}+0.498\,27\mathcal{M}_{45}-0.303\,01\mathcal{M}_{46}-0.180\,07\mathcal{M}_{47}-0.033\,02\mathcal{M}_{48},$$

$$L_{37}=0.261\,97\mathcal{M}_{18}+0.508\,51\mathcal{M}_{29}-0.492\,35\mathcal{M}_{30}-0.571\,08\mathcal{M}_{45}+0.053\,61\mathcal{M}_{46}-0.180\,26\mathcal{M}_{47}-0.262\,45\mathcal{M}_{48},$$

$$L_{38}=0.131\,64\mathcal{M}_{18}-0.160\,51\mathcal{M}_{29}-0.242\,72\mathcal{M}_{30}+0.311\,01\mathcal{M}_{45}+0.410\,04\mathcal{M}_{46}-0.767\,51\mathcal{M}_{47}+0.209\,91\mathcal{M}_{48},$$

$$L_{39}=0.033\,89\mathcal{M}_{18}-0.053\,86\mathcal{M}_{29}+0.159\,09\mathcal{M}_{30}-0.420\,84\mathcal{M}_{45}-0.525\,95\mathcal{M}_{46}-0.306\,94\mathcal{M}_{47}+0.650\,15\mathcal{M}_{48},$$

$$L_{40}=0.257\,51\mathcal{M}_{18}+0.291\,34\mathcal{M}_{29}+0.515\,03\mathcal{M}_{46}+0.430\,90\mathcal{M}_{47}+0.630\,77\mathcal{M}_{48}.$$

We can now express the decay amplitudes to each decay mode in terms of the linear combinations of reduced matrix elements. They appear below. Here $\Phi_{(XX)_R}$ is the phase of the products XX in the irreducible SU(3) representation R. These phases are introduced in Sec. III B. We have defined the symbol α_R^{XX} to represent $\exp i\Phi_{(XX)_R}$. The Cabibbo factors are abbreviated by $c\equiv\cos\theta_C$, $s\equiv\sin\theta_C$. The amplitudes to the physical mixtures of η_1, η_8, ω_1, ω_8 are mixtures of the relevant amplitudes with the mixing angles given in [8,9]:

$$\mathcal{A}(D^0\to\eta_1\eta_1)=A_{PP}cs\times[0.295\,80L_1\alpha^{\eta_1\eta_1}],$$

$$\mathcal{A}(D^0\to\eta_1\overline{K^0})=A_{PP}c^2\times[0.283\,82L_2\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^0\to\eta_1\pi^0)=A_{PP}cs\times[0.148\,79L_2\alpha^{\eta_1P}+0.175\,04L_3\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^0\to\eta_1K^0)=A_{PP}s^2\times[0.166\,38L_2\alpha^{\eta_1P}+0.032\,50L_3\alpha^{\eta_1P}-0.029\,59L_4\alpha^{\eta_1P}+0.129\,40L_5\alpha^{\eta_1P}$$
$$-0.039\,46L_6\alpha^{\eta_1P}+0.180\,67L_7\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^0\to\eta_1\eta_8)=A_{PP}cs\times[0.281\,68L_2\alpha^{\eta_1P}-0.019\,55L_3\alpha^{\eta_1P}-0.005\,04L_4\alpha^{\eta_1P}+0.061\,41L_5\alpha^{\eta_1P}+0.022\,76L_6\alpha^{\eta_1P}$$
$$+0.146\,79L_7\alpha^{\eta_1P}-0.052\,37L_8\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^0\to K^-\pi^+)=A_{PP}c^2\times[0.219\,85L_{10}\alpha_8^{PP}+0.070\,52L_{34}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to K^-K^+)=A_{PP}cs\times[0.147\,90L_9\alpha_1^{PP}+0.170\,57L_{10}\alpha_8^{PP}+0.104\,43L_{11}\alpha_8^{PP}+0.028\,74L_{34}\alpha_{27}^{PP}+0.069\,54L_{35}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\overline{K^0}\pi^0)=A_{PP}c^2\times[-0.155\,46L_{10}\alpha_8^{PP}+0.066\,84L_{34}\alpha_{27}^{PP}-0.004\,52L_{35}\alpha_{27}^{PP}+0.043\,59L_{36}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\overline{K^0}K^0)=A_{PP}cs\times[-0.147\,90L_9\alpha_1^{PP}-0.007\,58L_{10}\alpha_8^{PP}+0.060\,26L_{11}\alpha_8^{PP}+0.098\,21L_{12}\alpha_8^{PP}-0.036\,32L_{34}\alpha_{27}^{PP}$$
$$+0.018\,74L_{35}\alpha_{27}^{PP}+0.057\,33L_{36}\alpha_{27}^{PP}+0.032\,46L_{37}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\overline{K^0}\eta_8)=A_{PP}c^2\times[0.089\,75L_{10}\alpha_8^{PP}-0.056\,97L_{34}\alpha_{27}^{PP}-0.007\,83L_{35}\alpha_{27}^{PP}+0.075\,51L_{36}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\pi^-\pi^+)=A_{PP}cs\times[-0.147\,90L_9\alpha_1^{PP}+0.178\,15L_{10}\alpha_8^{PP}+0.044\,17L_{11}\alpha_8^{PP}-0.098\,21L_{12}\alpha_8^{PP}+0.043\,06L_{34}\alpha_{27}^{PP}$$
$$+0.030\,83L_{35}\alpha_{27}^{PP}+0.015\,57L_{36}\alpha_{27}^{PP}-0.002\,02L_{37}\alpha_{27}^{PP}+0.034\,06L_{38}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\pi^-K^+)=A_{PP}s^2\times[0.128\,88L_{10}\alpha_8^{PP}+0.100\,71L_{11}\alpha_8^{PP}-0.119\,74L_{12}\alpha_8^{PP}+0.085\,11L_{13}\alpha_8^{PP}+0.039\,01L_{34}\alpha_{27}^{PP}$$
$$+0.044\,98L_{35}\alpha_{27}^{PP}-0.001\,09L_{36}\alpha_{27}^{PP}+0.009\,16L_{37}\alpha_{27}^{PP}+0.000\,17L_{38}\alpha_{27}^{PP}+0.036\,65L_{39}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\pi^0\pi^0)=A_{PP}cs\times[0.104\,58L_9\alpha_1^{PP}-0.125\,97L_{10}\alpha_8^{PP}-0.031\,23L_{11}\alpha_8^{PP}+0.069\,44L_{12}\alpha_8^{PP}+0.037\,90L_{34}\alpha_{27}^{PP}$$
$$+0.025\,64L_{35}\alpha_{27}^{PP}+0.042\,29L_{36}\alpha_{27}^{PP}+0.008\,62L_{37}\alpha_{27}^{PP}+0.048\,17L_{38}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\pi^0K^0)=A_{PP}s^2\times[-0.091\,13L_{10}\alpha_8^{PP}-0.071\,21L_{11}\alpha_8^{PP}+0.084\,67L_{12}\alpha_8^{PP}-0.060\,18L_{13}\alpha_8^{PP}+0.033\,42L_{34}\alpha_{27}^{PP}$$
$$+0.043\,19L_{35}\alpha_{27}^{PP}+0.042\,44L_{36}\alpha_{27}^{PP}+0.009\,72L_{37}\alpha_{27}^{PP}+0.000\,18L_{38}\alpha_{27}^{PP}+0.038\,87L_{39}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\pi^0\eta_8)=A_{PP}cs\times[-0.094\,10L_{10}\alpha_8^{PP}-0.095\,08L_{11}\alpha_8^{PP}-0.056\,70L_{12}\alpha_8^{PP}-0.006\,56L_{34}\alpha_{27}^{PP}+0.076\,45L_{35}\alpha_{27}^{PP}$$
$$+0.049\,65L_{36}\alpha_{27}^{PP}+0.028\,11L_{37}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to K^0\eta_8)=A_{PP}s^2\times[0.052\,61L_{10}\alpha_8^{PP}+0.041\,12L_{11}\alpha_8^{PP}-0.048\,88L_{12}\alpha_8^{PP}+0.034\,75L_{13}\alpha_8^{PP}-0.037\,67L_{34}\alpha_{27}^{PP}$$
$$-0.035\,37L_{35}\alpha_{27}^{PP}+0.076\,17L_{36}\alpha_{27}^{PP}-0.005\,61L_{37}\alpha_{27}^{PP}-0.000\,10L_{38}\alpha_{27}^{PP}-0.022\,44L_{39}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^0\to\eta_8\eta_8)=A_{PP}cs\times[0.104\,58L_9\alpha_1^{PP}+0.125\,97L_{10}\alpha_8^{PP}+0.031\,23L_{11}\alpha_8^{PP}-0.069\,44L_{12}\alpha_8^{PP}-0.069\,00L_{34}\alpha_{27}^{PP}-0.053\,88L_{35}\alpha_{27}^{PP}+0.060\,81L_{36}\alpha_{27}^{PP}+0.034\,43L_{37}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\eta_1\pi^+)=A_{PP}cs\times[0.088\,08L_2\alpha^{\eta_1P}+0.160\,77L_3\alpha^{\eta_1P}+0.249\,45L_4\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^+\to\eta_1K^+)=A_{PP}s^2\times[0.068\,51L_2\alpha^{\eta_1P}+0.003\,48L_3\alpha^{\eta_1P}+0.157\,30L_4\alpha^{\eta_1P}+0.226\,07L_5\alpha^{\eta_1P}],$$

$$\mathcal{A}(D^+\to\overline{K^0}\pi^+)=A_{PP}c^2\times[0.066\,58L_{34}\alpha_{27}^{PP}+0.114\,20L_{35}\alpha_{27}^{PP}-0.000\,15L_{36}\alpha_{27}^{PP}+0.100\,50L_{37}\alpha_{27}^{PP}+0.049\,55L_{38}\alpha_{27}^{PP}-0.032\,47L_{39}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\overline{K^0}K^+)=A_{PP}cs\times[0.068\,23L_{10}\alpha_8^{PP}+0.104\,02L_{11}\alpha_8^{PP}+0.068\,72L_{12}\alpha_8^{PP}-0.041\,59L_{13}\alpha_8^{PP}+0.188\,61L_{14}\alpha_8^{PP}+0.027\,38L_{34}\alpha_{27}^{PP}+0.066\,76L_{35}\alpha_{27}^{PP}+0.020\,50L_{36}\alpha_{27}^{PP}+0.078\,45L_{37}\alpha_{27}^{PP}-0.008\,44L_{38}\alpha_{27}^{PP}+0.013\,60L_{39}\alpha_{27}^{PP}+0.022\,43L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\pi^0\pi^+)=A_{PP}cs\times[0.076\,92L_{34}\alpha_{27}^{PP}+0.030\,08L_{35}\alpha_{27}^{PP}+0.028\,54L_{36}\alpha_{27}^{PP}+0.035\,57L_{37}\alpha_{27}^{PP}+0.056\,80L_{38}\alpha_{27}^{PP}+0.020\,91L_{39}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\pi^0K^+)=A_{PP}s^2\times[0.037\,52L_{10}\alpha_8^{PP}+0.023\,35L_{11}\alpha_8^{PP}-0.035\,43L_{12}\alpha_8^{PP}+0.045\,71L_{13}\alpha_8^{PP}+0.099\,64L_{14}\alpha_8^{PP}+0.094\,55L_{15}\alpha_8^{PP}+0.036\,21L_{34}\alpha_{27}^{PP}+0.055\,22L_{35}\alpha_{27}^{PP}+0.020\,30L_{36}\alpha_{27}^{PP}+0.000\,62L_{37}\alpha_{27}^{PP}+0.006\,88L_{38}\alpha_{27}^{PP}+0.038\,81L_{39}\alpha_{27}^{PP}-0.007\,93L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\pi^+K^0)=A_{PP}s^2\times[-0.053\,07L_{10}\alpha_8^{PP}-0.033\,02L_{11}\alpha_8^{PP}+0.050\,11L_{12}\alpha_8^{PP}-0.064\,65L_{13}\alpha_8^{PP}-0.140\,91L_{14}\alpha_8^{PP}-0.133\,72L_{15}\alpha_8^{PP}+0.035\,07L_{34}\alpha_{27}^{PP}+0.027\,97L_{35}\alpha_{27}^{PP}+0.030\,22L_{36}\alpha_{27}^{PP}+0.022\,04L_{37}\alpha_{27}^{PP}-0.009\,31L_{38}\alpha_{27}^{PP}+0.036\,74L_{39}\alpha_{27}^{PP}+0.011\,21L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to\pi^+\eta_8)=A_{PP}cs\times[-0.055\,71L_{10}\alpha_8^{PP}-0.084\,93L_{11}\alpha_8^{PP}-0.056\,11L_{12}\alpha_8^{PP}+0.033\,96L_{13}\alpha_8^{PP}-0.154\,00L_{14}\alpha_8^{PP}+0.033\,54L_{34}\alpha_{27}^{PP}+0.081\,76L_{35}\alpha_{27}^{PP}+0.025\,11L_{36}\alpha_{27}^{PP}+0.096\,08L_{37}\alpha_{27}^{PP}-0.010\,33L_{38}\alpha_{27}^{PP}+0.016\,65L_{39}\alpha_{27}^{PP}+0.027\,47L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D^+\to K^+\eta_8)=A_{PP}s^2\times[0.021\,66L_{10}\alpha_8^{PP}+0.013\,48L_{11}\alpha_8^{PP}-0.020\,46L_{12}\alpha_8^{PP}+0.026\,39L_{13}\alpha_8^{PP}+0.057\,53L_{14}\alpha_8^{PP}+0.054\,59L_{15}\alpha_8^{PP}+0.023\,20L_{34}\alpha_{27}^{PP}-0.027\,14L_{35}\alpha_{27}^{PP}+0.038\,87L_{36}\alpha_{27}^{PP}+0.052\,91L_{37}\alpha_{27}^{PP}-0.034\,73L_{38}\alpha_{27}^{PP}+0.022\,78L_{39}\alpha_{27}^{PP}+0.041\,20L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to\eta_1\pi^+)=A_{PP}c^2\times[-0.097\,87L_2\alpha^{\eta_1P}-0.029\,02L_3\alpha^{\eta_1P}-0.197\,29L_4\alpha^{\eta_1P}-0.078\,37L_5\alpha^{\eta_1P}+0.158\,34L_6\alpha^{\eta_1P}],$$

$$\mathcal{A}(D_s\to\eta_1K^+)=A_{PP}cs\times[-0.078\,30L_2\alpha^{\eta_1P}-0.073\,71L_3\alpha^{\eta_1P}-0.175\,39L_4\alpha^{\eta_1P}-0.141\,85L_5\alpha^{\eta_1P}-0.043\,64L_6\alpha^{\eta_1P}+0.002\,65L_7\alpha^{\eta_1P}+0.192\,43L_8\alpha^{\eta_1P}],$$

$$\mathcal{A}(D_s\to\overline{K^0}K^+)=A_{PP}c^2\times[-0.075\,81L_{10}\alpha_8^{PP}-0.019\,81L_{11}\alpha_8^{PP}-0.010\,67L_{12}\alpha_8^{PP}-0.072\,59L_{13}\alpha_8^{PP}-0.172\,60L_{14}\alpha_8^{PP}-0.001\,32L_{15}\alpha_8^{PP}+0.083\,78L_{16}\alpha_8^{PP}+0.037\,51L_{34}\alpha_{27}^{PP}+0.048\,71L_{35}\alpha_{27}^{PP}-0.022\,43L_{36}\alpha_{27}^{PP}-0.005\,53L_{37}\alpha_{27}^{PP}+0.033\,69L_{38}\alpha_{27}^{PP}-0.026\,97L_{39}\alpha_{27}^{PP}+0.019\,62L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to\pi^0K^+)=A_{PP}cs\times[-0.042\,88L_{10}\alpha_8^{PP}-0.065\,38L_{11}\alpha_8^{PP}+0.030\,81L_{12}\alpha_8^{PP}+0.054\,10L_{13}\alpha_8^{PP}-0.088\,32L_{14}\alpha_8^{PP}-0.095\,49L_{15}\alpha_8^{PP}+0.059\,24L_{16}\alpha_8^{PP}+0.025\,60L_{34}\alpha_{27}^{PP}-0.007\,19L_{35}\alpha_{27}^{PP}-0.001\,76L_{36}\alpha_{27}^{PP}+0.029\,67L_{37}\alpha_{27}^{PP}+0.039\,75L_{38}\alpha_{27}^{PP}-0.008\,98L_{39}\alpha_{27}^{PP}+0.037\,66L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to\pi^+K^0)=A_{PP}cs[0.060\,65L_{10}\alpha_8^{PP}+0.092\,46L_{11}\alpha_8^{PP}-0.043\,57L_{12}\alpha_8^{PP}-0.076\,50L_{13}\alpha_8^{PP}+0.124\,90L_{14}\alpha_8^{PP}$$
$$+0.135\,04L_{15}\alpha_8^{PP}-0.083\,78L_{16}\alpha_8^{PP}+0.069\,77L_{34}\alpha_{27}^{PP}+0.028\,19L_{35}\alpha_{27}^{PP}-0.000\,43L_{36}\alpha_{27}^{PP}$$
$$-0.008\,10L_{37}\alpha_{27}^{PP}+0.027\,90L_{38}\alpha_{27}^{PP}-0.003\,04L_{39}\alpha_{27}^{PP}+0.051\,86L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to\pi^+\eta_8)=A_{PP}c^2[0.061\,90L_{10}\alpha_8^{PP}+0.016\,18L_{11}\alpha_8^{PP}+0.008\,71L_{12}\alpha_8^{PP}+0.059\,27L_{13}\alpha_8^{PP}+0.140\,92L_{14}\alpha_8^{PP}$$
$$+0.001\,08L_{15}\alpha_8^{PP}-0.068\,41L_{16}\alpha_8^{PP}+0.045\,94L_{34}\alpha_{27}^{PP}+0.059\,66L_{35}\alpha_{27}^{PP}-0.027\,47L_{36}\alpha_{27}^{PP}$$
$$-0.006\,77L_{37}\alpha_{27}^{PP}+0.041\,26L_{38}\alpha_{27}^{PP}-0.033\,03L_{39}\alpha_{27}^{PP}+0.024\,03L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to K^0K^+)=A_{PP}s^2[0.083\,47L_{34}\alpha_{27}^{PP}+0.081\,54L_{35}\alpha_{27}^{PP}+0.015\,64L_{36}\alpha_{27}^{PP}+0.006\,45L_{37}\alpha_{27}^{PP}+0.004\,20L_{38}\alpha_{27}^{PP}$$
$$+0.046\,31L_{39}\alpha_{27}^{PP}+0.105\,13L_{40}\alpha_{27}^{PP}],$$

$$\mathcal{A}(D_s\to K^+\eta_8)=A_{PP}cs[-0.024\,76L_{10}\alpha_8^{PP}-0.037\,75L_{11}\alpha_8^{PP}+0.017\,79L_{12}\alpha_8^{PP}+0.031\,23L_{13}\alpha_8^{PP}-0.050\,99L_{14}\alpha_8^{PP}$$
$$-0.055\,13L_{15}\alpha_8^{PP}+0.034\,20L_{16}\alpha_8^{PP}+0.126\,57L_{34}\alpha_{27}^{PP}+0.081\,50L_{35}\alpha_{27}^{PP}+0.001\,99L_{36}\alpha_{27}^{PP}$$
$$-0.071\,24L_{37}\alpha_{27}^{PP}-0.000\,51L_{38}\alpha_{27}^{PP}+0.008\,11L_{39}\alpha_{27}^{PP}+0.061\,80L_{40}\alpha_{27}^{PP}].$$

The amplitudes for the VV modes have the same coefficients as the corresponding PP modes. They are found by relabeling $PP\to VV$ in the above expressions. The PV modes do not obey Bose symmetry and therefore have antisymmetric parts. Only the antisymmetric parts are given below. The symmetric parts have coefficients that are $1/\sqrt{2}$ times the corresponding coefficients of the PP amplitudes, with the exception of $\eta_1 V$, $\omega_1 P$, $\pi^0\rho^0$, and $\eta_8\omega_8$ which do not involve $1/\sqrt{2}$. The antisymmetric parts of $\mathcal{A}(D_i\to XY^*)$ are $-\mathcal{A}(D_i\to YX^*)|_{\rm asy}$, where an asterisk denotes spin-1. The expressions given for VV are for the S-wave amplitudes. We have assumed that the D-wave amplitudes are given by the S-wave amplitude times a constant factor times a kinematic factor (phase space). The best-fit value of the constant factor is in Table VII (also see the text):

$$\mathcal{A}(D^0\to K^-\rho^+)_{\rm asy}=A_{PV}c^2[0.115\,87L_{17}\alpha_{8'}^{PV}+0.060\,86L_{24}\alpha_{10}^{PV}+0.071\,36L_{29}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to K^-K^{*+})_{\rm asy}=A_{PV}cs[0.097\,89L_{17}\alpha_{8'}^{PV}+0.103\,86L_{18}\alpha_{8'}^{PV}-0.041\,84L_{24}\alpha_{10}^{PV}+0.040\,07L_{25}\alpha_{10}^{PV}$$
$$+0.051\,90L_{29}\alpha_{\overline{10}}^{PV}+0.025\,75L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\overline{K^0}\rho^0)_{\rm asy}=A_{PV}c^2[-0.081\,93L_{17}\alpha_{8'}^{PV}+0.086\,07L_{24}\alpha_{10}^{PV}-0.043\,17L_{29}\alpha_{\overline{10}}^{PV}-0.014\,69L_{30}\alpha_{\overline{10}}^{PV}+0.022\,58L_{31}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\overline{K^0}K^{*0})_{\rm asy}=A_{PV}cs[-0.183\,79L_{17}\alpha_{8'}^{PV}-0.045\,19L_{18}\alpha_{8'}^{PV}+0.082\,28L_{19}\alpha_{8'}^{PV}-0.041\,84L_{24}\alpha_{10}^{PV}+0.040\,07L_{25}\alpha_{10}^{PV}$$
$$+0.051\,90L_{29}\alpha_{\overline{10}}^{PV}+0.025\,75L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\overline{K^0}\omega_8)_{\rm asy}=A_{PV}c^2[-0.141\,91L_{17}\alpha_{8'}^{PV}+0.074\,78L_{29}\alpha_{\overline{10}}^{PV}+0.025\,44L_{30}\alpha_{\overline{10}}^{PV}-0.039\,11L_{31}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\pi^-\rho^+)_{\rm asy}=A_{PV}cs[0.085\,90L_{17}\alpha_{8'}^{PV}-0.058\,68L_{18}\alpha_{8'}^{PV}-0.082\,28L_{19}\alpha_{8'}^{PV}-0.041\,84L_{24}\alpha_{10}^{PV}$$
$$+0.040\,07L_{25}\alpha_{10}^{PV}+0.051\,90L_{29}\alpha_{\overline{10}}^{PV}+0.025\,75L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\pi^-K^{*+})_{\rm asy}=A_{PV}s^2[0.067\,92L_{17}\alpha_{8'}^{PV}+0.058\,56L_{18}\alpha_{8'}^{PV}-0.058\,06L_{19}\alpha_{8'}^{PV}+0.044\,86L_{20}\alpha_{8'}^{PV}+0.038\,04L_{24}\alpha_{10}^{PV}$$
$$-0.006\,50L_{25}\alpha_{10}^{PV}+0.060\,03L_{26}\alpha_{10}^{PV}+0.032\,44L_{29}\alpha_{\overline{10}}^{PV}+0.051\,49L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\pi^0K^{*0})_{\rm asy}=A_{PV}s^2[-0.048\,03L_{17}\alpha_{8'}^{PV}-0.041\,41L_{18}\alpha_{8'}^{PV}+0.041\,05L_{19}\alpha_{8'}^{PV}-0.031\,72L_{20}\alpha_{8'}^{PV}-0.026\,90L_{24}\alpha_{10}^{PV}$$
$$+0.004\,50L_{25}\alpha_{10}^{PV}-0.042\,45L_{26}\alpha_{10}^{PV}+0.045\,87L_{29}\alpha_{\overline{10}}^{PV}+0.072\,82L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to\pi^0\omega_8)_{\rm asy}=A_{PV}cs[0.072\,47L_{24}\alpha_{10}^{PV}-0.069\,41L_{25}\alpha_{10}^{PV}+0.089\,89L_{29}\alpha_{\overline{10}}^{PV}+0.044\,59L_{30}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D^0\to K^0\omega_8)_{\rm asy}=A_{PV}s^2[0.083\,19L_{17}\alpha_{8'}^{PV}+0.071\,72L_{18}\alpha_{8'}^{PV}-0.071\,11L_{19}\alpha_{8'}^{PV}+0.054\,94L_{20}\alpha_{8'}^{PV}-0.046\,58L_{24}\alpha_{10}^{PV}$$
$$+0.007\,96L_{25}\alpha_{10}^{PV}-0.073\,52L_{26}\alpha_{10}^{PV}],$$

$$\mathcal{A}(D^+\to\overline{K^0}\rho^+)_{\rm asy}=A_{PV}c^2[-0.125\,52L_{24}\alpha_{10}^{PV}+0.094\,23L_{25}\alpha_{10}^{PV}-0.037\,52L_{26}\alpha_{10}^{PV}+0.085\,40L_{27}\alpha_{10}^{PV}],$$

$$\begin{aligned}\mathcal{A}(D^+\to\overline{K^0}K^{*+})_{\rm asy}=A_{PV}cs[&-0.035\,96L_{17}\alpha_{8'}^{PV}+0.040\,58L_{18}\alpha_{8'}^{PV}+0.051\,68L_{19}\alpha_{8'}^{PV}+0.021\,92L_{20}\alpha_{8'}^{PV}+0.099\,40L_{21}\alpha_{8'}^{PV}\\&-0.079\,88L_{24}\alpha_{10}^{PV}+0.072\,57L_{25}\alpha_{10}^{PV}-0.022\,51L_{26}\alpha_{10}^{PV}+0.012\,20L_{27}\alpha_{10}^{PV}+0.039\,84L_{28}\alpha_{10}^{PV}\\&+0.048\,66L_{29}\alpha_{\overline{10}}^{PV}+0.036\,78L_{30}\alpha_{\overline{10}}^{PV}+0.008\,22L_{31}\alpha_{\overline{10}}^{PV}+0.042\,04L_{32}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D^+\to\pi^0\rho^+)_{\rm asy}=A_{PV}cs[&0.050\,85L_{17}\alpha_{8'}^{PV}-0.057\,39L_{18}\alpha_{8'}^{PV}-0.073\,09L_{19}\alpha_{8'}^{PV}-0.031\,00L_{20}\alpha_{8'}^{PV}-0.140\,58L_{21}\alpha_{8'}^{PV}\\&-0.056\,48L_{24}\alpha_{10}^{PV}+0.051\,31L_{25}\alpha_{10}^{PV}-0.015\,92L_{26}\alpha_{10}^{PV}+0.008\,63L_{27}\alpha_{10}^{PV}+0.028\,17L_{28}\alpha_{10}^{PV}\\&+0.034\,41L_{29}\alpha_{\overline{10}}^{PV}+0.026\,01L_{30}\alpha_{\overline{10}}^{PV}+0.005\,82L_{31}\alpha_{\overline{10}}^{PV}+0.029\,73L_{32}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D^+\to\pi^0K^{*+})_{\rm asy}=A_{PV}s^2[&0.019\,78L_{17}\alpha_{8'}^{PV}+0.017\,61L_{18}\alpha_{8'}^{PV}-0.013\,79L_{19}\alpha_{8'}^{PV}+0.024\,09L_{20}\alpha_{8'}^{PV}-0.052\,51L_{21}\alpha_{8'}^{PV}\\&+0.049\,83L_{22}\alpha_{8'}^{PV}+0.029\,59L_{24}\alpha_{10}^{PV}-0.009\,96L_{25}\alpha_{10}^{PV}+0.026\,53L_{26}\alpha_{10}^{PV}+0.008\,63L_{27}\alpha_{10}^{PV}\\&+0.028\,17L_{28}\alpha_{10}^{PV}+0.045\,87L_{29}\alpha_{\overline{10}}^{PV}+0.072\,82L_{30}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D^+\to\pi^+K^{*0})_{\rm asy}=A_{PV}s^2[&-0.027\,97L_{17}\alpha_{8'}^{PV}-0.024\,90L_{18}\alpha_{8'}^{PV}+0.019\,50L_{19}\alpha_{8'}^{PV}-0.034\,07L_{20}\alpha_{8'}^{PV}+0.074\,27L_{21}\alpha_{8'}^{PV}\\&-0.070\,48L_{22}\alpha_{8'}^{PV}-0.041\,84L_{24}\alpha_{10}^{PV}+0.014\,08L_{25}\alpha_{10}^{PV}-0.037\,52L_{26}\alpha_{10}^{PV}-0.012\,20L_{27}\alpha_{10}^{PV}\\&-0.039\,84L_{28}\alpha_{10}^{PV}+0.032\,44L_{29}\alpha_{\overline{10}}^{PV}+0.051\,49L_{30}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D^+\to\pi^+\omega_8)_{\rm asy}=A_{PV}cs[&0.097\,83L_{24}\alpha_{10}^{PV}-0.088\,87L_{25}\alpha_{10}^{PV}+0.027\,57L_{26}\alpha_{10}^{PV}-0.014\,94L_{27}\alpha_{10}^{PV}-0.048\,79L_{28}\alpha_{10}^{PV}\\&+0.059\,59L_{29}\alpha_{\overline{10}}^{PV}+0.045\,05L_{30}\alpha_{\overline{10}}^{PV}+0.010\,07L_{31}\alpha_{\overline{10}}^{PV}+0.051\,49L_{32}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D^+\to K^+\omega_8)_{\rm asy}=A_{PV}s^2[&0.034\,25L_{17}\alpha_{8'}^{PV}+0.030\,50L_{18}\alpha_{8'}^{PV}-0.023\,89L_{19}\alpha_{8'}^{PV}+0.041\,73L_{20}\alpha_{8'}^{PV}-0.090\,96L_{21}\alpha_{8'}^{PV}\\&+0.086\,32L_{22}\alpha_{8'}^{PV}-0.051\,24L_{24}\alpha_{10}^{PV}+0.017\,24L_{25}\alpha_{10}^{PV}-0.045\,95L_{26}\alpha_{10}^{PV}\\&-0.014\,94L_{27}\alpha_{10}^{PV}-0.048\,80L_{28}\alpha_{10}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D_s\to\overline{K^0}K^{*+})_{\rm asy}=A_{PV}c^2[&0.039\,96L_{17}\alpha_{8'}^{PV}-0.006\,46L_{18}\alpha_{8'}^{PV}-0.009\,95L_{19}\alpha_{8'}^{PV}+0.038\,26L_{20}\alpha_{8'}^{PV}-0.090\,97L_{21}\alpha_{8'}^{PV}\\&+0.000\,70L_{22}\alpha_{8'}^{PV}+0.044\,16L_{23}\alpha_{8'}^{PV}-0.038\,04L_{24}\alpha_{10}^{PV}+0.023\,83L_{25}\alpha_{10}^{PV}-0.035\,02L_{26}\alpha_{10}^{PV}\\&+0.016\,27L_{27}\alpha_{10}^{PV}-0.039\,84L_{28}\alpha_{10}^{PV}-0.051\,90L_{29}\alpha_{\overline{10}}^{PV}-0.012\,26L_{30}\alpha_{\overline{10}}^{PV}+0.008\,77L_{31}\alpha_{\overline{10}}^{PV}\\&-0.030\,03L_{32}\alpha_{\overline{10}}^{PV}+0.035\,63L_{33}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D_s\to\pi^0\rho^+)_{\rm asy}=A_{PV}c^2[&-0.056\,51L_{17}\alpha_{8'}^{PV}+0.009\,13L_{18}\alpha_{8'}^{PV}+0.014\,07L_{19}\alpha_{8'}^{PV}-0.054\,11L_{20}\alpha_{8'}^{PV}+0.128\,65L_{21}\alpha_{8'}^{PV}\\&-0.000\,99L_{22}\alpha_{8'}^{PV}-0.062\,45L_{23}\alpha_{8'}^{PV}-0.026\,90L_{24}\alpha_{10}^{PV}+0.016\,85L_{25}\alpha_{10}^{PV}-0.024\,76L_{26}\alpha_{10}^{PV}\\&+0.011\,50L_{27}\alpha_{10}^{PV}-0.028\,17L_{28}\alpha_{10}^{PV}-0.036\,70L_{29}\alpha_{\overline{10}}^{PV}-0.008\,67L_{30}\alpha_{\overline{10}}^{PV}+0.006\,20L_{31}\alpha_{\overline{10}}^{PV}\\&-0.021\,24L_{32}\alpha_{\overline{10}}^{PV}+0.025\,20L_{33}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\begin{aligned}\mathcal{A}(D_s\to\pi^0K^{*+})_{\rm asy}=A_{PV}cs[&-0.022\,60L_{17}\alpha_{8'}^{PV}-0.013\,37L_{18}\alpha_{8'}^{PV}+0.035\,67L_{19}\alpha_{8'}^{PV}+0.028\,51L_{20}\alpha_{8'}^{PV}+0.046\,55L_{21}\alpha_{8'}^{PV}\\&-0.050\,33L_{22}\alpha_{8'}^{PV}-0.031\,22L_{23}\alpha_{8'}^{PV}+0.061\,86L_{24}\alpha_{10}^{PV}-0.037\,53L_{25}\alpha_{10}^{PV}+0.019\,46L_{26}\alpha_{10}^{PV}\\&+0.031\,63L_{27}\alpha_{10}^{PV}-0.028\,17L_{28}\alpha_{10}^{PV}-0.050\,46L_{29}\alpha_{\overline{10}}^{PV}-0.038\,14L_{30}\alpha_{\overline{10}}^{PV}+0.024\,04L_{31}\alpha_{\overline{10}}^{PV}\\&+0.016\,99L_{32}\alpha_{\overline{10}}^{PV}+0.050\,40L_{33}\alpha_{\overline{10}}^{PV}],\end{aligned}$$

$$\mathcal{A}(D_s\to\pi^+K^{*0})_{\rm asy}=A_{PV}cs[0.031\,96L_{17}\alpha_{8'}^{PV}+0.018\,91L_{18}\alpha_{8'}^{PV}-0.050\,44L_{19}\alpha_{8'}^{PV}-0.040\,32L_{20}\alpha_{8'}^{PV}-0.065\,83L_{21}\alpha_{8'}^{PV}$$
$$+0.071\,17L_{22}\alpha_{8'}^{PV}+0.044\,16L_{23}\alpha_{8'}^{PV}-0.087\,48L_{24}\alpha_{10}^{PV}+0.053\,07L_{25}\alpha_{10}^{PV}-0.027\,51L_{26}\alpha_{10}^{PV}$$
$$-0.044\,73L_{27}\alpha_{10}^{PV}+0.039\,84L_{28}\alpha_{10}^{PV}-0.035\,68L_{29}\alpha_{\overline{10}}^{PV}-0.026\,97L_{30}\alpha_{\overline{10}}^{PV}+0.017\,00L_{31}\alpha_{\overline{10}}^{PV}$$
$$+0.012\,01L_{32}\alpha_{\overline{10}}^{PV}+0.035\,63L_{33}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D_s\to\pi^+\omega_8)_{\rm asy}=A_{PV}c^2[0.046\,58L_{24}\alpha_{10}^{PV}-0.029\,18L_{25}\alpha_{10}^{PV}+0.042\,89L_{26}\alpha_{10}^{PV}-0.019\,92L_{27}\alpha_{10}^{PV}+0.048\,80L_{28}\alpha_{10}^{PV}$$
$$-0.063\,56L_{29}\alpha_{\overline{10}}^{PV}-0.015\,02L_{30}\alpha_{\overline{10}}^{PV}+0.010\,74L_{31}\alpha_{\overline{10}}^{PV}-0.036\,78L_{32}\alpha_{\overline{10}}^{PV}+0.043\,65L_{33}\alpha_{\overline{10}}^{PV}],$$

$$\mathcal{A}(D_s\to K^0K^{*+})_{\rm asy}=A_{PV}s^2[0.102\,70L_{24}\alpha_{10}^{PV}-0.100\,73L_{25}\alpha_{10}^{PV}+0.097\,55L_{26}\alpha_{10}^{PV}+0.085\,39L_{27}\alpha_{10}^{PV}],$$

$$\mathcal{A}(D_s\to K^+\omega_8)_{\rm asy}=A_{PV}cs[-0.039\,15L_{17}\alpha_{8'}^{PV}-0.023\,15L_{18}\alpha_{8'}^{PV}+0.061\,78L_{19}\alpha_{8'}^{PV}+0.049\,38L_{20}\alpha_{8'}^{PV}+0.080\,62L_{21}\alpha_{8'}^{PV}$$
$$-0.087\,17L_{22}\alpha_{8'}^{PV}-0.054\,08L_{23}\alpha_{8'}^{PV}-0.107\,14L_{24}\alpha_{10}^{PV}+0.065\,00L_{25}\alpha_{10}^{PV}-0.033\,70L_{26}\alpha_{10}^{PV}$$
$$-0.054\,78L_{27}\alpha_{10}^{PV}+0.048\,79L_{28}\alpha_{10}^{PV}].$$

[1] A. J. Buras *et al.*, Nucl. Phys. **B268**, 16 (1986).
[2] D. Lee, Phys. Lett. B **275**, 469 (1992).
[3] I. I. Bigi, in *Heavy Flavours* Proceedings of the Advanced Study Conference, Pavia, Italy, 1993, edited by G. Bellini *et al.* (Editions Frontieres, Gif-sur-Yvette, 1994).
[4] M. Bauer *et al.*, Phys. Lett. **152B**, 380 (1985); Z. Phys. C **34**, 103 (1987).
[5] L.-L. Chau *et al.*, Phys. Rev. D **36**, 137 (1987); Phys. Lett. B **222**, 285 (1989); Phys. Rev. D **42**, 1837 (1990); Z. Phys. C **53**, 413 (1992); Phys. Lett. B **280**, 281 (1992).
[6] C. Quigg, Z. Phys. C **4**, 55 (1980).
[7] M. J. Savage, Phys. Lett. B **257**, 414 (1991); **259**, 135 (1991).
[8] Crystal Barrel Collaboration, C. Amsler *et al.*, Phys. Lett. B **294**, 451 (1992).
[9] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
[10] M. B. Einhorn *et al.*, Phys. Rev. D **12**, 2015 (1975).
[11] F. J. Gilman and M. B. Wise, Phys. Rev. D **20**, 2392 (1979).
[12] T. A. Kaeding, Comput. Phys. Commun. **85**, 82 (1995).
[13] MINUIT, CERN Program Library, D506, 1993.
[14] E791 Collaboration, M. V. Purohit *et al.*, in *Proceedings of the 27th International Conference on High Energy Physics*, Glasgow, Scotland, 1994, edited by P. F. Bussey and I. G. Knowles (IOP, London, 1995).
[15] E691 Collaboration, J. C. Anjos *et al.*, Phys. Rev. D **46**, 1941 (1992).
[16] CLEO Collaboration, P. Avery *et al.*, Phys. Rev. Lett. **68**, 1279 (1992).
[17] CLEO Collaboration, D. Cinabro *et al.*, Phys. Rev. Lett. **72**, 1406 (1994).
[18] R. L. Kingsley *et al.*, Phys. Rev. D **11**, 1919 (1975).
[19] S. P. Rosen, Phys. Lett. B **218**, 353 (1989).