Meson-cloud contributions in $K \to \pi$ weak transitions

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Contributions of low-energy ''eye'' and ''figure-eight'' quark diagrams to the $K \to \pi$ weak transitions are studied in a hadron-level phenomenological approach in which they originate from meson-cloud effects. If contributions from cloud mesons exhibit SU(6) symmetry, only the ''eye'' (low-energy penguin) diagram is nonvanishing. When the symmetry between contributions from intermediate pseudoscalar and vector mesons is broken, the $\Delta I = \frac{1}{2}$ ($\Delta I = \frac{3}{2}$) amplitudes are enhanced (suppressed). The overall long-distance-induced enhancement of the ratio of the $\Delta I = \frac{1}{2}$ amplitudes over the $\Delta I = \frac{3}{2}$ amplitudes is estimated at around 2. [S0556-2821(96)03413-3]

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I. INTRODUCTION

After almost 40 years since the discovery of the $\Delta I = 1/2$ rule in strangeness-changing weak hadronic decays, its origin still eludes our understanding (for a recent review see Ref. [1]). The dominance of the $\Delta I = 1/2$ amplitudes over those with $\Delta I = 3/2$ requires a significant enhancement of the former and/or suppression of the latter. While for nonleptonic baryon decays at least part of the effect stems from the Pati-Woo theorem [2], according to which the symmetry of baryon wave functions ensures the vanishing of the $\Delta I = 3/2$ amplitude, no such symmetry-based mechanism is available for kaon decays.

The required effects can be obtained to some extent from perturbative QCD. Short-distance QCD corrections modify the effective weak Hamiltonian and lead to an enhancement of the $\Delta I = 1/2$ (suppression of the $\Delta I = 3/2$) operators [3]. In addition, a new purely $\Delta I = 1/2$ mechanism — the so-called penguin operator — appears. Its contributions add constructively to those of standard $\Delta I = 1/2$ operators. Detailed studies [4] show, however, that the original claim of a large penguin contribution is incorrect. This contribution remains small even if one takes into account the increase, over the value quoted in Ref. [5], of the real part of the penguin Wilson coefficient due to the incomplete Glashow-Iliopoulos-Maiani (GIM) cancellation above the charm quark mass [6].

Dropping the so-called Fierz contributions (which has been argued to be justified in the $1/N$ expansion [7]) does help a little, but a large discrepancy still remains [1]. In fact, for consistency with the spirit of the $1/N$ expansion, the Fierz terms should be considered along with nonfactorizable terms of the same order. Starting from an effective chiral Lagrangian, such subleading $1/N$ contributions have been calculated in Ref. [8] as nonfactorizable pseudoscalar meson loop corrections to $K \to 2\pi$. Their contribution has been found to be of the same order as that of the genuine factorizable terms. In Ref. [1] the following effects are mentioned as contributing to the subleading terms of the $1/N$ approach: the Fierz-transformed contributions, final-state interactions, low-energy ''eye'' graphs, and soft gluon exchanges between two quark loops in ''figure-eight'' graphs.

Because of the long-distance nature of the last three mechanisms, their evaluation from the first principles of QCD is possible on the lattice only. In practice it is the $K \to \pi$ matrix elements that are more amenable to such calculations. From these, the $K \to 2\pi$ amplitudes are then obtained by means of current algebra. Within very large statistical and systematic uncertainties the lattice calculations [9] support the $\Delta I = 1/2$ enhancement and indicate that the purely $\Delta I = 1/2$ ''eye'' graphs dominate over the ''figure-eight'' graphs.

The contribution from the ''eye'' and ''figure-eight'' graphs of the quark-level description must be contained in the meson-cloud (or unitarity) effects of the hadron level (as these include all confinement effects; see also Ref. [10]). In fact, it has been found repeatedly by many authors that such meson-cloud effects are very important in many areas of hadron physics, improving the predictions of the standard quark model. For a unitarity-oriented view of hadron spectroscopy see Refs. [11–13]. Meson-cloud effects have also been found to be instrumental in several other places [14,15]. Consideration of their effects in weak nonleptonic hyperon decays yields an explanation of the deviation of the f/d ratio from the naive valence quark model value of -1 to its observed values of about -2 [16]. It is therefore of great interest to perform a similar phenomenological analysis of meson-cloud effects in $K \to 2\pi$ decays to see whether and how they may help to explain relative sizes of the relevant $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes. In this paper an analysis of this type is carried out. We study the $K \to \pi$ transition matrix elements and show in detail how hadron-level effects from various two-meson intermediate states contributing to these transitions build up the ''eye'' (low-energy penguin) and the ''figure-eight'' diagrams of the quark level. An estimate of the absolute size of the total hadron-level-induced contribution to the $A_{1/2}/A_{3/2}$ enhancement is also given.

II. HADRONIC LOOP CONTRIBUTIONS TO THE $K \to \pi$ TRANSITIONS

The effect of pseudoscalar meson loop contributions to $K \to 2\pi$ was studied in a dispersion relation framework [14]

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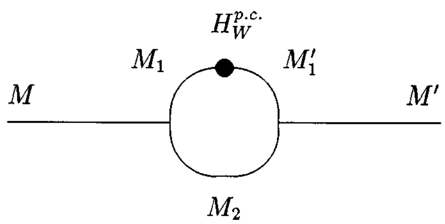

FIG. 1. Weak transition in hadronic loop.

and in a chiral approach [8,17]. In a more phenomenological way such meson rescattering final-state interaction (FSI) effects are discussed in Ref. [18]. In this paper we are concerned with meson loop (hadron sea) effects in $K\to\pi$ transitions themselves (see Fig. 1). If only ground-state mesons are permitted in the loop, at least one of them must be a vector meson (the allowed intermediate states are $M_1M_2=PV$, VP, and VV) (P denotes pseudoscalar, and V vector mesons). Although all these two-particle states are much heavier than the PP ones that were considered in Refs. [8,14,17,18], their contribution is expected to be significant as evidenced by estimates of their effects in hadron spectroscopy [11,13]. A transparent way to include both pseudoscalar and vector mesons in the intermediate state is to use general ideas of the unitarized quark model of Ref. [11].

What we want to estimate here is, in essence, the contribution from virtual two-meson continuum states admixed into the wave functions of the standard quark model. We shall disregard the virtual states composed of charmed mesons as such states lie much higher (by about 2 GeV) than those built of light flavors.

In the unitarized quark model the mass shifts due the influence of virtual two-meson states are calculated from [11,19]

$$\mathrm{Re}\Pi_{ij}(q^2)=-\frac{1}{\pi}\int_{\mathrm{thr}}^{\infty}\frac{\mathrm{Im}\Pi_{ij}(s')}{q^2-s'}ds', \tag{1}$$

where, for $P\to PV$, $VV\to P$ loops, one has

$$\mathrm{Im}\Pi_{ij}(q^2)=-g_{iM_1M_2}g_{jM_1M_2}\frac{k_{M_1}}{\sqrt{q^2}}k_{M_1}^2F_{iM_1M_2}(q^2)\times F_{jM_1M_2}(q^2), \tag{2}$$

where g denotes coupling constants and $F(q^2)$ hadronic form factors.

One can always perform a subtraction at $q^2=0$ and consider the constant term in Eq. (1) to be absorbed into the bare mass so that the resulting pseudoscalar meson mass is small. The shift function

$$\mathrm{Re}\Pi_{ij}(q^2)=-\frac{q^2}{\pi}\int_{\mathrm{thr}}^{\infty}\frac{\mathrm{Im}\Pi_{ij}(s')}{s'(q^2-s')}ds' \tag{3}$$

then takes into account those contributions from intermediate virtual two-meson states that vanish at $q^2=0$. Estimates of the size of the hadron-level effects depend mainly on the choice of hadronic form factors F. One expects the dominant contribution to come from the nearest-lying thresholds, i.e., from the $PV+VP$ intermediate states. The relevant form factor squared is then

$$F_{P,PV}^2(s)=\frac{s}{m_V^2}G^2(s), \tag{4}$$

where s/m_V^2 is a Lorentz boost factor resulting from the phenomenological form of the interaction Lagrangian (see Ref. [19]):

$$T=g_{PP'V}G\epsilon_\mu(p_P-p_{P'})^\mu. \tag{5}$$

Model dependence of the estimates of the size of hadronic loop effects is mainly through the assumed shape of $G(s)$. In the unitarized quark model one accepts the form

$$G^2(s)=\exp\left[-\left(\frac{k}{k_{\mathrm{cutoff}}}\right)^2\right], \tag{6}$$

where k_{cutoff} describes the (harmonic oscillator) meson size: $R_M=\sqrt{6}/k_{\mathrm{cutoff}}$.

For $P\to VV$ transitions one uses in the unitarized quark model the form [19]

$$F_{P,VV}^2(s)=G^2(s). \tag{7}$$

In the approach of Ref. [11] the admixture probability $|c_{M_1M_2}|^2$ of the $|M_1M_2\rangle$ two-particle state relative to the ''pure'' quark-model state for meson M is given by [20]

$$|c_{M_1M_2}|^2=L(M\to M_1M_2)[\mathrm{Tr}(F_M^\dagger F_{M_1}F_{M_2})+C_MC_{M_1}C_{M_2}\mathrm{Tr}(F_M^\dagger F_{M_2}F_{M_1})]^2, \tag{8}$$

where, for ground-state mesons M_1M_2, we have

$$\begin{aligned}L(M\to M_1M_2)&\equiv q^2 l(M\to M_1M_2)\\&\equiv q^2S(M\to M_1M_2)I(M\to M_1M_2)\\&\equiv q^2S(M\to M_1M_2)\frac{1}{\pi}\frac{f^2}{\pi}\\&\quad\times\int_{\mathrm{thr}}^{\infty}\frac{(k^3/\sqrt{s'})F_{M,M_1M_2}^2(s')}{s'(q^2-s')^2}ds',\end{aligned} \tag{9}$$

which vanishes with $q^2\to0$ and hence is adequate for an estimate of hadronic loop contribution to the (vanishing at $q^2=0$) matrix elements of the $K\to\pi$ transitions [21,22,1].

The trace factor in Eq. (8) [F_M is the SU(3) matrix corresponding to meson M] gives F- or D-type flavor couplings (see Fig. 2) depending on the sign of $C_MC_{M_1}C_{M_2}$ (where C_M is the charge conjugation quantum number of meson M). The spin-weight factors $S(M\to M_1M_2)$ are equal to $\frac{1}{4},\frac{1}{4},\frac{1}{2}$ for (M,M_1M_2) being (P,PV), (P,VP), (P,VV), respectively, and they sum up to 1. The overall size of the two-meson admixture is fixed by the size of the coupling constant $f=f_{\rho NN}=5.14$ [Eq. (9)], and by the shape of form factors [Eqs. (4) and (7)].

There is some uncertainty as to at what value of q^2 we should compare the matrix element $\langle\pi(q)|H_W|K(q)\rangle$ with

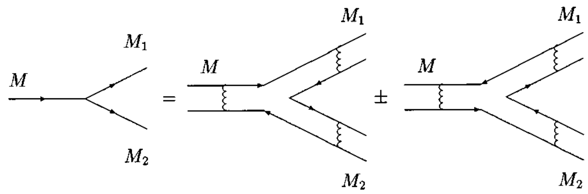

FIG. 2. Diagrammatic representation of F- and D-type strong vertices.

physical $K\to 2\pi$ amplitude. Values from $q^2=m_K^2/2$ to $q^2=m_K^2$ have been proposed [1,23]. We shall use $q^2=(m_K^2+m_\pi^2)/2\approx 0.132\ \mathrm{GeV}^2$. Using $m_P=(m_K+m_\pi)/2=0.32$ GeV and $m_V=(m_{K*}+m_\rho)/2=0.82$ GeV for the masses of the loop mesons we obtain for $I(P\to M_1M_2)$ the values given in Table I [for different values of $k_{\mathrm{cutoff}}=0.6,0.7,0.8,0.95$ GeV ($R_M=0.80,0.69,0.60,0.51$ fm)]. For $q^2=m_K^2$ the entries in Table I are larger by 5–10 %. In the unitarized quark model of Refs. [11,12] the value of $k_{\mathrm{cutoff}}=0.7$ GeV gives the best description of meson spectra.

Since, according to Eq. (8), admixtures of two-meson $|\rho\pi\rangle$, $|\rho\eta\rangle$, etc., states to π meson ($|\rho K\rangle$, etc., to K) are all to be considered, we will have to deal with the $K\to\eta$ transitions as well. With the Fierz terms dropped and small short-distance penguin contributions neglected, standard QCD-corrected short-distance calculations give the following predictions for the amplitudes:

$$\langle\pi^+|H_w|K^+\rangle=[c_1-(c_2+c_3+c_4)]X,$$

$$\langle\pi^0|H_w|K^0\rangle=\frac{1}{\sqrt{2}}[c_1-(c_2+c_3-2c_4)]X,$$

$$\langle\eta_8|H_w|K^0\rangle=\frac{1}{\sqrt{6}}[c_1-c_2+9c_3]X,$$

$$\langle\eta_1|H_w|K^0\rangle=\frac{1}{\sqrt{3}}[c_1+5c_2]X, \qquad (10)$$

where c_i are Wilson coefficients and

$$X=\langle\pi^+|-(d\overline{u})|0\rangle\langle 0|(u\overline{s})|K^+\rangle, \qquad (11)$$

with the notation

$$(q_1\overline{q_2})\equiv\overline{q_2}\gamma_\mu(1-\gamma_5)q_2. \qquad (12)$$

Let us express the matrix elements of the parity-conserving part of the weak Hamiltonian between pseudoscalar meson states through amplitudes of definite isospin:

TABLE I. Dependence of $I(P\to M_1M_2)$ on k_{cutoff}.

R (fm)	0.8	0.7	0.6	0.5
k_{cutoff} (GeV^2)	0.6	0.7	0.8	0.95
$I(P\to PV)$	0.075	0.106	0.139	0.192
$I(P\to VV)$	0.0042	0.0061	0.008	0.011

$$\langle\pi^+|H_w|K^+\rangle=\sqrt{\frac{2}{3}}A_{1/2}-\frac{1}{\sqrt{3}}A_{3/2},$$

$$\langle\pi^0|H_w|K^0\rangle=\frac{1}{\sqrt{3}}A_{1/2}+\sqrt{\frac{2}{3}}A_{3/2},$$

$$\langle\eta_8|H_w|K^0\rangle=B,\quad \langle\eta_1|H_w|K^0\rangle=C. \qquad (13)$$

Using the Gilman-Wise values [5]

$$c_1=-2.11,\quad c_2=0.12,\quad c_3=0.09,\quad c_4=0.45, \qquad (14)$$

for the Wilson coefficients, we obtain, from Eqs. (10) and (13),

$$\frac{A_{3/2}}{A_{1/2}}=-0.28,\quad \frac{B}{A_{1/2}}=0.20,\quad \frac{C}{A_{1/2}}=0.31. \qquad (15)$$

The experimental value for $|A_{1/2}/A_{3/2}|$ is around 22, 6 times larger than the theoretical value from Eq. (15) ($|A_{1/2}/A_{3/2}|=3.6$). When the short-distance penguin contribution is included (with $c_5\approx-0.06$) one obtains [1] $|A_{1/2}/A_{3/2}|=4.3$; i.e.,

$$\left(\frac{A_{1/2}}{A_{3/2}}\right)_{\mathrm{out}}=1.2\left(\frac{A_{1/2}}{A_{3/2}}\right)_{\mathrm{fact}}, \qquad (16)$$

an enhancement factor of 1.2 only. The remaining discrepancy by a factor of around 5 constitutes the $\Delta I=1/2$ puzzle.

The hadron-sea-generated corrections to the matrix elements of Eq. (13) are due to the weak Hamiltonian acting in one of M_1, M_2 mesons. Let the meson in which such a transition occurs be labeled M_1 (see Fig. 1). We restrict our considerations to the case when M_1 is in the ground state (i.e., $M_1=P,V$). For the sake of our discussion this should be a reasonable approximation: Quark-antiquark annihilation into a W boson is expected to be weaker for excited mesons.

In the normalization of Eq. (8) the contributions from the $M_1M_2=\underline{P}V$ two-meson states (the meson undergoing weak transition underlined for clarity) are easily calculable to be [with SU(3) classification of amplitudes on the left]

$$(\mathbf{27})A_{3/2,\mathrm{loop}}=-2L(P\to PV)A_{3/2},$$

$$(\mathbf{27})\frac{1}{\sqrt{10}}(A_{1/2,\mathrm{loop}}-3B_{\mathrm{loop}})$$

$$=-2L(P\to PV)\frac{1}{\sqrt{10}}(A_{1/2}-3B),$$

$$(\mathbf{8})\frac{1}{\sqrt{10}}(3A_{1/2,\mathrm{loop}}+B_{\mathrm{loop}})=+3L(P\to PV)\frac{1}{\sqrt{10}}(3A_{1/2}+B),$$

$$(\mathbf{8})C_{\mathrm{loop}}=0. \qquad (17)$$

Meson M_1 need not be a pseudoscalar meson. It may be a vector meson as well. For weak transitions in vector mesons we introduce notation analogous to that of Eq. (13): The $K^*\to\rho$ transitions are described by amplitudes $A_{1/2}^V,A_{3/2}^V$ of definite isospin, etc. When $M_1=V$ we have contributions from $\underline{V}P$ and $\underline{V}V$ two-meson states. They are (a) for $\underline{V}P$ diagrams,

$$A_{3/2,\text{loop}}=-2L(P\rightarrow VP)A^V_{3/2},$$

$$\frac{1}{\sqrt{10}}(A_{1/2,\text{loop}}-3B_{\text{loop}})=-2L(P\rightarrow VP)\frac{1}{\sqrt{10}}(A^V_{1/2}-3B^V),$$

$$\frac{1}{\sqrt{10}}(3A_{1/2,\text{loop}}+B_{\text{loop}})=+3L(P\rightarrow PV)\frac{1}{\sqrt{10}}(3A^V_{1/2}+B^V),$$

$$C_{\text{loop}}=0, \tag{18}$$

and (b) for $\underline{V}V$ diagrams,

$$A_{3/2,\text{loop}}=+2L(P\rightarrow VV)A^V_{3/2},$$

$$\frac{1}{\sqrt{10}}(A_{1/2,\text{loop}}-3B_{\text{loop}})=+2L(P\rightarrow VV)\frac{1}{\sqrt{10}}(A^V_{1/2}-3B^V),$$

$$\frac{1}{\sqrt{10}}(3A_{1/2,\text{loop}}+B_{\text{loop}})=+\frac{1}{3}L(P\rightarrow VV)\left(\frac{1}{\sqrt{10}}(3A^V_{1/2}+B^V)-4\sqrt{5}C^V\right),$$

$$C_{\text{loop}}=+\frac{1}{3}L(P\rightarrow VV)\left(-4\sqrt{5}\frac{1}{\sqrt{10}}(3A^V_{1/2}+B^V)+8C^V\right). \tag{19}$$

Summing the contributions from all $PV+VP$ and VV intermediate states we get

$$A_{3/2,\text{loop}}=-2L(P\rightarrow PV)(A_{3/2}+A^V_{3/2})+2L(P\rightarrow VV)A^V_{3/2},$$

$$\frac{1}{\sqrt{10}}(A_{1/2,\text{loop}}-3B_{\text{loop}})=-2L(P\rightarrow PV)\frac{1}{\sqrt{10}}(A_{1/2}-3B+A^V_{1/2}-3B^V)+2L(P\rightarrow VV)\frac{1}{\sqrt{10}}(A^V_{1/2}-3B^V),$$

$$\frac{1}{\sqrt{10}}(3A_{1/2,\text{loop}}+B_{\text{loop}})=+3L(P\rightarrow PV)\frac{1}{\sqrt{10}}(3A_{1/2}+B+3A^V_{1/2}+B^V)+\frac{1}{3}L(P\rightarrow VV)\left(\frac{1}{\sqrt{10}}(3A^V_{1/2}+B^V)-4\sqrt{5}C^V\right),$$

$$C_{\text{loop}}=+\frac{1}{3}L(P\rightarrow VV)\left(-4\sqrt{5}\frac{1}{\sqrt{10}}(3A^V_{1/2}+B^V)+8C^V\right). \tag{20}$$

If we had a fully P-V symmetric situation, i.e., $L(P\rightarrow PV)=L(P\rightarrow VV)/2$ and $A=A^V$, $B=B^V$, we would get, from Eq. (20),

$$A_{3/2,\text{loop}}=0,$$

$$\frac{1}{\sqrt{10}}(A_{1/2,\text{loop}}-3B_{\text{loop}})=0,$$

$$\sqrt{2}(3A_{1/2,\text{loop}}+B_{\text{loop}})+5C_{\text{loop}}=0,$$

$$\frac{1}{\sqrt{2}}(3A_{1/2,\text{loop}}+B_{\text{loop}})-2C_{\text{loop}}=12L(P\rightarrow PV)\left(\frac{1}{\sqrt{2}}(3A_{1/2}+B)-2C\right). \tag{21}$$

Thus, we see that in the symmetry limit the two-meson states give no contribution to the **27**-plet $\Delta I=1/2$ and 3/2 transition amplitudes. Furthermore, only one of the two combinations of octet ($\Delta I=1/2$) transition amplitudes receives contributions from such states.

From Eqs. (10),(13) we get

$$A_{1/2}=\sqrt{\frac{3}{2}}(c_1-c_2-c_3)X,\quad A_{3/2}=\sqrt{3}c_4X, \tag{22}$$

with similar formulas for $A^V_{1/2}$, $A^V_{3/2}$ in which X is replaced by X^V — the factorization contribution from a weak transition in the intermediate vector meson:

$$X^V=\langle\rho^+|-(\overline{du})|0\rangle\langle 0|(\overline{us})|K^{*+}\rangle. \tag{23}$$

The matrix elements of currents in Eqs. (11), (23) are given by

$$\langle\pi^+|A^\mu|0\rangle=f_\pi q^\mu,\quad \langle\rho^+|V^\mu|0\rangle=f_\rho\varepsilon^\mu, \tag{24}$$

where $f_\pi=0.13$ GeV, $f_\rho=0.17$ GeV^2. For the sake of the order-of-magnitude estimate and in accordance with the SU(3) symmetry used elsewhere in this paper we assume that $f_K=f_\pi$, $f_{K^*}=f_\rho$.

Using Eqs. (20), (22), (24) and taking into account the fact that a weak transition may occur in any one of the two intermediate mesons one derives

$$A_{3/2}=A_{3/2}^{\rm fact}\left\{1-4l(P\to PV)\left[\tilde{q}^2+\left(\frac{f_\rho}{f_\pi}\right)^2\right]+4l(P\to VV)\left(\frac{f_\rho}{f_\pi}\right)^2\right\},$$

$$A_{1/2}=A_{1/2}^{\rm fact}\left\{1+l(P\to PV)\left[5+\frac{c_1-c_2+9c_3}{c_1-c_2-c_3}\right]\left[\tilde{q}^2+\left(\frac{f_\rho}{f_\pi}\right)^2\right]-2l(P\to VV)\frac{c_1+7c_2+2c_3}{c_1-c_2-c_3}\left(\frac{f_\rho}{f_\pi}\right)^2\right\}, \quad (25)$$

where $\tilde{q}^2=0.132\ \text{GeV}^2$ and $(f_\rho/f_\pi)^2=1.71\ \text{GeV}^2$. From Eq. (25) it is clear that contributions from transitions in intermediate pseudoscalar mesons (corresponding to the $\tilde{q}^2$ piece) are negligible with respect to those in intermediate vector mesons [the $(f_\rho/f_\pi)^2$ terms]. Inserting the numerical values of the c_i coefficients we get

$$\frac{c_1-c_2+9c_3}{c_1-c_2-c_3}=0.61,\quad \frac{c_1+7c_2+2c_3}{c_1-c_2-c_3}=0.47. \quad (26)$$

Since $l(P\to VV)\ll l(P\to PV)$ (Table I), one concludes that contributions from VV loops are much smaller than those from PV loops. The factor of ''5'' in the second of Eqs. (25) is due to A $[K^*(K)\to\rho(\pi)]$ amplitudes while the fraction 0.61 added to it comes from B amplitudes. In other words the dominant contribution to hadronic loop enhancement of the isospin 1/2 amplitude comes from the $K^*\to\rho$ transitions in virtual intermediate states. Plugging in the numerical values into Eqs. (25) one obtains results gathered in Table II.

So far we have considered intermediate states composed of ground-state mesons. In strong virtual decays $M\to M_1M_2$ the p wave (that must appear somewhere to ensure parity conservation in the production of $q\bar{q}$ pair out of the vacuum) may reside either between mesons M_1M_2 or within meson M_2. The contributions from these two possibilities may be comparable. The spin-flavor factors $([\text{Tr}(F_M^\dagger F_{M_1}F_{M_2})+C_MC_{M_1}C_{M_2}\text{Tr}(F_M^\dagger F_{M_2}F_{M_1})]^2$ $*S(P\to M_1M_2)$ corresponding to the total contribution from all possible intermediate states under consideration are gathered in Table III.

When the p-wave excitation resides in the M_2 meson, the total contribution from the S- and D- wave two-meson states $\underline{P}V^*$ [$V^*=S$(scalar, $J^{PC}=0^{++}$], A(axial, 1^{++}), T(tensor, 2^{++}) mesons) is

$$A_{3/2,\rm loop}=+2L(P\to PV^*)A_{3/2},$$

$$A_{1/2,\rm loop}=L(P\to PV^*)\frac{1}{2}(A_{1/2}-B-4\sqrt{2}C). \quad (27)$$

In writing Eq. (27) we summed the contributions from the S and D waves by assuming that they are equal apart from their difference in weight. This should be a reasonable assumption since, at small values of q^2, we are away from thresholds where such differences might be important. In the $^3P_0$ model, factors $L(P\to PV^*)$ are given by a formula similar to Eq. (9).

The contributions from the $\underline{V}P^*$ [$P^*=B$ (axial $J^{PC}=1^{+-}$) meson] two-meson states are

$$A_{3/2,\rm loop}=+2L(P\to VP^*)A_{3/2}^V,$$

$$A_{1/2,\rm loop}=L(P\to VP^*)\frac{1}{2}(A_{1/2}^V-B^V-4\sqrt{2}C^V). \quad (28)$$

Finally, contributions from the $\underline{V}V^*$ diagrams are

$$A_{3/2,\rm loop}=-2L(P\to VV^*)A_{3/2}^V,$$

$$A_{1/2,\rm loop}=L(P\to VV^*)\frac{1}{2}(5A_{1/2}^V+3B^V). \quad (29)$$

The size of contributions from MM^* states may be estimated by looking at Eqs. (27), (28), (29). If we assume that the size of $I(P\to MM^*)$ depends mainly on the position of the MM^* threshold (with otherwise similar shapes of integrands), we should have $L(P\to VV^*)\approx 2L(P\to VP^*)$ and $L(P\to PV^*)>L(P\to VP^*)$. Then

$$A_{3/2,\rm loop}\approx 2L(P\to PV^*)A_{3/2}-2L(P\to VP^*)A_{3/2}^V, \quad (30)$$

leading to a small resultant negative contribution to $A_{3/2,\rm loop}$ ($A_{3/2}\ll A_{3/2}^V$). For the $A_{1/2}$ amplitudes, on the other hand, the dominant (positive) contribution comes from VV^* loops (large numerical factor of ''5''). Although $I(P\to PV^*)>I(P\to VV^*)$, the contribution from $P\to PV^*$ is small since $A_{1/2}\ll A_{1/2}^V$. Contributions from $P\to MM^*$ differ from previous estimates of contributions from intermediate ground-state mesons by factors of the order of $I(P\to MM^*)/I(P\to VP)$. It is notoriously difficult to estimate numerical values of such ratios since they depend on the poorly known shape of form factors. Taking into account the difference in threshold positions only [i.e., using the form of Eq. (9) and the shape of F_{PVV} in place of F_{PMM^*} so that in the limit of the $MM-MM^*$ threshold degeneracy one recovers the Zweig rule as expected in the unitarized quark model] results in a value of the order of 0.03 for these ratios. This yields an additional enhancement of the ratio

TABLE II. Loop contributions to isospin amplitudes.

$k_{\rm cutoff}$ (GeV^2)	0.6	0.7	0.8	0.95
$A_{3/2}^{\rm loop}/A_{3/2}^{\rm fact}$	-0.12	-0.17	-0.23	-0.32
$A_{1/2}^{\rm loop}/A_{1/2}^{\rm fact}$	+0.19	+0.27	+0.35	+0.49
$(A_{1/2}/A_{3/2})^{\rm total}$	1.35	1.53	1.75	2.19

TABLE III. Spin-flavor factors for $P\to M_1M_2$ loops (summed over flavor).

PV	VP	VV		PS	PA	PT	VB	VS	VA	VT
$\frac{3}{2}$	$\frac{3}{2}$	3	S wave	$\frac{1}{2}$	0	0	$\frac{1}{2}$	0	1	0
			D wave	0	0	1	1	0	$\frac{1}{2}$	$\frac{3}{2}$

$A_{1/2}/A_{3/2}$ by some 10%. The total enhancement factor for the $A_{1/2}/A_{3/2}$ ratio is then close to 2 (recall also the 5–10 % additional enhancement of the values of the loop integrals if $q^2=m_K^2$ is used).

III. DISCUSSION

Let us see what types of quark-level diagrams are generated by hadron-level loops under discussion. Consider $\underline{P}V$, $\underline{V}P$, and $\underline{V}V$ intermediate states. In the contribution from the $\underline{P}V$ and $\underline{V}P$ loops [Eq. (17)], strong vertices are described by F-type flavor factors, while for the $\underline{V}V$ loop the corresponding couplings are of D type [see Eq. (8)]. The flavor structure of these strong vertices may be represented diagrammatically as in Fig. 2. The wavy lines symbolize confining strong forces.

The structure of the product of flavor factors corresponding to two strong vertices of the loop is then (a) for $P\rightarrow\underline{P}V\rightarrow P$ or $P\rightarrow\underline{V}P\rightarrow P$ loops,

$$\mathrm{Tr}(F_M[F^{\dagger}_{M_1},F^{\dagger}_{M_2}])\mathrm{Tr}(F^{\dagger}_{M'}[F_{M'_1},F_{M_2}]), \tag{31}$$

and (b) for $P\rightarrow\underline{V}V\rightarrow P$ loops,

$$\mathrm{Tr}(F_M\{F^{\dagger}_{M_1},F^{\dagger}_{M_2}\})\mathrm{Tr}(F^{\dagger}_{M'}\{F_{M'_1},F_{M_2}\}). \tag{32}$$

Using the equality $\Sigma_{M=1\oplus 8}\mathrm{Tr}(AM)\mathrm{Tr}(AM^{\dagger})=\mathrm{Tr}(AB)$, summation over all intermediate mesons M_2 may be performed, giving the expression

$$\begin{aligned}&\mathrm{Tr}(F_{M_1^{\dagger}}F_M F_{M'^{\dagger}}F_{M'_1})+\mathrm{Tr}(F_M F_{M_1^{\dagger}}F_{M'_1}F_{M'^{\dagger}})\\&\mp\mathrm{Tr}(F_{M_1^{\dagger}}F_M F_{M'_1}F_{M'^{\dagger}})\mp\mathrm{Tr}(F_M F_{M_1^{\dagger}}F_{M'^{\dagger}}F_{M'_1}),\end{aligned} \tag{33}$$

with a $-(+)$ sign for F (D), respectively. Flavor contractions implicit in the first and the second term of Eq. (33) are visualized in Fig. 3(a), while those of the remaining two terms in Fig. 3(b). The black blob in Fig. 1 is replaced in Fig. 3 with boxes marked with dashed lines. Inside the boxes the diagrammatic representation of the genuine factorization prescription is drawn.

Figure 3(a) represents the familiar low-energy penguin (''eye'') diagram, while Fig. 3(b) is easily recognizable as the ''figure-eight''-type diagram with soft gluon exchanges between two quark loops. When the internal organization of the weak-interaction box is taken into account, the ''figure-eight'' diagram of Fig. 3(b) is actually equivalent to the W-exchange diagram with all possible soft gluon exchanges between an initial (anti)quark and a final (anti)quark [24–26].

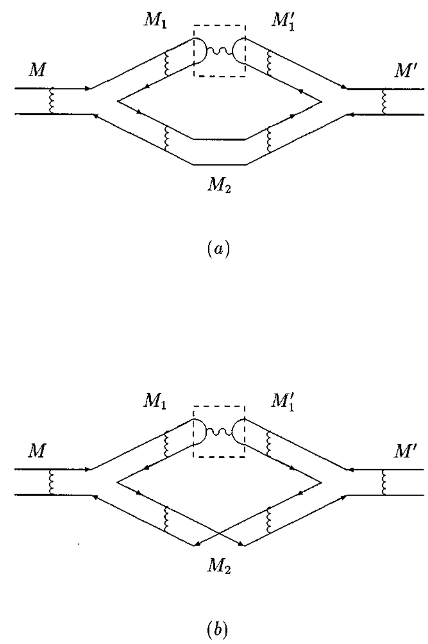

FIG. 3. Quark-level diagrams generated by hadronic loops of Fig. 1: (a) ''eye'' (low-energy penguin), (b) ''figure-eight.''

When one assumes a fully $P-V$ symmetric situation ($A=A^V$, $B=B^V$) and the summation of contributions from $PV+VP$ and VV configurations [Eq. (33)] is performed (note the relative spin-weight factors $PV:VP:VV=1/4:1/4:1/2$), the ''figure-eight'' contribution drops out totally from the final formulas [Eq. (21)] and, consequently, expressions in Eq. (21) correspond to the low-energy penguin interaction with a u-quark loop.

When $P-V$ symmetry is broken (by nondegenerate threshold positions and in input weak amplitudes) the effective contribution from the ''figure-eight'' diagram reappears suppressing (enhancing) the $A_{3/2}$ $(A_{1/2})$ amplitude.

Although it is notoriously difficult to make a reliable estimate of the size of hadron loop effects and our calculations involve significant simplifications, it should be obvious that the contribution from two-meson intermediate states is substantial and may be responsible for a part of the $\Delta I=1/2$ over $\Delta I=3/2$ enhancement. In our estimates we obtained for $R\approx 2/3$ fm an overall enhancement factor close to 2.

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