CP violation for $B \to X_s l^+ l^-$ including long-distance effects

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We consider the CP-violating effects for the $B \to X_s l^+ l^-$ process, including both short- and long-distance effects. We obtain the CP asymmetry parameter and present its variation over the dilepton mass. [S0556-2821(96)04611-5]

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As is well known, the flavor-changing process $b \to s l^+ l^-$ can serve as an excellent ''window'' for precisely testing the standard model or for finding new physics beyond it. This process occurs through one-loop diagrams. There are three types of Feynman diagrams for the $b \to s l^+ l^-$ transition: they are electromagnetic (photonic) penguin diagrams, weak (Z^0 boson) penguin diagrams, and box diagrams [1,2]. These diagrams produce the short-distance contributions to this process. The short-distance contribution to the branching ratio of the inclusive process $B \to X_s l^+ l^-$ is estimated to be about 10^{-5} at the large mass of the top quark [2,3]. In addition to the short-distance contributions, there are long-distance contributions to $b \to s l^+ l^-$ through physical intermediate states:

$$b \to s(u\overline{u}, c\overline{c}) \to s l^+ l^- .$$

The intermediate states can be vector mesons such as ρ, ω, J/ψ, and ψ'. The long-distance contribution to the branching ratio of $b \to s l^+ l^-$ is calculated to be as large as 10^{-3} [4,5]. So the long-distance effect is not negligible. In this paper, we study the long-distance effect in the CP violation of the inclusive process $B \to X_s l^+ l^-$. Our work is different from previous ones in two aspects. First, in Ref. [1], the authors studied the CP violation effect of $B \to X_s l^+ l^-$ by considering only photonic penguin diagrams; here, we consider all three types of diagrams (electromagnetic, weak, and box diagrams) and include QCD corrections within the leading logarithmic approximation [6]. Secondly, we consider both short- and long-distance contributions.

The effective Hamiltonian relevant to $b \to s l^+ l^-$ transitions is [3,6–8]

$$\begin{aligned}\mathcal{H}_{\rm eff}=&\frac{G_F}{\sqrt{2}}\left(\frac{\alpha}{4\pi S_W^2}\right)\sum_i V_i[A_i\overline{s}\gamma_\mu(1-\gamma_5)b\overline{l}\gamma^\mu(1-\gamma_5)l\\&+B_i\overline{s}\gamma_\mu(1-\gamma_5)b\overline{l}\gamma^\mu(1+\gamma_5)l\\&-2im_bS_W^2F_2^i\overline{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)/q^2b\overline{l}\gamma^\mu\gamma_5 l],\end{aligned}\qquad(1)$$

where $V_i = U_{is}^* U_{ib}$ $(i=u,c,t)$ is the product of Cabibbo-Kobayaski-Maskawa (CKM) matrix elements, $S_W=\sin\theta_W$, where θ_W is the Weinberg angle, $l=e,\ \mu$, and q is the momentum of the lepton pair.

At the scale $\mu \approx M_W$, the coefficients A_t and B_t take the forms

$$\begin{aligned}A_t&=-2B(x)+2C(x)-S_W^2[4C(x)+D(x)-4/9],\\B_t&=-S_W^2[4C(x)+D(x)-4/9],\end{aligned}\qquad(2)$$

where $x=m_t^2/M_W^2$,

$$B(x)=\frac{1}{4}\left[\frac{-x}{x-1}+\frac{x}{(x-1)^2}\ln x\right],$$

$$C(x)=\frac{x}{4}\left[\frac{x/2-3}{x-1}+\frac{3x/2+1}{(x-1)^2}\ln x\right],$$

$$\begin{aligned}D(x)=&\left[\frac{-19x^3/36+25x^2/36}{(x-1)^3}\right.\\&\left.+\frac{-x^4/6+5x^3/3-3x^2+16x/9-4/9}{(x-1)^4}\ln x\right].\end{aligned}\qquad(3)$$

Here, $B(x)$ arises from the box diagram, and $C(x)$ from the Z^0 penguin diagram, while $D(x)$ is contributed from the γ penguin diagram. We can see from Eq. (3) that with increasing x, the contribution from the box and γ penguin diagrams will decline, while $C(x)$ will become dominant. Then using the renormalization group equation to scale the effective Hamiltonian down to the order of the b quark mass, one obtains

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$$A_t(x,\xi)=A_t(x)+\frac{4\pi}{\alpha_s(M_W)}\left\{-\frac{4}{33}(1-\xi^{-11/23})+\frac{8}{87}(1-\xi^{-29/23})\right\}S_W^2,$$

$$B_t(x,\xi)=B_t(x)+\frac{4\pi}{\alpha_s(M_W)}\left\{-\frac{4}{33}(1-\xi^{-11/23})+\frac{8}{87}(1-\xi^{-29/23})\right\}S_W^2, \quad (4)$$

where $\xi=\alpha_s(m_b)/\alpha_s(M_W)=1.75$.

Moreover, the coefficient for the magnetic-moment operator is given by

$$F_2^t(x,\xi)=\xi^{-16/23}\left[-\frac{1}{12}\frac{8x^3+5x^2-7x}{(x-1)^3}+\frac{3x^3/2-x^2}{(x-1)^4}\ln x-\frac{116}{135}(\xi^{10/23}-1)-\frac{58}{189}(\xi^{28/23}-1)\right]. \quad (5)$$

In our numerical calculation, we take $m_t=174$ GeV [9]. Furthermore, the nonresonant coefficients A_i and B_i $(i=u,c)$ are represented by

$$A_i=B_i=a_2S_W^2 g\left(\frac{m_i^2}{m_b^2},\frac{q^2}{m_b^2}\right), \quad (6)$$

with [6,8]

$$g(r_i,s)=\begin{cases}\frac{4}{3}\ln r_i-\frac{8}{9}-\frac{4}{3}\frac{4r_i}{s}+\frac{2}{3}\sqrt{1-\frac{4r_i}{s}}\left(2+\frac{4r_i}{s}\right)\left(\ln\frac{1+\sqrt{1-4r_i/s}}{1-\sqrt{1-4r_i/s}}+i\pi\right) & \left(\frac{4r_i}{s}<1\right),\\ \frac{4}{3}\ln r_i-\frac{8}{9}-\frac{4}{3}\frac{4r_i}{s}+\frac{4}{3}\sqrt{\frac{4r_i}{s}-1}\left(2+\frac{4r_i}{s}\right)\arctan\frac{1}{\sqrt{4r_i/s-1}} & \left(\frac{4r_i}{s}>1\right).\end{cases} \quad (7)$$

Here $a_2=C_-+C_+/3$ is the coupling for the neutral $b\overline{s}\overline{q}q$ $(q=u,c)$ four-quark operator.

In addition to the short-distance contribution, the inclusive decay $B\to X_s l^+ l^-$ involves the long-distance contributions arising from $u\overline{u}$ and $c\overline{c}$ resonances, such as $\rho(770)$, $\omega(782)$, $J/\psi(3100)$, and $\psi'(3700)$, etc. The long-distance contribution to the coefficients A and B in Eq. (1) can be taken as [4,5,10,11]

$$A_V=B_V=\frac{16\pi^2}{3}\left(\frac{f_V}{M_V}\right)^2\frac{a_2S_W^2}{q^2-M_V^2+iM_V\Gamma_V}e^{2i\phi_V}, \quad (8)$$

where M_V and Γ_V are the mass and width of the relevant vector meson ρ, ω, J/ψ, and ψ', respectively. $e^{2i\phi}$ is the relevant phase between the resonant and nonresonant amplitudes. The decay constant f_V is defined as

$$\langle 0|\overline{c}\gamma_\mu c|V(\epsilon)\rangle=f_V\epsilon_\mu. \quad (9)$$

We can determine f_V through the measured partial width for the decays of the mesons to lepton pairs [12]:

$$\Gamma(v\to l^+l^-)=\frac{4\pi}{3}\frac{(Q_c\alpha)^2}{M_V^3}f_V^2, \quad (10)$$

with $Q_c=\frac{2}{3}$. For the parameter a_2, there is the CLEO value $|a_2|=0.26\pm0.03$ [13]. In this work, a_2 should be taken as $a_2=-(0.26\pm0.03)$ and $\phi_V=0$ or $a_2=0.26\pm0.03$, $\phi_V=\pi/2$ [11].

The differential decay width of the inclusive process $B\to X_s l^+ l^-$ over the dilepton mass is given by [4]

$$\frac{d}{dz}\Gamma(B\to X_s l^+l^-)=\frac{G_F^2m_b^5}{192\pi^3}\left[\frac{\alpha}{4\pi S_W^2}\right]^2F_b(z), \quad (11)$$

where $z=q^2/m_b^2$,

$$F_b(z)=[|V_iA_i(z)|^2+|V_iB_i(z)|^2]f_1^b(z)+S_W^2\{V_i^*V_j[A_i(z)+B_i(z)]^*F_2^j+\text{H.c.}\}f_{12}^b(z)+2S_W^4|V_iF_2^i|^2f_2^b(z), \quad (12)$$

and

$$f_1^b(z)=2(1-z)(1+z-2z^2),$$

$$f_{12}^b(z)=6(1-z)^2, \quad (13)$$

TABLE I. CP asymmetries for some ‘‘best values’’ of (ρ,η). $\mathcal{A}_{CP}^S$ denotes the cases without long-distance contributions, $\mathcal{A}_{CP}^{S+L}$ with long-distance contributions.

(ρ,η)	$\mathcal{A}_{CP}^S$	$\mathcal{A}_{CP}^{S+L}$
(-0.48, 0.10)	1.81×10^{-4}	1.60×10^{-5}
(-0.44, 0.12)	2.17×10^{-4}	1.92×10^{-5}
(-0.40, 0.15)	2.71×10^{-4}	2.40×10^{-5}
(-0.36, 0.18)	3.25×10^{-4}	2.88×10^{-5}
(-0.32, 0.21)	3.79×10^{-4}	3.35×10^{-5}
(-0.28, 0.24)	4.34×10^{-4}	3.83×10^{-5}
(-0.23, 0.27)	4.88×10^{-4}	4.31×10^{-5}
(-0.17, 0.29)	5.24×10^{-4}	4.63×10^{-5}
(-0.11, 0.32)	5.78×10^{-4}	5.11×10^{-5}
(-0.04, 0.33)	5.96×10^{-4}	5.27×10^{-5}
(+0.03, 0.33)	5.96×10^{-4}	5.27×10^{-5}
(-0.12, 0.34)	6.14×10^{-4}	5.43×10^{-5}

$$f_2^b(z)=4(1-z)\left(1/z-\frac{1}{2}-z/2\right).$$

We define the CP violating asymmetry through the rate difference between B and $\bar{B}$:

$$\mathcal{A}_{CP}=\frac{\Gamma_{\bar{b}}-\Gamma_b}{\Gamma_{\bar{b}}+\Gamma_b} \tag{14}$$

where Γ_b is obtained by integrating Eq. (11) over the dilepton mass squared z from $z_{\min}=(2m_l/m_b)^2$ to $z_{\max}=(1-m_s/m_b)^2$. The CKM matrix in Eq. (12) can be written in terms of four parameters λ, A, ρ, and η in the Wolfenstein parametrization [14]. There have been definite results for λ and A, which are $\lambda=0.2205\pm0.0018$ [15] and $A=0.80\pm0.12$ [16]. But for ρ and η, there are no definite results. So we express the CP-violating parameter for $B\to X_s e^+e^-$ in terms of ρ and η,

$$\mathcal{A}_{CP}^{S+L}=\frac{7.618\times10^{-3}\eta}{1.799+2.991\times10^{-3}\rho+45.876(1+0.0484^2\eta^2)+1.4718\times10^{-4}(\rho^2+\eta^2)} \tag{15}$$

for the case of including long-distance effects, and

$$\mathcal{A}_{CP}^{S}=\frac{3.1389\times10^{-3}\eta}{1.702+3.345\times10^{-3}\rho+3.607\times10^{-2}(1+0.0484^2\eta^2)+1.4373\times10^{-4}(\rho^2+\eta^2)} \tag{16}$$

for the case without long-distance effects. Equations (15) and (16) indicate that η affects the CP asymmetry mainly, and ρ does not.

In Table I, we give the results of $\mathcal{A}_{CP}$ for some ''best values'' of (ρ,η) [16]. We can see the following. (i) Without the long-distance effects, the CP violating asymmetry $\mathcal{A}_{CP}$ is about $(1.8-6.1)\times10^{-4}$, while, in Ref. [1], the relevant CP asymmetry is about 1.3×10^{-2}. Our result is about 20 times smaller than theirs. The reason is that in Ref. [1] only the photonic penguin diagram is considered. But in fact the Z^0 penguin diagram will give a big contribution to the amplitude at large $m_t(\sim 174\text{ GeV})$ [2]; at the same time, it does not provide a large CP-nonconserving phase. (ii) Including the long-distance effects, the result of the CP asymmetry parameter $\mathcal{A}_{CP}$ is about $(1.6-5.4)\times10^{-5}$. It is reduced about one order of magnitude by the resonant effects. The main difference between the cases with and without long-distance effects resides in the third term of the denominator of Eqs. (15) and (16), which comes from the integration of the first term of Eq. (12), i.e., $\int_{z_{\min}}^{z_{\max}}dz\{|V_c|^2|(A_c(z)+B_c(z))|^2f_1^b(z)\}$. Without resonant contributions,

$$\int_{z_{\min}}^{z_{\max}}dz\{|V_c|^2|[A_c(z)+B_c(z)]^S|^2f_1^b(z)\}=0.12|V_c|^2, \tag{17}$$

while with resonant contributions,

$$\int_{z_{\min}}^{z_{\max}}dz\{|V_c|^2|[A_c(z)+B_c(z)]^{S+L}|^2f_1^b(z)\}=152.9|V_c|^2. \tag{18}$$

Because the total decay width of J/ψ or ψ' is narrow ($\Gamma_{J/\psi}=88$ KeV, $\Gamma_{\psi'}=277$ KeV), when the dilepton mass squared z is near the mass squared of J/ψ or ψ', the resonance will give a big contribution. At the same time, the first term of Eq. (12) only contributes to the decay width Γ_b and $\Gamma_{\bar{b}}$; it does not give a contribution to the CP violation. So with the resonant effects the CP violation will be greatly reduced.

We also calculated the distribution of the CP asymmetry over the dilepton mass for (ρ,η) taking the ''preferred value'' of $(-0.12,0.34)$ [16]:

$$a_{CP}=\frac{F_{\bar{b}}(z)-F_b(z)}{F_{\bar{b}}(z)+F_b(z)}. \tag{19}$$

The result is plotted in Fig. 1. The solid line is for the case without resonances and the dotted line for the case with resonances. We can see that, in general, the CP asymmetry is

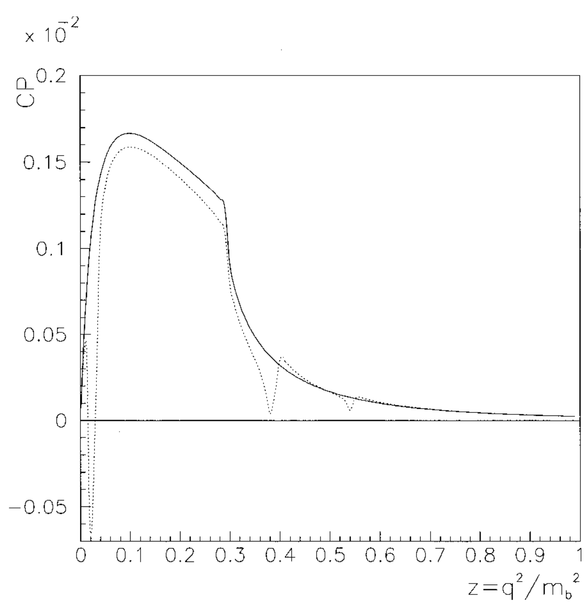

FIG. 1. The dilepton mass distribution of the CP asymmetry parameter for the $B\to X_s e^+e^-$ process without resonances (solid line) and with resonances (dotted line).

suppressed by the resonance effect, and in the region near the resonances, the CP violating parameter is severely suppressed.

Golowich and Pakvasa have discussed the long-range effects in $B\rightarrow K^*\gamma$ [17], which are relevant to the condition of the squared mass $q^2=0$. They found a small effect by respecting gauge invariance. It should be noted that there is no controversy between their results and ours. That is, in Fig. 1, it is shown that when $q^2\rightarrow 0$ the long-distance effect is also very small in the case of $B\rightarrow X_s l^+ l^-$.

Finally, we want to point out that for the case of $l=\mu$, the CP asymmetry parameter is smaller than in the $l=e$ case.

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