Radiative $B^*\to B\gamma$ and $D^*\to D\gamma$ decays in light-cone QCD sum rules

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The radiative decays $B^*(D^*)\to B(D)\gamma$ are investigated in the framework of light-cone QCD sum rules. The transition amplitude and decay rates are estimated. It is shown that our results for the branching ratios of D meson decays are in good agreement with the existing experimental data. [S0556-2821(96)03913-6]

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I. INTRODUCTION

The experimental and theoretical investigations of the heavy-flavored hadron physics constitute one of the most interesting research areas in particle physics. This is due to their outstanding role in the precise determination of the fundamental parameters of the standard model, and so in development of a deeper understanding of the dynamics of QCD. However, the theoretical interpretation of the experimental results is not always easy; the main problem is the influence of long distance dynamics. Therefore, the extraction of fundamental parameters from the data of heavy-flavored hadrons inevitably requires some information about large distance physics. In the literature, there exist numerous theoretical works trying to make this extraction as reliable as possible. While inclusive B and D decays are better understood theoretically, exclusive decays are often much easier to measure experimentally. However, their interpretation requires accurate estimates of the decay form factors and the other hadronic matrix elements which can only come from nonperturbative approaches. Among such nonperturbative approaches the QCD sum rule method [1,2] occupies an exceptional place, since it is based on the first principles of QCD, and the nonperturbative (i.e., long distance) effects are parametrized only in terms of the vacuum condensates and these parameters are process independent.

Nowadays, QCD sum rules based on light-cone expansion are widely exploited as an alternative to the ''classical QCD sum rule method.'' The main features of this version are that it is based on the approximate conformal nonperturbative invariance of QCD, and instead of many vacuum condensate parameters in classical ''QCD sum rules,'' it involves a new universal nonperturbative parameter, namely the wave function [3]. Light-cone sum rules were successfully applied to calculating the decay amplitude $\Sigma\to p\gamma$ [4], the nucleon magnetic moment, the $g_{\pi NN}$ and $g_{\rho\omega\pi}$ couplings [5], form factors of semileptonic and radiative decays [6–9], the $\pi A\gamma^*$ form factor [10], $B\to\rho\gamma$ and $D\to\rho\gamma$ decays [11,12], $B^*B\pi$ and $D^*D\pi$ coupling constants [13], etc.

In this work we study the radiative $B^*(D^*)\to B(D)\gamma$ decays in the framework of the light-cone QCD sum rules. Note that these decays have been previously investigated [14,15], in the framework of a classical QCD sum rule method.

The paper is organized as follows. In Sec. II, we derive the sum rule which describes $B^*(D^*)\to B(D)\gamma$ in the framework of the light-cone sum rules. In the last section we present the numerical analysis.

II. THE RADIATIVE $B^*\to B\gamma$ DECAY

According to the general strategy of QCD sum rules, we will calculate the transition amplitude for $B^*\to B\gamma$ decay, by equating the representation of a suitable correlator function in hadronic and quark-gluon languages. To this aim, we consider the correlator

$$\Pi_\mu(p,q)=i\int d^4x e^{ipx}\langle 0|T[\overline{q(x)}\gamma_\mu b(x),\overline{b(0)}i\gamma_5 q(0)]|0\rangle_F \tag{1}$$

in the external electromagnetic field

$$F_{\alpha\beta}(x)=i(\epsilon_\beta q_\alpha-\epsilon_\alpha q_\beta)e^{iqx}. \tag{2}$$

Here q is the momentum, and ϵ_μ is the polarization vector of the electromagnetic field. The Lorentz decomposition of the correlator is

$$\Pi_\mu=\epsilon_{\mu\nu\alpha\beta}p_\nu q_\alpha\epsilon_\beta\Pi. \tag{3}$$

Our main problem is to calculate Π in Eq. (3). This problem can be solved in the deep Euclidean space where both p^2 and $p'^2=(p+q)^2$ are negative and large. In this deep Euclidean region, the photon interacts with the heavy quark perturbatively. The various contributions to the correlator function, Eq. (1), are depicted in Fig. 1, where Figs. 1(a) and 1(b) represent the perturbative contributions, Fig. 1(c) the quark condensate, Figs. 1(c) and 1(d) the five-dimensional operator, Fig. 1(e) is the photon interaction with a soft quark line, and Fig. 1(f) the three-particle high twist contributions. A part of the calculation of these diagrams was performed in [12,14,15].

First, let us calculate the perturbative contributions, namely the contributions of Figs. 1(a) and 1(b). For the contribution of Fig. 1(b) we get

$$\Pi_1=\frac{Q_q}{4\pi^2}N_c\int_0^1 xdx\int_0^1 dy\,\frac{m_b\bar{x}+m_a x}{m_q^2x+m_b^2\bar{x}-p^2x\bar{x}\bar{y}-p'^2x\bar{x}\,y} \tag{4}$$

where $N_c=3$ is the color factor, $\bar{x}=1-x$,$\bar{y}=1-y$, $p'=p+q$, and Q_q and m_q are, respectively, the charge and the mass of the light quarks. The next step is to use the exponential representation for the denominator:

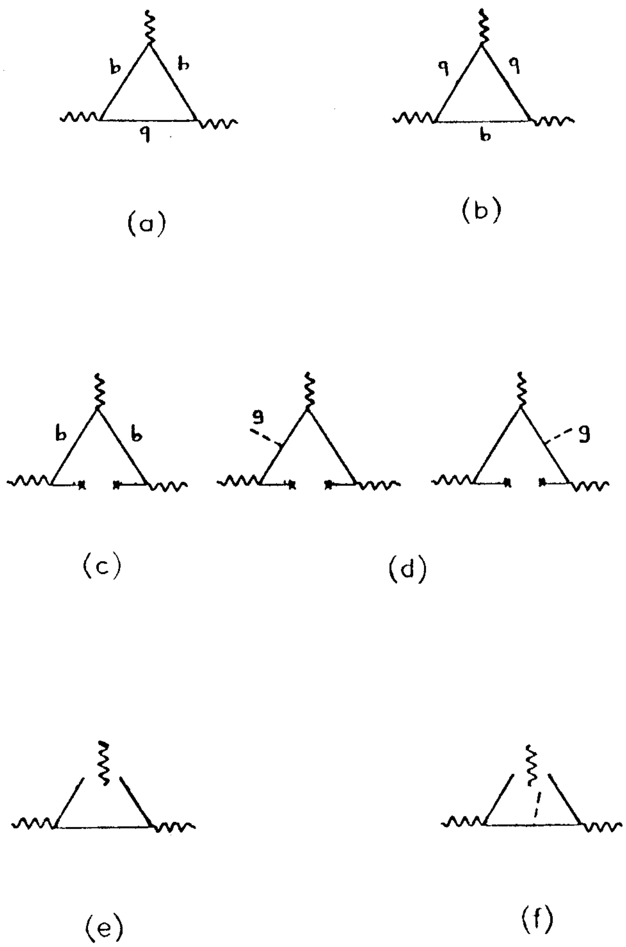

FIG. 1. Diagrams contributing to the correlation function 1. Solid lines represent quarks, wavy lines external currents.

$$\frac{1}{C^n}=\frac{1}{(n-1)!}\int_0^\infty d\alpha\alpha^{n-1}e^{-\alpha C}.$$

Then

$$\Pi_1=\frac{Q_qN_c}{4\pi^2}\int_0^1 x dx\int_0^1 dy[m_b\overline{x}+m_q x]$$
$$\times\int_0^\infty d\alpha e^{-\alpha(m_a^2x+m_b^2\overline{x}-p^2x\overline{x}y-p'^2x\overline{x}\overline{y})}. \qquad (5)$$

Application of the double Borel operator $\hat{B}(M_1^2)\hat{B}(M_2^2)$ on Π_1 gives

$$\widetilde{\Pi}_1=\frac{Q_qN_c}{4\pi^2}\,\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}\int_0^1 dx\,\frac{1}{\overline{x}}[m_b\overline{x}+m_qx]$$
$$\times\exp\left(-\frac{m_q^2x+m_b^2\overline{x}}{x\overline{x}}(\sigma_1+\sigma_2)\right), \qquad (6)$$

where $\sigma_1=1/M_1^2$ and $\sigma_2=1/M_2^2$. In deriving Eq. (6) we have used the definition

$$\hat{B}(M^2)e^{-\alpha p^2}=\delta(1-\alpha M^2). \qquad (7)$$

We next consider the spectral density, which can be shown [16] to be given by

$$\rho_1(s,t)=\frac{1}{st}\hat{B}\left(\frac{1}{s},\sigma_1\right)\hat{B}\left(\frac{1}{t},\sigma_2\right)\frac{\widetilde{\Pi}_1}{\sigma_1\sigma_2}. \qquad (8)$$

Using Eq. (6) and Eq. (8), for the spectral density, we get

$$\rho_1(s,t)=\frac{Q_qN_c}{4\pi^2}\int_{x_0}^{x_1}dx\,\delta(s-t)\,\theta(s-(m_b+m_q)^2)$$
$$\times\theta(t-(m_b+m_q)^2)\frac{m_b\overline{x}+m_qx}{\overline{x}}, \qquad (9)$$

where the integration region is determined by the inequality

$$sx\overline{x}-(m_b^2\overline{x}+m_q^2x)\geqslant 0. \qquad (10)$$

Carrying out the integration over x in (9), we get

$$\rho_1^q(s,t)=\frac{Q_qN_c}{4\pi^2}\,\delta(s-t)\,\theta(s-(m_b+m_q)^2)$$
$$\times\theta(t-(m_b+m_q)^2)\bigg((m_b-m_q)\lambda(1,\kappa,l)$$
$$+m_b\ln\frac{1+\kappa-l+\lambda(1,\kappa,l)}{1+\kappa-l-\lambda(1,\kappa,l)}\bigg), \qquad (11)$$

where $\kappa=m_q^2/s, l=m_b^2/s$ and,

$$\lambda(1,\kappa,l)=\sqrt{1+\kappa^2+l^2-2\kappa-2l-2\kappa l}. \qquad (12)$$

The contribution of Fig. 1(a) can be obtained by making the following replacements in Eq. (4): $m_b\leftrightarrow m_q, e_q\leftrightarrow e_Q$, which yields

$$\rho_2(s,t)=\rho_1(q\leftrightarrow b, m_q\leftrightarrow m_b, Q_q\leftrightarrow Q_b). \qquad (13)$$

Finally, for the perturbative part of the correlator we have

$$\Pi^{\rm per}=\frac{N_cm_b}{4\pi^2}\int ds\,\frac{1}{(s-p^2)(s-p'^2)}$$
$$\times\left[(Q_q-Q_b)\left(1-\frac{m_b^2}{s}\right)+Q_b\ln\frac{s}{m_b^2}\right]. \qquad (14)$$

Here we have neglected the mass of the light quark. Finally, applying the double Borel transformation to Eq. (14) for the bare-loop contribution, we get

$$\widetilde{\Pi}^{\rm per}=\frac{N_cm_b}{M_1^2M_2^2 4\pi^2}\int ds\exp\left[-s\left(\frac{1}{M_1^2}+\frac{1}{M_2^2}\right)\right]$$
$$\times\left[(Q_a-Q_b)\left(1-\frac{m_b^2}{s}\right)+Q_b\ln\frac{s}{m_b^2}\right]. \qquad (15)$$

After a simple calculation, for the double Borel transformed quark condensate contribution, corresponding to Fig. 1(c), we get

$$\tilde{\Pi}^{\overline{q}q} = -Q_b\langle\overline{q}q\rangle\frac{1}{M_1^2M_2^2}\exp\left(-\frac{m_b^2}{M_1^2+M_2^2}\right). \tag{16}$$

Similarly, the result for the five-dimensional operator contribution, corresponding to Figs. 1(c) and 1(d), is

$$\tilde{\Pi}^{d=5} = -Q_b\langle\overline{q}q\rangle\frac{1}{M_1^2M_2^2}e^{-m_b^2/(M_1^2+M_2^2)}\times\left[-\frac{m_0^2m_b^2}{4}\left(\frac{1}{M_1^2}+\frac{1}{M_2^2}\right)^2+\frac{1}{3}\frac{m_0^2}{M_2^2}\right]. \tag{17}$$

Here $m_0^2=(0.8\pm0.2)$ GeV^2 and is defined by $g_s\langle\overline{q}\sigma_{\alpha\beta}G_{\alpha\beta}q\rangle=m_0^2\langle\overline{q}q\rangle$. For the calculation of Fig. 1(e) corresponding to the propagation of the soft quark in the external electromagnetic field, we use the light-cone expansion for nonlocal operators. After contracting the b-quark line in Eq. (1) we get

$$\Pi_\mu = i\int d^4x\frac{d^4k}{(2\pi)^4}\frac{e^{i(p-k)x}}{m_b^2-k^2}\times\langle 0|\overline{q(x)}\gamma_\mu(m_b+\not{k})\gamma_5q(0)|0\rangle_F. \tag{18}$$

Using the identity, $\gamma_\mu\gamma_\alpha\gamma_5=g_{\mu\alpha}\gamma_5-(1/2)\sigma_{\rho\beta}\epsilon_{\mu\alpha\rho\beta}$, Eq. (18) can be rewritten as

$$\Pi_\mu = i\int d^4x\frac{d^4k}{(2\pi)^4}\frac{e^{i(p-k)x}}{m_b^2-k^2}\{m_b\langle 0|\overline{q(x)}\gamma_\mu\gamma_5q|0\rangle_F - (1/2)\epsilon_{\mu\alpha\rho\beta}k_\alpha\langle 0|\overline{q(x)}\sigma_{\rho\beta}q|0\rangle_F\}. \tag{19}$$

The leading twist $\tau=2$ contribution to this matrix element in the presence of an external electromagnetic field is defined as [4,17]

$$\langle\overline{q(x)}\sigma_{\rho\beta}q\rangle_F = Q_q\langle\overline{q}q\rangle\int_0^1 du\,\phi(u)F_{\rho\beta}(ux),$$

$$\langle\overline{q(x)}\gamma_\mu\gamma_5q\rangle_F = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}x_\nu q_\alpha\epsilon_\beta f\int_0^1 du\,e^{iuqx}g_\perp(u). \tag{20}$$

Here the functions $\phi(u)$, $g_\perp(u)$ are the photon wave functions. The asymptotic form of the wave function $\phi(u)$ is well known [4,18,19]:

$$\phi(u)=6\chi u(1-u), \tag{21}$$

where χ is the magnetic susceptibility. In [7], it is shown that $f=(Q_q/g_\rho)f_\rho m_\rho$ where $f_\rho=200$ MeV and $g_\rho=5.5$ [1].

The most general decomposition of the relevant matrix elements, up to twist-four terms, involves two new invariant functions (see, for example, [11,12]):

$$\langle\overline{q(x)}\sigma_{\rho\beta}q\rangle_F = Q_q\langle\overline{q}q\rangle\left(\int_0^1 du\,x^2\phi_1(u)F_{\rho\beta}(ux)+\int_0^1 du\,\phi_2(u)[x_\beta x_\eta F_{\rho\eta}(ux)-x_\rho x_\eta F_{\beta\eta}(ux)-x^2F_{\rho\beta}(ux)]\right). \tag{22}$$

The two new invariant functions entering (22) are given by [11]

$$\phi_1(u)=-\frac{1}{8}(1-u)(3-u),$$

$$\phi_2(u)=-\frac{1}{4}(1-u)^2. \tag{23}$$

Using Eqs. (19), (20), and (22), for the twist $\tau=2$ and $\tau=4$ contributions we get

$$\Pi^{(\tau=2)+(\tau=4)} = Q_q\langle\overline{q}q\rangle\left[\int_0^1\frac{\phi(u)du}{m_b^2-(p+uq)^2}-4\int_0^1\frac{[\phi_1(u)-\phi_2(u)]du}{[m_b^2-(p+uq)^2]^2}\left(1+\frac{2m_b^2}{m_b^2-(p+uq)^2}\right)\right]+\int_0^1 du\frac{fg_\perp(u)m_b}{2[m_b^2-(p+uq)^2]^2}. \tag{24}$$

In order to perform the double Borel transformation we rewrite the denominator in the following manner:

$$m_b^2-(p+uq)^2=m_b^2-(1-u)p^2-(p+q)^2u.$$

After Wick rotation this becomes

$$m_b^2-(p+uq)^2\to m_b^2+(1-u)p^2+(p+q)^2u.$$

Using the exponential representation for the denominator and performing the double Borel transformation for the twist $\tau=2$ and $\tau=4$ contributions we get

$$\tilde{\Pi}^{(\tau=2)+(\tau=4)} = e^{-m_b^2(1/M_1^2+1/M_2^2)}\left(Q_q\langle\overline{q}q\rangle\left\{\phi\left(\frac{M_1^2}{M_1^2+M_2^2}\right)\frac{1}{M_1^2+M_2^2}-4\left[\phi_1\left(\frac{M_1^2}{M_1^2+M_2^2}\right)-\phi_2\left(\frac{M_1^2}{M_1^2+M_2^2}\right)\right]\times\left(\frac{1}{M_1^2M_2^2}+\frac{m_b^2(M_1^2+M_2^2)}{M_1^4M_2^4}\right)\right\}+\frac{m_b}{2}fg_\perp\left(\frac{M_1^2}{M_1^2+M_2^2}\right)\frac{1}{M_1^2M_2^2}\right). \tag{25}$$

The masses of the $B^*(D^*)$ and $B(D)$ mesons are practically equal. So, it is natural to take $M_1^2=M_2^2$, and introduce a new Borel parameter M^2 such that $M_1^2=M_2^2=2M^2$. In this case the theoretical part of the sum rule becomes

$$\tilde{\Pi}^{\rm theor}=\frac{3m_b}{4\pi^2}\int_{m_b^2}^{s_0}ds\,e^{-s(1/M^2)}\left[(Q_q-Q_b)\left(1-\frac{m_b^2}{s}\right)+Q_b\ln\frac{s}{m_b^2}\right]\frac{1}{4M^4}-Q_b\langle\bar{q}q\rangle e^{-m_b^2/M^2}\left(1-\frac{m_0^2m_b^2}{M^4}+\frac{m_0^2}{6M^2}\right)\frac{1}{4M^4}$$
$$+(e^{-m_b^2/M^2}-e^{-s_0/M^2})\left\{Q_q\langle\bar{q}q\rangle\left[\frac{1}{4M^2}\phi\left(\frac{1}{2}\right)\right]-4\left(1+\frac{m_b^2}{M^2}\right)[\phi_1(1/2)-\phi_2(1/2)]\frac{1}{4M^4}+\frac{m_b}{8M^4}fg_\perp(1/2)\right\}. \quad (26)$$

In deriving Eq. (26), we have subtracted the continuum and the higher resonance states contributions from the double spectral density. The details of this procedure are given in [13].

To construct the sum rules we need the expression for the physical part as well. Saturating Eq. (1) by the lowest-lying meson states, we have

$$\Pi_\mu^{\rm phys}=\frac{\langle 0|\bar{q}\gamma_\mu b|B^*\rangle\langle B^*|B\gamma\rangle\langle B|\bar{b}i\gamma_5 q|0\rangle}{(m_{B^*}^2-p^2)[m_B^2-(p+q)^2]}. \quad (27)$$

These matrix elements are defined as

$$\langle 0|\bar{q}\gamma_\mu b|B^*\rangle=\epsilon_\mu f_{B^*}m_{B^*}, \quad (28)$$

$$\langle B|\bar{b}i\gamma_5 q|0\rangle=\frac{f_Bm_B^2}{m_b}, \quad (29)$$

$$\langle B^*|B\gamma\rangle=\varepsilon_{\alpha\beta\rho\sigma}p_\alpha\epsilon_\beta q_\rho\epsilon_\sigma^{(\gamma)}h/m_B. \quad (30)$$

Here h is the dimensionless amplitude for the transition matrix element; ϵ_μ and m_{B^*} are the polarization four-vector and the mass of the vector particle, respectively; f_B is the leptonic decay constant and m_B is the mass of the pseudoscalar particle; q_β and $\epsilon_\sigma^{(\gamma)}$ are the photon momentum and the polarization vector. Applying the double Borel transformation we get for the physical part of the sum rules

$$\Pi^{\rm phys}=f_{B^*}m_{B^*}f_Bm_B\frac{h}{m_b}\frac{e^{-(m_{B^*}^2+m_B^2)/2M^2}}{4M^4}. \quad (31)$$

Note that the contribution of three-particle twist-four operators are very small [4], and thus we neglect them [Fig. 1(f)]. From Eqs. (26)–(30) we finally get the dimensionless coupling constant h as

$$f_{B^*}f_Bh=\frac{m_b}{m_{B^*}m_B}e^{(m_{B^*}^2+m_B^2)/2M^2}\left\{\frac{3m_b}{4\pi^2}\int_{m_b^2}^{s_0}ds\,e^{-s(1/M^2)}\left[(Q_q-Q_b)\left(1-\frac{m_b^2}{s}\right)+Q_b\ln\frac{s}{m_b^2}\right]\right.$$
$$-\langle\bar{q}q\rangle e^{-m_b^2/M^2}\left[Q_b\left(1-\frac{m_0^2m_b^2}{M^4}+\frac{m_0^2}{6M^2}\right)\right]$$
$$\left.+(e^{-m_b^2/M^2}-e^{-s_0/M^2})\left\{Q_q\langle\bar{q}q\rangle\left[\phi\left(\frac{1}{2}\right)M^2-4\left(1+\frac{m_b^2}{M^2}\right)[\phi_1(1/2)-\phi_2(1/2)]\right]+\frac{1}{2}m_bfg_\perp(1/2)\right\}\right\}. \quad (32)$$

III. NUMERICAL ANALYSIS OF THE SUM RULES

The main issue concerning Eq. (32) is the determination of the dimensionless transition amplitude, h. First, we give a summary of the parameters entering in Eq. (32). The value of the magnetic susceptibility of the medium, in the presence of an external field, was determined in [20,21]:

$$\chi(\mu^2=1\ \text{GeV}^2)=-4.4\ \text{GeV}^{-2}.$$

If we include the anomalous dimension of the current $\bar{q}\sigma_{\alpha\beta}q$, which is equal to $(-4/27)$ at the $\mu=m_b$ scale, we get

$$\chi(\mu^2=m_b^2)=-3.4\ \text{GeV}^{-2}$$

and

$$\langle\bar{q}q\rangle=-(0.24\ \text{GeV})^3.$$

The leptonic decay constants $f_{B(D)}$ and $f_{B^*(D^*)}$ are known from two-point QCD sum rules: $f_{B(D)}=0.14(0.17)$ GeV [13,22], $f_{B^*(D^*)}=0.16(0.24)$ GeV [13,23–25], $m_b=4.7$ GeV, $m_u=m_d=0$, $m_0^2=(0.8\pm0.2)$ GeV^2, $m_{B^*(D^*)}=5.324(2.007)$ GeV, and $m_{B(D)}=5.279(1.864)$ GeV, and for the continuum threshold we choose $s_0^B(s_0^D)=36\,(6)$ GeV^2.

The value $\phi_1(u)-\phi_2(u)$ is calculated in [11]:

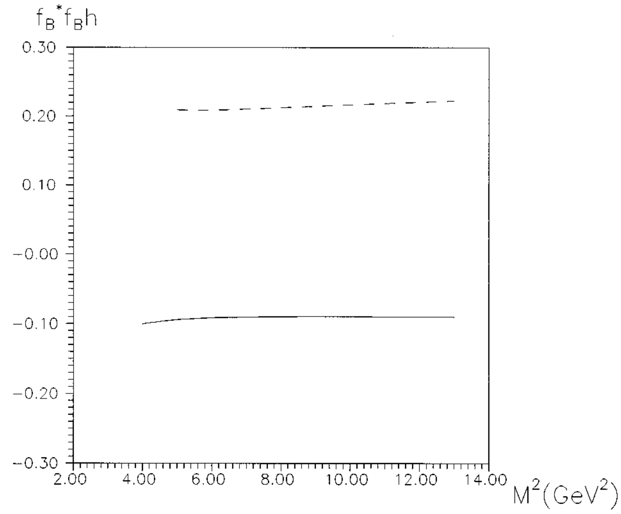

FIG. 2. The dependence of the transition amplitude h on the square of the Borel parameter M^2. The solid line corresponds to B^0 and the dashed line to B^+ meson cases.

$$\phi_1(u)-\phi_2(u)=\frac{-1}{8}(1-u^2).$$

From the asymptotic form of the photon wave function, given in Eq. (21), we get

$$\phi(1/2)=3/2\chi. \tag{33}$$

Following [17], we will use $g_\perp(u)=1$, i.e., to the leading twist accuracy, in the numerical calculations below.

Having fixed the input parameters, it is necessary to find a range of M^2 for which the sum rule is reliable. The lowest value of M^2, according to the QCD sum rule ideology, is determined by requiring that the power corrections are reasonably small. The upper bound is determined by imposing the condition that the continuum and the higher states contributions remain under control.

In Fig. 2 we presented the dependence of h on M^2. From this figure it follows that the best stability region for h is $6\leqslant M^2\leqslant 12$ GeV^2, and, thus we obtain

$$f_{B^{0*}}f_{B^0}h=(-0.1\pm 0.02)\ \text{GeV}^2,$$

$$f_{B^{+*}}f_{B^+}h=(0.2\pm 0.02)\ \text{GeV}^2. \tag{34}$$

Note that the variation of the threshold value from 36 to 40 GeV^2 changes the result by only a few percent. We see that the sign of the amplitudes for B^0 and B^+ are different. This is due to the fact that the main contributions to the theoretical part of the sum rules comes from the bare loop, and the quark condensate in the external field [last term in Eq. (32)]. In the B^0 (B^+) case, both contributions have negative (positive) signs, and therefore the sign of h is negative (positive). To get the dimensionless transition amplitude for the decay $D^*\to D\gamma$, it is sufficient to make the following replacements in Eq. (32):

$$m_b\to m_c,$$

$$f_{B^*(B)}\to f_{D^*(D)},\quad Q_b\to Q_c,$$

$$s_{0B}\to s_{0D}. \tag{35}$$

Performing similar calculations for the $D^*\to D\gamma$ decay, we get the best stability region for h as $2\leqslant M^2\leqslant 4$ GeV^2, and we find

$$f_{D^{0*}}f_{D^0}h=(0.12\pm 0.02)\ \text{GeV}^2,$$

$$f_{D^{+*}}f_{D^+}h=(-0.04\pm 0.01)\ \text{GeV}^2. \tag{36}$$

The signs of the transition amplitudes for D_0 and D^+ meson decays are different in the B-meson case.

Using the transition amplitude h, one can calculate the decay rates for $B^*(D^*)\to B(D)\gamma$, which can be tested experimentally. For the decay width we get

$$\Gamma(B_0^*\to B_0\gamma)=0.16\ \text{keV}, \tag{37}$$

$$\Gamma(B^{+*}\to B^+\gamma)=0.63\ \text{keV}, \tag{38}$$

and

$$\Gamma(D_0^*\to D_0\gamma)=14.40\ \text{keV}, \tag{39}$$

$$\Gamma(D^{+*}\to D^+\gamma)=1.50\ \text{keV}. \tag{40}$$

In order to compare these theoretical results with experimental data for D-meson decays, we need the theoretical values of the $D^*\to D\pi$ decay widths. We take these values from [13]:

$$\Gamma(D^{*+}\to D^0\pi^+)=32\pm 5\ \text{keV}, \tag{41}$$

$$\Gamma(D^{*+}\to D^+\pi^0)=15\pm 2\ \text{keV}, \tag{42}$$

$$\Gamma(D^{*0}\to D_0\pi^0)=22\pm 2\ \text{keV}. \tag{43}$$

From Eqs. (39)–(43), for the branching ratios (BR's), we obtain

$$B(D_0^*\to D_0\gamma)=39\%,$$

$$B(D^{+*}\to D^+\gamma)=3\%. \tag{44}$$

These results are in agreement with the CLEO data [26], which are

$$B(D_0^*\to D_0\gamma)=(36.4\pm 2.3\pm 3.3)\%,$$

$$B(D^{+*}\to D^+\gamma)=(1.1\pm 1.4\pm 1.6)\%.$$

We see that our predictions on branching ratio are in reasonable agreement with experimental results.

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[1] M.A. Shifman, A.I. Vainstein, and V.I. Zakharov, Nucl. Phys. **B147**, 385 (1979).

[2] L.J. Reinders, H.R. Rubinstein, and S. Yazaki, Phys. Rep. **127**, 1 (1985).

[3] V.L. Chernyak and A.R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[4] I.I. Balitsky, V.M. Braun, and A.V. Kolesnichenko, Nucl. Phys. **B312**, 509 (1989).

[5] V.M. Braun and I.B. Filyanov, Z. Phys. C **48**, 239 (1990).

[6] V.M. Belyaev, A. Khodjamirian, and R. Ruckl, Z. Phys. C **60**, 349 (1993).

[7] V.L. Chernyak and I.R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990).

[8] P. Ball, V.M. Braun, and H.G. Dosch, Phys. Rev. D **44**, 3567 (1991).

[9] A. Ali, V.M. Braun, and H. Simma, Z. Phys. C **63**, 437 (1994).

[10] V. M. Belyaev, Z. Phys. C **65**, 93 (1995).

[11] A. Ali and V.M. Braun, Phys. Lett. B **359**, 223 (1995).

[12] A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. B **358**, 129 (1995).

[13] V.M. Belyaev, V.M. Braun, A. Khodjamirian, and R. Ruckl, Phys. Rev. D **51**, 6177 (1995).

[14] T.M. Aliev, E. Iltan, and N.K. Pak, Phys. Lett. B **334**, 169 (1994).

[15] T.M. Aliev, E. Iltan, and N.K. Pak (unpublished).

[16] V.A. Nesterenko and A.V. Radyushkin, Sov. J. Nucl. Phys. **39**, 811 (1984).

[17] G. Eilam, I. Halperin, and R.R. Mendel, Phys. Lett. B **361**, 137 (1995).

[18] I.I. Balitsky and V.M. Braun, Nucl. Phys. **B311**, 541 (1988).

[19] V.M. Braun and I.E. Filyanov, Z. Phys. C **44**, 157 (1989).

[20] V.M. Belyaev and Ya.I. Kogan, Sov. J. Nucl. Phys. **40**, 659 (1984).

[21] I.I. Balitsky, A.V. Kolesnicenko, and A.V. Yung, Sov. J. Nucl. Phys. **41**, 178 (1985).

[22] T.M. Aliev and V.L. Eletsky, Sov. J. Nucl. Phys. **38** 936 (1983).

[23] P. Colangelo, G. Nardulli, A.A. Ovchinnikov, and N. Paver, Phys. Lett. B **269**, 204 (1991).

[24] E. Bagan *et al.*, Phys. Lett. B **278**, 457 (1992).

[25] M. Neubert, Phys. Rev. D **45**, 2451 (1992).

[26] CLEO Collaboration, F. Butler *et al.*, Phys. Rev. Lett. **69**, 2041 (1992).