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From vacuum fluctuations to radiation. I. Accelerated detectors

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In this article, the properties of the mean fluxes emitted by an accelerated two level atom are analyzed in detail. In spite of the fact that the mean flux vanishes once thermal equilibrium is reached, we show that each transition of the atom is nevertheless accompanied by the emission of one Minkowski quantum. Furthermore, we prove that the Minkowski energy emitted is equal to the sum of the Doppler shifted energies of each transition. Both results are first derived to second order in the coupling between the atom and the radiation by explicitly introducing a switch on and off function whose virtue is to regularize the fluxes on the horizon. Then we generalize these results to arbitrary coupling. In the second part of the paper, the mean fluxes are decomposed according to the final state of the atom and the notion of conditional flux is introduced. This approach sheds light on the properties of the mean fluxes and gives the energy content of the vacuum fluctuations that shall induce the transitions of the accelerated atom. These conditional energies are expressed in terms of off-diagonal matrix elements and are generically complex. Finally, the dynamical relevance of these conditional fluxes is proven. This last point is further developed in a companion article and allows the evaluation of gravitational back reaction effects induced by black hole radiation. [S0556-2821(96)04424-4]

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I. INTRODUCTION

It is now well known that a uniformly accelerated system thermalizes in Minkowski vacuum at temperature $a/2\pi$ [1]. But it is much more complicated to obtain a complete description of the fluxes emitted by this system. Controversial debates have arisen in the literature over whether such systems still radiate once they have reached equilibrium [2–11].

The origin of the difficulties are twofold. First, the modes of the radiation field coupled to the accelerated system (Rindler modes) are highly singular on the horizon. This singular behavior is due to the exponentially growing Doppler shift relating Rindler frequencies to inertial (Minkowski) ones. Second, the fact that the trajectory of the accelerated detector is classical implies that the energy emitted during successive transitions interfere perfectly. As first pointed out by Grove [3], these interferences lead to the vanishing of the mean energy flux when the accelerated system has reached equilibrium. Furthermore, the singularity of the modes on the horizon and the destructive interferences conspire intimately to render the emitted energy localized and singular on the horizon. Thus global quantities such as the total number of emitted particles are ill defined as well.

In this paper, we circumvent these difficulties, which hindered previous work, by introducing a switch off function which specifies how and when the accelerated system is coupled to radiation. This regulator leads to finite expressions for both the energy density and global quantities. Moreover, the interferences are also regularized by its introduction. In spite of the fact that the flux is entirely located in transients which occur when the detector is coupled to the field [5,8], we are now able to prove that the total Minkowski energy emitted is equal to the sum of the Doppler shifted energies associated to each transition of the detector. Similarly, the total number of quanta emitted is equal to the number of transitions and therefore grows linearly with the duration of proper time wherein the interaction is turned on. This demonstrates that the transients incorporate the entire history of the coupling between the detector and the radiation field.

For simplicity and clarity, we have taken the accelerated system to be a two level atom and work in perturbation theory to second order in the interaction with the radiation. However we have included two appendices in order to generalize our results. In Appendix B, we consider the accelerated oscillator model introduced by Raine, Sciama, and Grove [4–7]. We evaluate the emitted fluxes to all order in the coupling constant and prove that all the results found to second order obtain in this case as well. In Appendix C, we prove that, irrespectively of the interaction considered, the scattering of Rindler modes by an accelerated system leads *inevitably* to the production of Minkowski quanta.[1]

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[1]This latter generalization therefore applies in the case of accelerated black holes in thermal equilibrium which was recently considered by Yi [10]. Yi argued that when the Hawking temperature of the black holes equals their Unruh temperature they no longer radiate. Our analysis shows that his conclusions are incorrect and that the accelerated black holes will emit a steady flux of Doppler shifted Minkowski quanta, see [11].

To clarify the properties of the mean fluxes is the first aim of this paper. The second one is to obtain a description of the fluctuations around this mean value. To this end, we address the following issue. The final state of the detector, after having switched off the interaction, is either excited or not. In addition, the final configurations of the radiation field are entangled with the final detector state. Therefore, one can ask what are the properties of the emitted fluxes *conditional* to the fact that the final state of the detector is either excited or not. By answering this question, one obtains a description of the energy density emitted correlated to a single transition of the accelerated detector, see [2,3,8].

We then generalize this procedure in order to obtain a description of the energy density of the vacuum configurations which *shall* induce a transition of the accelerated detector. We show that this conditional energy density is given in terms of a normalized off-diagonal matrix element of the energy density operator. It reveals the pattern of the Einstein-Podolsky-Rosen (EPR) correlations present in the wave function of the coupled system atom plus radiation. In particular, we obtain an explicit description of the modifications of these correlated field configurations when considered before and after the interaction has occurred.

This analysis is a particular case of a general approach to describe the vacuum fluctuations which materialize, in the presence of external fields, into pairs of asymptotic on mass shell quanta. It was applied in the context of pair production in an external electric field to isolate the vacuum fluctuations which give rise to a specific pair [12]. In [13], we apply this analysis of vacuum fluctuations through normalized off-diagonal matrix elements of $T_{\mu\nu}$ to black hole radiation. In that article, we show how these matrix elements encode the correlations between the field configurations corresponding to the creation of a specific asymptotic Hawking quantum and the field configurations at earlier times. They furnish a historical description of the emergence of that quantum from vacuum fluctuations. Furthermore, they control back reaction effects which cannot be described by the mean field theory wherein only the expectation value of $T_{\mu\nu}$ acts as a source for gravity (in a similar manner as final state interactions are introduced upon studying strong interactions that appear in a particular weak channel). Finally, we recall that there this approach is closely related to (and inspired by) the work of Aharonov *et al.* concerning measurements on pre- and post-selected systems [14], see [15] for a discussion of this correspondence.

Having understood the physical meaning of these off diagonal elements and justified their dynamical relevance, we display the properties of the conditional value of $T_{\mu\nu}$. For the nonce, let us mention one of these properties. Owing to the free and massless propagation of the radiation, the conditional value of $T_{\mu\nu}(x)$ extends from the past to the future null infinities. Furthermore, owing to the absence of back reaction (i.e., the neglection of recoils), this function is automatically boost covariant. Thus, after a proper time lapse of the order of $a\Delta\tau = \ln(M/a)$, the frequencies involved in the fluxes are bigger than M owing to the exponentially growing Doppler effect relating Minkowski frequencies to the accelerated ones. This is similar to the ''trans-Planckian'' problem emphasized by 't Hooft and Jacobson in the black hole evaporation context [16,17]. Therefore, if one attributes a finite mass M to the detector, the recoils cannot be neglected after that proper time lapse. The consequences of the recoils induced by these exponentially growing frequencies have been analyzed in [9]. They nicely confirm the fact that each transition of the accelerated system leads to the production of one Minkowski quantum.

II. MEAN ENERGY EMITTED BY AN ACCELERATED ATOM

This part is devoted to an analysis of the mean fluxes emitted by an accelerated detector. In Sec. II A we present the model of the accelerated two level atom with an explicit switch off function. In Sec. II B we obtain formal expressions for the energy emitted. The mean fluxes emitted as the atom thermalizes are discussed in Sec. II C and then in thermal equilibrium in Sec. II D. In these sections, we insist on the role of the transients in guaranteeing that global properties are respected.

A. The uniformly accelerated two level atom

We consider, following Unruh [1], a uniformly accelerated two level atom coupled to a massless field ϕ. Contrary to the usual treatment [3,5,19], we couple the atom to the field only for a finite time by introducing a switch on and off function, $f(\tau)$, and we pay special attention to the new aspects introduced by this time dependence. The reason why we have introduced this function is that we want finite energy densities everywhere including the horizon. Then, the global properties of the fluxes such as the total energy and the total number of quanta emitted are finite and can be related to the period wherein the interaction is turned on. We show in the next section that the singular behavior of the Rindler modes imposes severe restrictions on $f(\tau)$ if one requires finite energy densities.

We briefly review the Rindler quantization of massless scalar field. The reader unfamiliar with the properties of the Rindler modes might consult [18,1,19,15]. Let us just recall here their salient features while insisting on their singular behavior.

The conformal invariance of the massless field in $1+1$ dimensions is best exploited by using the lightlike coordinates U,V defined by $U=t-z$, $V=t+z$, whereupon the general solution of the Klein-Gordon equation is

$$\Phi = f(U) + g(V). \tag{1}$$

The Minkowski modes, of energy $i\partial_V = \omega$ are given by

$$\varphi_\omega(V) = \frac{e^{-i\omega V}}{\sqrt{4\pi\omega}}. \tag{2}$$

Similarly the Rindler modes, of Rindler energy $iV\partial_V = \lambda$ are given by

$$\varphi_{\lambda,R}(v) = \theta(V)\,\frac{(aV)^{-i\lambda/a}}{\sqrt{4\pi\lambda}} = \frac{e^{-i\lambda v}}{\sqrt{4\pi\lambda}} \tag{3}$$

where we have introduced the Rindler lightlike coordinate $av = \theta(V)\ln(aV)$. Since the $\varphi_{\lambda,R}$ constitute a complete set in R $(V>0)$ only, they cannot be related to the Minkowski

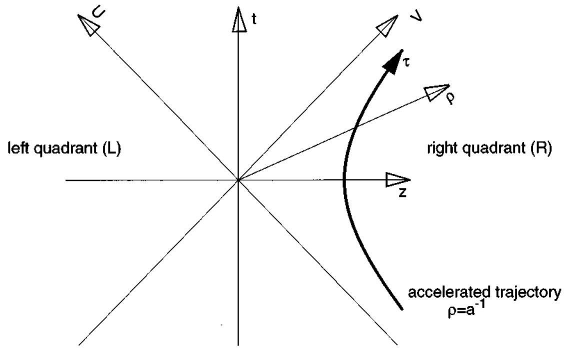

FIG. 1. The Minkowski coordinates t,z and U,V. The left (L) and right (R) Rindler quadrants. The Rindler coordinates τ,ρ in R and the trajectory of a uniformly accelerated atom.

basis by a unitary transformation. One must also introduce Rindler modes living in the left quadrant, $\varphi_{\lambda,L}(V)=\varphi^*_{\lambda,R}(-V)$. But since both Rindler modes are singular at $V=0$, care must be taken to define the Bogoljubov transformation relating Minkowski modes to these Rindler modes. To obtain regular expressions on the horizon, it is useful to consider a new basis of positive frequency Minkowski modes, eigenmodes of $iV\partial_V$ and defined for all V [1]:

$$\begin{aligned}\varphi_{\lambda,M}(V)&=\int_0^\infty d\omega\,\gamma_{\lambda,\omega}\varphi_\omega(V)\\&=_{\epsilon\to 0^+}\frac{[a(\epsilon+iV)]^{-i\lambda/a}}{\sqrt{(e^{\pi\lambda/a}-e^{-\pi\lambda/a})4\pi\lambda}}\\&=\frac{1}{\sqrt{|e^{\pi\lambda/a}-e^{-\pi\lambda/a}|}}\left[e^{\pi\lambda/2a}\frac{\theta(V)(aV)^{-i\lambda/a}}{\sqrt{4\pi|\lambda|}}\right.\\&\qquad\left.+e^{-\pi\lambda/2a}\frac{\theta(-V)(-aV)^{-i\lambda/a}}{\sqrt{4\pi|\lambda|}}\right]\end{aligned}\tag{4}$$

where

$$\gamma_{\lambda,\omega}=\left(\frac{1}{\Gamma(i\lambda/a)}\sqrt{\frac{a\pi}{\lambda\,\sinh\pi\lambda/a}}\right)\frac{1}{\sqrt{2\pi a\omega}}\left(\frac{\omega}{a}\right)^{i\lambda/a}e^{-\omega\epsilon}.\tag{5}$$

The factor $e^{-\omega\epsilon}$ defines the integral equation (4), regularizes the modes $\varphi_{\lambda,M}(V)$ at $V=0$, and ensures the correct Minkowski properties of the theory, see [20]. We shall see in the next section that the window function $f(\tau)$ plays a role similar to the cutoff ϵ in that it leads also to well defined expressions on the horizon.

The trajectory of the uniformly accelerated atom is given by (see Fig. 1)

$$\begin{aligned}t_a(\tau)&=a^{-1}\sinh a\tau,\quad x_a(\tau)=a^{-1}\cosh a\tau,\\V_a(\tau)&=a^{-1}e^{a\tau},\quad U_a(\tau)=-a^{-1}e^{-a\tau},\end{aligned}\tag{6}$$

where τ is the proper time and a the acceleration. The interaction Hamiltonian between the atom and the field is

$$\begin{aligned}\int dtdxH_{\rm int}(t,x)=gm\int d\tau\{[f(\tau)e^{-im\tau}A\\+f^*(\tau)e^{im\tau}A^\dagger]\phi(t_a(\tau),x_a(\tau))\},\end{aligned}\tag{7}$$

where g is a dimensionless coupling constant that shall be taken for simplicity small enough that second order perturbation theory be valid and m is the difference of energy between the ground ($|-\rangle$) and the excited state ($|+\rangle$) of the atom. A is the lowering operator that induces a transition from the excited state to the ground state of the atom and $f(\tau)$ is the dimensionless function that governs when and how the interaction is turned on and off.

We shall be most interested in the situation where $f(\tau)=1$ inside a long interval $\tau_i<\tau<\tau_f$ and $f(\tau)$ tends to zero outside this interval. In the ''golden rule'' limit (i.e., in the limit $\tau_f-\tau_i=T\to\infty$ with g^2T finite) the concept of a transition rate emerges. This rate comes from the resonance of the Doppler shifted Minkowski vacuum fluctuations with the fixed Rindler frequency m [21]. In addition, we shall assume that the V part of the ϕ field only is coupled to the atom. This is a legitimate truncation owing to Eq. (1). For simplicity of notation it is convenient to introduce

$$\phi_m=\int_{-\infty}^{+\infty}d\tau e^{+im\tau}f^*(\tau)\phi(V_a(\tau)).\tag{8}$$

Let us consider first the situation in which both the atom and the field are initially in their ground state. The state $|\psi_-\rangle$ at $t=-\infty$ is thus

$$|\psi_-(t=-\infty)\rangle=|0_M\rangle\,|-\rangle\tag{9}$$

where $|0_M\rangle$ is Minkowski vacuum. At $t=+\infty$, when the interaction has been switched off, the state can again be expressed in terms of the uninteracting states. To order g^2, it is given by

$$\begin{aligned}|\psi_-(t=+\infty)\rangle=|0_M\rangle\,|-\rangle-igm\phi_m\,|0_M\rangle\,|+\rangle\\-\frac{g^2m^2}{2}[\phi_m^\dagger\phi_m+\mathcal{D}]\,|0_M\rangle\,|-\rangle\end{aligned}\tag{10}$$

where

$$\begin{aligned}\mathcal{D}=\int d\tau_2\int d\tau_1[\theta(\tau_2-\tau_1)-\theta(\tau_1-\tau_2)]\\\times e^{-im\tau_2}f(\tau_2)\phi(\tau_2)e^{+im\tau_1}f^*(\tau_1)\phi(\tau_1).\end{aligned}\tag{11}$$

We have split the g^2 term in two pieces in order to isolate the steady regime from transitory periods associated with the switch on and off. The first regime is controlled by the term proportional to $\phi_m^\dagger\phi_m$ whereas the second is concerned by the $\mathcal{D}$ term which comes from the time ordering contained in $\exp(-i\int dtdxH_{\rm int})$. Indeed as proven in Appendix A, $\mathcal{D}$ does not contribute to the energy density emitted in the steady state regime: its energy density scales like g^2/T rather than like mg^2. Furthermore it carries no Minkowski nor Rindler energy. We shall therefore drop this term in the rest of the paper.

In the ‘‘golden rule’’ limit, the operator $\phi_m^\dagger \phi_m$ tends towards the counting operator for Rindler quanta of energy m $(=a_{m,R}^\dagger a_{m,R})$ multiplied by π/m and the probability P_e for the two level atom to get excited becomes

$$P_e = g^2 m^2 \langle 0_M | \phi_m^\dagger \phi_m | 0_M \rangle = (1/2) g^2 m T N_m \tag{12}$$

where $N_m = 1/(e^{2\pi m/a} - 1)$ is the mean number of Rindler quanta present in Minkowski vacuum, see [1,18]. This proves that the atom maintained on an accelerated trajectory reacts to the mean number of Rindler quanta as the same atom, put on an inertial trajectory, would have reacted to the mean number of Minkowski quanta.

Similarly, when the initial state is the product of Minkowski vacuum and the excited state $|+\rangle$, at $t=+\infty$ the state is

$$|\psi_+(t=+\infty)\rangle = |0_M\rangle |+\rangle - igm\phi_m^\dagger |0_M\rangle |-\rangle - \frac{g^2 m^2}{2} [\phi_m \phi_m^\dagger - \mathcal{D}] |0_M\rangle |+\rangle \tag{13}$$

where the operator $\mathcal{D}$ is the same as in Eq. (10). In the ‘‘golden rule’’ limit, the probability to be found in the ground state at $t=+\infty$ is

$$P_d = g^2 m^2 \langle 0_M | \phi_m \phi_m^\dagger | 0_M \rangle = (1/2) g^2 m T (N_m + 1). \tag{14}$$

Hence, at equilibrium, by Einstein’s famous argument, the probabilities P_+, P_- to be in the excited or ground states are given by

$$\frac{P_+}{P_-} = \frac{P_e}{P_d} = \frac{N_m}{N_m + 1} = e^{-2\pi m/a} \tag{15}$$

that is a thermal distribution at temperature $a/2\pi$. Had we coupled the atom to both the U and V parts, the probabilities P_e and P_d would have been multiplied by 2 but the thermal ration equation (15) would remain unaffected.

B. The mean fluxes to order g^2

In this section, we introduce the central notion of conditional energy emitted by decomposing the mean value of the flux according to the final state of the atom. Then we obtain the necessary condition that the switch off function $f(\tau)$ should satisfy in order to have finite densities on the horizon. Finally, we shall express the conditional energy densities in terms of the Fourier transform of $f(\tau)$ in order to obtain analytical expressions which will serve in Sec. II C for the thermalization period and for the equilibrium situation in Sec. II D.

When the initial state is $|\psi_-\rangle$ defined in Eq. (9), to order g^2, the mean flux emitted on the left of the atom is

$$\begin{aligned} \langle T_{VV}(V) \rangle_{\psi_-} &= \langle \psi_-(t=+\infty) | T_{VV}(V, U > U_a(\tau)) \\ &\quad \times |\psi_-(t=+\infty)\rangle \\ &= g^2 m^2 \langle 0_M | \phi_m^\dagger T_{VV} \phi_m | 0_M \rangle \\ &\quad - g^2 m^2 \mathrm{Re}[\langle 0_M | T_{VV} \phi_m^\dagger \phi_m | 0_M \rangle] \end{aligned} \tag{16}$$

where we have used Eq. (10) and dropped the contribution of the $\mathcal{D}$ term. The physical meaning of the two terms on the right-hand side (RHS) of Eq. (16) was first discussed in [2], see also [3,8]. To prepare the discussion of the next part, devoted to the analysis of the fluctuations, we rewrite Eq. (16) as

$$\langle T_{VV}(V) \rangle_{\psi_-} = P_e \langle T_{VV} \rangle_e + P_g \langle T_{VV} \rangle_g \tag{17}$$

where we have defined

$$\begin{aligned} \langle T_{VV} \rangle_e &= g^2 m^2 \langle 0_M | \phi_m^\dagger T_{VV} \phi_m | 0_M \rangle / P_e, \\ \langle T_{VV} \rangle_g &= - g^2 m^2 \mathrm{Re}[\langle 0_M | T_{VV} \phi_m^\dagger \phi_m | 0_M \rangle] / P_g. \end{aligned} \tag{18}$$

P_e and P_g are the probabilities to find the atom in the excited or ground state at $t=+\infty$. P_e is given in Eq. (12) and $P_g = 1 - P_e$.

The interpretation of the two quantities $\langle T_{VV} \rangle_e$ and $\langle T_{VV} \rangle_g$ is clear when one recalls their origin. $\langle T_{VV} \rangle_e$ comes from the square of the second term of Eq. (10) (linear in g) whereas $\langle T_{VV} \rangle_g$ comes from an interference between the first unperturbed term and the third term in which the interaction has acted twice. Hence $\langle T_{VV} \rangle_e$ is the energy emitted if the atom is found excited at $t=+\infty$ whereas $\langle T_{VV} \rangle_g$ is the energy emitted if the atom is found in the ground state. These fluxes have been normalized so as to express the RHS of Eq. (17) as the probability of finding the atom in a final state times the energy emitted if that final state is realized. Thus $\langle T_{VV} \rangle_g$ and $\langle T_{VV} \rangle_e$ are the conditional ‘‘mean’’ energy emitted. The word ‘‘mean’’ is understood here in its quantum sense, i.e., as the average over repeated realizations of the same situation: the same initial state $|\psi_-\rangle$ *and* the same final state of the atom, see [15] for further comments on this point.

Similarly, when the initial state of the system is $|\psi_+\rangle = |0_M\rangle |+\rangle$, the mean energy emitted is

$$\begin{aligned} \langle T_{VV}(V) \rangle_{\psi_+} &= g^2 m^2 \langle 0_M | \phi_m T_{VV} \phi_m^\dagger | 0_M \rangle \\ &\quad - g^2 m^2 \mathrm{Re}[\langle 0_M | T_{VV} \phi_m \phi_m^\dagger | 0_M \rangle] \end{aligned} \tag{19}$$

where we have used Eq. (13) and dropped the $\mathcal{D}$ term as well. As in Eq. (17), we rewrite this flux as

$$\langle T_{VV} \rangle_{\psi_+} = P_d \langle T_{VV} \rangle_d + P_h \langle T_{VV} \rangle_h \tag{20}$$

where P_d is the deexcitation probability given in Eq. (14) and where $P_h = 1 - P_d$ is the probability to be found in the excited state at $t=+\infty$. The conditional fluxes $\langle T_{VV} \rangle_d$ and $\langle T_{VV} \rangle_h$ are given by

$$\begin{aligned} \langle T_{VV} \rangle_d &= g^2 m^2 \langle 0_M | \phi_m T_{VV} \phi_m^\dagger | 0_M \rangle / P_d \\ \langle T_{VV} \rangle_h &= - g^2 m^2 \mathrm{Re}[\langle 0_M | T_{VV} \phi_m \phi_m^\dagger | 0_M \rangle] / P_h \end{aligned} \tag{21}$$

and are interpreted as the energy emitted when the atom is found in the ground state (deexcitation d) or in the excited state at $t=+\infty$ knowing that the atom was prepared in the excited state at $t=-\infty$.

At this point, two properties which will play an important role in what follows should be pointed out. First, the matrix elements $\langle T_{VV}\rangle_i$ (where i stands for $e,\ g,\ d$, and h) are acausal. For instance, they are nonvanishing in the left Rindler quadrant $V<0$, $U>0$ which is causally disconnected. However the mean energies $\langle T_{VV}\rangle_{\psi_j}$ (where $j=+,-$) are causal. Indeed, when V,U is separated from the trajectory of the atom by a space like distance then $T_{VV}(V,U)$ commute with $H_{\rm int}$ [2]. Thus the mean value vanishes:

$$\begin{aligned}\langle T_{VV}(V,U)\rangle_{\psi_j}&=\langle \psi_j|\exp\left(+i\int dtH_{\rm int}\right)T_{VV}(V,U)\\&\quad\times\exp\left(-i\int dtH_{\rm int}\right)|\psi_j\rangle\\&=\langle \psi_j|T_{VV}(V,U)\exp\left(+i\int dtH_{\rm int}\right)\\&\quad\times\exp\left(-i\int dtH_{\rm int}\right)|\psi_j\rangle\\&=\langle \psi_j|T_{VV}(V,U)|\psi_j\rangle=\langle 0_M|T_{VV}(V,U)\,|0_M\rangle\\&=0.\end{aligned}\tag{22}$$

The very same causality argument applies in regions where $\langle T_{VV}(V,U)\rangle_{\psi_j}\neq 0$ to guarantee that it only depends on $H_{\rm int}(\tau)$ for τ's such that $V(\tau)<V$, i.e., that it only depends on the form of $H_{\rm int}(\tau)$ in the past light cone of (V,U).

Second, the total Minkowski energy carried by $\langle T_{VV}\rangle_e$, Eq. (18), is strictly positive,

$$\int_{-\infty}^{+\infty}dV\langle T_{VV}\rangle_e=g^2m^2\,\langle 0_M|\phi_m^\dagger H_M\phi_m\,|0_M\rangle/P_e>0\tag{23}$$

since it is the expectation value of the Hamiltonian $H_M=\int_0^\infty d\omega\omega a_\omega^\dagger a_\omega$ in a state which is not Minkowski vacuum. On the other hand, the Minkowski energy carried by $\langle T_{VV}\rangle_g$ vanishes identically,

$$\int_{-\infty}^{+\infty}dV\langle T_{VV}\rangle_g=-g^2m^2{\rm Re}[\,\langle 0_M|H_M\phi_m^\dagger\phi_m\,|0_M\rangle]/P_g=0,\tag{24}$$

since $H_M\,|0_M\rangle=0$. Similarly the integral of $\langle T_{VV}\rangle_d$ is positive whereas the integral of $\langle T_{VV}\rangle_h$ vanishes.

In preparation for the next sections and in order to obtain explicit expressions for $\langle T_{VV}\rangle_i$, it is appropriate to work out certain technicalities.

First we note that the Wick contractions which arise upon evaluating $\langle T_{VV}\rangle_i$ $(i=e,g,d,h)$ [Eqs. (18) and (21)] are given in terms of the two functions

$$\begin{aligned}\mathcal{C}_+(V)&=\langle 0_M|\phi(V)\phi_m^\dagger\,|0_M\rangle\\&=\int d\tau e^{-im\tau}f(\tau)G_+(V,V_a(\tau)),\\\mathcal{C}_-(V)&=\langle 0_M|\phi(V)\phi_m\,|0_M\rangle\\&=\int d\tau e^{+im\tau}f^*(\tau)G_+(V,V_a(\tau)),\end{aligned}\tag{25}$$

where $G_+(V,V')$ is the Wightman function in Minkowski vacuum. Thus we obtain

$$\begin{aligned}\langle T_{VV}\rangle_e&=2\left(\frac{g^2m^2}{P_e}\right)(\partial_V\mathcal{C}_-)(\partial_V\mathcal{C}_-^*),\\\langle T_{VV}\rangle_d&=2\left(\frac{g^2m^2}{P_d}\right)(\partial_V\mathcal{C}_+)(\partial_V\mathcal{C}_+^*),\\\langle T_{VV}\rangle_g&=-2\left(\frac{g^2m^2}{P_g}\right){\rm Re}[(\partial_V\mathcal{C}_-)(\partial_V\mathcal{C}_+)]=\left(\frac{P_h}{P_g}\right)\langle T_{VV}\rangle_h.\end{aligned}\tag{26}$$

For these matrix elements of T_{VV} not to be singular the functions $\partial_V\mathcal{C}_+(V)$ and $\partial_V\mathcal{C}_-(V)$ must be regular. From the second equality of Eq. (25) we obtain

$$\partial_V\mathcal{C}_+(V)=-\frac{1}{4\pi}\int d\tau\frac{1}{V-a^{-1}e^{a\tau}-i\epsilon}f(\tau)e^{-im\tau}\tag{27}$$

which can be singular only for $V=0$ where it takes the form

$$\begin{aligned}\partial_V\mathcal{C}_+(V)&=-\frac{1}{4\pi}\int d\tau\frac{1}{-a^{-1}e^{a\tau}-i\epsilon}f(\tau)e^{-im\tau}\\&\simeq\frac{a}{4\pi}\int d\tau e^{-a\tau}f(\tau)e^{-im\tau}.\end{aligned}\tag{28}$$

The last integral is finite if and only if $f(\tau)$ decreases for $\tau\to-\infty$ quicker than $e^{a\tau}$. Similarly if we had considered right movers, the condition for finiteness on the future horizon would have been sufficient rapid decrease of f for $\tau\to+\infty$. Thus the condition to have regular expressions on both horizons is that $f(\tau)$ decreases faster than $e^{-a|\tau|}$. This leads to

$$\int d\tau\frac{dt}{d\tau}|f(\tau)|=\int dt|f(\tau(t))|<\infty,\tag{29}$$

i.e., the interaction of the atom with the field must last a finite Minkowski time.

Secondly, it is appropriate to reexpress $f(\tau)e^{-im\tau}$ in Fourier transform

$$f(\tau)e^{-im\tau}=\int_{-\infty}^{+\infty}d\lambda\frac{c_\lambda}{2\pi}e^{-i\lambda\tau}.\tag{30}$$

The normalization is

$$\begin{aligned}\int d\tau|f(\tau)|^2=\int d\lambda\frac{|c_\lambda|^2}{2\pi}&=T\\&=\text{ total proper time of interaction.}\end{aligned}\tag{31}$$

When $e^{-im\tau}f(\tau)$ contains no negative frequency, i.e., $c_\lambda=0$ for $\lambda<0$, Eq. (7) defines a Lee model: were the detector inertial it would only respond to the presence of Minkowski particles. However the regularity condition [Eq. (29)] implies that c_λ be an analytic function in the strip $-a<{\rm Im}\lambda<a$. Hence in order to have regular energy densities, we shall be obliged to work with non-Lee models which

can spontaneously excite. Nevertheless, by choosing an $f(\tau)$ which satisfies Eq. (29), but such that the negative components of c_λ are exponentially small, the spontaneous excitation probability is exponentially small as well. Then spontaneous transitions occur only at switch on and off transitory periods and do not contribute to rates.

More precisely, when c_λ is peaked around $\lambda=m$ and T satisfies both $T \gg m^{-1}$ and $T \gg a^{-1}$, the ''golden rule'' probability Eq. (12) is recovered. The first of these conditions is that $f(\tau)$ be spread over a distance at least equal to the inverse frequency m^{-1} (the time-energy uncertainty condition). The second condition, which corresponds to T being greater than the Euclidean tunneling time $2\pi a^{-1}$ [21], is required for the probability P_e to be linear in time and proportional to the Bose distribution N_m.

In terms of c_λ the functions $\mathcal{C}_\pm$ read

$$\mathcal{C}_+(V)=\int_{-\infty}^{+\infty} d\lambda c_\lambda \frac{1}{\sqrt{4\pi\lambda(e^{\pi\lambda/a}-e^{-\pi\lambda/a})}} e^{\pi\lambda/2a}\varphi_{\lambda,M}(V)$$
$$=\int_{-\infty}^{+\infty} d\lambda c_\lambda \frac{1}{4\pi\lambda}[(\tilde{n}_\lambda+1)(aV)^{-i\lambda/a}\theta(V)+\tilde{n}_\lambda e^{\pi\lambda/a}|aV|^{-i\lambda/a}\theta(-V)],$$

$$\mathcal{C}_-(V)=\int_{-\infty}^{+\infty} d\lambda c_\lambda^* \frac{1}{\sqrt{4\pi\lambda(e^{\pi\lambda/a}-e^{-\pi\lambda/a})}} \times e^{-\pi\lambda/2a}\varphi_{-\lambda,M}(V)$$
$$=\int_{-\infty}^{+\infty} d\lambda c_\lambda^* \frac{1}{4\pi\lambda}[\tilde{n}_\lambda(aV)^{i\lambda/a}\theta(V)+\tilde{n}_\lambda e^{\pi\lambda/a}|aV|^{i\lambda/a}\theta(-V)] \quad (32)$$

where we have used Eq. (4) for the expression of $\varphi_{\lambda,M}(V)$ and where $\tilde{n}_\lambda=1/(e^{2\pi\lambda/a}-1)$. Upon inserting Eq. (30) into Eq. (12), the probability P_e to be found excited can then be written as

$$P_e=g^2m^2\int_{-\infty}^{+\infty} d\lambda \frac{|c_\lambda|^2}{4\pi\lambda}\tilde{n}_\lambda. \quad (33)$$

As one picture is worth a thousand words, we take a particular form for c_λ such that all the integrals above are Gaussian and can be evaluated explicitly. This form is

$$c_\lambda=D\frac{\lambda}{m}e^{-(\lambda-m)^2T^2/2}(1-e^{-2\pi\lambda/a}) \quad (34)$$

where D is a normalization constant taken such as to verify Eq. (31). We shall give throughout the text the exact expressions followed by the approximate expressions valid when $T>>m^{-1}$ and $T>>a^{-1}$. In this golden rule limit, the approximate expressions are particularly easy to interpret. These shall be preceded by the symbol $\simeq$. For instance, the switch off function f is equal to (see Fig. 2)

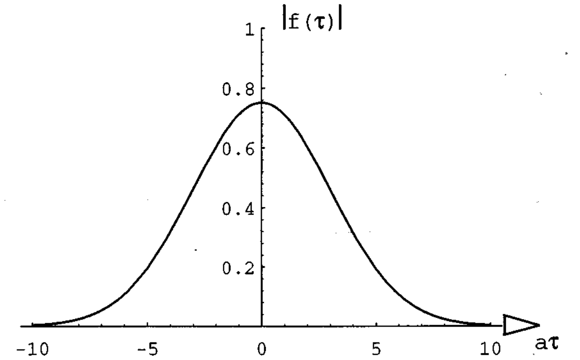

FIG. 2. The absolute value of the switch function $f(\tau)$ given in Eq. (35) for $m=2a$ and $T=3a^{-1}$. τ is given in units of a^{-1}.

$$f(\tau)=\frac{D}{\sqrt{2\pi}T}e^{-\tau^2/2T^2}\left[\left(1-i\frac{\tau}{mT^2}\right)-e^{-2\pi m/a}e^{i2\pi\tau/aT^2}e^{2\pi^2/a^2T^2}\left(1-i\frac{\tau}{mT^2}-\frac{\pi}{amT^2}\right)\right]$$
$$\simeq\pi^{-1/4}e^{-\tau^2/2T^2}[1+N_m(1-e^{i2\pi\tau/aT^2})] \quad (35)$$

where the constant D takes the form $D\simeq 2^{1/2}\pi^{1/4}T(N_m+1)$ and $N_m=(e^{2\pi m/a}-1)^{-1}$. Equation (35) shows the almost Gaussian character of the switch off function whose width is T. The plateau of the Gaussian gives a good approximation of the steady state regime which we intend to study.

C. Fluxes and particles to order g^2 during thermalization

The main results of this section are the following.

(1) During thermalization a steady flux of negative Rindler energy is emitted. This is understood from the isomorphism [3] with the thermal bath: as the atom gets exited it absorbs energy from the thermal bath.

(2) Notwithstanding this negative energy density, the integrated total Minkowski energy is positive and grows with the probability to find the atom excited at $t=\infty$.

(3) The transcription of the negative flux in terms of Minkowski quanta requires to consider the oscillatory tails of this flux since they are enhanced by the Jacobian that converts from Rindler to Minkowski energy. In the Minkowski description, the steady negative flux is due to a ''repolarization'' of the atom corresponding to the fact that the probability of finding the atom in its exited level *decreases* with time. This repolarization is similar (CPT conjugate) with what occurs when negative energy is absorbed by an inertial detector [22].

To reveal the structure of the oscillatory tails and to display the properties in the stationary regime, both the adiabatic switch off controlled by $f(\tau)$ and a sudden switch off model shall be worked out.

We start with the adiabatic switch off. The Minkowski energy density radiated by the two level atom initially in its ground state is given by Eq. (16). In terms of the Fourier components c_λ, the Rindler density defined by $T_{vv}=T_{VV}(dV/dv)^2=T_{VV}e^{2av}$ is

$$\langle T_{vv}(v)\rangle_{\psi_-} = -g^2m^2\int d\lambda\int d\lambda' c_\lambda c^*_{\lambda'}\frac{1}{(4\pi)^2}\times(\tilde{n}_\lambda+\tilde{n}_{\lambda'})e^{-i(\lambda-\lambda')v}$$
$$\simeq \frac{-g^2m^2}{2}N_m\frac{e^{-v^2/T^2}}{\pi^{1/2}}[(N_m+1)\times\cos(2\pi v/aT^2)-N_m]. \quad (36)$$

We recall that the Rindler coordinate v is $v=a^{-1}\ln(aV)$. In Eq. (36), the first line is the exact expression valid for all c_λ, see Eqs. (26) and (32). The second line is the approximate expression, valid in the limit $T\gg m^{-1}$ and $T\gg a^{-1}$ when c_λ is given by Eq. (34). As announced, T_{vv} carries negative Rindler energy

$$\int_{-\infty}^{+\infty}dv\,\langle T_{vv}(v)\rangle_{\psi_-} = -\frac{g^2m^2}{4\pi}\int_{-\infty}^{+\infty}d\lambda|c_\lambda|^2\tilde{n}_\lambda$$
$$\simeq -\frac{1}{2}g^2m^2N_mT = -mP_e \quad (37)$$

equal to the probability P_e, Eq. (33), times the absorbed Rindler energy $-m$.

The total Minkowski energy radiated is

$$\langle H_M\rangle_e = \int_0^{+\infty}dV\,\langle T_{VV}(V)\rangle_{\psi_-} = \int_{-\infty}^{+\infty}dv\,e^{-av}\,\langle T_{vv}(v)\rangle_{\psi_-}$$
$$\simeq \tfrac{1}{2}g^2m^2N_mTe^{a\tau_0}(1+2N_m)$$
$$= mP_ee^{a\tau_0}(1+2N_m). \quad (38)$$

We have used Eq. (22) and $e^{a\tau_0}$ is the mean Doppler effect associated with the window function $f(\tau)$, Eq. (35), defined by

$$\int_{-\infty}^{+\infty}dv\,e^{-av}\frac{e^{-v^2/T^2}}{T\pi^{1/2}}\cos(2\pi v/aT^2) = -e^{a\tau_0}. \quad (39)$$

The Minkowski energy is positive contrary to the total Rindler, Eq. (37). The flip in sign is due to the effect of the transients around $v=aT^2$ where the cosine is negative. Indeed whereas these transients are negligible upon computing the Rindler energy, upon computing the Minkowski energy they are enhanced by the Jacobian $dv/dV=e^{-av}$ and give rise to the sign flip. Thus, it is the same exponential Doppler effect, e^{-av}, which leads both to the thermalization through the nontrivial Bogolyubov transformation and to the compatibility of *absorbing* Rindler energy while emitting Minkowski energy. [Note that this sign flip can also be conceived as arising from the imaginary part of the saddle point of Eq. (39): $v_{sp}=-aT^2/4+i\pi/a$ and stands therefore exactly on the same footing as the flip of frequency which leads to a nonvanishing β coefficient at the saddle point approximation, see [21].] The additional factor $1+2N_m$ in Eq. (38) comes from the inherent ambiguity in defining $e^{a\tau_0}$ as the mean Doppler shift associated to the switch function $f(\tau)$.

We notice that the total Minkowski energy radiated can also be expressed as

$$\langle H_M\rangle_e = \int_0^{+\infty}dV\,\langle T_{VV}(V)\rangle_{\psi_-} = P_e\int_{-\infty}^{+\infty}dV\langle T_{VV}\rangle_e \quad (40)$$

because of Eq. (24). But $\langle T_{VV}\rangle_e$ is located essentially in the region $V<0$. Thus the $\langle T_{VV}\rangle_g$ term, defined in Eq. (17), ''restores'' causality, see Eq. (22), and localizes all the energy in the right quadrant. This shall be explained with care in Sec. III B, after Eq. (70).

Another case of interest is the golden rule limit for which $c_\lambda=2\pi\delta(\lambda-m)$ corresponding to $f(\tau)=1$ for all τ. In this case there is a constant negative flux for all $V>0$ which can be seen by taking the limit $T\to\infty$ at fixed v in Eq. (36). The transients are located on the past horizon $V=0$ where they consist of a singular positive flux [5]. Rather than this case we shall analyze the case where the time dependent coupling is $f(\tau)=\theta(\tau)\theta(T-\tau)$ in order to prove point (3) mentioned above. With this time dependence, the transients are also singular and will not be studied here because the divergent behavior is already present in the inertial case. On the contrary, the steady part is easily computed and gives a differential version of the relation between the probability and the total Rindler energy, see Eq. (37).

The probability of spontaneous emission is given by

$$P_e(T) = g^2m^2\int_0^T d\tau_1\int_0^T d\tau_2 e^{-im(\tau_2-\tau_1)}\,\langle\phi(\tau_2)\phi(\tau_1)\rangle$$
$$\simeq \tfrac{1}{2}g^2mN_mT. \quad (41)$$

The second line contains the golden rule result valid when $aT\to\infty$ with g^2T finite. It is useful to introduce the rate of transition, the derivative of $P_e(T)$:

$$\dot{P}_e(T) = \frac{dP_e(T)}{dT}$$
$$= g^2m^22\,\mathrm{Re}\left[\int_0^T d\tau e^{-im(T-\tau)}\,\langle\phi(T)\phi(\tau)\rangle\right]$$
$$\simeq \frac{1}{2}g^2mN_m. \quad (42)$$

This rate is related to the steady part of the stress energy tensor. Indeed one finds

$$\langle T_{vv}(v=T)\rangle_{\psi_-} = g^2m^22\,\mathrm{Re}\left[\int_0^T d\tau_2\int_0^{\tau_2}d\tau_1 e^{-im(\tau_2-\tau_1)}\times\langle[\phi(\tau_2),T_{vv}(T)]_-\phi(\tau_1)\rangle\right]$$
$$= g^2m^22\,\mathrm{Re}\left[\int_0^T d\tau e^{-im(T-\tau)}\times\langle i\partial_v\phi(T)\phi(\tau)\rangle\right]$$
$$= -m\dot{P}_e(T) + g^2m^22\,\mathrm{Re}[ie^{-imT}\,\langle\phi(T)\phi(0)\rangle]. \quad (43)$$

The first equality follows straightforwardly from the expansion of the evolution operator $\exp(-i\int H_{\rm int} d\tau)$ in g^2. The second equality is obtained using the commutator relation: $[\phi(\tau_2), T_{vv}(\tau_1)]_- = i\partial_v \phi \delta(\tau_1-\tau_2)$ which itself follows from the fundamental commutator $[\phi(v'), \partial_v\phi(v)] = (i/2)\delta(v'-v)$ evaluated on the accelerated trajectory. The third equality follows by integration by parts. The final result contains a steady part proportional to $-m\dot{P}_e(T)$ which tends to $-\frac{1}{2}g^2m^2N_m$ in the golden rule limit and an oscillatory term which is damped if one adds a term in $(\mu^2+i\epsilon)\phi^2$ to the action of ϕ. The steady piece simply indicates that to an *increase* of the probability to make a transition corresponds the *absorption* of the necessary Rindler energy to provoke this increase, i.e., the local version of Eq. (37).

We now turn to the Minkowski description of this steady piece. We first rewrite Eqs. (41)–(43) in terms of the Minkowski basis $e^{-i\omega V}/\sqrt{4\pi\omega}$, Eq. (2). The probability of the transition equation (41) reads

$$P_e(T) = \int_0^\infty d\omega g^2 m^2 \left| \int_0^T d\tau e^{-im\tau} \frac{\exp\left(-i\dfrac{\omega}{a}e^{a\tau}\right)}{\sqrt{4\pi\omega}} \right|^2 = \int_0^\infty d\omega P_{e,\omega}(T) \tag{44}$$

where $P_{e,\omega}(T)$ is the probability to have emited a Minkowski quantum of energy ω at time T [since we are working in g^2, i.e., emission of *one* quantum, $P_{e,\omega}(T)$ can be expressed as $P_{e,\omega}(T) = \langle\psi_-||+\rangle a_\omega^\dagger a_\omega \langle +||\psi_-\rangle$]. Similarly the transition rate, Eq. (42), and the total Minkowski energy can be expressed as

$$\dot{P}_e(T) = \int_0^\infty d\omega \dot{P}_{e,\omega}(T), \tag{45}$$

$$\langle H_M(T)\rangle_e = \int_{-\infty}^{+\infty} dv e^{av} \langle T_{vv}\rangle_{\psi_-} = \int_0^\infty d\omega \omega P_{e,\omega}(T). \tag{46}$$

The second equality follows from the diagonal character of the Hamiltonian: $H_M = \int_0^\infty d\omega \omega a_\omega^\dagger a_\omega$ and the matrix element which defines $P_{e,\omega}(T)$. The positivity of $\langle H_M(T)\rangle_e$ is manifest since all the $P_{e,\omega}(T)$ are positive definite. Nevertheless, within the steady regime, the time derivative of $\langle H(T)\rangle_e$ is negative:

$$\begin{aligned}\frac{d\langle H_M(T)\rangle_e}{dT} &= e^{av(T)} \langle T_{vv}(v(T))\rangle_{\psi_-} \\ &= -me^{av(T)}[\dot{P}_e(T) + \text{``damped'' term}] \\ &= \int_0^\infty d\omega \omega \dot{P}_{e,\omega}(T).\end{aligned} \tag{47}$$

Since $\dot{P}_e(T)>0$, $d\langle H_M\rangle/dT$ negative implies that, for large ω, some $\dot{P}_{e,\omega}$ are negative. This corresponds to a ''repolarization'' since all $P_{e,\omega}$ are positive definite and vanish for $\tau\leqslant 0$. This repolarization is exactly the inverse process of the absorption of negative energy by an atom described in [22].

D. Fluxes and particles to order g^2 at equilibrium

The important result of this section lies in Eq. (53) for the energy density and Eq. (55) for the total Minkowski radiated by the accelerated atom. These equations prove how the vanishing of the flux during the whole equilibrium regime but nevertheless preceded and followed by small oscillatory tails is perfectly coherent with the naive guess that each internal transition of the atom should be accompanied by the emission of a Minkowski quantum.

Before studying the equilibrium situation it behooves us first to consider the flux emitted by an atom that makes a transition from excited to ground state. The mean energy emitted when the initial state is $|\psi_+\rangle$ is

$$\begin{aligned}\langle T_{vv}\rangle_{\psi_+} &= g^2m^2 \int d\lambda \int d\lambda' c_\lambda c_{\lambda'}^* \frac{1}{(4\pi)^2} \\ &\quad \times (\tilde{n}_\lambda + \tilde{n}_{\lambda'} + 2) e^{-i(\lambda-\lambda')v} \\ &\simeq \frac{g^2m^2}{2\sqrt{\pi}}(N_m+1)e^{-v^2/T^2} \\ &\quad \times [1 - N_m\{\cos(2\pi v/aT^2) - 1\}]\end{aligned} \tag{48}$$

and the total Rindler energy radiated is, compared with Eq. (37),

$$\begin{aligned}\int dv \,\langle T_{vv}(v)\rangle_{\psi_+} &= \frac{g^2m^2}{4\pi}\int d\lambda |c_\lambda|^2(\tilde{n}_\lambda+1) \\ &\simeq \tfrac{1}{2}g^2m^2(N_m+1)T = mP_d.\end{aligned} \tag{49}$$

In the example for which the time-dependent coupling is $f(\tau) = \theta(\tau)\theta(T-\tau)$, the relation between the derivative of the probability, $\dot{P}_d(T)$, and the flux $\langle T_{vv}\rangle_{\psi_+}$ is

$$\langle T_{vv}(T)\rangle_{\psi_+} = +m\dot{P}_d(T) + \text{``damped'' term.} \tag{50}$$

Contrary to the sign in Eq. (43), the relative sign between $\langle T_{vv}(T)\rangle_{\psi_+}$ and $\dot{P}_d(T)$ is now positive: Deexcitation consists in emitting the energy stored in the atom. Similarly, the total Minkowski energy emitted is

$$\begin{aligned}\int_0^{+\infty} dV \,\langle T_{VV}\rangle_{\psi_+} &\simeq \frac{g^2m^2}{2}(N_m+1)Te^{a\tau_0}(2N_m+1) \\ &= mP_d e^{a\tau_0}(2N_m+1).\end{aligned} \tag{51}$$

For deexcitation, the integrated Rindler and Minkowski energies have the same sign and are related by the mean Doppler shift $e^{a\tau_0}$ times $(2N_m+1)$.

We now turn to the thermal equilibrium situation. The energy radiated is the weighted sum of the fluxes $\langle T_{vv}\rangle_{\psi_-}$ and $\langle T_{vv}\rangle_{\psi_+}$. This stems from the fact that the energy momentum operator changes the photon number by an even

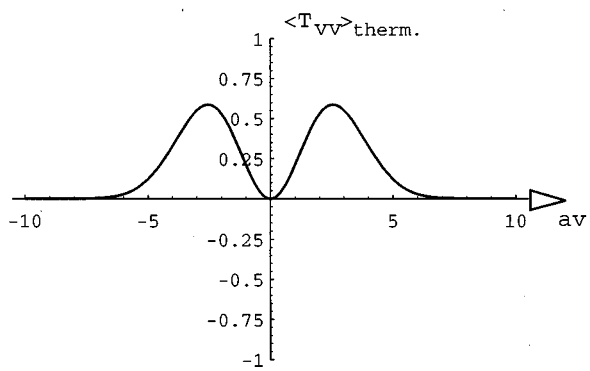

FIG. 3. The mean Rindler energy density ($\langle T_{vv}(v)\rangle_{\rm therm}$) emitted to order g^2 at thermal equilibrium is represented for $m=2a$ and $T=3a^{-1}$. v is given in units of a^{-1} and T_{vv} in arbitrary units since the flux is proportional to the coupling g. One sees the vanishing of the flux in the steady regime and the positivity of the transients. In the Minkowski description they are enhanced by the Jacobian dV/dv to make the total Minkowski energy emitted positive.

number and that the interaction Hamiltonian changes the photon number by an odd number while changing the state of the atom. Hence one has

$$\langle T_{vv}\rangle_{\rm therm}=P_-\,\langle T_{vv}\rangle_{\psi_-}+P_+\,\langle T_{vv}\rangle_{\psi_+}$$
$$\simeq -mP_-\dot P_e+mP_+\dot P_d=0. \tag{52}$$

The steady fluxes given in Eqs. (43) and (50) cancel each other exactly because at thermal equilibrium $P_\pm$ satisfy Eq. (15). This is Grove theorem to order g^2 [3,7]. Only the oscillatory transients remain. Using Eqs. (36) and (48) and $P_-+P_+=1$, they read

$$\langle T_{vv}\rangle_{\rm therm}=g^2m^2\int d\lambda\int d\lambda'\,\frac{c_\lambda c^*_{\lambda'}}{(4\pi)^2}\Bigg[\frac{N_m(\tilde n_\lambda+\tilde n_{\lambda'}+2)}{2N_m+1}$$
$$-\frac{(N_m+1)(\tilde n_\lambda+\tilde n_{\lambda'})}{2N_m+1}\Bigg]e^{-i(\lambda-\lambda')v}$$
$$\simeq\frac{g^2m^2}{\sqrt{4\pi}}N_m(N_m+1)e^{-v^2/T^2}[1-\cos(2\pi v/aT^2)]. \tag{53}$$

To illustrate these transients, we have plotted $\langle T_{vv}\rangle_{\rm therm}$ in Fig. 3. The total Rindler energy emitted is

$$\int_{-\infty}^{\infty}dv\,\langle T_{vv}\rangle_{\rm therm}=\frac{g^2m^2}{4\pi}\,\frac{1}{2N_m+1}\int d\lambda|c_\lambda|^2(N_m-\tilde n_\lambda)$$
$$\simeq\frac{g^2m^2}{2}N_m(N_m+1)\,\frac{\pi^2}{a^2T^2}. \tag{54}$$

It tends to zero as the time of interaction T tends to ∞, i.e., as c_λ tends to a δ function. In this limit, the two level atom tends to Lee model. This can be seen in Eq. (34) where the negative frequencies are exponentially suppressed.

However, the total Minkowski energy *increases* with the interaction time T and is given by

$$\int_0^{+\infty}dV\,\langle T_{VV}\rangle_{\rm therm}=P_-\int_0^{+\infty}dV\,\langle T_{VV}\rangle_{\psi_-}$$
$$+P_+\int_0^{+\infty}dV\,\langle T_{VV}\rangle_{\psi_+}\simeq m(P_-\dot P_e$$
$$+P_+\dot P_d)Te^{a\tau_0}(2N_m+1). \tag{55}$$

The Minkowski energy of the two fluxes coincide, by virtue of Eq. (15) and sum up. This result is what one might have ''naively'' guessed: The total energy is the integral over the interacting period of the weighted sum of the rates of transition times the varying Doppler shift times the energy gap m.

This is nicely confirmed by evaluating the total number of Minkowski quanta emitted by the atom. One has

$$\langle N\rangle_{\rm therm}=\int_0^\infty d\omega\,\langle a^\dagger_\omega a_\omega\rangle_{\rm therm}=\int_{-\infty}^\infty d\lambda\,\langle a^\dagger_{\lambda,M}a_{\lambda,M}\rangle_{\rm therm}$$
$$=\int_{-\infty}^\infty d\lambda g^2m^2\frac{|c_\lambda|^2}{4\pi\lambda}[\tilde n_\lambda P_-+(\tilde n_\lambda+1)P_+]$$
$$=T(P_-\dot P_e+P_+\dot P_d) \tag{56}$$

where $a_{\lambda,M}$ is the destruction operator associated to the Unruh mode equation (4). We have used the expression (33) for P_e in terms of c_λ and a similar expression for P_g. Equation (56) proves that the mean number of Minkowski quanta is *equal* to the mean number of transitions, i.e., the total duration of interaction times the weighted sum of the transition rates.

In Appendix B, we generalize these properties to all order in g in order to prove that the emission of Minkowski quanta we just found is not an artifact of the second order perturbation theory.

In Appendix C, we prove in full generality that the scattering of Rindler modes by an accelerated system leads inevitably to the *production* of Minkowski quanta. The key point lies in the noncommutativity of the scattering matrix with the matrix which describes the Bogoljubov transformation from Rindler modes to Minkowski modes.

In [9], upon taking into account the recoils of the atom induced by the transitions, it is proven that both Eqs. (55) and (56) perfectly hold. On the contrary, the local flux, $\langle T_{VV}\rangle_{\psi_-}$ is drastically modified since it no longer vanishes in the equilibrium regime.

III. THE CONDITIONAL VALUES OF $T_{\mu\nu}$

In Sec. II D, the mean energy radiated by the atom was decomposed into two contributions according to the final state of the two level atom. This decomposition was performed in the future of the atom's trajectory only, i.e., for $U>U_a(V)$, where $U_a(V)$ is the trajectory of the atom. In this part, we generalize this decomposition for *all* points (U,V) so as to obtain as well the energy density of the vacuum field configurations which are correlated to the final state of the atom. Then we prove the dynamical relevance of this generalized decomposition by considering a perturbation of the system treated quantum mechanically.

In Sec. III A, the generalized decomposition according to the final state is performed and the modified action is introduced. In Sec. III B, the resulting conditional values of the energy correlated to the transitions of the atom are calculated and interpreted.

A. The conditional energy correlated to a transition of the accelerated atom

In Eq. (17), the mean energy emitted to the left was written as

$$\langle T_{VV}(U,V)\rangle_{\psi_-} = P_e\langle T_{VV}(U,V)\rangle_e + P_g\langle T_{VV}(U,V)\rangle_g . \tag{57}$$

This decomposition was discussed for (U,V) in the future of the accelerated trajectory, $U>U_a(V)$, as well as in the left quadrant, for $V<0$, all U. On the basis of this decomposition and of the structure of the two terms to order g^2, we argued that $\langle T_{VV}\rangle_e$ ($\langle T_{VV}\rangle_g$) should be interpreted as the energy emitted if the atom has (has not) gotten excited.

We shall now generalize this decomposition to a form valid for all U,V rather than $U>U_a(V)$ only. To this end we introduce the projectors $\Pi_+ = |+\rangle\langle+|$ and $\Pi_- = |-\rangle\langle-|$ onto the excited and ground state of the atom. In order not to encumber the notation with exponents of $H_{\rm int}$ we shall work in Heisenberg representation rather than the interaction representation used so far. In this representation, the state of the system is $|\psi_-\rangle = |0_M\rangle|-\rangle$ and the projector is a time dependent operator given by

$$\Pi_+(t) = \exp\left(i\int_{-\infty}^{t} dt H_{\rm int}\right)\Pi_+ \exp\left(-i\int_{-\infty}^{t} dt H_{\rm int}\right). \tag{58}$$

The probability to be found in the excited state at $t=+\infty$ can then be written as

$$P_e = \langle\psi_-|\Pi_+(t=+\infty)|\psi_-\rangle. \tag{59}$$

The conservation of probability $P_e+P_g=1$ is realized through the completeness of the projectors $\Pi_+(t)+\Pi_-(t)=I$.

The conditional energies are now defined by decomposing the mean energy using the projectors $\Pi_\pm(t)$ at $t=\infty$:

$$\begin{aligned}\langle T_{VV}(U,V)\rangle_{\psi_-} &= \langle\psi_-|[\Pi_+(\infty)+\Pi_-(\infty)]T_{VV}(U,V)|\psi_-\rangle\\ &= P_e\frac{\langle\psi_-|\Pi_+(\infty)T_{VV}(U,V)|\psi_-\rangle}{\langle\psi_-|\Pi_+(\infty)|\psi_-\rangle}\\ &\quad + P_g\frac{\langle\psi_-|\Pi_-(\infty)T_{VV}(U,V)|\psi_-\rangle}{\langle\psi_-|\Pi_-(\infty)|\psi_-\rangle}\\ &= P_e\langle T_{VV}(U,V)\rangle_e + P_g\langle T_{VV}(U,V)\rangle_g .\end{aligned} \tag{60}$$

In the future of the accelerated trajectory, when $U>U_a(V)$, the explicit expression for $\langle T_{VV}[U>U_a(V),V]\rangle_e$, obtained by going back to interaction representation, is

$$\begin{aligned}\langle T_{VV}[U>U_a(V),V]\rangle_e &= \frac{1}{P_e}\langle\psi_-|\exp\left(i\int dt H_{\rm int}\right)\\ &\quad\times\Pi_+ T_{VV}(U,V)\\ &\quad\times\exp\left(-i\int dt H_{\rm int}\right)|\psi_-\rangle\\ &= \left(\frac{g^2m^2}{P_e}\right)(\partial_V\mathcal{C}_-)(\partial_V\mathcal{C}_-^*).\end{aligned} \tag{61}$$

The relative ordering of the evolution operator $\exp(-i\int dt H_{\rm int})$ and of Π_+ and T_{VV} is dictated by the fact that both Π_+ and T_{VV} act in the future of the accelerated trajectory (to which is confined $H_{\rm int}$). To order g^2, Eq. (61) coincides with the expression previously obtained in Eqs. (18) and (26).

In the past of the accelerated trajectory, when $U<U_a(V)$, these matrix elements are the desired expressions of the conditional energy if the atom shall be found at $t=+\infty$ in the excited (ground) state. This results from the fact that the decomposition, Eq. (60), *is exactly the same as in usual conditional probabilities*: to wit, the mean is expressed as the sum over possible outcomes of the probability for each outcome to be realized times the value of T_{VV} if that outcome is realized.

The explicit expression for $\langle T_{VV}[U<U_a(V),V]\rangle_e$ is

$$\begin{aligned}\langle T_{VV}[U<U_a(V),V]\rangle_e &= \frac{1}{P_e}\langle\psi_-|\exp\left(i\int dt H_{\rm int}\right)\Pi_+\\ &\quad\times\exp\left(-i\int dt H_{\rm int}\right)\\ &\quad\times T_{VV}(U,V)|\psi_-\rangle\\ &= \frac{g^2m^2}{P_e}\langle 0_M|\phi_m^\dagger\phi_m T_{VV}(U,V)|0_M\rangle\\ &= \frac{g^2m^2}{P_e}\partial_V\mathcal{C}_+^*(V)\partial_V\mathcal{C}_-^*(V),\end{aligned} \tag{62}$$

where in the second line we have given the expression valid to order g^2 and used Eq. (10) and Eq. (25). We emphasize that the difference between Eq. (61) and Eq. (62) lies in the relative order of T_{VV} and $\exp(i\int dt H_{\rm int})$. This ordering encodes the fact that T_{VV} in Eq. (61) is evaluated in the future of the trajectory while it is evaluated in the past in Eq. (62).

Two important properties of the conditional fluxes when it is evaluated in the past, for $U<U_a(V)$, should be noted. First

$$\langle T_{VV}[U<U_a(V),V]\rangle_e = -\frac{P_g}{P_e}\langle T_{VV}[U<U_a(V),V]\rangle_g \tag{63}$$

since the mean flux $\langle T_{VV}[U<U_a(V),V]\rangle_{\psi_-}$ vanishes identically (the interaction with the accelerated atom has not yet perturbed Minkowski vacuum).

Secondly, $\langle T_{VV}[U<U_a(V),V]\rangle_e$ is complex whereas $\langle T_{VV}[U>U_a(V),V]\rangle_e$ is real as can be seen from the explicit expressions (61) and (62). Note that the relative time ordering of T_{VV} and $H_{\rm int}$ ensures that the first is real whereas the second is complex.

Being complex when evaluated in the past of the atom's trajectory, the interpretation of $\langle T_{VV}\rangle_e$ requires some care. In what follows, we shall prove that both the real and imaginary part of $\langle T_{VV}\rangle_e$ intervene directly into dynamical processes and have therefore an intrinsic physical meaning. To this end, we shall perturb the action and introduce an additional quantum system coupled to the operator T_{VV}, following the approach of [14].

For definiteness, we take the additional system to be a quantum oscillator sitting at $x=x_0$ and coupled to T_{VV} by the interaction Hamiltonian

$$\int dtH_{\rm osc}=\int dtg^{VV}(t)p(t)T_{VV}(t,x_0) \tag{64}$$

where $p(t)$ is the momentum conjugate to the position $q(t)$ of the oscillator and $g^{VV}(t)$ is a switch function with the correct Lorentz variance, i.e., $g^{VV}T_{VV}$ is a scalar. When the initial state of the oscillator is $|{\rm osc}\rangle$, the state of the entire system (i.e., field + two level atom + oscillator) is simply the product $|\Psi_-\rangle=|\psi_-\rangle\,|{\rm osc}\rangle$.

We work in the interaction picture with respect to $H_{\rm osc}$ and we stay in the Heisenberg representation for the interaction between the field and the two level atom. Then to first order in $g^{VV}(t)$, the mean position of the oscillator is given by

$$\begin{aligned}\langle q(t)\rangle_{\Psi_-}=&\langle\Psi_-|\left(1+i\int_{-\infty}^{t}dt'H_{\rm osc}\right)q(t)\\&\times\left(1-i\int_{-\infty}^{t}dt'H_{\rm osc}\right)|\Psi_-\rangle\\=&\langle{\rm osc}|q(t)|{\rm osc}\rangle\\&-\int_{-\infty}^{t}dt'g^{VV}(t')\,\langle{\rm osc}\,|\,i[q(t),p(t')]_-\,|{\rm osc}\rangle\\&\times\langle T_{VV}(t',x_0)\rangle_{\psi_-}.\end{aligned} \tag{65}$$

That is, the *mean* change of the position is driven by the *mean* value of $T_{VV}(t,x_0)$ in the state $|\psi_-\rangle$. It corresponds to the response of $q(t)$ to a classical but fluctuating driving force. Notice that in this first order approximation there is no back reaction of the Hamiltonian $H_{\rm osc}$, Eq. (64), while computing $\langle T_{VV}(t,x_0)\rangle_{\psi_-}$.

But, one can also investigate the *correlations* among the oscillator state and the atom by asking more detailed questions such as: What is the position of the oscillator when the two level atom is found in its excited state? Exactly as in Eq. (60), the answer is the conditional value of q obtained by decomposing the mean position according to the final state of the atom at $t=\infty$

$$\langle q(t)\rangle_{\Psi_-}=P_e\langle q(t)\rangle_e+P_g\langle q(t)\rangle_g. \tag{66}$$

To first order in g^{VV}, the conditional value $\langle q(t)\rangle_e$ is given by

$$\begin{aligned}\langle q(t)\rangle_e=&\frac{\langle\Psi_-|\left(1+i\int dtH_{\rm osc}\right)\Pi_+(\infty)q(t)\left(1-i\int dtH_{\rm osc}\right)|\Psi_-\rangle}{\langle\Psi_-|\left(1+i\int dtH_{\rm osc}\right)\Pi_+(\infty)\left(1-i\int dtH_{\rm osc}\right)|\Psi_-\rangle}\\=&\langle{\rm osc}|q(t)|{\rm osc}\rangle-\int_{-\infty}^{t}dt'g^{VV}(t')\langle{\rm osc}|i[q(t),p(t')]_-|{\rm osc}\rangle\,{\rm Re}\langle T_{VV}(t',x_0)\rangle_e\\&+\int_{-\infty}^{t}dt'g^{VV}(t')\langle{\rm osc}|\{q(t),p(t')\}_+|{\rm osc}\rangle\,{\rm Im}\langle T_{VV}(t',x_0)\rangle_e.\end{aligned} \tag{67}$$

The conditional value $\langle T_{VV}\rangle_e$ is the source which drives the conditional value of the oscillator position. Both its real and imaginary part control the conditional position.[2] Note that ${\rm Re}\langle T_{VV}\rangle_e$ enters exactly in the same way in the integrals giving rise to $\langle q(t)\rangle_e$ as the mean value $\langle T_{VV}\rangle_{\psi_-}$ drove the mean $q(t)$ in Eq. (65). Instead, the imaginary part of $\langle T_{VV}\rangle_e$ appears in an unusual way through an anticommutator which depends explicitly on the state of the oscillator. Note also that the complex ''driving force'' of the conditional $\langle q(t)\rangle_e$ is the *normalized* matrix element of T_{VV}. This legitimates dynamically the decomposition in Eq. (60).

In quantum mechanics therefore, by coupling an additional system to the operator T_{VV}, one can isolate in a well-defined manner both the energy content of the particle correlated to a transition of the atom and the energy content of the vacuum fluctuations that shall induce the transition of the atom at later times. This procedure wherein an external quantum system is introduced to reveal the physical significance of matrix elements like $\langle T_{VV}\rangle_e$ is displayed in more details in Appendix C of Ref. [15] wherein it is put in parallel with the treatment of Aharonov *et al.* [14]. We shall use the same

[2]For the reader interested by these aspects, we note that this was not the case in the original work of Aharonov *et al.* since they considered the simplified case in which the free Hamiltonian of the oscillator vanishes. This corresponds to the large mass limit of our case. We hope to return to the new aspects brought in by this additional dependence.

procedure in the black hole situation for evaluating the conditional value of the metric correlated to a particular final state of the radiation, see [13].

B. The properties of the conditional energy

Having indicated by an example how both the real and imaginary parts of $\langle T_{VV}\rangle_e$ intervene in physical processes, we now display the properties of the conditional values.

By virtue of Eq. (63), we shall discuss $\langle T_{VV}\rangle_e$ only. In order to obtain exact expressions, we use again the Fourier components c_λ introduced in Eq. (34). We obtain three expressions for $\langle T_{VV}\rangle_e$. Three because the point (U,V) can be in the causal past of the atom's trajectory $V>0$, $U<U_a(V)$ or in its causal future $V>0$, $U>U_a(V)$, i.e., before or after the interaction occurs, or even in the causally disconnected region, $V<0$ all for U's:

$$\begin{aligned}\langle T_{vv}(U<U_a,V>0)\rangle_e &= \frac{g^2m^2}{P_{e,v}}\int d\lambda\int d\lambda' c_\lambda c^*_{\lambda'} \\ &\quad\times\frac{1}{(4\pi)^2}\tilde{n}_\lambda(\tilde{n}_{\lambda'}+1)e^{-i(\lambda-\lambda')v} \\ &= \frac{m(N_m+1)}{2\sqrt{\pi}TC_0}\left(1-\frac{iv+2\pi/a}{mT^2}\right) \\ &\quad\times\left(1+\frac{iv}{mT^2}\right)e^{-(v-i\pi/a)^2/T^2} \\ &\simeq \frac{m(N_m+1)}{2\sqrt{\pi}T}e^{-(v-i\pi/a)^2/T^2}, \qquad (68)\end{aligned}$$

$$\begin{aligned}\langle T_{vv}(U>U_a,V>0)\rangle_e &= \frac{g^2m^2}{P_{e,v}}\left|\int d\lambda c_\lambda \frac{1}{4\pi}\tilde{n}_\lambda e^{-i\lambda v}\right|^2 \\ &= \frac{mN_m}{2\sqrt{\pi}TC_0}\left|1-\frac{iv+2\pi/a}{mT^2}\right|^2 \\ &\quad\times e^{-v^2/T^2}e^{3\pi^2/a^2T^2} \\ &\simeq \frac{mN_m}{2\sqrt{\pi}T}e^{-v^2/T^2}, \qquad (69)\end{aligned}$$

$$\begin{aligned}\langle T_{v_Lv_L}(U,V<0)\rangle_e &= \frac{g^2m^2}{P_{e,v}}\left|\int d\lambda c_\lambda \frac{1}{4\pi}\tilde{n}_\lambda e^{\pi\lambda/a}e^{-i\lambda v_L}\right|^2 \\ &= \frac{m(N_m+1)}{2\sqrt{\pi}TC_0}\left|1-\frac{iv_L+\pi/a}{mT^2}\right|^2 e^{-v_L^2/T^2} \\ &\simeq \frac{m(N_m+1)}{2\sqrt{\pi}T}e^{-v_L^2/T^2}, \qquad (70)\end{aligned}$$

where $v_L=\theta(-V)a^{-1}\ln(-aV)$ is the Rindler coordinate in the left quadrant. The second equalities in Eqs. (68)–(70) give the exact expressions if c_λ is given by Eq. (34). The last equalities furnish the approximate expressions valid for $T\gg m^{-1}$, $T\gg a^{-1}$. In this limit $C_0\simeq 1$. We now present the complementary Rindler and Minkowski properties of these conditional values of T_{vv}. These functions are presented in Fig. 4.

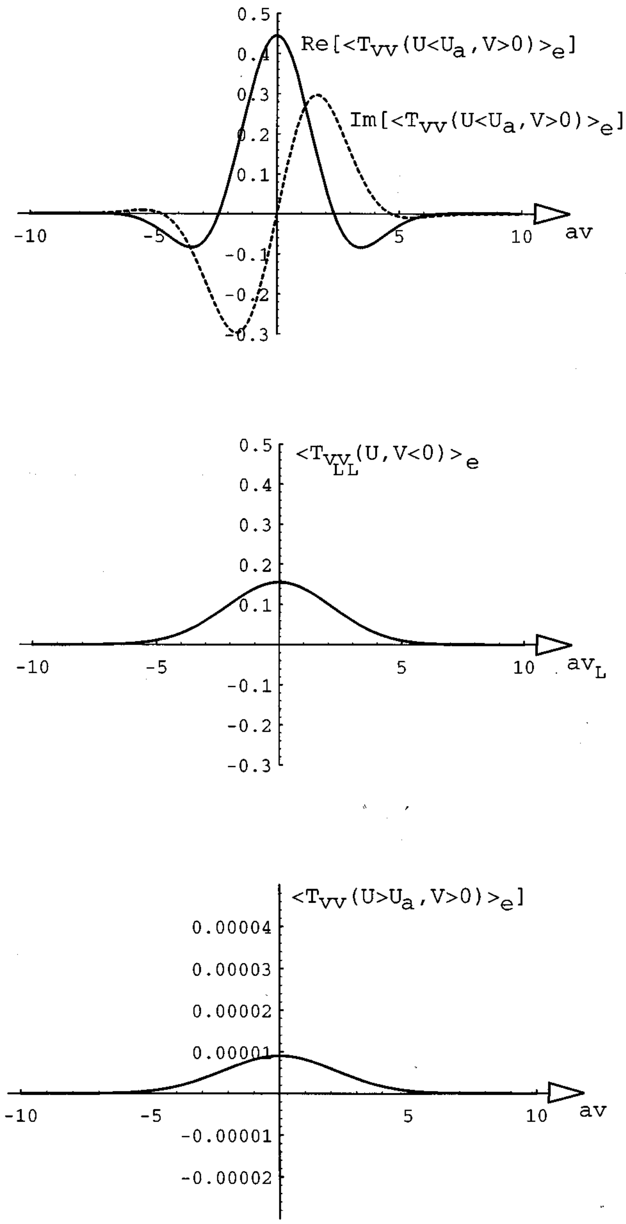

FIG. 4. The conditional value $\langle T_{vv}\rangle_e$ if the two level atom is initially in its ground state and ends up in its excited state. The parameters are the same as in Figs. 2 and 3: $m=2a$ and $T=3a^{-1}$. The v axis is given in units of a^{-1} and T_{vv} in units of a^2. For $U<U_a,V>0$, $\langle T_{vv}\rangle_e$ is complex and oscillates. The real part has a central positive bump which encodes that their is a rindleron carrying positive energy which will induce the transition of the atom. For $V<0$, $\langle T_{vv}\rangle_e$ is real and positive. It describes the partner of the rindleron which will be absorbed by the atom. The oscillations of $\langle T_{vv}(U<U_a,\ V>0)\rangle_e$ are such that the total Minkowski energy of the vacuum fluctuation vanishes. For $U>U_a$, $V>0$, $\langle T_{vv}\rangle_e$ is positive and of order N_m. In order to represent it we have had to change the vertical scale.

The Rindler description is that used by a uniformly accelerated observer in the same quadrant as the two level atom. It is best understood by making appeal to the isomorphism with an inertial thermal bath.

By specifying that the two level atom shall be found excited, one imposes that, in the past, for $U<U_a$, the thermal state contains at least one particle in the mode created by $\phi_m^\dagger$. Furthermore since energy flows along the lines $v=cst$, $\langle T_{vv}(U<U_a,V>0)\rangle_e$ is centered around $v=0$ with at spread $\Delta v=T$. It carries a Rindler energy obtained by integrating Eq. (68):

$$\int dv\, \langle T_{vv}(U<U_a,V>0)\rangle_e = \frac{\int d\lambda |c_\lambda|^2 \tilde{n}_\lambda(\tilde{n}_\lambda+1)}{\int d\lambda |c_\lambda|^2 (1/\lambda)\tilde{n}_\lambda} \simeq m(N_m+1). \tag{71}$$

The factor N_m+1 takes correctly into account the Bose statistics of the field since Eq. (71) corresponds to evaluating $\langle n^2\rangle/\langle n\rangle$ in a thermal distribution.

When $U>U_a$, the two level atom has absorbed *one* quantum and the residual energy is [see Eq. (69)]

$$\int dv\, \langle T_{vv}(U>U_a,V>0)\rangle_e = \frac{\int d\lambda |c_\lambda|^2 \tilde{n}_\lambda^2}{\int d\lambda |c_\lambda|^2 (1/\lambda)\tilde{n}_\lambda} \simeq m N_m . \tag{72}$$

We now consider what is ''seen'' by a uniformly accelerated in the left Rindler quadrant, $V<0$, i.e., what is the nature of the correlations between the transition of the atom and an additional system uniformly accelerated in the left Rindler quadrant. Because of the strict correlations between the left and right quadrants in Minkowski vacuum, to the (N_m+1) Rindler quanta present in the past on the right, correspond (N_m+1) Rindler quanta on the left. Indeed, since Minkowski vacuum $|0_M\rangle$ is annihilated by the boost generator $H_R=\int dV aV T_{VV}$, the Rindler energy in the left quadrant is equal to the energy in the right quadrant. This can be verified by integrating Eq. (70) and using the relation $\tilde{n}_\lambda(\tilde{n}_\lambda+1)=\tilde{n}_\lambda^2 e^{2\pi\lambda/a}$.

Furthermore, the symmetry between the left and the right Rindler quadrants results in $\langle T_{v_L v_L}\rangle_e$ being centered around $v_L=0$ with the same width $\Delta v_L=T$. Thus $\langle T_{v_L v_L}\rangle_e$ is almost exactly the symmetric of $\langle T_{vv}(U<U_a,V>0)\rangle_e$ except for small transient oscillations present for $V>0$, see the explicit expressions (68) and (70) and Fig. 4. Notice however that $\langle T_{v_L v_L}\rangle_e$ is real whereas $\langle T_{vv}(U<U_a,V>0)\rangle_e$ is complex. This results from causality and can be proven in complete generality by making appeal to a reasoning similar to that in Eq. (22). This has important consequences in the black hole problem, see [13].

The Minkowski description, i.e., that used by an inertial observer, is best understood by rewriting the conditional value of T_{VV} in terms of the $\varphi_{\lambda,M}(V)$ modes, Eq. (4).

For $U<U_a(\tau)$ and all V, one finds

$$\begin{aligned}\langle T_{VV}(U<U_a,V)\rangle_e &= \frac{1}{a^2V^2}\frac{g^2m^2}{P_e}\int d\lambda \int d\lambda' c_\lambda^* c_{\lambda'} \frac{1}{4\pi} \\ &\quad\times \sqrt{\lambda\lambda' \tilde{n}_{\lambda'}(\tilde{n}_\lambda+1)}\,\varphi_{\lambda,M}^*\varphi_{-\lambda',M}^* \\ &= \frac{1}{a^2V^2}\frac{m(N_m+1)}{2\sqrt{\pi}TC_0}\left(1+\frac{i}{maT^2}\right. \\ &\quad\left.\times \ln(-aV-i\epsilon)-\frac{\pi}{maT^2}\right)\left(1-\frac{i}{maT^2}\right. \\ &\quad\left.\times \ln(-aV-i\epsilon)-\frac{\pi}{maT^2}\right) \\ &\quad\times e^{-[\ln(-aV-i\epsilon)]^2/a^2T^2}. \end{aligned} \tag{73}$$

The $i\epsilon$ defines $\ln(-aV-i\epsilon)$ as $\ln|aV|$ for $V<0$ and as $\ln|aV|-i\pi$ for $V>0$. In the limit $\epsilon\to 0$ at fixed T, $\langle T_{VV}(U,V)\rangle_e$ stays finite. The $i\epsilon$ prescription controls also the analyticity of the modes $\varphi_{\lambda,M}$, Eq. (4), in the lower half complex plane which in turn leads to the vanishing of the integral

$$\int_{-\infty}^{+\infty} dV\, \langle T_{VV}(U<U_a,V)\rangle_e = 0 \tag{74}$$

by contour integration. This reflects the fact that $|0_M\rangle$ is the ground state of H_M. In other words, vacuum fluctuations carry no energy. Finally, the $i\epsilon$ controls the above mentioned slight asymmetry between the left and right quadrants: $\langle T_{VV}(U<U_a,V)\rangle_e$ is real and positive for $V<0$ whereas it is complex and oscillates for $V>0$.

In view of the vanishing of the total Minkowski energy and of the positivity in the region $V<0$, the energy in the region $V>0$ must integrate to an exactly compensating real and negative value. This is not in contradiction with the positivity of Rindler energy in the right quadrant, Eq. (71), since the expressions for the Rindler and the Minkowski energy differ by the Jacobian $dv/dV=1/aV$. The oscillations of T_{vv} for $V>0$ that occur in Eq. (68) as $v\to-\infty$ are negligible in the Rindler description but are dramatically enhanced by the Jacobian in such a way that the Minkowski energy in the right quadrant becomes negative, c.f. Eq. (39).

For $U>U_a$, all V, after the atom has made a transition, the conditional Minkowski energy takes the form

$$\begin{aligned}\langle T_{VV}(U>U_a,V)\rangle_e &= \frac{1}{a^2V^2}\frac{g^2m^2}{P_{e,v}}\left|\int d\lambda\, c_\lambda \sqrt{\frac{\lambda\tilde{n}_\lambda}{4\pi}}\varphi_{-\lambda,M}^*\right|^2 \\ &= \frac{1}{a^2V^2}\frac{m(N_m+1)}{2\sqrt{\pi}TC_0} \\ &\quad\times\left|1-\frac{i}{maT^2}\ln(-aV-i\epsilon)-\frac{\pi}{maT^2}\right|^2 \\ &\quad\times\left|e^{-[\ln(-aV-i\epsilon)]^2/a^2T^2}e^{-im\ln(-aV-i\epsilon)/a}\right|^2. \end{aligned} \tag{75}$$

It is real and positive because we are calculating the mean energy density in a state that contains one Minkowski quantum. Then the integral $\int dV\,\langle T_{VV}(U>U_a,V)\rangle_e$ is strictly positive, cf. Eq. (23). Notice also how the $i\epsilon$ prescription in

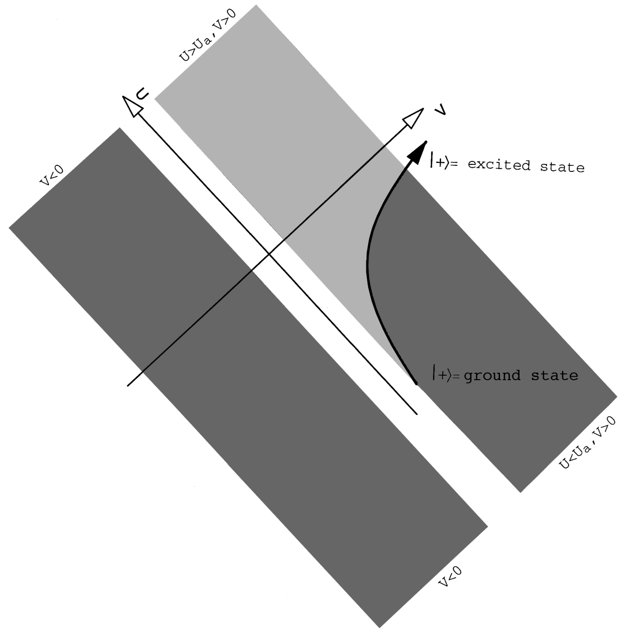

FIG. 5. A schematic picture of the energy fluxes $\langle T_{vv}\rangle_e$. We have represented in dark grey the regions where $\langle T_{vv}\rangle_e$ is $O(N_m+1)$ and in light grey the regions where it is $O(N_m)$.

Eq. (75) encodes the asymmetry between the left quadrant (proportional to N_m+1) and the right quadrant (proportional to N_m).

The interaction with the accelerated atom transforms the field configurations in such a way that the conditional value Eq. (73) which was complex and carried no energy in the past, becomes real and carries positive energy. Therefore, one can say that by absorbing the positive Rindler energy m, the two level atom has reduced the negative Minkowski energy on the right and has converted a vacuum fluctuation into a quantum. This conversion is summarized in Fig. 5. It is worth pointing out that this conclusion was anticipated by Unruh and Wald [2] on the basis of their analysis of $\langle T_{VV}(U>U_a,V)\rangle_e$. Indeed in the last paragraph of their article they state: ''But our analysis suggests a rather surprising viewpoint on this radiation process: it seems as though the detector is excited by swallowing part of the vacuum fluctuation of the field in the region of spacetime containing the detector. This liberates the correlated fluctuation in a noncausally related region of the spacetime to become a real particle.'' By introducing the notion of generalized conditional values, we have shown in this section how to give precise physical meaning to Unruh and Wald's qualitative picture.

C. Conclusions

The main properties of the conditional values of the energy distribution correlated to an excitation of the atom are the following.

(1) Owing to the free character of the propagation of the massless ϕ field everywhere but on the accelerated trajectory, the conditional values of the energy density form a pattern which extends through all space, from past null infinity to future null infinity.

(2) In the past infinity, the total conditional Minkowski energy, i.e., the integral of the density on a Cauchy surface, vanishes identically: vacuum configurations carry no energy. This can be deduced from the fact that the total energy does not fluctuate in Minkowski vacuum even though the density does so.

(3) In the past infinity, the total conditional Rindler energy vanishes as well. This is due to the fact that Minkowski vacuum is an eigenstate of the boost operator with zero eigenvalue.

(4) In the future infinity, the conditional energy density is real, contrariwise to what happens in the past. It encodes a positive conditional Minkowski energy, since a Minkowski quantum has been produced, but a negative Rindler energy since a Rindler quantum has been absorbed in the right sector.

Because of the close formal and physical analogies between black hole radiation and the Unruh process, the conditional values of the energy density correlated to the emission of a quantum by a black hole present similar properties. This is the subject of the next article [13]. In that paper, the gravitational back reaction to black hole evaporation engendered by these conditional energy fluxes are also discussed.

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APPENDIX A: THE $\mathcal{D}$ TERM

We recall that this term arises from the following decomposition of the second g^2 Born term in $|\psi_-(t=+\infty)\rangle = \exp(-i\int dtdxH_{\rm int})\,|0_M\rangle\,|-\rangle$, see Eq. (10):

$$\begin{aligned}
-g^2m^2\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{\tau}d\tau' f(\tau)e^{-im\tau}\phi(\tau)f^*(\tau')e^{+im\tau'}\phi(\tau')\\
\times|0_M\rangle\,|-\rangle\\
=-\frac{g^2m^2}{2}\int_{-\infty}^{+\infty}d\tau\int_{-\infty}^{+\infty}d\tau' f(\tau)f^*(\tau')e^{-im(\tau-\tau')}\\
\times\phi(\tau)\phi(\tau')[1+\epsilon(\tau-\tau')]\,|0_M\rangle\,|-\rangle\\
=-\frac{g^2m^2}{2}[\phi_m^\dagger\phi_m+\mathcal{D}]\,|0_M\rangle\,|-\rangle \qquad \text{(A1)}
\end{aligned}$$

where

$$\begin{aligned}
\mathcal{D}=\int_{-\infty}^{+\infty}d\tau_2\int_{-\infty}^{+\infty}d\tau_1 f(\tau_2)f^*(\tau_1)\epsilon(\tau_2-\tau_1)\\
\times e^{-im(\tau_2-\tau_1)}\phi(\tau_2)\phi(\tau_1) \qquad \text{(A2)}
\end{aligned}$$

and where $\epsilon(\tau_2-\tau_1)=\theta(\tau_2-\tau_1)-\theta(\tau_1-\tau_2)$.

To explicitize the role of the $\mathcal{D}$ term, it is appropriate to compute the energy density carried by it when the initial state is $|0_M\rangle\,|-\rangle$. One finds, to order g^2,

$$\langle T_{VV}(V)\rangle_{\mathcal{D}}=-g^2m^2\ \mathrm{Re}[\ \langle 0_M|T_{VV}(V)\mathcal{D}\,|0_M\rangle]$$
$$=-\frac{g^2m^2}{2}\langle 0_M|[T_{VV}(V),\mathcal{D}]_-|0_M\rangle \quad \text{(A3)}$$

where we have used the anti-Hermitian property of $\mathcal{D}$: $\mathcal{D}^\dagger=-\mathcal{D}$.

$\langle T_{VV}(V)\rangle_{\mathcal{D}}$ enjoys the following properties.

(1) Being a commutator, $\langle T_{VV}(V)\rangle_{\mathcal{D}}$ is causal [see Eq. (22)], and vanishes in the left quadrant $V<0$ contrary to $\langle T_{VV}(V)\rangle_e$ and $\langle T_{VV}(V)\rangle_g$.

(2) $\langle T_{VV}(V)\rangle_{\mathcal{D}}$ carries no Minkowski energy since the Hamiltonian H_M [see the paragraph after Eq. (23)], annihilates Minkowski vacuum.

(3) $\langle T_{VV}(V)\rangle_{\mathcal{D}}$ carries no Rindler energy since H_R [the boost generator —see the paragraph after Eq. (72)] annihilates Minkowski vacuum. Therefore by virtue of 1, the Rindler energy in the right quadrant $(V>0)$ vanishes:

$$\int_{-\infty}^{+\infty}dv\ \langle T_{vv}(v)\rangle_{\mathcal{D}}=0. \quad \text{(A4)}$$

Thus $\langle T_{vv}(v)\rangle_{\mathcal{D}}$ is, at most, an energy density repartition.

(4) When $f(\tau)=1$ for all τ, $\langle T_{vv}(v)\rangle_{\mathcal{D}}$ vanishes identically. To prove this one evaluates the commutator in Eq. (A3) and one finds

$$\langle T_{vv}(v)\rangle_{\mathcal{D}}=2g^2m^2\int_{-\infty}^{+\infty}d\tau_2\epsilon(\tau_2-v)\ \mathrm{Re}[f(\tau_2)$$
$$\times f^*(v)e^{-im(\tau_2-v)}\langle 0_M|\phi(\tau_2)i\partial_v\phi(v)\ |0_M\rangle] \quad \text{(A5)}$$

where we have used the commutation relation

$$[T_{vv}(v),\phi(\tau_2)\phi(\tau_1)]_-=-2i\,\delta(\tau_1-v)\phi(\tau_2)\partial_v\phi(v)$$
$$-2i\,\delta(\tau_2-v)\partial_v\phi(v)\phi(\tau_1) \quad \text{(A6)}$$

and the antisymmetric character of $\epsilon(\tau_2-\tau_1)$. Since the expectation value $\langle 0_M|\phi(\tau_2)\phi(v)|0_M\rangle$ is evaluated along the accelerated trajectory equation (6), it is a function of τ_2-v only. Therefore the integrand of Eq. (A5) is an odd function of τ_2-v and the integral vanishes. Hence $\langle T_{vv}(v)\rangle_{\mathcal{D}}$ is an energy repartition which is concerned only with the transients induced by the switch on and off effects.

(5) When $f(\tau)$ is a slowly varying function with respect to both $1/m$ and $1/a$ [cf. the discussion associated with Eq. (31)], $\langle T_{vv}(v)\rangle_{\mathcal{D}}$ is smaller than the contribution of $\mathrm{Re}[\langle T_{vv}(v)\phi_m\phi_m^\dagger\rangle]$ by a factor $1/aT$ except near the edges of the interaction period where $f(\tau)$ almost vanishes. This can be seen by developing $f(\tau_2)$ given in Eq. A5 in a series around $\tau_2=v$ and evaluating the magnitude of the first non-vanishing term, i.e., one treats the variations of the switch off function $f(\tau)$ as an adiabatic effect. One finds that indeed the $\mathcal{D}$ is smaller than $\mathrm{Re}[\langle T_{vv}(v)\phi_m\phi_m^\dagger\rangle]$ except when $\tau>aT^2$.

APPENDIX B: FLUXES TO ALL ORDER IN *g*

We use the exactly solvable model, used by Raine, Sciama, and Grove (RSG) [4–7], to prove that one does recover, to all order in g, that every quantum jump of the accelerated oscillator leads to the emission of a Minkowski quantum even when the oscillator has reached the stationary state characterized by the Unruh temperature. Hence, the rate of production of the Minkowski quanta is simply the rate of the thermal internal transitions of the oscillator. But, as in second order perturbation theory, these quanta interfere and their energy content is found at the edges of the interacting period only. This is due to the complete neglection of the recoils of the oscillator. Indeed upon taking into account the recoils by giving the oscillator a finite mass and by quantizing the position of its center of mass, one proves that the Minkowski quanta no longer interfere after a short time (a few $1/a$) [9].

We first recall the main properties of the RSG model and then analyze the particle content of the emitted fluxes. The system consists of a massless field coupled to a harmonic oscillator maintained in constant acceleration. Its action is

$$S=\int dtdx\left[\frac{1}{2}[(\partial_t\phi)^2-(\partial_x\phi)^2]+\int d\tau\left[\frac{1}{2}[(\partial_\tau q)^2-m^2q^2]\right.\right.$$
$$\left.\left.+e(\partial_\tau q)\phi\right]\delta^2(X^\mu-X^\mu_a(\tau))\right] \quad \text{(B1)}$$

where $X^\mu(\tau)$ is the accelerated trajectory equation (6) and $e=g\sqrt{2m}$ is a rescaled coupling constant. Since this action is quadratic, the Heisenberg equations are identical to the classical Euler Lagrange ones. They read

$$\partial_u\partial_v\phi=\frac{e}{4}\theta(V)\delta(\rho-1/a)\partial_\tau q, \quad \text{(B2)}$$

$$\partial_\tau^2q+m^2q=-e\partial_\tau\phi(X^\mu(\tau)). \quad \text{(B3)}$$

The left part of the field (i.e., for $V<0$) is, by causality, identically free. And, for $V>0$, on the left of the accelerated oscillator trajectory, the v-part of the field only is scattered. There the general solution is

$$\tilde{\phi}(u,v)=\phi(u)+\phi(v)+\frac{e}{2}\tilde{q}(v), \quad \text{(B4)}$$

$$\tilde{q}(v)=q(v)+i\int_{-\infty}^{+\infty}d\lambda\,\psi_\lambda e^{-i\lambda v}[\phi_{\lambda,R,v}+\phi_{\lambda,R,u}], \quad \text{(B5)}$$

where $\phi(u)$ and $\phi(v)$ are the homogeneous free solutions of Eq. (B2); where the operator $\phi_{\lambda,R,v}$ is defined by

$$\phi_{\lambda,R,v}=\int\frac{dv}{2\pi}e^{i\lambda v}\phi(v)=\frac{1}{\sqrt{4\pi|\lambda|}}[\theta(\lambda)a_{\lambda,R}$$
$$+\theta(-\lambda)a^\dagger_{-\lambda,R}] \quad \text{(B6)}$$

(a similar equation defines $\phi_{\lambda,R,u}$); where ψ_λ is given by

$$\psi_\lambda = \frac{e\lambda}{m^2 - \lambda^2 - ie^2\lambda/2} \tag{B7}$$

and where $q(v)$ is a solution of

$$\partial_\tau^2 q + m^2 q + \frac{e^2}{2}\partial_\tau q = 0. \tag{B8}$$

The two independent solutions of Eq. (B8) are exponentially damped as τ increases. Being interested by the properties at equilibrium, we drop $q(v)$ from now on. Then, the remaining part of $\tilde{q}(v)$ is a function of the free field only. Hence, in Fourier transform, Eq. (B4) reads

$$\tilde{\phi}_{\lambda,R,u} = \phi_{\lambda,R,u},$$

$$\tilde{\phi}_{\lambda,R,v} = \phi_{\lambda,R,v}\left(1 + i\frac{e}{2}\psi_\lambda\right) + \left(i\frac{e}{2}\psi_\lambda\right)\phi_{\lambda,R,u}. \tag{B9}$$

The second term in Eq. (B9) mixes u and v modes. It encodes the static Rindler polarization cloud (see [5–7]) which accompanies the oscillator and carries neither Minkowski nor Rindler energy. In order to simplify the following equations, we drop it and multiply the other scattered term by two for unitary reason—see below. (By a simple algebra, one can explicitly verify that this modification does not affect the main properties of the emitted fluxes.) Then Eq. (B9) becomes

$$\tilde{\phi}_{\lambda,R,v} = \phi_{\lambda,R,v}(1 + ie\psi_\lambda). \tag{B10}$$

It is useful, for future discussions, to introduce explicitly the scattered operators $\tilde{a}_{\lambda,R}$, and the scattered modes $\tilde{\varphi}_{\lambda,R}(v)$,

$$\tilde{a}_{\lambda,R} = \langle \varphi_{\lambda,R} | \tilde{\phi} \rangle = a_{\lambda,R}(1 + ie\psi_\lambda), \tag{B11}$$

$$\tilde{\varphi}_{\lambda,R}(v) = -[a^\dagger_{\lambda,R}, \tilde{\phi}(v)]_- = (1 + ie\psi_\lambda)\varphi_{\lambda,R}(v) \tag{B12}$$

whereupon the scattered field operator $\tilde{\phi}(v)$ may be written as

$$\tilde{\phi}(v) = \int_0^\infty d\lambda [\tilde{a}_{\lambda,R}\varphi_{\lambda,R} + \text{H.c.}] = \int_0^\infty d\lambda [a_{\lambda,R}\tilde{\varphi}_{\lambda,R} + \text{H.c.}]. \tag{B13}$$

It is now straightforward to obtain the scattered Green function. If the initial state is Minkowski vacuum, the v part of the scattered Green function is, for $V, V' > V_a(U)$,

$$\begin{aligned}\tilde{G}_+(v,v') &= \langle 0_M | \tilde{\phi}(v)\tilde{\phi}(v') | 0_M \rangle \\ &= \int_0^\infty d\lambda |1 + ie\psi_\lambda|^2 \varphi_{\lambda,M}(v)\varphi^*_{\lambda,M}(v') \\ &= G_+(v,v'),\end{aligned} \tag{B14}$$

where we have used Eq. (4). $G_+(v,v')$ is the (unperturbed) Minkowski Green function and we have availed ourselves of the identity [see Eq. (B7)]

$$|1 + ie\psi_\lambda|^2 = 1. \tag{B15}$$

This unitary relation expresses the conservation of the number of Rindler particles. Indeed there is no mixing of positive and negative frequencies in Eq. (B11); in other words, the β term of the ''Bogolyubov'' transformation equation (B11) vanishes. The identity of the Green functions, Eq. (B14), proves that, once the steady regime is established, the mean flux vanishes, see Refs. [4,7] for more details.

We now examine how this stationary scattering of Rindler modes is perceived in Minkowski terms. The Minkowski scattered modes $\tilde{\varphi}_{\lambda,M}$ are defined by

$$\begin{aligned}\tilde{\varphi}_{\lambda,M} &= [\tilde{\phi}(V), a^\dagger_{\lambda,M}]_- \\ &= \varphi_{\lambda,M}(1 + ie\alpha_\lambda^2\psi_\lambda) - ie\alpha_\lambda\beta_\lambda\psi_\lambda\varphi^*_{-\lambda,M} \\ &= \tilde{\alpha}_\lambda\varphi_{\lambda,M} + \tilde{\beta}_\lambda\varphi^*_{-\lambda,M},\end{aligned} \tag{B16}$$

$$\begin{aligned}\tilde{\varphi}_{-\lambda,M} &= [\tilde{\phi}(V), a^\dagger_{-\lambda,M}]_- \\ &= \varphi_{-\lambda,M}(1 - ie\beta_\lambda^2\psi_{-\lambda}) - ie\alpha_\lambda\beta_\lambda\psi_{-\lambda}\varphi^*_{\lambda,M} \\ &= \tilde{\alpha}_{-\lambda}\varphi_{-\lambda,M} + \tilde{\beta}_{-\lambda}\varphi^*_{\lambda,M},\end{aligned} \tag{B17}$$

where $\lambda > 0$ and where we have introduced the scattered Bogolyubov coefficients:

$$\tilde{\alpha}_\lambda = 1 + ie\alpha_\lambda^2\psi_\lambda, \quad \tilde{\alpha}_{-\lambda} = 1 + ie\beta_\lambda^2,$$

$$\tilde{\beta}_\lambda = -ie\alpha_\lambda\beta_\lambda\psi^*_\lambda, \quad \beta_{-\lambda} = -ie\alpha_\lambda\beta_\lambda\psi_\lambda. \tag{B18}$$

One verifies that the unitary relation is satisfied: $|\tilde{\alpha}_\lambda|^2 - |\tilde{\beta}_\lambda|^2 = 1$. The fact that the $\tilde{\beta}$ are different from zero indicates that each couple of jumps of the oscillator, i.e., the absorption and subsequent emission of a Rindler quantum, leads, in Minkowski vacuum, to the production of two Minkowski quanta. The member $\varphi_{-\lambda,M}$ is emitted when the oscillator absorbs a rindleron and jumps into a higher level and the other one, $\varphi_{\lambda,M}$ is emitted during the inverse process. This is apparent in the mean energy flux

$$\begin{aligned}\langle \tilde{T}_{VV} \rangle &= \lim_{V' \to V} \partial_V \partial_{V'} \langle [\tilde{\phi}(V)\tilde{\phi}(V') - \phi(V)\phi(V')] \rangle \\ &= 2\int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_\lambda|^2 |\partial_V\varphi_{\lambda,M}|^2 \\ &\quad + \text{Re}[\tilde{\alpha}_\lambda\tilde{\beta}^*_\lambda \partial_V\varphi_{\lambda,M}\partial_V\varphi_{-\lambda,M}]\end{aligned} \tag{B19}$$

whereupon the total Minkowski energy is

$$\langle \tilde{H}_M \rangle = \int_0^{+\infty} d\lambda \lambda (|\tilde{\beta}_\lambda|^2 + |\tilde{\beta}_{-\lambda}|^2) \int_{-\infty}^{+\infty} \frac{dV}{2\pi} \frac{1}{a^2|V + i\epsilon|^2} \tag{B20}$$

since the integral of the second term of $\langle \tilde{T}_{VV} \rangle$ vanishes.

Exactly as in second order perturbation theory, there is a steady regime during which all the emitted quanta interfere destructively leaving no contribution to the *mean* flux [see Eq. (B14)]. But all nondiagonal matrix elements will be sensitive to the created pairs. This is also the case for the the total energy equation (B20) since being diagonal in ω it ignores the destructive interferences. The second term of Eq. (B19) whose role is to make the mean flux vanishing during the steady regime gives no contribution to $\langle \tilde{H}_M \rangle$.

In order to prove that Eq. (B20) corresponds to a steady production of Minkowski quanta during the whole interacting period $\Delta\tau = T$ [infinite in Eq. (B20)] we evaluate how many quanta are produced. Contrary to the energy, the total number of Minkowski quanta is a scalar under the Lorentz group, hence not affected by the exponentially growing Doppler shift present in the energy:

$$\begin{aligned}\langle \tilde{N}(\Delta\tau) \rangle &= \int_0^{+\infty} d\omega \ \langle \tilde{0}_M | a_\omega^\dagger a_\omega | \tilde{0}_M \rangle \\ &= \int_0^{+\infty} d\omega \ \langle 0_M | \tilde{a}_\omega^\dagger \tilde{a}_\omega | 0_M \rangle \\ &= \int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} d\lambda |\gamma_{\lambda,\omega}(\Delta\tau)|^2 |\tilde{\beta}_\lambda|^2 \end{aligned} \quad \text{(B21)}$$

where $|\tilde{0}_M>$ is the scattered (Schrödinger) state.[3] As in Eq. (4), the $\tilde{a}_\omega$ are related to the $\tilde{a}_{\lambda,M}$ by

$$\tilde{a}_\lambda = \int_0^\infty d\lambda \, \gamma_{\lambda,\omega}(\Delta\tau) \tilde{a}_{\omega,M} \quad \text{(B22)}$$

where $\gamma_{\lambda,\omega}(\Delta\tau)$ takes into account the time dependence of the coupling. As shown in [21,15], $\gamma_{\lambda,\omega}(\Delta\tau)$ is nonvanishing only for the ω which enter into resonance with the oscillator frequency m during the interaction period $\tau_i < \tau < \tau_f = \tau_i + T$. When these frequencies belong to

$$\omega_i = m e^{-a\tau_i} < \omega < m e^{-a\tau_f} = \omega_f \quad \text{(B23)}$$

$\gamma_{\lambda,\omega}(\Delta\tau)$ may be replaced by $\gamma_{\lambda,\omega}$ [given in Eq. (5)]. Hence $\tilde{N}(\Delta\tau)$ reads

$$\langle \tilde{N}(\Delta\tau) \rangle = \int_{\omega_i}^{\omega_f} \frac{d\omega}{2\pi a \omega} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_\lambda|^2 = \frac{T}{2\pi} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_\lambda|^2. \quad \text{(B24)}$$

The total energy emitted obtained from Eq. (B24) is

$$\begin{aligned}\langle \tilde{H}_M(\Delta\tau) \rangle &= \int_{\omega_i}^{\omega_f} \frac{d\omega}{2\pi a} \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_\lambda|^2 \\ &= \int_{\tau_i}^{\tau_f} \frac{d\tau}{2\pi} e^{-a\tau} m \int_{-\infty}^{+\infty} d\lambda |\tilde{\beta}_\lambda|^2, \end{aligned} \quad \text{(B25)}$$

in perfect agreement with Eq. (B20) if the frequency width of the oscillator in small compared to m. In that case, the rate of production, Eq. (B24) divided by T, is $e^2 \alpha_m^2 \beta_m^2$. This is the rate of jumps for an inertial oscillator in a bath at temperature $a/2\pi$, exactly as in Eq. (56). Therefore, to all order in g as well, the number of Minkowski quanta produced by the thermalized oscillator equals the number of internal jumps.

[3]The simplest way to obtain this state is to find the scattering operator U such that $\tilde{a}_{\lambda,M} = U^\dagger a_{\lambda,M} U$ where $\tilde{a}_{\lambda,M} = \langle \varphi_{\lambda,M} | \tilde{\phi} \rangle$. Then $|\tilde{0}_M\rangle = U|0_M\rangle$.

APPENDIX C: THE QUANTA EMITTED BY ACCELERATED SYSTEMS

We generalize the results of Sec. II D and Appendix B by proving that, for any accelerated system coupled to the radiation in such a way that the scattered radiation modes are linearly related to the ingoing modes, the elastic character of the scattering process in the accelerated frame, i.e., the absence of creation of Rindler quanta, implies a production of Minkowski quanta. This general proof is therefore applicable to the accelerated mirror considered by Davies and Fulling [23], as explained in [24], as well as to accelerated black holes [10,11], see the footnote in the Introduction. In addition, we believe that it can be further generalized, using the same type of argumentation, to nonlinear scattering processes.

The proof goes as follow. Any linear scattering of Rindler quanta by an accelerated system which does not lead to the production of Rindler quanta can be described, as in Eq. (B11), by

$$\tilde{a}_{\lambda,R} = S_{\lambda\lambda'} a_{\lambda',R}, \quad \text{(C1)}$$

where repeated indices are summed over and where the summation over λ' includes both u and v modes as in Eq. (B9). The matrix S satisfy the unitary relation

$$S_{\lambda\lambda''} S^\dagger_{\lambda''\lambda'} = \delta_{\lambda\lambda'} \quad \text{(C2)}$$

which express the conservation of the number of Rindler quanta: $S_{\lambda\lambda'}$ mixes positive Rindler frequencies only. It is convenient to introduce the matrix T (from now on we do not write the indices)

$$S = 1 + iT \quad \text{(C3)}$$

which satisfies

$$2 \, \mathrm{Im} T = T T^\dagger. \quad \text{(C4)}$$

We introduce also the vector operator $b = (a_{\lambda,R}; a_{\lambda,L}; a^\dagger_{\lambda,R}; a^\dagger_{\lambda,L})$. Then Eq. (C1) can be written as

$$\tilde{b} = \mathcal{S} b \quad \text{(C5)}$$

where $\mathcal{S}$ has the block structure

$$\mathcal{S} = \begin{pmatrix} 1+iT & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1-iT^\dagger & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{(C6)}$$

since the u and v modes on the left quadrant are still free.

On the other hand, the Bogolyubov transformation which relates Minkowski and Rindler quanta reads in this notation

$$c=\mathcal{B}b \tag{C7}$$

where $c=(a_{\lambda,M};a_{-\lambda,M};a^{\dagger}_{\lambda,M};a^{\dagger}_{-\lambda,M})$ are the Minkowski operators associated to the Unruh modes, Eq. (4), and where $\mathcal{B}$ is

$$\mathcal{B}=\begin{pmatrix} \alpha & 0 & 0 & -\beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ -\beta & 0 & 0 & \alpha \end{pmatrix}. \tag{C8}$$

The diagonal matrices (in λ) α and β have been taken real.

Then, from Eq. (C5) and Eq. (C7), the scattered Minkowski operators $\tilde{c}$ are given in terms of the ingoing operators c by the following matrix relation

$$\tilde{c}=\mathcal{B}\mathcal{S}\mathcal{B}^{-1}c=(\mathcal{S}+\mathcal{B}[\mathcal{S},\mathcal{B}^{-1}]_{-})c=\mathcal{S}_M c. \tag{C9}$$

Since $\mathcal{S}$ and $\mathcal{B}$ do not commute, $\mathcal{S}_M$ has nondiagonal elements:

$$\mathcal{S}_M=\begin{pmatrix} \tilde{\alpha}_1 & 0 & 0 & -\tilde{\beta}_1 \\ 0 & \tilde{\alpha}_2 & \tilde{\beta}_1^{\dagger} & 0 \\ 0 & \beta_2^{\dagger} & \alpha_1^{\dagger} & 0 \\ -\tilde{\beta}_2 & 0 & 0 & \tilde{\alpha}_2^{\dagger} \end{pmatrix}. \tag{C10}$$

$\tilde{\alpha}$ $\tilde{\beta}$ are given in terms of T by [see Eq. (B18)]

$$\tilde{\alpha}_1=1+i\alpha T\alpha, \quad \tilde{\alpha}_2=1+i\beta T^{\dagger}\beta,$$

$$\tilde{\beta}_1=-i\alpha T\beta, \quad \beta_2=i\beta T\alpha. \tag{C11}$$

The nondiagonal matrix elements, the $\tilde{\beta}$'s mix creation and destruction operators, and encode as usual the amplitudes of pair creation.

Therefore, the noncommutativity of $\mathcal{S}$ and $\mathcal{B}$ is sufficient to deduce that any scattering giving rise to no production of Rindler quanta necessarily induces pair production of Minkowski quanta.

If furthermore, the Rindler scattering is stationary during a lapse of proper time much greater than $1/a$, that is, $S_{\lambda\lambda'}$ is, to a good approximation, diagonal in λ, then, the number of created pairs of Minkowski quanta is proportional to the interval of proper time. See Eq. (B24) for the proof.

[1] W. G. Unruh, Phys. Rev. D **14**, 287 (1976).
[2] W. G. Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).
[3] P. G. Grove, Class. Quantum Grav. **3**, 801 (1986).
[4] D. Raine, D. Sciama, and P. Grove, Proc. R. Soc. London, Ser. A **435**, 205 (1991).
[5] W. G. Unruh, Phys. Rev. D **46**, 3271 (1992).
[6] F. Hinterleitner, Ann. Phys. (N.Y.) **226**, 165 (1993).
[7] S. Massar, R. Parentani, and R. Brout, Class. Quantum Grav. **10**, 385 (1993).
[8] J. Audretsch and R. Müller, Phys. Rev. D **49**, 4056 (1994); **49**, 6566 (1994); Phys. Rev. A **50**, 1755 (1994).
[9] R. Parentani, Nucl. Phys. **B454**, 227 (1995).
[10] P. Yi, Phys. Rev. Lett. **75**, 382 (1995); Phys. Rev. D **53**, 7041 (1996).
[11] S. Massar and R. Parentani, Comment on ''Vanishing Hawking Radiation from a Uniformly Accelerated Black Hole,'' Report No. LPTENS 96/17 TAU 2325-96 gr-qc/9603018 (unpublished); P. Yi, Phys. Rev. Lett. **75**, 382 (1995).
[12] R. Brout, S. Massar, R. Parentani, S. Popescu, and Ph. Spindel, Phys. Rev. D **52**, 1119 (1995).
[13] S. Massar and R. Parentani, following paper, Phys. Rev. D **54**, 7444 (1996).
[14] Y. Aharonov, D. Albert, A. Casher, and L. Vaidman, Phys. Lett. A **124**, 199 (1987); Y. Aharonov and L. Vaidman, Phys. Rev. A **41**, 11 (1990).
[15] R. Brout, S. Massar, R. Parentani, and Ph. Spindel, Phys. Rep. **260**, 329 (1995).
[16] G. 't Hooft, Nucl. Phys. **B256**, 727 (1985).
[17] T. Jacobson, Phys. Rev. D **44**, 1731 (1991); **48**, 728 (1993).
[18] S. A. Fulling, Phys. Rev. D **7**, 2850 (1973).
[19] N.D. Birrel and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
[20] R. Parentani, Class. Quantum Grav. **10**, 1409 (1993).
[21] R. Parentani and R. Brout, Int. J. Mod. Phys. D **1**, 169 (1992).
[22] P. G. Grove, in *The Origin of Structure in the Universe*, edited by E. Gunzig and P. Nardone (Kluwer Academic, Dordrecht, 1993); L. H. Ford, P. G. Grove, and A. C. Ottewill, Phys. Rev. D **46**, 4566 (1992).
[23] P. C. W. Davies and S. A. Fulling, Proc. R. Soc. London, Ser. A **356**, 237 (1977).
[24] R. Parentani, Nucl. Phys. **B456**, 175 (1996).