Hyperextended scalar-tensor gravity

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We study a general scalar-tensor theory with an arbitrary coupling function $\omega(\phi)$ but also an arbitrary dependence of the *gravitational constant* $G(\phi)$ in the cases in which either one of them, or both, do not admit an analytical inverse, as in the hyperextended inflationary scenario. We present the full set of field equations and study their cosmological behavior. We show that different scalar-tensor theories can be grouped in classes with the same solution for the scalar field. [S0556-2821(96)00424-9]

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I. INTRODUCTION

Scalar-tensor theories of gravity have an interesting physical embodiment which makes them a natural generalization of general relativity (GR). This provides a convenient framework for the study of observational limits on possible deviation of Einstein's theory, making them a profitable arena for cosmology.

The archetypical and best known case of scalar-tensor theory is Brans-Dicke (BD) gravity [1] where there is a coupling function $\omega(\phi)$ equal to a constant. More general cases with more complicated couplings have also been studied [2]. In any case, in order to evaluate the cosmological scenario and to test the predictable force of any scalar-tensor theory, it is necessary to have exact analytical solutions of the field equations. Once having these solutions, simultaneous constraints arising from different epochs of cosmic history must be set up. That is the case for primordial nucleosynthesis [3] and the weak-field solar system test [4]. It has also been shown that scalar-tensor theories may drive new forms of inflation [5,6] and that unusual physical effects arise on black hole physics if the *gravitational constant* becomes a scalar-field-dependent magnitude [7]. On the other hand, perhaps a more philosophical way of thinking about scalar-tensor theories of gravity is related to the Mach's principle and the nature of space and the inertial properties of the bodies. Comparatively, little advance has been reached in this area up to date [8]. Scalar-tensor theories have also been related with strings, in which a dilaton field coupled to the curvature appears in the low energy effective action [9].

Recently, a great improvement in the search of solutions of the field equations has been given in the form of methods that allow analytical integration through suitable changes of variables. Barrow [10] presented a method which enables exact solutions to be found for vacuum and radiation dominated Friedmann universes of all curvatures in arbitrary coupling scalar-tensor theories. Then, and also for arbitrary $\omega(\phi)$, Barrow and Mimoso [11] and Mimoso and Wands [12] derived exact Friedmann-Robertson-Walker (FRW) cosmological solutions in models with a perfect fluid satisfying the equation of state $p=(\gamma-1)\rho$ (with γ a constant and $0\leqslant\gamma\leqslant2$).

However, scalar-tensor theories have been formulated in two different ways depending on the choice of the basic action or, equivalently, of the Lagrangian density for the field. Via a field redefinition one can establish the equivalence between these Lagrangians (see below) and so between the theories of gravitation they lead. But, as was clearly remarked by Liddle and Wands [13] this is not always possible. So, we have two physically different theories arising from the fact that, in the general case, we have two nonrelated functions of the field ϕ; i.e., $G(\phi)$ and the coupling $\omega(\phi)$; where $G(\phi)$ is not limited to the form $1/\phi$ but it is an arbitrary function of the field. Since there is a deep connection between these models and hyperextended inflation we propose to call hyperextended scalar-tensor gravity to these kinds of two free functions theories.

In this work, we study the equivalence among the different scalar Lagrangian densities coupled to gravity that may be constructed retaining only a term proportional to the curvature scalar. We present the field equations for the more general scalar-tensor theory, i.e., with arbitrary dependence of $\omega(\phi)$, $G(\phi)$, and eventually a potential term $V(\phi)$ and show how to extend the procedure described in [12] to analytically solve the system of the field equations in any of the geometries of space time. As in [10–12] the solutions will be given in terms of a single integral over ϕ which may be performed exactly in many cases (namely, in the cases of vacuum, radiation and stiff filled universes) and numerically in all cases.

This paper is organized as follows. In Sec. II we describe the equivalence problems among Lagrangians; Sec. III presents the field equations, and in Sec. IV the FRW models are introduced together with a convenient choice of variables. The procedure to obtain cosmological solutions is shown in Sec. V. Finally, our conclusions are sketched in Sec. VI.

II. EQUIVALENCE AMONG SCALAR LAGRANGIAN DENSITIES

The more general Lagrangian density for a scalar field coupled to gravity in the usual way; i.e., it has only a term proportional to the curvature scalar, is

$$L=16\pi L_M+\frac{K(\phi)}{2}\phi_{,\mu}\phi^{,\mu}+G(\phi)^{-1}R+V(\phi), \qquad (1)$$

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where L_M represents the Lagrangian density for the matter content of the space-time with no dependence on ϕ and $K, G^{-1}=1/G$ and V are arbitrary functions of the field.

In general, it is common to find in the literature only one of the two following Lagrangian densities:

$$L_1=16\pi L_M+\phi R-\frac{\omega(\phi)}{\phi}\phi_{,\mu}\phi^{,\mu}-V(\phi), \tag{2}$$

$$L_2=16\pi L_M+f(\phi)R+\frac{1}{2}\phi_{,\mu}\phi^{,\mu}-V(\phi). \tag{3}$$

L_1 leads to the well-known generalized Brans-Dicke theories of gravity [2] while L_2 is referred to as a nonminimally coupled gravity. As particular cases of L we shall have L_1 reproduced when $K(\phi)=-2\,\omega(\phi)/\phi$ and $G(\phi)=1/\phi$ simultaneously (the BD cases) and L_2 when $K(\phi)=1$. $L_1$and L_2 are related through a scalar field transformation which may be completed defining another field ψ by

$$\psi=f(\phi). \tag{4}$$

Defining also the coupling as

$$\omega=-\frac{1}{2}\frac{f(\phi)}{(df/d\phi)^2}, \tag{5}$$

L_2 may be transformed to the form of L_1 for the new field ψ. This kind of transformation was first noted by Nordtvedt [2] and usually recalled by almost all the workers in the area. In particular, Steinhardt and Ascetta [6] used this transformation to study the mechanism of hyperextended inflation. However, it is easy to see that we have here a dependence on the *simplicity* of the coupling or the functional form of $G(\phi)$. As it is noted in [13] if one takes $\omega(\phi)=\omega_0+\omega_m\phi^m$ (as in [14]) or $f(\phi)$ as a truncated Taylor series (as in [6]) one cannot write down the equivalence between L_1 and L_2; in fact, to do such a thing one has to ask for the existence of the analytical inverse of $f(\phi)$ (note that $f\equiv 1/G$). So, the choice of Steinhardt and Ascetta leads to a singularity in the ϕ-ψ transformation and this constitutes the representation of a physical difference between the two Lagrangian densities. In these cases, and in general, in all cases in which $G(\phi)$ is not an analytically invertible function of ϕ the basic actions differs and so the theory of gravity they lead and the cosmological effects of it. A similar situation comes down when one tries to establish an equivalence between L and L_1-L_2.

III. HYPEREXTENDED SCALAR-TENSOR THEORIES

From now on, we shall call $K(\phi)=-2\omega(\phi)/\phi$ to facilitate comparison with the BD cases by only particularizing the dependence of $G(\phi)$. Anyway, this does not represent a loss of generality but only a change in the names of the functions. Taking variational derivatives of the action constructed using the Lagrangian density (1) with respect to the dynamical variables $g^{\mu\nu}$ and ϕ yields to the field equations [15]

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=G(\phi)\bigg[8\pi T_{\mu\nu}+\frac{\omega}{\phi}\phi_{,\mu}\phi^{,\mu}-\frac{\omega}{2\phi}\phi_{,\alpha}\phi^{,\alpha}g_{\mu\nu}-\frac{V}{2}g_{\mu\nu}+(G^{-1})_{,\mu;\nu}-g_{\mu\nu}\Box(G^{-1})\bigg], \tag{6}$$

$$R\frac{dG^{-1}}{d\phi}+\frac{1}{\phi}\frac{d\omega}{d\phi}\phi_{,\mu}\phi^{,\mu}-\frac{\omega}{\phi^2}\phi_{,\mu}\phi^{,\mu}+\frac{2\omega}{\phi}\Box\phi-\frac{dV}{d\phi}=0. \tag{7}$$

The second equation may be written down in a more usual way which involves the trace of the stress-energy tensor of matter fields instead of the curvature scalar.

It is very important to remark that the usual relation $T^{\mu\nu}_{\ ;\nu}=0$ establishing the conservation laws (in the meaning of GR) of the matter fields holds true. This may be seen by direct differentiation from Eq. (6) recalling the identities of the curvature tensor as a commutator of covariant derivatives.

IV. FRIEDMANN-ROBERTSON-WALKER MODELS

We shall consider homogeneous and isotropic models with the metric given by the Friedmann-Robertson-Walker (FRW) line element:

$$ds^2=dt^2-a(t)^2\bigg[\frac{dr^2}{1-kr^2}+r^2(d\theta^2+\sin^2\theta d\Phi^2)\bigg]. \tag{8}$$

In this framework, all the scalars are functions only of time and not of the space coordinates. As an equation of state we shall use that of a perfect fluid $p=(\gamma-1)\rho$ (with γ a constant and $0\leqslant\gamma\leqslant 2$). The field equations become

$$\left(\frac{\dot a}{a}\right)^2-\left(\frac{\dot a}{a}\right)\frac{1}{G}\frac{dG}{d\phi}\dot\phi-\frac{\omega}{6}\frac{\dot\phi^2}{\phi}G+\frac{k}{a^2}=\frac{8\pi}{3}G\rho, \tag{9}$$

$$\dot\phi^2\left[\frac{1}{\phi}\frac{d\omega}{d\phi}-\frac{\omega}{\phi^2}-\frac{1}{G}\frac{dG}{d\phi}\frac{\omega}{\phi}-\frac{6}{G^4}\left(\frac{dG}{d\phi}\right)^3+\frac{3}{G^3}\frac{dG}{d\phi}\frac{d^2G}{d\phi^2}\right]+\Box\phi\left[\frac{2\omega}{\phi}+\frac{3}{G^3}\left(\frac{dG}{d\phi}\right)^2\right]=-\frac{1}{G}\frac{dG}{d\phi}8\pi\rho(4-3\gamma), \tag{10}$$

$$2\frac{d}{dt}\left(\frac{\dot a}{a}\right)+3\left(\frac{\dot a}{a}\right)^2+\frac{k}{a^2}-\left(\frac{\dot a}{a}\right)\frac{2}{G}\frac{dG}{d\phi}\dot\phi=-G\,8\pi\rho(\gamma-1)-\frac{\omega}{2}\frac{\dot\phi^2}{\phi}G-2\left(\frac{1}{G}\frac{dG}{d\phi}\right)^2\dot\phi+\frac{1}{G}\frac{d^2G}{d\phi^2}\dot\phi^2+\frac{1}{G}\frac{dG}{d\phi}\ddot\phi. \tag{11}$$

Note that the solutions of these equations, as remarked by Weimberg [16] in the case of Brans-Dicke theory, are defined by four integration constants. It is useful to have the spatial equation in alternative forms: for instance,

$$\dot H+H^2+H\frac{1}{G}\frac{dG}{d\phi}\dot\phi+\frac{\omega}{3}\frac{\dot\phi^2}{\phi}G=\left\{G\frac{8\pi\rho}{3}\left[(2-3\gamma)\frac{\omega}{\phi}-\frac{3}{G^3}\left(\frac{dG}{d\phi}\right)^2\right]+\frac{1}{2}\dot\phi^2\Delta\right\}\frac{1}{[2\omega/\phi+(3/G^3)(dG/d\phi)^2]} \tag{12}$$

where we have defined H as usual and

$$\Delta = -\frac{3}{G^2}\left(\frac{dG}{d\phi}\right)^2\frac{\omega}{\phi} + \frac{1}{G}\frac{d^2G}{d\phi^2}\frac{2\omega}{\phi} - \frac{1}{G}\frac{dG}{d\phi}\frac{1}{\phi}\frac{d\omega}{d\phi} + \frac{1}{G}\frac{dG}{d\phi}\frac{\omega}{\phi^2}. \tag{13}$$

The derivation of general barotropic solutions were done only for the case of generalized BD theories, i.e., $G(\phi)=1/\phi$. The most salient ones were derived by Nariai [17], O'Hanlon and Tupper [18], Gurevich, Finkelstein, and Ruban [19], Lorenz-Petzold [20], Barrow [10], Barrow and Mimoso [11], and Mimoso and Wands [12]. Recently, a complete cualitative study of the behavior of scalar-tensor theories was also presented [21]. In what follows we generalize the method described in [12] for Eqs. (9), (10), and (11). In generalized BD cases it was shown that the change of variables

$$X = a^2\phi, \tag{14}$$

$$Y = \int \sqrt{\frac{2\omega(\phi)+3}{3}}\frac{d\phi}{\phi}, \tag{15}$$

together with the introduction of the conformal time defined by the differential relation

$$dt = a\,d\eta, \tag{16}$$

allows one to rewrite the BD field equations as

$$(X')^2 + 4kX^2 - (Y'X)^2 = 4MXa^{4-3\gamma}, \tag{17}$$

$$(Y'X)' = M(4-3\gamma)\sqrt{\frac{3}{2\omega+3}}a^{4-3\gamma}, \tag{18}$$

$$X'' + 4kX = 3(2-\gamma)Ma^{4-3\gamma}, \tag{19}$$

where the density of the barotropic fluid has been written as $\rho = 3M/8\pi a^{3\gamma}$ and the prime denotes differentiation with respect to η. In the general case given by the system (9-10-11) the leading idea is to retain the simplicity of the transformed system by asking for a suitable choice of new variables. So we propose them in the form

$$X = \frac{a^2}{G}j(\phi), \tag{20}$$

$$Y = \int \alpha(\phi)\frac{d\phi}{\phi}, \tag{21}$$

where j and α ought to be selected in order to maintain the form of (17-18-19) and have to reduced to their particular values for BD theories ($j=1$ and $\alpha=\sqrt{2\omega(\phi)+3/3}$) when $G=1/\phi$. So, computing all the necessary terms of the transformed system we obtain two constraint equations (we shall show them in the vacuum case):

$$\frac{1}{\alpha}\frac{d\alpha}{d\phi} - \frac{1}{\phi} - \frac{1}{G}\frac{dG}{d\phi} + \frac{1}{j}\frac{dj}{d\phi} = \frac{[d\omega/d\phi - \omega/\phi^2 - (1/G)(dG/d\phi)(\omega/\phi) - (6/G^4)(dG/d\phi)^3 + (3/G^3)(dG/d\phi)(d^2G/d\phi^2)]}{[2\omega/\phi + (3/G^3)(dG/d\phi)^2]}, \tag{22}$$

$$\dot\phi^2\left[\left(\frac{1}{G}\frac{dG}{d\phi}\right)^2 - \left(\frac{\alpha}{\phi}\right)^2 + \left(\frac{1}{j}\frac{dj}{d\phi}\right)^2 - \frac{2}{G}\frac{dG}{d\phi}\frac{1}{j}\frac{dj}{d\phi}\right] + \dot\phi\left[4\frac{\dot a}{a}\frac{1}{j}\frac{dj}{d\phi}\right] = -\frac{2}{3}\omega G\frac{\dot\phi^2}{\phi}, \tag{23}$$

which allow for a solution to be found in the form

$$j = 1, \tag{24}$$

$$\alpha = \sqrt{\left(\frac{\phi}{G}\right)^2\left(\frac{dG}{d\phi}\right)^2 + \frac{2}{3}\omega G\phi}. \tag{25}$$

So, defining the variables X and Y as in Eqs. (20) and (21) and the conformal time as in Eq. (16) the system of field equations simplifies to a form analogous to the generalized BD cases. As a matter of fact, the function $\alpha(\phi)$ becomes the same as in Eq. (15) for $G(\phi)=1/\phi$. In the general hyperextended scalar-tensor formalism it is necessary to ask for the positivity of the term under the square root in the definition (25). That was also the case in BD theories [12] where ω must be greater than $-3/2$. The final expression of the system is then

$$(X')^2 + 4kX^2 - (Y'X)^2 = 4MX(XG)^{(4-3\gamma)/2}, \tag{26}$$

$$(Y'X)' = -M(4-3\gamma)\frac{1}{\alpha}(XG)^{(4-3\gamma)/2}\frac{1}{G}\frac{dG}{d\phi}\phi, \tag{27}$$

$$X'' + 4kX = 3(2-\gamma)M(XG)^{(4-3\gamma)/2}. \tag{28}$$

V. COSMOLOGICAL SOLUTIONS

In this section we sketch how to analytically obtain cosmological solutions for different perfect fluid universes. We follow, using the exact reproduction of the form of the field equations obtained in the previous section, the work of [12], which may be seen for further details.

A. Vacuum solutions

Let us first consider the simplest case. In a vacuum model, the right-hand sides of Eqs. (26)–(28) are equal to zero. Now, we use the fact that the new equations have the same form as the generalized BD ones. So, the work made in [12], i.e., the solutions of the system, is completely applicable here, except for the different meaning of the variables. From Eq. (27) we have $Y'X=c$, constant, and so the solutions for X may be obtained using Eq. (26). They are given by Eq. (3.20) of [12]. Note $X(\eta)$ is independent of the particular form of ω and of G. As $Y'X=c$, this implies that

$$Y=\int \sqrt{\left(\frac{\phi}{G}\right)^2\left(\frac{dG}{d\phi}\right)^2+\frac{2}{3}\omega G\phi}\frac{d\phi}{\phi}=\int \frac{c}{X}d\eta=I(\eta). \tag{29}$$

We can compute this integral because of our knowledge of the dependence of X over η. So, given the functions $G(\phi)$ and $\omega(\phi)$, we can compute $Y(\phi)$ and invert it using our knowledge of the right side of Eq. (29) to obtain $\phi(\eta)$. Together with $a^2=XG$, this yields the solution of the problem.

Even without solving these equations for particular values of $G(\phi)$ and $\omega(\phi)$ it is possible to obtain some general conclusions about the nature of the singularity in these vacuum models. When $X\to 0$ and $(X'/X)^2\to\infty$, it can be seen that $X'/X\to\pm Y'$. Using the definition of the variables it is easy to show that

$$\dot{a}\to\frac{1}{2}\left[1\mp\frac{1}{\alpha}\frac{\phi}{G}\frac{dG}{d\phi}\right]\frac{X'}{X} \tag{30}$$

and the initial singularity, which is produced when $\dot{a}\to\pm\infty$ can only be avoided in these cases when $\omega\to 0$ or $(dG/d\phi)^2\geqslant(2\omega/3)(G^3/\phi)$. Note that in the generalized BD cases only the first condition is obtained [12].

B. Nonvacuum solutions: Radiation

With $\gamma=4/3$ the equation of state becomes that of a radiation fluid. The two first field equations read, in this case as

$$(X')^2+4kX^2-(Y'X)^2=4MX, \tag{31}$$

$$(Y'X)'=0. \tag{32}$$

Note that the second equation retains its form from the vacuum case and this implies again that $Y'X=c$. Using this in Eq. (31) it is possible to integrate for the variable X and then obtain as above the function $I(\eta)$. Once again, due to the exact reproduction of the form of the equations, we have the same solutions as in the BD case but in the new variables, Eq. (3.70) of [12]. It can be seen in this case that at early times all solutions approach the vacuum ones. Thus, defining $G(\phi)$ and $\omega(\phi)$ we can follow again the same logical steps to obtain a^2 and ϕ as functions of η.

C. Nonvacuum solutions: Stiff matter fluid

Let us finally consider case in which $\gamma=2$. That election represents a barotropic equation of state given by $p=\rho$. The field equation becomes in this case,

$$(X')^2+4kX^2-(Y'X)^2=\frac{4M}{G}, \tag{33}$$

$$(Y'X)'=-2M\frac{1}{\alpha}\frac{1}{X}\frac{1}{G^2}\frac{dG}{d\phi}\phi, \tag{34}$$

$$X''+4kX=0. \tag{35}$$

The last equation is identical to the corresponding vacuum equations and so $X(\eta)$ is given by the same expressions as in the vacuum case. In addition, we have a useful relation:

$$Y'X=\pm\sqrt{A-4\frac{M}{G}} \tag{36}$$

with A a constant of integration. This requires that

$$\frac{A}{4M}\geqslant\frac{1}{G}. \tag{37}$$

It can be seen that only for $k=-1$ could A be negative. This means that G is a negative function. In this case an extra solution for $X(\eta)$ arise in addition to the vacuum ones. From Eq. (36) it can be shown that defining

$$Z(\phi)=\int \sqrt{\left(\frac{\phi}{G}\right)^2\left(\frac{dG}{d\phi}\right)^2+\frac{2}{3}\omega G\phi}\frac{d\phi}{\phi\sqrt{A-4(M/G)}}=\pm\int\frac{1}{X}d\eta \tag{38}$$

and

$$\sqrt{\left(\frac{\phi}{G}\right)^2\left(\frac{dG}{d\phi}\right)^2+\frac{2}{3}\omega G\phi}=\sqrt{\left(\frac{\phi}{G_{\mathrm{vac}}}\right)^2\left(\frac{dG_{\mathrm{vac}}}{d\phi}\right)^2+\frac{2}{3}\omega_{\mathrm{vac}}G_{\mathrm{vac}}\phi}\left[A-4\frac{M}{G}\right]\frac{1}{c}, \tag{39}$$

the vacuum solutions for ω_{vac} and G_{vac} carry with the $\gamma=2$ solutions for ω and G. The behavior of the scale factor and of the scalar field in the stiff matter universe with coupling ω and *gravitational constant* G are the same of those of the vacuum universe with ω_{vac} and G_{vac}. In this general theory and as we have two generic functions instead of one in the leading Lagrangian we can put all the dependence on ϕ in only one vacuum function if convenient. Then, proceeding as previously done, we can obtain $\phi(\eta)$ and $a(\eta)^2$.

VI. CONCLUSIONS

We have shown how to extend the recently presented procedure by Mimoso and Wands [12] to obtain the solutions for a generic coupling simultaneously with a generic dependence of the *gravitational constant* on the field ϕ, reducing the whole problem to the solution of a single integral over the field like in [10–12]. This can be done for all curvatures in vacuum, radiation and stiff matter universes.

The particular case in which the leading Lagrangian density of the theory is Eq. (3) may be exploited in this general formalism defining $\omega(\phi)=-\phi/2$ for all the $G(\phi)$'s that still retain the positivity of the expression under the square root in α. That case seems to be clearly important since only for particular choices of $G(\phi)$ an analytical solution is known [22]. Examples of the kind of results that may be obtained in that way, together with other couplings, will be presented in a forthcoming work.

A crucial point is to note that in this formalism, to equal α [Eq. (25)] correspond equal solution for the field ϕ. This point actually means that if a solution for a particular $\omega(\phi)$ in a BD-like theory (say $\omega_{\rm BD}$) is known, and we have as result the ϕ and a^2 dependences on η, we can use the $\phi(\eta)$ as a solution for a class of hyperextended scalar-tensor theories, i.e., those which have

$$\left(\frac{\phi}{G}\right)^2\left(\frac{dG}{d\phi}\right)^2+\frac{2}{3}\omega G\phi=\frac{2\omega_{\rm BD}+3}{3} \tag{40}$$

and obtain the a^2 dependence in each member of the class by using the X definition. In this way, we could speak of equivalence classes of scalar-tensor gravitation, that may, in principle, be formed by an infinite set of members. In addition, all members of a given class will predict the same results for all observable quantities that are functions of ϕ and X. So, if we were able to prove that for a given set (ϕ,X) or equivalently (ω,G), a correct behavior in the gravity tests is obtained, we were proving that not only is there not a unique theory of gravity with equal predictive observational verified power but an infinite set of them.

Let us finally comment on the overall feeling that one has after the development of the theory concerning how much it is like generalized BD cases. It can be seen that, for instance, in the vacuum cases the solutions behave as a whole like in BD theory with respect to the initial singularity provided $G(\phi)$ satisfies mildly restrictive conditions. In the radiation case, the solutions behave like in vacuum in exactly the same way as in BD. And finally, we have also shown that the solutions for a stiff matter universe are contained in those of vacuum through a convenient choice of the functions. We believe that the correct way of thinking in these similarities is to understand that generalized Brans-Dicke theories stand as a particular case of the formalism presented in this paper and so, the cualitative behavior must be expected as similar.

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