Reconstruction of the bubble nucleating potential

Luca Amendola,[1] Carlo Baccigalupi,[1,2] Rostislav Konoplich,[3] Franco Occhionero,[1] and Sergei Rubin[3]
[1]*Osservatorio Astronomico di Roma, Viale del Parco Mellini, 84, 00136 Roma, Italy*
[2]*Dipartimento di Fisica, Università di Ferrara, Via del Paradiso 12, 44100 Ferrara, Italy*
[3]*Moscow Engineering Physics Institute, Kashirskoe sh, 31, 115409 Moscow, Russia*

(Received 29 July 1996)

We calculate analytically the bubble nucleation rate in a model of first-order inflation which is able to produce the large-scale structure. The computation includes the first-order departure from the thin-wall limit, the explicit derivation of the preexponential factor, and the gravitational correction. The resulting bubble spectrum then is compared with constraints from the large-scale structure and the microwave background. We show that there are models which pass all the constraints and produce bubblelike perturbations of interesting size. Furthermore, we show that it is in principle, possible to reconstruct completely the inflationary two-field potential from observations. [S0556-2821(96)06324-2]

PACS number(s): 98.80.Cq, 98.80.Es

I. INTRODUCTION

One of the most interesting ideas introduced in inflationary cosmology in recent years is the possibility of performing a phase transition *during* inflation. In such scenarios, two fields act on stage: one, say ω, slow rolls, driving enough inflation to solve the standard problems; the second field, say ψ, tunnels from a false-vacuum state to an energetically favored true-vacuum state, producing *bubbles* of the new phase embedded in the old one. Both processes are governed by a two-field potential $U(\omega,\psi)$. To avoid the graceful exit problem, the true vacuum state has to allow for a period of inflation on its own. We then can speak of a true-vacuum *channel* over which the bubbles slow roll until inflation ends, and reheating takes over. Depending on the potential, three classes of first-order inflation models have been proposed so far. The first is the classical extended inflation [1–3]: the bubbles are produced in a copious quantity, so that they fill eventually the space and complete the transition. To avoid too large distortions on the cosmic microwave background (CMB), this scenario must produce very small bubbles [4,5], so that they are thermalized rapidly after inflation. No trace of the bubbles is left in our Universe, and from this point of view, such scenarios do not lead to new predictions over inflation without bubble production. The second class is the $\Omega<1$ inflation [6–8]: here the transition is never completed, so that each bubble resembles an open Universe to inside observers. Therefore if the bubbles inflate for less than the canonical $N_T=60$ or so e-foldings, they will approach an $\Omega<1$ Universe. Here the effect of the nucleation process is observable, although it is indistinguishable from a slow roll inflation just shorter than N_T e-foldings and with no first-order phase transition at all. Finally, in [9] a third class of models has been proposed, following an early suggestion of La [10]. In such models, the phase transition is completed *before* the end of inflation. Then it has been shown that the primordial bubbles can be large enough to drive structure formation, and still be below the CMB level of detection [11–14]. In such a scenario, the present large-scale structure is a direct outcome of the first-order transition, which is, therefore, observable and testable.

Several deep redshift surveys detected large voids in the galaxy distribution [15–17], although it is still not clear if they are really empty of matter or just lack luminous galaxies [18]. Standard models of galaxy formation can barely account for these structure, and do so only at the price of adjusting the parameters to get very large-scale power (see, e.g., Ref. [19]). Therefore just as we associate matter clumps to primordial fluctuations, it appears worth trying to associate the present voids to primordial bubblelike fluctuations, produced during a first-order phase transition. Within different contexts, the idea of the voids as separate dynamical entities has been investigated several times in earlier literature [20].

A crucial aspect of bubble inflation is the calculation of the bubble spectrum $n_B(L)$, defined as the number of bubbles per horizon with comoving size larger than L. In [9] we calculated n_B in a specific model, built on fourth-order gravity [23], which we found to possess the requested features. We found that $n_B(L)$ can be approximated by a power law,

$$n_B=(L_m/L)^p, \tag{1}$$

and that L_m can be as large as the observed voids in the Universe.

The central quantity needed to evaluate n_B is the nucleation rate in the semiclassical limit [21]

$$\Gamma=\mathcal{M}^4\exp(-B), \tag{2}$$

where $\mathcal{M}$ is a constant with a dimension of mass, and B is the Euclidean least action minus the action for the external de Sitter space-time solution. The calculation of Γ in [9] (and in most other papers on the topic) was based on the thin-wall limit, on neglecting the gravitational correction, and on a dimensional argument for evaluating $\mathcal{M}$. In this paper we remove, at least partially, all three approximations. This calculation allows us to reconstruct completely the inflationary two-field potential from the determination of four observable quantities: the slope of the bubble spectrum, its amplitude, the density contrast inside the bubbles, and the amplitude of the ordinary slow-rolling fluctuations. We remark that only in a model, such as ours, in which the bubbles are directly observable, it is possible to reconstruct the tunneling sector of the primordial potential.

The scheme of the paper is as follows. In Sec. II we present the class of potentials we are going to investigate. In Sec. III we give the basic formulas. In Sec. IV, V, and VI we go through the detailed calculation of Γ, taking into account deviations from the thin-wall limit, inserting the gravitational correction, and calculating explicitly the factor $\mathcal{M}$. In Sec. VII we introduce our model of first-order inflation in a fourth-order gravity theory [23] and calculate the time-dependent nucleation rate. In Sec. VIII we write down the bubble spectrum and in Sec. IX, finally, we compare it with constraints from CMB and the large-scale structure. In the last section we draw our conclusions.

II. THE MODEL

We consider the scalar field theory described by the action (hereinafter, $\hbar=c=1$)

$$S=\int d^4x\sqrt{-g}\left(-\frac{\mathcal{R}}{16\pi G}+\frac{1}{2}\psi_{;\mu}\psi^{;\mu}-U(\psi)\right), \quad (3)$$

where g is the metric determinant and $\mathcal{R}$ is the curvature scalar. The potential is a generic quartic function with non-degenerate minima which allows for tunneling. We can write it very generally in the form

$$U(\psi)=\Lambda+V_1(\psi)+V_2(\psi), \quad (4)$$

where Λ is a cosmological constant, V_1 is a quartic with two equal-energy minima, and V_2 is a symmetry-breaking potential which brings the energy of one minimum, the false vacuum (subscript F), to a value larger than the other, the true vacuum (subscript T). In the two minima, the Einstein equation reduces simply to

$$H^2=8\pi GU/3. \quad (5)$$

We will denote with H_T, U_T the Hubble constant and the potential energy of the true vacuum, and with H_F, U_F the same quantities for the false vacuum. We wish to calculate the tunneling rate (2) where $B=S_E(\psi)-S_E(\psi_F)$, and $S_E(\psi)$ is the Euclidean least action, i.e., the Action of the ''bounce'' solution. We perform the computation in the thin-wall limit (actually, we go to a post-thin-wall limit calculation), according to which the $O(4)$ bubbles nucleated have four-radius $R\gg\Delta$, where Δ is the wall thickness. Further, we include the gravitational term in the action: as we will show, this term is important when the parameter $g=RH_T$ (not to be confused with the metric determinant) is much larger than unity. This limit amounts in fact to a bubble approaching the space curvature radius $1/H$. It is convenient to write the potential (4) in the form

$$U(\psi)=\frac{3g^2}{8\pi GR^2}+\frac{1}{2\Delta^2\psi_0^2}(\psi^2-\psi_0^2)^2+\frac{1}{R\Delta}(\psi+\psi_0)^2. \quad (6)$$

Then, the true-vacuum state is $\psi=-\psi_0$, and the false vacuum is $\psi=\psi_0$. The potential (6), therefore, is defined by four physical parameters: g, R, Δ, ψ_0.

III. THIN-WALL LIMIT

The Euclidean action of the scalar theory (3) is

$$S_E=\int d^4x\sqrt{-g}\left(-\frac{\mathcal{R}}{16\pi G}+\frac{1}{2}\psi_{;\mu}\psi^{;\mu}+U(\psi)\right). \quad (7)$$

In the Euclidean metric for an O(4) space $ds^2=dr^2+a^2(r)d\Omega_3^2$, one has $\mathcal{R}=-6(aa''+a'^2-1)/a^2$ and

$$S_E=2\pi^2\int dr\left[\frac{3}{8\pi G}(a^2a''+aa'^2-a)+a^3\left(\frac{1}{2}\psi'^2+U\right)\right]$$
$$=-\frac{3\pi}{2G}\int dra(1-a^2H^2), \quad (8)$$

where the prime denotes derivation with respect to the four-radius r. The Euclidean Klein-Gordon equation for ψ is

$$\psi''+3\frac{a'}{a}\psi'=dU/d\psi \quad (9)$$

and the Euclidean Friedmann equation is

$$a'^2=1+\frac{8\pi G}{3}a^2\left(\frac{1}{2}\psi'^2-U\right). \quad (10)$$

In the zero-gravity limit, $G\to 0$, the latter equation gives $a'=\text{const}$, so that Eq. (9) reduces to

$$\psi''+\frac{3}{r}\psi'=dU/d\psi. \quad (11)$$

In the thin-wall limit, in which $R\gg\Delta$, one can assume that the second term in Eq. (11) can be neglected, and that $dU/d\psi=2\psi(\psi^2/\psi_0^2-1)/\Delta^2$. The solution which interpolates between false and true vacuum is then

$$\psi^{(0)}=\psi_0\tanh\left(\frac{r-R_w}{\Delta}\right), \quad (12)$$

where R_w is an integration constant that will be determined later. To integrate the action over the bounce solution, we consider that outside the bubble, i.e. in the false vacuum, $\psi=\psi_0$, so that $B_{\text{ext}}=S_E(\psi_0)-S_E(\psi_0)=0$. On the wall, at distance R_w, we have

$$B_{\text{wall}}=2\pi^2R_w^3S_1, \quad (13)$$

where

$$S_1=\int_{-\psi_0}^{\psi_0}d\psi[2(U(\psi)-U_F)]^{1/2}=\frac{4\psi_0^2}{3\Delta}. \quad (14)$$

Finally, inside the bubble, $\psi=-\psi_0$, and since [from Eq. (10), and neglecting ψ'] $da=dr[1-a^2H_T^2]^{1/2}$, we have

$$B_{\rm int}=-\frac{3\pi}{2G}\int_0^{R_w} a\,da[(1-a^2H_T^2)^{1/2}-(1-a^2H_F^2)^{1/2}]. \tag{15}$$

The general expression is, therefore,

$$B(R_w)=2\pi^2R_w^3S_1+\frac{\pi}{2G}\{H_T^{-2}[(1-R_w^2H_T^2)^{3/2}-1] -H_F^{-2}[(1-R_w^2H_F^2)^{3/2}-1]\}. \tag{16}$$

Let us note that $R=3S_1/\varepsilon$, where $\varepsilon=U_F-U_T=4\psi_0^2/R\Delta$. Then we see that $B(R_w)$ is minimized by [24]

$$R_w=R(1+g^2+4\pi GRS_1+12\pi^2G^2R^2S_1^2)^{-1/2}. \tag{17}$$

Then, for $G\rightarrow 0$, which implies $g=RH_T\rightarrow 0$, one has $R_w=R$. Notice that the parameter R_wH_T which appears in Eq. (16) equals g, since $H_T^2=g^2/R^2$ for the potential (6). This shows explicitly the role played by the constants R and g. Finally, we obtain the usual (zero-gravity, thin-wall) result [21]

$$B_0=27\pi^2\frac{S_1^4}{2\varepsilon^3}=\frac{2\pi^2}{3}\frac{R^3\psi_0^2}{\Delta}. \tag{18}$$

IV. THE GRAVITATIONAL CORRECTION

Now to the general case, $G\neq 0$. To the purpose of this paper, we simplify our problem by putting $(H_F^2-H_T^2)/H_T^2\ll 1$, i.e. assuming that the vacuum energy difference is much smaller than the true-vacuum energy. Our results will be consistent with this approximation. This is equivalent to neglecting the last two terms in parentheses in Eq. (17). Then we have that $B(R_w)$ is minimized by a bubble radius $R_w=R/(1+g^2)^{1/2}$. The bounce action is $B=B_0f(g)$ where B_0 is the no-gravity action (18) and where

$$f(g)=4(1+g^2)^{-3/2}\{1+g^{-4}[2+3g^2-2(1+g^2)^{3/2}]\}. \tag{19}$$

For $g\rightarrow 0$, $f(g)\rightarrow 1$, as expected. To the lowest nontrivial order in g,

$$B=B_0(1-g^2). \tag{20}$$

Gravitational effects, then, increase the nucleation rate Γ [21].

V. POST-THIN-WALL CORRECTION

Let us come back to the Euclidean spherically symmetric equation of motion in absence of gravity, Eq. (11). While the trivial true-vacuum solution $\psi_-=-\psi_0$ holds also when the symmetry-breaking term in the potential is considered, the false-vacuum solution is displaced slightly:

$$\psi_+=\psi_0\left(1-\frac{\Delta}{R}-\frac{\Delta^2}{R^2}-\cdots\right). \tag{21}$$

It is useful now to expand the equation of motion around $r=R$. Introducing the variable $z=(r-R)/\Delta$, we obtain

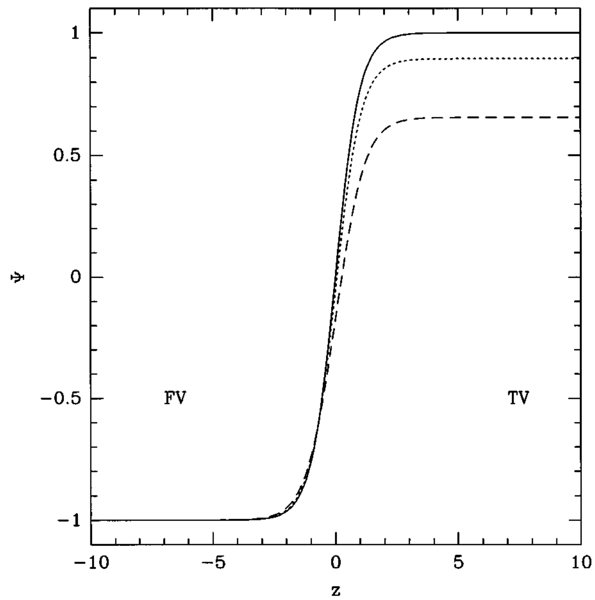

FIG. 1. The bounce solution, interpolating between false vacuum FV and true vacuum TV. The solid line corresponds to $\Delta/R=0$, the dotted line to $\Delta/R=0.1$, and the dashed line to $\Delta/R=0.3$.

$$\psi''+\frac{3\Delta}{R}\psi'\left(1-\frac{z\Delta}{R}+\frac{z\Delta^2}{R^2}+\cdots\right) =\frac{2\psi}{\Delta^2}(\psi^2/\psi_0^2-1)+\frac{2}{R\Delta}(\psi-\psi_0), \tag{22}$$

where now the prime denotes derivation with respect to z. We search solutions of Eq. (22) to the first order in Δ/R:

$$\psi=\psi^{(0)}+\psi^{(1)}(\Delta/2R)+\cdots. \tag{23}$$

We know already that to the zeroth order $\psi^{(0)}=\psi_0\tanh(z)$. Subtracting the zeroth solution from Eq. (22) we get, for $i\geqslant 1$,

$$\psi''^{(i)}+\psi^{(i)}\left(-4+\frac{6}{\cosh^2 z}\right)=f_i(z), \tag{24}$$

where $f_i(z)$ is defined order by order from Eq. (22) and depends only on z and not on $\psi^{(i)}$. At first order, then, the solution is

$$\psi=\psi_0\left\{\tanh z+\left(\frac{\Delta}{2R}\right)\left(-\frac{z}{\cosh^2 z}-\tanh z-1\right) -2\left(\frac{\Delta}{2R}\right)^2(\tanh z+1)\right\}. \tag{25}$$

It can be noticed, from Fig. 1, that the post-thin-wall correction moves the large z value of ψ from ψ_0 to ψ_+. In Eq. (25) we included the part of the correction at second order which also contributes to the leading classical correction to the action after integration over z.

The Euclidean action, finally, is calculated as

$$B=S(\psi)-S(\psi_0)=B_0(1-9\Delta/2R). \tag{26}$$

The term $9\Delta/2R$ is, therefore, the post-thin-wall correction term, which again increases the tunneling rate.

VI. THE PREEXPONENTIAL FACTOR

In semiclassical approximation in four dimensions, the rate of false vacuum decay is given by Coleman's formula [21]

$$\Gamma=\left(\frac{B}{2\pi}\right)^2\left|\frac{\det^{(4)}(-\partial^2+U''(\psi))}{\det(-\partial^2+U''(\psi_+))}\right|^{-1/2}e^{-B}, \tag{27}$$

where the functional determinant is computed with the four zero eigenvalues omitted, ψ is the classical solution of Eq. (22), ψ_+ is the trivial false-vacuum solution. It was shown in [22] that in the thin-wall limit, the functional determinant can be expanded in power series of $2R/\Delta$ as

$$\ln\frac{\det^{(5)}D}{\det^{(5)}D_0}=C_l\ln(2R/\Delta)+C_3(2R/\Delta)^3+C_2(2R/\Delta)^2+\cdots. \tag{28}$$

The parts of this expression, which behave as powers of $2R/\Delta$ have no universal significance, for they are adjusted or even concealed completely by ultraviolet renormalizations and we will include them in the overall multiplicative factor $\mathcal{M}'$. The logarithmic part, on the other hand, is not expected to be affected by short distance structure. The universal infrared logarithm $\ln(2R/\Delta)$ is associated with the geometrical features of the bubble and does not depend on the specific form of the double well potential. Since the logarithmic term appears due to the infrared region, we have to consider the low-lying levels of the operator of the small fluctuations around the bounce solution

$$D=-\partial^2+U''(\psi). \tag{29}$$

This operator is rotationally invariant and, thus, its eigenfunctions, are, in their angular dependence, four-dimensional scalar spherical harmonics. In terms of radial eigenfunctions, the eigenfunction equation becomes

$$\left(-\frac{d^2}{dr^2}+\frac{l(l+2)+3/4}{r^2}+U''(\psi)\right)f_{nl}(r)=\lambda_{nl}f_{nl}(r). \tag{30}$$

Expanding the centrifugal potential in power series around $r=R$ (it is justified when we deal with the low-lying states $l\ll 2R/\Delta$ since bound states are localized at the wall of the bubble), keeping only the leading term, and replacing r with z, we can rewrite Eq. (30) for $n=0$ as

$$\left(-\frac{d^2}{dz^2}+\Delta^2U''(\psi)-2E_0\right)f_{0l}=0, \tag{31}$$

where

$$E_0=\frac{1}{2}\left(\Delta^2\lambda_{0l}-4-\frac{4l(l+2)+3}{(2R/\Delta)^2}\right) \tag{32}$$

does not depend on the quantum number l. Taking E_0 in the form of expansion

$$E_0=2\left(a+\frac{b}{2R/\Delta}+\frac{c}{(2R/\Delta)^2}\right), \tag{33}$$

we obtain the spectrum of the eigenvalues in the form

$$\lambda_{0l}=\frac{4}{\Delta^2}\left(a+\frac{b}{2R/\Delta}+\frac{c}{(2R/\Delta)^2}+1+\frac{4l(l+2)+3}{4(2R/\Delta)^2}\right). \tag{34}$$

We know that in the spectrum of the eigenvalues, there is one negative eigenvalue λ_{00} corresponding to uniform expansion of the bubble, which is just vacuum instability. There is also the 4-fold degenerate eigenvalue $\lambda_{10}=0$ describing translations of the bubble center in the Euclidean space in four directions. Therefore substituting $l=1$ into Eq. (34), we obtain

$$a=-1,\quad b=0,\quad c=-15/4 \tag{35}$$

and find the lowest band of the eigenvalues

$$\lambda_{0l}=\frac{(l-1)(l+3)}{R^2}. \tag{36}$$

It is important to know that in deriving Eq. (36) for λ_{0l}, we assumed the existence of nontrivial classical solution of the equation of motion which breaks a translational invariance (zero modes) and the thin-wall limit in order to justify the expansion in powers of $2R/\Delta$. Therefore this equation does not depend on the specific form of the potential $U(\psi)$.

Calculating the product of λ_{0l} for $l\ll 2R/\Delta$ and keeping only the logarithmic term, it is straightforward to show that $c_l=-10$ (see Eq. 28). Then it follows immediately from the general formula (27) that in the thin-wall limit, and taking into account the quantum corrections, the rate of false-vacuum decay is given by

$$\Gamma=\frac{\mathcal{M}'^4B^2}{(2R/\Delta)^4}e^{-B} \tag{37}$$

independently of the specific form of the potential $U(\psi)$.

VII. A FOURTH-ORDER GRAVITY, FIRST-ORDER INFLATION MODEL

As we mentioned in the Introduction, astrophysically interesting bubbles can be produced only in a two field model, in which one has slow-rolling along a field, ω, while the second field, ψ, performs a phase transition in an orthogonal direction.

We introduced already in [9] a specific model of first-order inflation able to generate large bubbles in fourth-order gravity. Once a conformal transformation is applied, the model is described by the action (we put hereinafter also $G=1$)

$$S=\int\sqrt{-g}d^4x\left(-\frac{\mathcal{R}}{16\pi}+(3/4\pi)\omega_{;\mu}\omega^{;\mu}+\frac{1}{2}e^{-2\omega}\psi_{;\mu}\psi^{;\mu}+U(\psi,\omega)\right), \tag{38}$$

where the potential is

$$U(\psi,\omega)=e^{-4\omega}\left(V(\psi)+\frac{3M^2}{32\pi}W(\psi)(1-e^{2\omega})^2\right), \quad (39)$$

and where

$$W(\psi)=1+\frac{8\lambda}{\psi_0^4}\psi^2(\psi-\psi_0)^2, \quad V(\psi)=\frac{1}{2}m^2\psi^2. \quad (40)$$

The slow-roll inflation driven by ω takes place at $\omega\gg1$, and is over when ω approaches zero. At large ω, the potential U is dominated by $W(\psi)$, and, thus, the false-vacuum minimum at $\psi\approx\psi_0$, for which $U_F=e^{-4\omega}[(3M^2/32\pi)(1-e^{2\omega})^2+V(\psi_0)]$ is unstable with respect to tunneling toward the true vacuum $\psi=0$ (for which $U_T\approx3M^2/32\pi$). At small ω, U is dominated by $V(\psi)$, and both the true and the false vacua converge to the global zero-energy minimum at $\omega=\psi=0$, where inflation ends and reheating takes place. The slow-roll solution in this model for $\omega\gg1$ can be written very conveniently as [9]

$$\tfrac{4}{3}N=e^{2\omega}, \quad (41)$$

where N is the number of e-foldings to the end of inflation. In the same limit, $H=M/2$. The potential (39) for $\omega\gg1$ takes the form (6) by the substitution

$$\psi\rightarrow\psi+\psi_0/2 \quad (42)$$

and, therefore, in the new notations we have

$$R=(3/2\pi)^{1/2}\frac{M\lambda^{1/2}}{m^2\psi_0}e^{4\omega}, \quad (43)$$

$$\Delta=(8\pi/3)^{1/2}\frac{\psi_0}{M\lambda^{1/2}}, \quad (44)$$

$$g=MR/2. \quad (45)$$

Notice that now R depends on the e-folding time as $R\sim N^2$. This is where the scale dependence of the bubble spectrum arises from, as we show below.

The Euclidean action is

$$S_E=\int\sqrt{-g}d^4x\left(-\frac{\mathcal{R}}{16\pi}+\frac{1}{2}e^{-2\omega}\psi_{;\mu}\psi^{;\mu}+U(\psi,\omega)\right) \quad (46)$$

(neglecting the kinetic energy of ω). To obtain a canonical kinetic term, we rescale the coordinates [25] as $x^\mu=e^{-\omega}\hat{x}^\mu$, so that, in the new coordinates

$$S_E=e^{-4\omega}\int\sqrt{-g}d^4\hat{x}\left(-\frac{\mathcal{R}}{16\pi}+\frac{1}{2}\psi_{;\mu}\psi^{;\mu}+U\right)$$
$$=e^{-4\omega}\hat{S}_E(\psi), \quad (47)$$

where finally $\hat{S}_E$ is canonical. Combining Eq. (47), Eq. (26), and Eq. (20), we obtain

$$B=e^{-4\omega}\frac{\pi^2}{6}\frac{R^3\psi_0^2}{\Delta}\left(1-\frac{9\Delta}{2R}\right)(1-g^2) \quad (48)$$

(there is an extra factor of 1/4 because now the vacuum states are at 0 and ψ_0 instead of at $\pm\psi_0$). In terms of N this is

$$B=\frac{N^4}{N_1^4}\left[1-\left(\frac{N_2}{N}\right)^2\right]\left[1-\left(\frac{N}{N_3}\right)^4\right], \quad (49)$$

where

$$N_1^2=\frac{3^{3/2}}{4}\frac{\psi_0 m^3}{M^2\lambda}, \quad (50)$$

$$N_2^2=\frac{27\pi}{8}\frac{\psi_0^2m^2}{M^2\lambda}, \quad (51)$$

$$N_3^2=\left(\frac{27\pi}{32}\right)^{1/2}\frac{\psi_0 m^2}{M^2\lambda^{1/2}}. \quad (52)$$

The two approximations we adopted are valid for $N>N_2$ (thin wall), and for $N<N_3$ (gravitational correction). It is useful now to introduce a fourth e-folding epoch, N_0, defined as the epoch at which the crucial quantity $Q=4\pi\Gamma/9H^4$ equals unity. This can be seen as the epoch at which one bubble for horizon volume for Hubble time (or 4-horizon) is nucleated, that is, the bubbles are saturating the false-vacuum space. The bulk of the nucleation occurs, therefore, just before N_0, and spans only few e-folding times. We, therefore, can approximate $Q(N)$ as

$$Q=\frac{4\pi\Gamma}{9H^4}=\exp\left\{\frac{(N_0^4-N^4)}{N_1^4}\left[1-\left(\frac{N_2}{N_0}\right)^2\right]\left[1-\left(\frac{N_0}{N_3}\right)^4\right]\right\}, \quad (53)$$

where

$$N_0^4=\frac{N_1^4}{[1-(N_2/N_0)^2][1-(N_0/N_3)^4]}\ln\left(\frac{64\pi\mathcal{M}^4}{9M^4}\right). \quad (54)$$

From Eq. (37) we can write the factor $\mathcal{M}$ as $\mathcal{M}'B^{1/2}(\Delta/2R)$.

The scalar and gravitational zero-point fluctuations generated during the ω-driven slow-rolling are proportional to M [26,27]. From the current microwave background measurements, we obtain $M\approx5\times10^{-6}$ (in Planck units), a value that we will adopt in the numerical results below. However since in our model one also should consider the contribution of the bubbles to the microwave background, this constraint is actually only an upper limit on M. Once M is fixed, the three e-folding constants N_{1-3} determine fully the inflationary potential, through the three remaining constants m,λ,ψ_0. A successful sequence of epochs is the following: at N_3, the gravitational correction ceases to be important; at N_0 the crucial parameter Q is of order unity, and the bubbles saturate the space; at N_2 the thin-wall approximation breaks down; and finally at N_1 the bounce action B equals unity, and, thus, the semiclassical approximation is no longer reliable. For traces of the phase transition to be visible today, we also must impose $N_0<N_T$, being N_T the observable last 60 e-foldings of the inflation. Then the full set of constraints is

$$N_3 > N_T > N_0 > N_1, N_2. \tag{55}$$

VIII. THE BUBBLE SPECTRUM

We now can calculate explicitly the bubble spectrum in our model. The number of bubbles nucleated in the interval dt is [28]

$$\frac{dn_B}{dt} = \Gamma a^3 V_{\rm in} \exp(-I),$$

$$I \equiv \left[-\frac{4\pi}{3} \int_0^t dt' \Gamma(t') a^3(t') \left(\int_{t'}^t \frac{d\tau}{a(\tau)} \right)^3 \right], \tag{56}$$

where $V_{\rm in}$ is the horizon volume at $N = N_T$, $V_{\rm in} = 4\pi/3H_{\rm in}^3$ and where the exponential factor accounts for the fraction of space which remains in the false vacuum. To get a manageable expression, we first change variable in Eq. (56) from the nucleation epoch t to the scale L in horizon crossing at t, by use of the relation $dL/dt \approx -H_{\rm in} L_h/a$ valid during slow roll. This gives

$$\frac{dn_B}{dL} = -3 L_h^3 L^{-4} Q(N) e^{-I}. \tag{57}$$

We approximate $Q(N)$ around $N = N_0$ as $Q \approx \exp[s(N_0 - N)]$, where

$$s = 4 \frac{N_0^3}{N_1^4} \left[1 - \left(\frac{N_2}{N_0} \right)^2 \right] \left[1 - \left(\frac{N_0}{N_3} \right)^4 \right]. \tag{58}$$

We make use of the relation between the e-folding time N and the bubble comoving size L

$$HL(N) = H_{\rm in} L_h \exp(N - N_T). \tag{59}$$

It follows $Q = e^{-s\Delta N} (L_h/L)^s$, where $\Delta N = N_T - N_0$ corresponds to the duration of the transition. For $I \ll 1$, i.e., far from the end of the transition, we obtain

$$\frac{dn_B}{dL} = A L^{-4-s} \tag{60}$$

and

$$n_B = (L_m/L)^p, \quad p = 3 + s, \tag{61}$$

where

$$L_m = L_h e^{(3-p)\Delta N/p} (3/p)^{1/p}. \tag{62}$$

In the thin-wall, zero-gravity limit, $s = 4N_0^3/N_1^4$ [9]. When I approaches unity, the nucleation process reaches a peak, and then declines rapidly, due to the fast decrease in the false-vacuum space available. In [9] we showed that the peak occurs, as expected, just after N_0, which we, therefore, consider the end of the nucleation.

As we discussed in Ref. [9], Eq. (62) differs from the analogous quantity in extended inflation [1,5,10] because here we have ΔN instead of $N_T \approx 60$. We show below, by a suitable choice of ΔN, i.e. of the duration of the transition, that we may have a bubble spectrum n_B which generates astrophysically large bubbles and escapes the CMB bounds, contrary to what occurs in the other models of first-order inflation.

IX. RESULTS AND DISCUSSION

We now wish to compare the bubble spectrum with the constraints from large-scale structure and the cosmic microwave background. It is not the aim of this work to determine the best model parameters, also because the statistics on observed voids is still at a very schematic level. We will focus instead on showing that there is a region of parametric space which gives astrophysically large bubbles, that can contribute significantly, if not exclusively, to structure formation, while passing the CMB constraints.

When a bubble nucleates, its density contrast can be estimated (in the thin-wall limit), as

$$\delta \equiv |\delta\rho/\rho| = \frac{U_F - U_T}{U_F} = [(N/N_4)^2 + 1]^{-1}, \tag{63}$$

where

$$N_4^2 = 3\pi \frac{\psi_0^2 m^2}{M^2}. \tag{64}$$

Since the bubbles cross out the horizon soon after their nucleation, the same density contrast will be found at reenter. The bubble density contrast increases to unity for $N \ll N_4$, i.e., toward the end of inflation. One can also write $N_4 \approx 1.5 M N_3^4/N_1^2$. For instance, taking acceptable values as $N_3 = 90$, $N_1 = 20$, and $M = 5 \times 10^{-6}$, one obtains $\delta \approx 6 \times 10^{-4}$ at $N \approx N_0 \approx 50$. Taking $N_3 \to \infty$ one would obtain completely void bubbles, $\delta \to 1$. Here we will consider instead only small values of δ [13]. From Eq. (59) and $N \gg N_4$, we have the density spectrum at reenter

$$\delta(L) = \left(\frac{N_4}{N_T} \right)^2 \left(1 - \frac{2}{N_T} \ln(L/L_h) \right). \tag{65}$$

Since $\delta \ll 1$, we do not consider (contrary to Refs. [9,11,14]) the overcomoving growth that takes place only if and when, in its late history, the density becomes nonlinear.

Let us discuss first the CMB constraints. A bubble of radius L at decoupling produces a Sachs-Wolfe distortion on the microwave temperature of $\Delta T/T \sim \delta(L) L^2/L_d^2$ if L_d denotes the horizon scale at decoupling. In reality, bubbles which reenter before decoupling have time to deepen; for the range of scales we are interested in, however, this is a minor effect and we neglect it. In a pixel corresponding to a size of $L_p > L$ at decoupling, a further factor of L^2/L_p^2 smears the signal [5]. There are two main CMB constraints arising from observational upper bounds to such Sachs-Wolfe effect. Full-sky, low-resolution surveys such as the Cosmic Background Explorer (COBE) can detect rare big bubbles as hot spots. On the other hand, a large number of small bubbles can be detected as Poissonian fluctuations in high resolution, small coverage experiments with antenna beam below 1°. Here we restrict our attention to the first kind of observational effect, as the second one depends on the lesser-known physics of the small bubbles nucleated near the end of the first-order transition. At any rate, the small bubble constraint is expected to be less severe than the hot-spot test for bubble spectrum slopes $p \leqslant 10$ [11]. In [13,14] we also analyzed the constraints on bubble models from Doppler effects on the last scattering surface, and found them to be generally smaller than the Sachs-Wolfe ones.

Assuming a power-law spectrum like Eq. (56), the constraint can be put in form of restrictions on the two parameters p and L_m for large p [11]. Let L_v denote the smallest bubble at decoupling that can give an observable 3σ signal ($\Delta T/T \approx 5\times 10^{-5}$) in a COBE pixel. We simply require that there are fewer than one bubble larger than L_v intersecting the last scattering surface. Then the hot-spot constraint amounts to [5,9]

$$L_m < L_v \left(\frac{(p-1)L_h}{pL_v} \right)^{1/p}. \tag{66}$$

L_v is calculated as

$$\Delta T/T \approx \frac{\delta(L_v)}{3} \frac{L_v^4}{L_d^2 L_p^2}. \tag{67}$$

We will take $L_d = 190h^{-1}$ Mpc, and $L_p = 300h^{-1}$ Mpc. The latter value corresponds to a beam angular opening of 3°, which is roughly the COBE beam opening.

Next we impose to our bubble spectrum the qualitative requirement that it be able to produce a significant large-scale structure. We realize this condition in a very simplified way: we just find the parametric region for which bubbles larger than $3h^{-1}$ Mpc fill the space for more than 50% [9], a constraint which can be compared, for instance, with the observed voids in the SSRS2 database [17]. Notwithstanding its simplicity, this minimal condition puts a strong constraint on the parameter space. If we had indications on the real matter content of large voids, we could put a further direct constraint on δ. For instance, if the voids of $20h^{-1}$ Mpc (which, therefore, are nucleated 5-6 e-foldings after N_T) that are detected currently on the large-scale surveys (see, e.g., [17]) are completely empty of matter, they should have $\delta \geqslant 10^{-3}$ at decoupling, in order for them to be cleared of matter by linear growth at the present. If some matter, maybe in the form of Lyα clouds [18], is found inside the voids, then one should impose on the contrary $\delta < 10^{-3}$. Here, to fix the ideas, we put $\delta = 10^{-3}$ on scales of $\approx 30h^{-1}$ Mpc. In any case, the results are not very sensitive to δ.

The main results are contained in Fig. 2. On the plane (p, L_m), we display as a shaded area the parametric region of cosmological interest, i.e., the models which satisfy the CMB constraint (66), and fill the space as requested. The CMB constraint is plotted for $N_3 = 90$ and $\delta = 10^{-3}$, which, along with $M = 5\times 10^{-6}$, implies $N_1 \approx 17$. These values satisfy all the conditions for a successful inflation. Then we plot two curves $L_m(p)$ given in Eq. (61) for $N_0 = 53{,}55$. It can be seen that these curves cross the acceptable region; our model is, therefore, capable of producing bubble spectra which pass the CMB tests and have interesting large scale features. Let us consider one pair of parameters for reference: $p = 9$, $L_m = 100h^{-1}$ Mpc, which lies on the curve $N_0 = 54$, and is

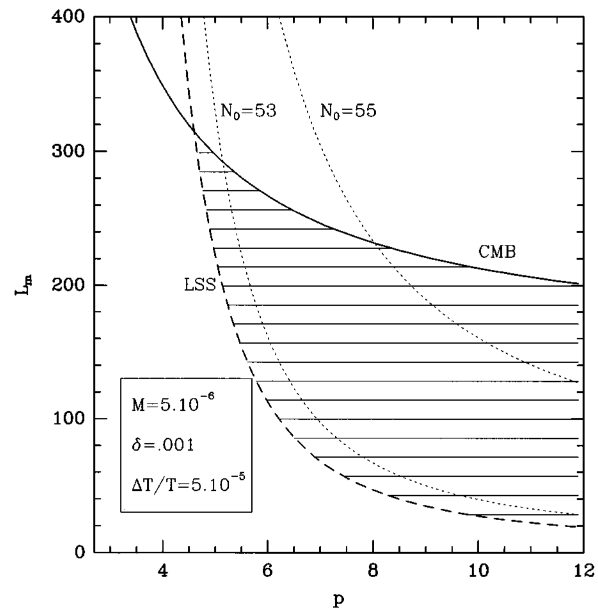

FIG. 2. Parametric region of the model. The shaded area is the region which passes the constraint from the microwave background isotropy (solid line labeled CMB) and the constraint from the large-scale structure (dashed line labeled LSS). The two dotted lines give $L_m = L_m(p)$ in our model for the two values of N_0 shown. The other parameters adopted are indicated in the box.

inside the acceptable region. Then, from Eq. (58), and putting $\delta = 0.001$ and $M = 5\times 10^{-6}$, we have that the full set of e-folding parameters is

$$N_0 = 54, \quad N_1 = 17, \quad N_2 = 16, \quad N_3 = 90, \tag{68}$$

which verifies Eq. (55); we have also $N_4 = 1.7$. Clearly, this is only one of an infinite set of parameters that fulfills all the conditions; however, the values are, in general, close to those quoted. We also may note that with such a bubble spectrum, $p = 9$ and $L_m = 100h^{-1}$ Mpc, one can estimate $(R_s/L_h)^3 n_B(>20h^{-1}\,\mathrm{Mpc}) \approx 70$ voids larger than 20 h^{-1} Mpc by radius in a spherical survey of depth $R_s = 200h^{-1}$ Mpc; this value, which of course is extremely approximate, is indicative that many, if not all, of the present voids can be explained by a first-order phase transition. Finally, the potential parameters, other than M, are (all in Planck units)

$$\lambda = 4\times 10^{-3}, \quad m = 3.3\times 10^{-3}, \quad \psi_0 = 1.8\times 10^{-3}. \tag{69}$$

Thus as anticipated in the Introduction, the observations of the two bubble spectrum parameters, L_m and p, of the matter content inside bubbles (so to fix δ), and of the ordinary fluctuations driven by ω on the microwave background, which give M, are in principle all what we need to reconstruct completely the primordial first-order potential. This would be impossible in the other models of first-order inflation, in which bubbles do not leave observable traces.

X. CONCLUSIONS

Large voids are ubiquitous in the Universe. They occupy most of the observable volume, and are probably the dominant contribution to the large-scale power spectrum [16]. The standard possibility is that they derive simply from the primordial underdensities which are expected in the ordinary Gaussian models of structure formation. However this scenario would be put in jeopardy if larger and larger voids continue to be discovered [29]. Moreover, if the voids are actually filled with unclustered, or mildly clustered, matter, it would be impossible to explain their roughly spherical shape: an underdensity, in fact, becomes more and more spherical only as it becomes nonlinear [30]. It is, therefore, worthy to investigate alternatives. We proposed here, based on earlier work [11,9], a first-order inflationary model which produces primordial nonempty bubbles, along with ordinary slow-rolling fluctuations. We calculated in detail the nucleation rate, including classical, quantum, and gravitational corrections, and showed that the model gives a strong contribution to large scale structure, while passing the microwave constraints. The determination of four observable quantities fixes completely the primordial potential, including the tunneling sector, which, on the contrary, is unobservable in the other models of first-order inflation.

[1] D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 376 (1989).
[2] D. La, P. J. Steinhardt, and E. Bertschinger, Phys. Lett. B **231**, 231 (1989); E. J. Weinberg, Phys. Rev. D **40**, 3950 (1989); T. Damour, G. W. Gibbons, and C. Gundlach, Phys. Rev. Lett. **64**, 123 (1990); P. J. Steinhardt and F. S. Accetta, *ibid.* **64**, 2740 (1990); R. Holman *et al.*, Phys. Lett. B **237**, 37 (1990).
[3] F. C. Adams and K. Freese, Phys. Rev. D **43**, 353 (1991).
[4] M. S. Turner, E. J. Weinberg, and L. M. Widrow, Phys. Rev. D **46**, 2384 (1992).
[5] A. R. Liddle and D. Wands, Mon. Not. R. Astron. Soc. **253**, 637 (1991).
[6] M. Bucher, A. S. Goldhaber, and N. Turok, Phys. Rev. D **52**, 3314 (1995); M. Bucher and N. Turok, *ibid.* **52**, 5538 (1995).
[7] A. Linde, Phys. Lett. B **351**, 99 (1995); A. Linde, Report No. SU-ITP-95-15, gr-qc/9508019 (unpublished); A. Linde and A. Mezhlumian, Phys. Rev. D **52,** 6789 (1995).
[8] L. Amendola, C. Baccigalupi, and F. Occhionero, Phys. Rev. D **54**, 4760 (1996).
[9] F. Occhionero and L. Amendola, Phys. Rev. D **50**, 4846 (1994).
[10] D. La, Phys. Rev. B **265**, 232 (1991).
[11] L. Amendola and F. Occhionero, Astrophys. J. **413**, 39 (1993).
[12] L. Amendola and S. Borgani, Mon. Not. R. Astron. Soc. **266**, 191 (1994).
[13] F. Occhionero, C. Baccigalupi, and L. Amendola, report, 1996 (unpublished).
[14] C. Baccigalupi, L. Amendola, and F. Occhionero, report, 1996 (unpublished).
[15] M. J. Geller and J. P. Huchra, Science **246**, 897 (1989); L. N. Da Costa *et al.*, Astrophys. J. **424**, L1 (1994).
[16] S. D. Landy, S. A. Schectman, H. Lin, R. P. Kirshner, A. A. Oemler, and D. Tucker, Astrophys. J. **456,** L1 ((1996).
[17] H. El-Ad, T. Piran, and L. N. Da Costa, Astrophys. J. **462**, 13L (1996).
[18] J. Stocke *et al.*, Astrophys. J. **451**, 24 (1995); P. A. Henning, *ibid.* **450**, 578 (1995); S. Pustil'nik *et al.*, *ibid.* **443**, 499 (1995); A. Szomoru *et al.*, Astron. J. **108**, 491 (1994).
[19] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
[20] J. P. Ostriker and L. N. Cowie, Astrophys. J. **243**, L127 (1981); F. Occhionero, P. Santangelo, and N. Vittorio, Astron. Astrophys. **117**, 365 (1983); P. Coles and J. Barrow, Mon. Not. R. Astron. Soc. **244**, 557 (1990); S. Yoshioka and S. Ikeuchi, Astrophys. J. **341**, 16 (1989); R. van de Weygaert and V. Icke, Astron. Astrophys. **213**, 1 (1989).
[21] S. Coleman, Phys. Rev. D **15**, 2929 (1977); C. Callan and S. Coleman, *ibid.* **16**, 1762 (1977); S. Coleman and F. De Luccia, *ibid.*, **21**, 3305 (1980).
[22] R. V. Konoplich, Theor. Math. Phys. **73**, 379 (1987).
[23] A. A. Starobinsky, JETP Lett. **30**, 682 (1979).
[24] S. Parke, Phys. Lett. **121B**, 313 (1983).
[25] E. W. Kolb, Phys. Scr. **T36**, 199 (1991).
[26] L. Amendola, M. Litterio, and F. Occhionero, Phys. Lett. B **237**, 348 (1990).
[27] A. A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).
[28] A. Guth and E. J. Weinberg, Nucl. Phys. **B212**, 321 (1983).
[29] J. G. Cohen, D. W. Hogg, M. A. Pahre, and R. Blandford, Astrophys. J. **462**, L9 (1996).
[30] V. Icke, Mon. Not. R. Astron. Soc. **206**, 1 (1984).