More varieties of hybrid inflation

David H. Lyth
School of Physics and Chemistry, Lancaster University, Lancaster, LA1 4YB, United Kindgom

Ewan D. Stewart
Research Center for the Early Universe, School of Science, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan

(Received 20 June 1996)

It is pointed out that hybrid inflation can be implemented with the inflaton field rolling away from the origin instead of towards it. This ''inverted'' hybrid inflation has a spectral index $n<1$, in contrast with ordinary hybrid inflation which has $n\gtrsim 1$, so a measured value of n substantially different from 1 would distinguish the two. Other generalizations of hybrid inflation are also considered. [S0556-2821(96)00824-7]

PACS number(s): 98.80.Cq

I. INTRODUCTION

The most attractive models of inflation at present are the ''hybrid'' inflation models [1–19], in which inflation ends due to the interaction of the inflaton field with other fields. As the very first models [20], but in contrast with those proposed in the intervening years, hybrid models have the virtue that the relevant fields can be small on the Planck scale. As a result it has become possible, even in the demanding context of supergravity [6,8,10,17–19], to construct reasonable-looking models of inflation which work with potentials similar to those routinely proposed to describe particle physics beyond the standard model. The purpose of the present paper is to propose some more versions of hybrid inflation satisfying these criteria, and to evaluate their predictions regarding large scale structure.

Inflation generates an adiabatic density perturbation and gravitational waves. The former is supposed to be the origin of large scale structure and, together with a possible contribution from the gravitational waves, of the cosmic microwave background (CMB) anisotropy. The spectrum of the density perturbation is conveniently specified by a quantity δ_H^2 whose scale dependence is $\delta_H^2\propto k^{n-1}$ where n is the spectral index. The spectrum of the gravitational waves is conveniently specified by their relative contribution R to the mean-square low multipoles of the CMB anisotropy seen by a randomly placed observer.

Within the usual paradigm of a single slow-rolling field[1] (which includes most versions of hybrid inflation considered up to now, and we shall also restrict ourselves to this simple case) the predictions for δ_H^2 [22,23], n [24,25], and R [26] are

$$\delta_H^2(k)=\frac{1}{75\pi^2}\frac{V^3}{V'^2}, \tag{1}$$

$$n-1=-3\left(\frac{V'}{V}\right)^2+2\frac{V''}{V}, \tag{2}$$

$$R=6\left(\frac{V'}{V}\right)^2. \tag{3}$$

The units are $\hbar=c=M_{\rm Pl}\equiv(8\pi G)^{-1/2}=1$, and the potential V and its derivatives are to be evaluated when cosmological scales leave the horizon N e folds before the end of inflation, where $N=50$–25 and is given by $N=\int(V/V')d\phi$. In models with a small field variation, including in particular hybrid inflation models, R is unobservably small [27] and n is either very close to 1 or is given by $n-1=2V''/V$.

At present, observation is consistent with $R=0$, and with this value the Cosmic Background Explorer (COBE) measurement of the low multipoles of the CMB anisotropy gives [28] $\delta_H=1.94\times10^{-5}$. The present constraint on n is only [29] $0.7<n<1.4$, though eventually one can expect a measurement with an accuracy $\Delta n\sim0.01$ [30]. A future measurement of n, and detection or nondetection of R, will discriminate sharply between models of inflation.

In hybrid inflation the other field can either be fixed, as in the original model, or it can move slowly as it adjusts to minimize the potential at a fixed value of the inflaton field. We shall call the former case ''ordinary'' hybrid inflation, and following [10] shall call the latter case ''mutated'' hybrid inflation.

In practically all versions[2] of ordinary hybrid inflation proposed so far, the inflaton field is near a minimum of the potential. In them the potential is convex and n can be significantly *bigger* than 1 up to and beyond the observational upper limit, though in most of parameter space it is actually indistinguishable from 1. The two versions [10,11] of mutated hybrid inflation proposed so far have a mildly concave potential leading to n in the range 0.93–00.97 (depending on the value of N). We are going to point out the existence of ''inverted'' ordinary hybrid inflation, which works near a maximum of the potential so that n can be substantially *less*

[1]A more general slow-roll paradigm [21] leaves the gravitational waves unchanged but increases δ_H, so that Eq. (1) becomes a lower bound, Eq. (3) an upper bound, and Eq. (2) is no longer valid.

[2]The exception is the polynomial potential studied in [12]. It is claimed there that a value of n substantially below 1 can be obtained, but as pointed out later in [13] the possible interruption of inflation is not studied, and will restrict the allowed parameter space.

than 1 (though again indistinguishable from it in most of parameter space). We are also going to write down a whole class of mutated hybrid models, and evaluate their predictions for n as well as the normalization of δ_H. In all cases, we are going to indicate how the models can be justified in the demanding context of supergravity, using the scheme proposed in [8].

The outline of the paper is as follows. In the next section we study inverted hybrid inflation, and in the following we mention some generalizations of it. In Sec. IV we give the mutated hybrid models, and in Sec. V our conclusions.

II. INVERTED HYBRID INFLATION

In the usual models of hybrid inflation ϕ is rolling towards zero. The potential is typically of the form

$$V=V_0+\frac{1}{2}m^2\phi^2+\cdots \tag{4}$$

and is dominated by the term V_0. This term arises because some other field ψ is held at the origin by its interaction with ϕ. When ϕ falls below some critical value ϕ_c, the other field rolls to its vacuum value so that V_0 disappears and inflation ends. This model gives negligible R and $n-1=2m^2/V_0$.

We consider instead the opposite case of ''inverted'' hybrid inflation, where ϕ rolls away from the origin and

$$V=V_0-\frac{1}{2}m^2\phi^2+\cdots \tag{5}$$

$$=V_0\left[1-\frac{1}{2}\left(\frac{\phi}{f}\right)^2+\cdots\right]. \tag{6}$$

We again assume that V_0 dominates, and have introduced the parameter $f^2=V_0/m^2$. This gives $n-1=-2m^2/V_0=-2/f^2$. The present observational lower bound $n>0.7$ [29] requires $f>2.6$ and any future tightening will increase the required value of f.

In this model, gravitational waves are negligible and the normalization is

$$V_0=7\times10^{-8}(1-n)^2\phi_c^2\exp^{[-(1-n)N]}, \tag{7}$$

where we have used the results of Sec. IV.

A complete model should specify the mechanism which ends inflation. In ordinary hybrid inflation one considers a potential

$$V=V_0+\frac{1}{2}m_\phi^2\phi^2-\frac{1}{2}m_\psi^2\psi^2+\frac{1}{2}\lambda\phi^2\psi^2+\cdots \tag{8}$$

with $m_\phi^2\ll V_0\lesssim m_\psi^2$ One possibility for inverted hybrid inflation is just to reverse the signs of all the terms

$$V=V_0-\frac{1}{2}m_\phi^2\phi^2+\frac{1}{2}m_\psi^2\psi^2-\frac{1}{2}\lambda\phi^2\psi^2+\cdots . \tag{9}$$

Then ψ will be constrained to zero for

$$\phi<\phi_c=\frac{m_\psi}{\sqrt{\lambda}} \tag{10}$$

and will roll away from zero when ϕ becomes larger than ϕ_c. This model could be realized in supersymmetry using the superpotential

$$W=\left(\Lambda^2+\frac{\lambda\Phi^2\Psi^2}{\Lambda^2}\right)\Xi. \tag{11}$$

The corresponding globally supersymmetric scalar potential is

$$V=\left|\Lambda^2+\frac{\lambda\Phi^2\Psi^2}{\Lambda^2}\right|^2+\frac{4\lambda^2}{\Lambda^4}(|\Phi|^2+|\Psi|^2)|\Phi|^2|\Psi|^2|\Xi|^2. \tag{12}$$

Writing $|\Phi|=\phi/\sqrt{2}$ and $|\Psi|=\psi/\sqrt{2}$, minimizing with respect to $\arg\Phi^2\Psi^2$, and assuming Ξ is held at zero, gives

$$V=\left(\Lambda^2-\frac{\lambda\phi^2\psi^2}{4\Lambda^2}\right)^2=\Lambda^4-\frac{1}{2}\lambda\phi^2\psi^2+\cdots. \tag{13}$$

Adding soft supersymmetry-breaking masses[3] of the required sign gives

$$V=\Lambda^4-\frac{1}{2}m_\phi^2\phi^2+\frac{1}{2}m_\psi^2\psi^2-\frac{1}{2}\lambda\phi^2\psi^2+\cdots. \tag{14}$$

An alternative way of implementing inverted hybrid inflation is given by

$$V=V_0-\frac{1}{2}m_\phi^2\phi^2-\frac{1}{2}m_\psi^2\psi^2-\frac{1}{2}m_\chi^2\chi^2+\frac{1}{2}\lambda_\phi^2\phi^2\chi^2+\frac{1}{2}\lambda_\psi^2\psi^2\chi^2+\frac{1}{4}\lambda_\chi^2\chi^4 \tag{15}$$

with $m_\phi^2\ll V_0\lesssim m_\psi^2, m_\chi^2$. Here, ψ will be constrained to zero if $\chi>m_\psi/\lambda_\psi$. The minimum of χ's potential is at

$$\chi=\frac{\sqrt{m_\chi^2-\lambda_\phi^2\phi^2}}{\lambda_\chi}, \tag{16}$$

assuming $\psi=0$ and $\phi<m_\chi/\lambda_\phi$. Therefore, for

$$\phi<\phi_c=\frac{\sqrt{\lambda_\psi^2m_\chi^2-\lambda_\chi^2m_\psi^2}}{\lambda_\phi\lambda_\psi}, \tag{17}$$

ψ will be constrained to zero. Clearly, we require $\lambda_\psi m_\chi>\lambda_\chi m_\psi$. Also, to ensure that the terms involving χ in Eq. (15) make a negligible contribution to the effective potential during inflation, we require $\lambda_\phi m_\chi\ll\lambda_\chi m_\phi$. The contribution of the ϕ dependence of χ to the effective kinetic terms during inflation can be neglected if $\lambda_\phi\lambda_\psi m_\chi\ll\lambda_\chi^2 m_\psi$.

[3]Note that one would in general expect $m_\phi\sim m_\psi\sim\Lambda^2$ but as we have $V=|W_\Xi|^2$ and $W_\Phi=W=\Xi=0$ we can use the method of Ref. [8] to allow $m_\phi\ll\Lambda^2$.

Previous authors [31–35] considered the potential (5) in the context of a single-field model, and assumed that it holds until $\phi \simeq f$, after which ϕ settles down to the minimum located at a value somewhat bigger than f. Since observation requires $f \gg 1$ (the Planck scale in our units), this places the model outside the regime of ordinary particle theory so that one can hardly justify the form of the potential. It has been suggested that ϕ might be identified with one of the superstring moduli [32,34,35], but attempts [32,34,36,37] to construct a specific model using this idea have not been successful.[4]

We have avoided these difficulties by ending inflation earlier using the hybrid inflation mechanism. An alternative would be to keep the single field ϕ, but to suppose that its potential steepens soon after cosmological scales leave the horizon so that inflation again ends at some value $\phi_c \ll f$. Such a proposal is not unreasonable, though it does postulate a lot of structure in the single potential $V(\phi)$.

III. GENERALIZATIONS OF INVERTED HYBRID INFLATION

Instead of assuming that the inflationary potential is quadratic, one can consider the possibility that it is of higher order. This might be because the quadratic term is absent, or it might be because one is not close to the origin though in that case there is no reason to suppose that a single term dominates. If the quadratic term is absent one might have

$$V = V_0 - \frac{1}{4}\lambda\phi^4 + \cdots. \tag{18}$$

This form could arise from one of the generalizations of mutated hybrid inflation considered in the next section, and we use the results derived there. For $\phi_c^2 \gg V_0/(2N\lambda)$ the predictions are independent of ϕ_c and are the same as those for the nonhybrid case; $1-n=3/N$ with negligible gravitational waves and the normalization of δ_H requiring $\lambda \sim 10^{-12}$ independently of V_0. In the opposite case, $1-n$ is reduced by a factor $\phi_c^2/[V_0/(2N\lambda)]$ and the value of λ is reduced by this factor cubed.

Including both a quadratic and a quartic term, one could have

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4. \tag{19}$$

If inflation (after the observable Universe leaves the horizon) takes place near the maximum it reduces to the quadratic inverted hybrid inflation that we started out with. If it takes place near the minimum it reduces to the usual version of hybrid inflation, and in the intermediate case one gets something different.

Finally, we should point out that a related model of inflation was proposed a long time ago [38]. In it, the inflaton is rolling away from the origin and it triggers the GUT Higgs transition at or before the end of inflation. The potential for this model is very complicated and it is not clear whether the Higgs potential is supposed to dominate the energy density. If there is a regime of parameter space in which it does, then the model would represent the first version of inverted hybrid inflation (and, in fact, the first version of hybrid inflation of any kind).

IV. GENERALIZATIONS OF MUTATED HYBRID INFLATION

Here, we consider potentials of the form

$$V = V_0 - \frac{\sigma}{p}\psi^p + \frac{\lambda}{q}\psi^q\phi^r \tag{20}$$

with $p \neq q$. The opposite case of $p=q$ simply gives an ordinary or inverted hybrid inflation model where ψ is held at zero during inflation. In mutated hybrid inflation [10], and its generalizations [11] considered here, ψ is held close to zero, but not at zero, during inflation. An effective potential for the inflaton $\simeq \phi$ is then generated from the couplings without requiring any additional term and, as we shall see, it can take unusual forms. In the case that this contribution to the effective potential is not the dominant one, it will not determine the spectrum but may still determine when inflation ends.

We assume $\psi > 0$ and $\phi > 0$. Then, to get a model which, for fixed ϕ, has a minimum at $\psi = \psi_*$ with $\psi_* > 0$, we require $\sigma\lambda > 0$ and $(q-p)\sigma > 0$. See Eqs. (21) and (23), respectively below. $\sigma > 0$ corresponds to a generalization of mutated hybrid inflation with the inflaton ϕ rolling towards zero, while $\sigma < 0$ corresponds to a general inverted mutated hybrid inflation model with ϕ rolling away from zero.

Now,

$$V_\psi = -\sigma\psi^{p-1} + \lambda\psi^{q-1}\phi^r, \tag{21}$$

and so $V_\psi = 0$ when $\psi = 0$ (for $p \geq 2$ and $q \geq 2$), or

$$\psi = \psi_* \equiv \left(\frac{\sigma}{\lambda}\right)^{1/(q-p)}\phi^{-r/(q-p)}. \tag{22}$$

Now,

$$V_{\psi\psi}|_{\psi=\psi_*} = (q-p)\sigma\psi_*^{p-2}. \tag{23}$$

For simplicity, we will assume that $V_{\psi\psi}|_{\psi=\psi_*} \gg V_0$ so that ψ is held firmly at $\psi = \psi_*$ during inflation. Then,

$$V|_{\psi=\psi_*} = V_0 - \left(\frac{q-p}{pq}\right)\sigma\psi_*^p \tag{24}$$

and the kinetic terms evaluated along $\psi = \psi_*$ are

$$\frac{1}{2}\left[1 + \left(\frac{r}{q-p}\right)^2\frac{\psi_*^2}{\phi^2}\right](\partial\phi)^2. \tag{25}$$

Assuming $\sigma\psi_*^p \ll V_0$ so that V_0 dominates the energy density and $\psi_* \ll \phi$ so that the kinetic terms are approximately canonical, we get the effective potential during inflation

[4]Reference [37] claims to have been successful but an analytic calculation shows that their Eq. (7) gives a potential $V(\phi) \simeq -4.3 - 0.01\cos(12\phi)$ for the canonically normalized field proportional to ImS after minimizing with respect to ReS.

$$V(\phi)=V_0(1-\mu\phi^{-\alpha}), \tag{26}$$

where

$$\mu=\left(\frac{q-p}{pq}\right)\frac{\sigma^{q/(q-p)}\lambda^{-p/(q-p)}}{V_0}>0 \tag{27}$$

and

$$\alpha=\frac{pr}{q-p}\neq 0. \tag{28}$$

Now,

$$\frac{V'}{V}=\alpha\mu\phi^{-\alpha-1} \tag{29}$$

and

$$\frac{V''}{V}=-\alpha(\alpha+1)\mu\phi^{-\alpha-2}. \tag{30}$$

The number of e folds to the end of inflation is given by

$$N=\int\frac{V}{V'}d\phi=\frac{\phi^{\alpha+2}}{\alpha(\alpha+2)\mu},\quad \alpha>0 \text{ or } \alpha<-2, \tag{31}$$

$$=\frac{1}{|\alpha|(2-|\alpha|)\mu}(\phi_c^{2-|\alpha|}-\phi^{2-|\alpha|}),\quad -2<\alpha<0, \tag{32}$$

$$=-\frac{1}{2\mu}\ln\frac{\phi}{\phi_c},\quad \alpha=-2. \tag{33}$$

The COBE normalization [29] gives

$$\begin{aligned}5.3\times10^{-4}=\frac{V^{3/2}}{V'}&=\frac{V_0^{1/2}\phi^{\alpha+1}}{\alpha\mu}\\ &=|\alpha|^{-1/(\alpha+2)}|\alpha+2|^{\alpha+1/(\alpha+2)}N^{(\alpha+1)/(\alpha+2)}\\ &\quad\times\mu^{-1/(\alpha+2)}V_0^{1/2},\end{aligned}$$

$$\alpha>0 \text{ or } \alpha<-2, \tag{34}$$

$$=\frac{V_0^{1/2}}{|\alpha|\mu}[\phi_c^{2-|\alpha|}-|\alpha|(2-|\alpha|)\mu N]^{-(|\alpha|-1)/(2-|\alpha|)},$$

$$-2<\alpha<0, \tag{35}$$

$$=\frac{e^{2\mu N}V_0^{1/2}}{2\mu\phi_c},\quad \alpha=-2. \tag{36}$$

The spectral index is given by

$$n\simeq 1+2\frac{V''}{V}=1-2\left(\frac{\alpha+1}{\alpha+2}\right)\frac{1}{N},\quad \alpha>0 \text{ or } \alpha<-2, \tag{37}$$

$$=1-\frac{2|\alpha|(|\alpha|-1)\mu}{\phi_c^{2-|\alpha|}-|\alpha|(2-|\alpha|)\mu N},\quad -2<\alpha<0, \tag{38}$$

$$=1-4\mu,\quad \alpha=-2. \tag{39}$$

Note that $n<1$ in all cases except $-1\leqslant\alpha<0$. However, to get α in this range would require either $r<1$ or at least one of p, q, or r to be negative. The case $\alpha=-2$ is the one considered already in Sec. II, and for this as well as other α in the range $-2\leqslant\alpha<0$ we needed to specify that inflation ends at $\phi=\phi_c$. If inflation ends because V_0 stops dominating the energy density, we will have $\phi_c\sim\mu^{1/\alpha}$, but higher-order terms neglected in Eq. (20) may end inflation before this which would permit the desirable $\phi_c\lesssim 1$.

Potentials of the form (20) can be straightforwardly derived from supersymmetry along the lines of Ref. [10] for the case $\sigma>0$ (corresponding to $\alpha>0$) and as in Sec. II for the opposite case. In both cases, the superpotentials involved will be compatible with the method of Ref. [8] for avoiding fatal supergravity corrections.

In supersymmetry, one would prefer q and r to be even if $\sigma>0$, and p to be even if $\sigma<0$. Particularly natural possibilities are $\sigma\sim V_0$ and $p=1$ or 2, and $\sigma\sim -V_0$ and $p=2$, with the $p=1$ and $p=2$ cases corresponding, respectively, to a generic soft supersymmetry, breaking term for a singlet and for a nonsinglet.

The three simplest possibilities compatible with these ideas are the following.

(1) $p=1$, $q=2$, $r=2$, leading to $\alpha=2$. This is the original mutated hybrid inflation model [10].

(2) $p=2$, $q=1$, $r=1$, leading to $\alpha=-2$ which is the inverted hybrid inflation model of Sec. II.

(3) $p=2$, $q=1$, $r=2$, leading to $\alpha=-4$ which is the quartic inverted hybrid inflation model of Sec. III.

V. CONCLUSION

The most important new model that we have discussed is inverted hybrid inflation. In contrast with all other known hybrid inflation models it can give a spectral index *significantly* below 1. The other models we have discussed either reproduce already-known possibilities (though with a different prescription for the field value at which inflation ends), or else add to the list of models which give a spectral index *slightly* below 1. A future measurement of n will be a powerful discriminator between hybrid inflation models.

ACKNOWLEDGMENTS

D.H.L. acknowledges support from PPARC, and from the European Commission under the Human Capital and Mobility program, Contract No. CHRX-CT94-0423. E.D.S. was supported by the JSPS at RESCEU, and the work of E.D.S. was supported by Monbusho JSPS Grant-in-Aid No. 95209.

[1] A. D. Linde, Phys. Lett. B **249**, 18 (1990).
[2] F. C. Adams and K. Freese, Phys. Rev. D **43**, 353 (1991).
[3] A. D. Linde, Phys. Lett. B **259**, 38 (1991).
[4] A. R. Liddle and D. H. Lyth, Phys. Rep. **231**, 1 (1993).
[5] A. D. Linde, Phys. Rev. D **49**, 748 (1994).
[6] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart, and D. Wands, Phys. Rev. D **49**, 6410 (1994).
[7] S. Molerach, S. Matarrese, and F. Lucchin, Phys. Rev. D **50**, 4835 (1994).
[8] E. D. Stewart, Phys. Rev. D **51**, 6847 (1995).
[9] G. Dvali, Q. Shafi, and R. Schaefer, Phys. Rev. Lett. **73**, 1886 (1994).
[10] E. D. Stewart, Phys. Lett. B **345**, 414 (1995).
[11] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D **52**, 559 (1995); G. Lazarides, C. Panagiotakopoulos, and N. D. Vlachos, *ibid.* **54**, 1369 (1996).
[12] Y. Wang, Phys. Rev. D **50**, 6135 (1994).
[13] A. R. Liddle, D. H. Lyth, and D. Roberts, Phys. Rev. D **51**, 4122 (1995).
[14] L. Randall, M. Soljacic, and A. H. Guth, Nucl. Phys. **B472**, 377 (1996); Report No. hep-ph/9601296 (unpublished).
[15] J. Garcia-Bellido, A. D. Linde, and D. Wands, Phys. Rev. D **54**, 6040 (1996).
[16] G. Dvali, Report No. hep-ph/9605445 (unpublished).
[17] E. D. Stewart, Report No. hep-ph/9606241 (unpublished).
[18] P. Binetruy and G. Dvali, Report No. hep-ph/9606342 (unpublished).
[19] E. Halyo, Report No. hep-ph/9606423 (unpublished).
[20] A. D. Linde, Phys. Lett. **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982); Q. Shafi and A. Vilenkin, *ibid.* **52**, 691 (1984).
[21] M. Sasaki and E. D. Stewart, Prog. Theor. Phys. **95**, 71 (1996); T. T. Nakamura and E. D. Stewart, Phys. Lett. B **381**, 413 (1996).
[22] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982); S. W. Hawking, *ibid.* **115B**, 295 (1982); A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); J. M. Bardeen, P. S. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[23] D. H. Lyth, Phys. Lett. **147B**, 403 (1984); **150B**, 465 (1985); Phys. Rev. D **31**, 1792 (1985).
[24] A. R. Liddle and D. H. Lyth, Phys. Lett. B **291**, 391 (1992).
[25] D. S. Salopek, Phys. Rev. Lett. **69**, 3602 (1992).
[26] A. A. Starobinsky, Sov. Astron. Lett. **11**, 133 (1985).
[27] D. H. Lyth, Lancaster Report No. hep-ph/9606387 (unpublished).
[28] E. F. Bunn, D. Scott, and W. White, Astrophys. J. **441**, L9 (1995).
[29] A. R. Liddle *et al.*, Report No. astro-ph/9511057 (unpublished); M. White *et al.*, Report No. astro-ph/9605057 (unpublished).
[30] D. H. Lyth, Report No. hep-ph/9609431 (unpublished).
[31] A. D. Linde, Phys. Lett. **132B**, 317 (1983).
[32] P. Binetruy and M. K. Gaillard, Phys. Rev. D **34**, 3069 (1986).
[33] K. Freese, J. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65**, 3233 (1990).
[34] F. C. Adams *et al.*, Phys. Rev. D **47**, 426 (1993).
[35] T. Banks *et al.*, Phys. Rev. D **52**, 3548 (1995).
[36] A. De la Macorra and S. Lola, Phys. Lett. B **373**, 299 (1996).
[37] M. C. Bento and O. Bertolami, Report No. gr-qc/9605070 (unpublished).
[38] K. Enqvist and D. V. Nanopoulos, Phys. Lett. **142B**, 349 (1984); Nucl. Phys. **B252**, 508 (1985); C. Kounnas and M. Quiros, Phys. Lett. **151B**, 189 (1985).