Inflation and inverse symmetry breaking

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An inflation model with inverse symmetry breaking of two scalar fields is proposed. Constraints on the parameters for successful inflation are obtained. In general the inequality $\lambda_1 \ll g < \lambda_2$ should be satisfied, where $\lambda_{1,2}$ and g are the coupling constants for self-interaction and mutual interaction of two scalar fields, respectively. An example with an SU(5) GUT phase transition and numerical study is presented. This model introduces a new mechanism for the onset of inflation. [S0556-2821(96)05224-1]

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I. INTRODUCTION

Various inflation models [1] have been proposed to solve the horizon and flatness problems of standard big bang cosmology. However, in relation to particle physics each model has its own problems to be solved. Therefore, reconciling the inflation models with particle physics is an important subject of modern cosmology.

Since the upper bound on the inflation energy scale is about the grand unified theory (GUT) scale [2], it is natural to search for inflation during the GUT phase transition. However, the original inflation model has the graceful-exit problem [1] and the ''new'' inflation model with the GUT nonsinglet fields leads to a too strong density fluctuation [3]. As a solution to this problem a model with the GUT singlet inflaton coupled with an SU(5) Higgs boson was suggested [4].

Generally, the smallness of the coupling constants required for the small density perturbation prevents inflaton fields from obtaining thermal equilibrium, while the chaotic inflation model [5] uses this nonequilibrium state to give the initial conditions for the inflaton fields.

Though many aspects of the phase transition theory have already been used for the various inflation models, field theory has still other mechanisms of the phase transition to be studied in the context of inflation.

In this paper, an inflation model with ''inverse symmetry breaking'' is investigated. Inverse symmetry breaking [6] is a phenomenon in which the symmetry broken at a higher temperature is restored at a lower temperature, contrary to the ordinary phase transitions. The phenomenon has been applied to solving the monopole problem by allowing the temporary breaking of the U(1) gauge symmetry [7]. A similar phenomenon, called antirestoration, appears in some global supersymmetry (SUSY) theories [8].

Our model is a type of two fields inflation models [9], where generally an additional scalar field besides inflaton is introduced to complete the inflation and/or give an appropriate density perturbation. For example, in the ''hybrid'' or ''false vacuum'' inflation model [10] the additional scalar field gives the inflaton extra masses, which make the inflaton roll down and end the inflation.

The inflaton potential in our model is similar to that in the hybrid model, but the detailed features of the phase transition are very different. In our model the phase transition of a scalar field (say, ϕ_2, for example a GUT Higgs boson) is responsible for the beginning of the inflation driven by a gauge singlet inflaton (say ϕ_1) rather than the end of the inflation. Moreover, the additional field (ϕ_2) is in the true vacuum rather than the false vacuum during the inflation.

In Sec. II, we review inverse symmetry breaking and derive the conditions for the phenomenon. In Sec. III, the constraints for successful inflation are derived. In Sec. IV, an application with an SU(5) GUT model and numerical study are presented. Section V contains discussions.

II. INVERSE SYMMETRY BREAKING

In this section we review inverse symmetry breaking and conditions required for it. Consider the following potential, which is a simple example of inverse symmetry breaking. Such a potential can appear in the approximation of the one-loop finite temperature effective potential of the two interacting massive scalar fields:

$$V(\phi_1,\phi_2,T)=(D_1T^2-\mu_1^2)\phi_1^2+\lambda_1\phi_1^4+(D_2T^2-\mu_2^2)\phi_2^2 +\lambda_2\phi_2^4+g\phi_1^2\phi_2^2+C, \tag{1}$$

where $-\mu_i^2\phi_i^2(i=1,2)$ is the bare mass term of ϕ_i, and D_i is the coefficient of the thermal mass correction term [11]. Here the constant C is introduced to make the cosmological constants zero.

The mutual interaction term

$$V_{\rm int}=g\phi_1^2\phi_2^2, \tag{2}$$

is essential for inverse symmetry breaking. We will consider the case where this term exists in the tree-level potential. This term may also arise via fermion exchange box diagrams, even if it is absent in the tree level potential [12].

When the fields have vacuum expectation value (VEV), they acquire additional masses through $V_{\rm int}$. Their effective masses squared at a temperature T without D_iT^2 terms are

$$m_{1\ \rm eff}^2(T)\equiv 2[-\mu_1^2+g\langle\phi_2(T)\rangle^2], \tag{3}$$

$$m_{2eff}^2(T)\equiv 2(-\mu_2^2+g\langle\phi_1(T)\rangle^2), \tag{4}$$

where $\langle\phi_i(T)\rangle$ is the VEV of ϕ_i at T.

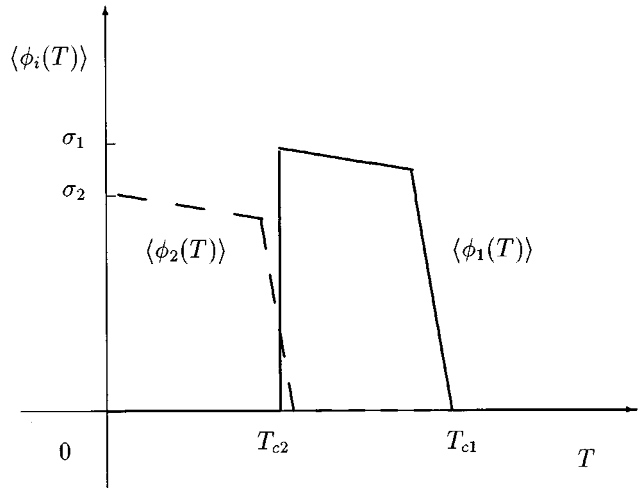

FIG. 1. Schematic diagram for inverse symmetry breaking. $\langle \phi_1(T)\rangle$(thick line) and $\langle \phi_2(T)\rangle$(dashed line) versus temperature T.

Then the phase transition temperature T_{ci} at which the coefficient of ϕ_i^2 vanishes can be defined: i.e.,

$$T_{ci}^2 \equiv -\frac{m_{i\mathrm{eff}}^2(T_{ci})}{2D_i}. \tag{5}$$

From now on we will consider the case

$$T_{c1} > T_{c2}, \tag{6}$$

which means that $\langle \phi_1(T)\rangle$ becomes nonzero at T_{c1}, and after the expansion of the universe $\langle \phi_2(T)\rangle$ becomes nonzero at the lower temperature T_{c2} in turn. If $\langle \phi_2(T)\rangle$ is sufficiently large at T_{c2}, the symmetry of ϕ_1 broken at T_{c1} can be restored due to the additional mass term from V_{int} [see Eq. (4) and Fig. 1]. This is so-called ''inverse symmetry breaking'' [6].

If at this temperature (T_{c2}) ϕ_1 rolls down slowly from $\langle \phi_1(T_{c2})\rangle$ to zero and its energy dominates others, we can expect a chaotic-type slow-rollover inflation and regard ϕ_1 as an inflaton field. Note that here we use the terminology ''chaotic'' to mean a kind of inflation potential and not a chaotic initial condition [13].

$\langle \phi_i(T)\rangle$ can be found from the relation $dV(\phi_1,\phi_2,T)/d\phi_i=0$. From Eq. (1) one can obtain

$$\langle \phi_1(T)\rangle = \sqrt{\frac{\mu_1^2 - g\langle \phi_2(T)\rangle^2 - D_1T^2}{2\lambda_1}} \simeq \sqrt{\frac{\mu_1^2}{2\lambda_1}} \equiv \sigma_1, \tag{7}$$

when $T_{c2}<T<T_{c1}$.

The above approximation is justified by the fact that $\langle \phi_2(T)\rangle = 0$ in this temperature range, and D_1T^2 term decreases rapidly after the phase transition at T_{c1} (see again Fig. 1).

Similarly, when $T<T_{c2}$,

$$\langle \phi_2(T)\rangle \simeq \sqrt{\frac{\mu_2^2}{2\lambda_2}} \equiv \sigma_2. \tag{8}$$

From now on, to simplify the calculation, we will use σ_i as an approximation of $\langle \phi_i(T)\rangle$ in the temperature region described above. It is a good enough approximation for the order of magnitude estimates.

Note that σ_1 and σ_2 minimize $V(\phi_1,0,0)$ and $V(0,\phi_2,0)$, respectively.

III. CONSTRAINTS FOR THE INFLATION

In this section the conditions for a successful inflation will be obtained. There are many constraints for the successful inflation models. The most significant one comes from the density perturbation:

$$\left[\frac{\Delta T}{T}\right]_Q^2 = \frac{32\pi V_{\mathrm{inf}}^3}{45 V_{\mathrm{inf}}'^2 M_P^6}, \tag{9}$$

where V_{inf}' is $dV_{\mathrm{inf}}/d\phi_1$ at the horizon crossing of the observed scale. We consider the quadratic-term-dominated inflaton potential $V_{\mathrm{inf}} \equiv m_1^2\phi_1^2/2$, which is the ϕ_1-dependent part of the approximation of $V(\phi_1,\phi_2,T\leqslant T_{c2})$. So $m_1^2/2 \simeq -\mu_1^2 + g\sigma_2^2$.

Cosmic Background Explorer (COBE) [14] observation, $[\Delta T/T]_Q \simeq 6\times 10^{-6}$, demands $m_1 \simeq 10^{13}$ GeV for our model.

The sufficient expansion condition requires [15]

$$\sigma_1 = \sqrt{\frac{N}{2\pi}} M_P \gtrsim 3M_P \tag{10}$$

for e^N expansion and $N \gtrsim 60$. Note that for the quadratic term dominated inflaton-potential slow-rolling condition $m_1 \ll H$ is automatically satisfied for $\sigma_1 \gtrsim M_P$. The above two constraints are common to many mass term dominated chaotic type inflation models.

Now we will investigate conditions specific to our model.

First, the condition for inverse symmetry breaking [Eq. (6)] is equal to

$$\frac{\mu_1^2}{D_1} > \frac{\mu_2^2 - g\sigma_1^2}{D_2}. \tag{11}$$

Second, the phase transition at T_{c2} must be energetically favorable to take place. It means that the free energy released by symmetry breaking by ϕ_2 must be larger than the free energy absorbed by symmetry restoration by ϕ_1. This implies

$$V(\langle \phi_1(T)\rangle,0,T\geqslant T_{c2}) - V(0,\langle \phi_2(T)\rangle,T\leqslant T_{c2}) > 0, \tag{12}$$

or approximately $V(\sigma_1,0,0)-V(0,\sigma_2,0)>0$, which is equivalent to

$$\mu_1^2\sigma_1^2 < \mu_2^2\sigma_2^2. \tag{13}$$

Third, restoring the symmetry of ϕ_1 implies $m_{1\,\mathrm{eff}}^2(0)>0$, or

$$\mu_1^2 < g\sigma_2^2. \tag{14}$$

Similarly, the broken symmetry of ϕ_2 implies $m_{2\,\mathrm{eff}}^2(0)<0$,

$$\mu_2^2 > g\sigma_1^2. \tag{15}$$

From Eq. (11) and Eq. (13) we obtain

$$D_1 < \frac{\mu_1^2}{\mu_2^2 - g\sigma_1^2} D_2 \simeq \frac{\mu_1^2}{\mu_2^2} D_2 < \left(\frac{\sigma_2}{\sigma_1}\right)^2 D_2, \tag{16}$$

where we have used Eq. (15) in the approximation.

Finally, we want the potential $V_{\rm inf}$ to be dominated by the ϕ_1^2 term rather than by the ϕ_1^4 term. So

$$\mu_1^2 < \frac{2}{3} g\sigma_2^2. \tag{17}$$

Using Eq. (7) and Eq. (8), one can rewrite the constraints [Eq. (11) and Eq. (13)] with λ_i instead of μ_i:

$$\lambda_1 \left(\frac{\sigma_1}{\sigma_2}\right)^4 < \lambda_2 < \lambda_1 \left(\frac{\sigma_1}{\sigma_2}\right)^2 \frac{D_2}{D_1} + \frac{g}{2}\left(\frac{\sigma_1}{\sigma_2}\right)^2. \tag{18}$$

Let us further consider miscellaneous constraints. The one-loop correction to λ_i should not be larger than itself, i.e., $\lambda_i \gtrsim 0.1 g^2$.

Whether ϕ_2 drives an inflation or not at T_{c2}, ϕ_2 oscillates around the potential minima (σ_2) with period $\sim 1/m_2$ after the phase transition (see Fig. 1), and its energy density ρ_{ϕ_2} decreases as $R^{-3}(t)$ such as the classical nonrelativistic matter field [16]. Here $m_2^2/2 \equiv -\mu_2^2 + g\sigma_1^2$ is an approximation of $m_{2\,{\rm eff}}^2(T)$ at $T_{c2} \leqslant T < T_{c1}$.

Since $R \propto t^{2/3}$ in the matter-dominated era, ρ_{ϕ_2} is proportional to t^{-2} during the oscillation. (Even if ρ_{ϕ_2} rapidly changes to radiation energy so that the universe is in the radiation-dominated era, the energy density is proportional to t^{-2} and the above arguments still hold.)

We need to know the time ($\triangle t_{\rm osc}$) when ρ_{ϕ_2} decreases to ρ_{ϕ_1} and the inflation by ϕ_1 begins. From the fact that $\rho_{\phi_2}(t) \simeq \rho_{\phi_2}(t_2) H(t_2)^{-2} t^{-2}$ this time scale is given by

$$\triangle t_{\rm osc} \simeq \frac{1}{H(t_2)} \left[\frac{\rho_{\phi_2}(t_2)}{\rho_{\phi_1}}\right]^{1/2} \sim \frac{M}{m_1 \sigma_1}, \tag{19}$$

where $M \equiv M_P/8\pi$ is the reduced Planck mass, t_2 is the time when the oscillation of ϕ_2 starts and $H(t_2) \sim m_2\sigma_2/M \sim \rho_{\phi_2}^{1/2}/M$. We have also used the fact that $\rho_{\phi_i} \sim m_i^2 \sigma_i^2$ before ϕ_i start to oscillate.

During $\triangle t_{\rm osc}$, ϕ_1 should not fall down too much. Since the equation for ϕ_1 is

$$3H\dot{\phi}_1 = -m_1^2 \phi_1, \tag{20}$$

whose solution is $\phi_1 = \sigma_1 - m_1 M_P t/2\sqrt{3}$ [15], the rolling time scale is $\triangle t_{\rm rol} \sim 1/m_1$ (the dots denote time derivatives.) Therefore one can know that if $\sigma_1 \gg M$, $\triangle t_{\rm osc} \ll \triangle t_{\rm rol}$ and ϕ_1 does not decrease too much during ϕ_2 oscillation, and one could expect inflation by ϕ_1.

IV. AN EXAMPLE WITH A SU(5) GUT AND A NUMERICAL STUDY

Let us apply our model to a SU(5) GUT. Consider the case where ϕ_2 is a SU(5) Higgs field [3]. Then the phase transition temperature $T_{c2} \simeq 10^{15}$ GeV $\simeq \sqrt{(\mu_2^2 - g\sigma_1^2)/D_2} \simeq \sqrt{\mu_2^2 - g\sigma_1^2}$, because $D_2 = \frac{75}{8} g_{\rm SU(5)}^2 \simeq 3$ with the unified gauge coupling $g_{\rm SU(5)}$.

We also know that $\sigma_2 \simeq M_X/g_{\rm SU(5)} \simeq 10^{15}$ GeV.

From Eq. (10) and Eq. (16) it is easy to find that

$$D_1 < \left(\frac{\sigma_2}{\sigma_1}\right)^2 D_2 \lesssim 10^{-8}. \tag{21}$$

From the density perturbation constraint $m_1^2/2 \simeq (10^{13}\ {\rm GeV})^2 \simeq -\mu_1^2 + g\sigma_2^2 \leqslant g\sigma_2^2$ we get $g \gtrsim 10^{-4}$. However, $D_1 \simeq 0.1g < 10^{-8}$, so $g < 10^{-7}$. Hence, g cannot satisfy both conditions. This problem is easily solved by considering the GUT models whose energy scale is larger ($T_{c2} \simeq 10^{16}$ GeV). In this case, using the same procedure we obtain $g \gtrsim 10^{-6}$ and $D_1 \lesssim 10^{-6}$, so all the conditions are satisfied within our approximation.

From Eq. (14) and Eq. (7) we obtain

$$\lambda_1 < \frac{g}{2} \left(\frac{\sigma_2}{\sigma_1}\right)^2 \lesssim 10^{-12}, \tag{22}$$

so $\lambda_1 \ll g$.

Such a small coupling constant is typical to many slow-rollover inflation models, and gives rise to a thermal non-equilibrium problem. Like many other slow-rollover inflation models (except for the chaotic inflation model), it is very hard to establish the initial thermal equilibrium required for our model.

For the following, we will assume that somehow this equilibrium is established and ϕ_i has the appropriate initial values. (The parametric resonance mechanism [17] might help good reheating, but it is still unclear whether produced light particles can obtain the thermal equilibrium before T_{c2}.)

If we want any inflation at T_{c2}, the vacuum energy of ϕ_1 or ϕ_2 must be larger than the radiation energy. In this case, from Eq. (13) the energy of ϕ_2 is larger than that of ϕ_1, so it is possible that there is a new inflation by "ϕ_2" before that by ϕ_1. So our model could be a kind of "double inflation" [18].

Whether the first inflation(by ϕ_2) can exist depends on the rolling speed of ϕ_2 at this phase transition. Since the number of e-foldings of expansion in the new inflation is given by $N \simeq (H/m_2)^2$, the first slow-rollover inflation is available only for $m_2 \ll H$.

However, from the fact that $m_2^2/2 = -\mu_2^2 + g\sigma_1^2$, one can know that $m_2 \gg H_2 \simeq 10^{13}$ GeV without fine tuning and there is no slow-rollover inflation driven by ϕ_2 preceding ϕ_1 inflation with a GUT.

Now we will discuss the numerical study of our model. The process of our inflation model seems to be rather complicated. To confirm the scenario we perform numerical study of the following equations for the evolution of the fields:

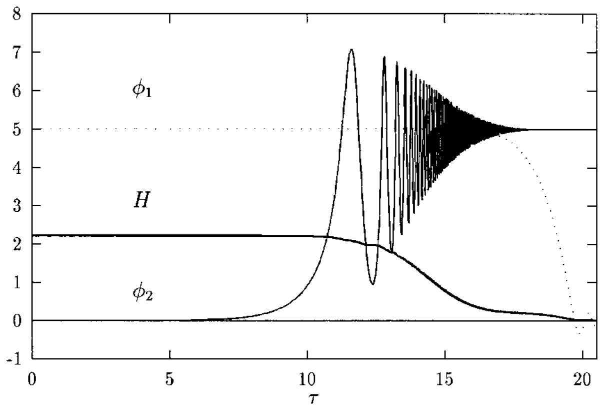

FIG. 2. The results of numerical study showing the evolution of ϕ_1, ϕ_2 and H versus time in log scale $\tau = ln(m_2 t)$. ϕ_1 is in units of M, ϕ_2 in units of $10^{-2}M$ and H in units of $10^{-2}m_2$.

$$H = \left[\frac{1}{3M^2} \left(\frac{\dot{\phi}_1{}^2}{2} + \frac{\dot{\phi}_2{}^2}{2} + V \right) \right]^{1/2}, \quad \ddot{\phi}_i + 3H\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0, \tag{23}$$

where V is $V(\phi_1, \phi_2, 0)$ in Eq. (1). We have ignored thermal contributions that may become small relatively when there is inflation or oscillation of ϕ_1, ϕ_2.

Figure 2 shows the results with $m_1 = 10^{13}$ GeV, $m_2 = 5 \times 10^{16}$ GeV, $\sigma_1 = 5M$, $\sigma_2 = 5 \times 10^{-2}M$, and $g = 10^{-7}$.

After the long oscillation of ϕ_2 for $\tau \gtrsim 11$ (in the realistic case, this oscillation disappears rapidly by producing particles), ρ_{ϕ_2} decreases and ϕ_1 rolls down and begins the inflation. The sign of the inflation by ϕ_1 can be identified by the flat region of the H graph ($\tau \gtrsim 16$). After inflation ends, ϕ_1 starts to oscillate when $\tau \simeq 19$.

Now let us consider the case where no initial thermal equilibrium state is established. It is well known that at the Planck scale the typical initial value of ϕ_1 could be about $\lambda_1^{-1/4} M_P \gg M_P$. Hence, generally there could be a chaotic inflation by ϕ_1 before the inflation by ϕ_1 and/or by ϕ_2 at the lower temperature.

Whether there has been a chaotic inflation or not, ϕ_1 field rolls down to σ_1 and starts to oscillate when $\phi_1 - \sigma_1$ becomes about M_P. Since $m_2 \gg m_1$, during the chaotic inflation ϕ_2 rolls down to σ_2 rapidly, then the effective mass of ϕ_1 becomes positive and ϕ_1 may roll down to zero again. In this case our scenario is hardly distinguishable from the ordinary chaotic inflation by ϕ_1. So it seems to be essential to assume the initial thermal equilibrium, if we consider our model with a GUT.

V. DISCUSSIONS

The most special feature of our model is that we can choose the initial value of the inflaton field (σ_1) by varying the parameters.

From Eq. (18) and Eq. (22) we know that the relation $\lambda_1 \ll g < \lambda_2$ should be satisfied for successful inflation.

For some parameter ranges our model could be a two-field double inflation model whose properties depend on the rolling speed of ϕ_2.

Our model with the GUT phase transition requires the GUT energy scale to be $\sim 10^{16}$ GeV, while the assumption of thermal equilibrium is needed like many other slow-rollover inflation models.

The numerical study indicates that in spite of the complexity of our model, inflation could occur with parameters constrained by many conditions.

This model may also be used to give the appropriate density perturbation to match COBE normalization with the galaxy-galaxy correlation function [19]. Note that for this purpose σ_1 [Eq. (10)] should be lowered so that we can observe the effect of the inflation by ϕ_2.

Many constraints on the masses and couplings of the fields for the successful inflation and inverse symmetry breaking are studied. However, some of the requirements can be abandoned. For example, ϕ_1 needs not have zero VEV after inflation and may have some finite VEV. In this case, ϕ_1 could be a scalar field responsible for the broken symmetry in some particle physics theories.

It is also possible that the inflaton potential is dominated by the quartic term, not by the quadratic term.

Furthermore, for more general case the potential $V(\phi_1, \phi_2, T)$ may have a small barrier term, such as $T\phi_i^3$. In this case, it is possible that there is a first-order inflation by ϕ_2, which is interesting, because it could be another mechanism for the recently proposed open-inflation models [20].

In a word, there still remain various scenarios to be studied in different parameter spaces in this model where our means of the onset of inflation is introduced.

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