PHYSICAL REVIEW D VOLUME 54, NUMBER 11 1 DECEMBER 1996

Lightest Higgs boson mass in the minimal supersymmetric standard model with four generations

Sin Kyu Kang
Center for Theoretical Physics, Seoul National University, Seoul, 151-742, Korea

(Received 18 June 1996)

We study a bound on the lightest Higgs boson mass in the minimal supersymmetric standard model (MSSM) with four generations by solving the one-loop renormalization group equations. We show how the bound depends on the fourth generation fermion masses as well as the top quark mass. We also briefly compare the bound with the one in the MSSM with three generations. [S0556-2821(96)02923-2]

PACS number(s): 14.80.Bn, 11.10.Hi

With the discovery of the top quark at the Fermilab Tevatron [1], the Higgs boson is now the only unknown sector in the context of the standard model (SM). Despite the remarkable successes of the SM in its excellent agreement with the precision measurements at present energies [2], it is generally believed that the SM is not the final theory of elementary particle interactions. The minimal supersymmetric standard model (MSSM) [3] is one of the most popular extensions of the SM. Because of the nature of supersymmetry (SUSY), the Higgs sector in the MSSM consists of two CP-conserving Higgs doublets with opposite hypercharge.

In the SM, the Higgs boson mass is usually considered as an adjustable parameter because the quartic coupling of the Higgs potential is arbitrary. Nevertheless if certain theoretical assumptions are imposed, upper and lower bounds on the Higgs boson mass can be obtained. The requirement of the vacuum stability yields a severe lower bound on the Higgs boson mass which depends on the top quark mass and the cutoff scale beyond which the SM is no longer valid [4], while an upper bound follows from the requirement that no Landau singularity appears up to a scale [5]. In the MSSM, an intrinsic upper bound on the lightest Higgs boson mass is obtained from the quartic Higgs coupling which is no longer arbitrary but is constrained by SUSY [6,7].

On the other hand, since there is still no experimental evidence for the absence of extra generations, it would also be interesting to study how the Higgs boson mass is limited in the presence of extra generations. In the SM with extra generations, several authors [8–10] have derived the upper and lower bounds on the Higgs boson mass as functions of the extra fermion masses.

In this paper, we study a bound on the lightest Higgs boson mass in the minimal supersymmetric standard model with four generations (MSSM4). In order to do this we adopt a basic assumption [11] that all super partners of the SM particles and another Higgs scalar orthogonal to the lightest one have masses of order of the supersymmetry breaking scale $M_{\rm SUSY}\geqslant 1$ TeV. Then, the effective low-energy theory below $M_{\rm SUSY}$ is equivalent to the SM with a single Higgs doublet and four generations. This is the main difference between Ref. [12] and our analysis.[1]

Now, we impose the possible constraints on the fourth generation fermion masses. The recent precision tests of the SM [14] and the good agreement between the direct measurements of the top quark mass at the Tevatron [1] and its indirect determination from the global fits to the electroweak data [14,15] demonstrate that no significant violation of the isospin symmetry for the extra generations is observed. Thus the masses of the fourth generation isopartners must be highly degenerate (see, e.g., [10]). To reduce the number of parameters we will consider the fourth generation fermions with the common mass m_4. Recently, the limit on the masses of the extra neutral and charged leptons, m_N and m_E, has been improved by the CERN e^+e^- collider LEP1.5 to $m_N > 59$ GeV and $m_E > 62$ GeV [16]. For the extra quarks, the direct lower limit on their masses is somewhere near 100 GeV [10]. On the other hand, the upper limits on the fourth generation fermion masses coming from vacuum stability are about 250 GeV for the scales of $\Lambda = 10^3 - 10^4$ GeV at which new physics begins [17,18]. Taking into account these observations, we will restrict the range of m_4 to $50 \leqslant m_4 \leqslant 250$ GeV in our analysis. The existence of the degenerate fourth generation with the range of mass as indicated above should not spoil the successful description of the electroweak data [19,9,10]. In addition, we will assume that the fourth generation quarks are not mixed with the known quarks. This is possible since the mixing angles are so small that the new particles leave Tevatron detectors without decays.

We denote the fourth generation by (T,B) for quarks and (N,E) for leptons and their Yukawa couplings by h_i $(i=T,B,N,E)$. Since the low energy effective theory below $M_{\rm SUSY}$ is nothing more than the SM with four generations, we can use the SM renormalization group equations (RGEs) for the gauge couplings and Yukawa couplings.

The one-loop RGEs for the gauge couplings are

$$\kappa\, dg_i/dt = b_i g_i^3 \quad (i=1,2,3) \tag{1}$$

with $b_1 = (\frac{20}{9}N + \frac{1}{6}),\ b_2 = (\frac{4}{3}N - \frac{43}{6}),\ b_3 = (\frac{4}{3}N - 11)$.

Here N is the number of generations of quarks and leptons, $\kappa = 16\pi^2$, $t = \ln(\mu/M_Z)$ and $g_i (i=1,2,3)$ are the gauge coupling constants of U(1), SU(2), and SU(3), respectively. Upon writing the one-loop RGEs for the Yukawa coupling,

---

[1] Recently, there are several attempts to regard the top quark discovered at the Tevatron as the fourth generation quark in order to resolve R_b anomaly [13]. In this paper we do not consider this possibility.

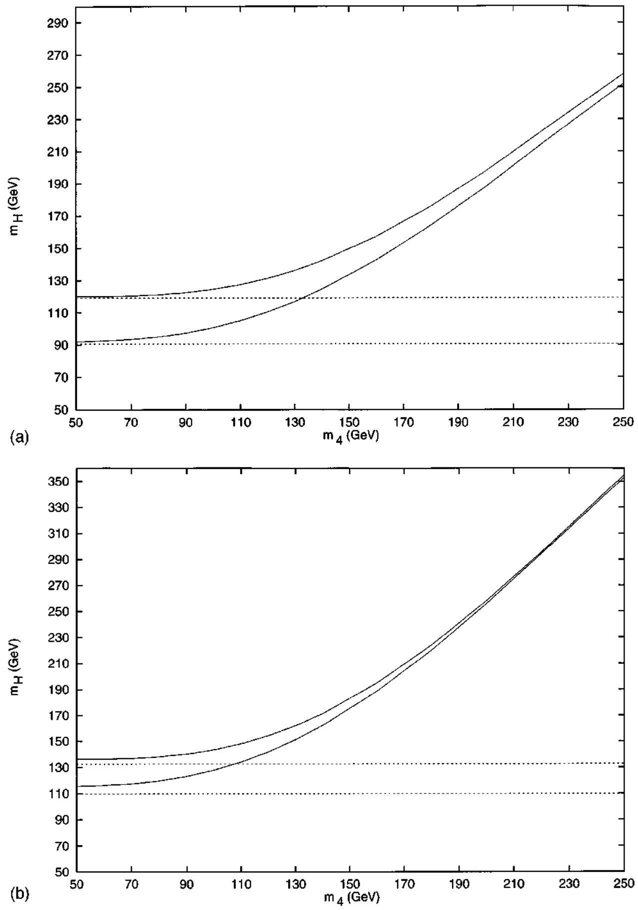

FIG. 1. Plots of the lightest Higgs boson mass m_H as a function of m_4 for $m_t = 170$ GeV; (a) at $M_{\mathrm{SUSY}} = 1$ TeV and (b) at $M_{\mathrm{SUSY}} = 10$ TeV. The solid and dotted lines correspond to the MSSM4 and the MSSM3, respectively. The upper and lower lines of each correspond to $|\cos 2\beta| = 1$ and 0, respectively.

we neglect the contributions of the first two generations and the bottom quark. Consequently, we can write [17,20]

$$\kappa\, dh_t/dt = h_t\left[\tfrac{3}{2}\, h_t^2 + \Sigma - \tfrac{17}{12}\, g_1^2 - \tfrac{9}{4}\, g_2^2 - 8 g_3^2\right], \tag{2}$$

$$\kappa\, dh_T/dt = h_T\left[\tfrac{3}{2}(h_T^2 - h_B^2) + \Sigma - \tfrac{17}{12}\, g_1^2 - \tfrac{9}{4} g_2^2 - 8 g_3^2\right], \tag{3}$$

$$\kappa\, dh_B/dt = h_B\left[\tfrac{3}{2}\,(h_B^2 - h_T^2) + \Sigma - \tfrac{5}{12} g_1^2 - \tfrac{9}{4} g_2^2 - 8 g_3^2\right], \tag{4}$$

$$\kappa\, dh_N/dt = h_N\left[\tfrac{3}{2}(h_N^2 - h_E^2) + \Sigma - \tfrac{3}{4} g_1^2 - \tfrac{9}{4} g_2^2\right], \tag{5}$$

$$\kappa\, dh_E/dt = h_E\left[\tfrac{3}{2}(h_E^2 - h_N^2) + \Sigma - \tfrac{15}{4} g_1^2 - \tfrac{9}{4} g_2^2\right], \tag{6}$$

with $\Sigma = 3(h_t^2 + h_T^2 + h_B^2) + h_N^2 + h_E^2$. For the Higgs coupling constant λ, the renormalization group equation (RGE) is

$$\begin{aligned}\kappa\, d\lambda/dt = {} & 12\lambda^2 + 4\lambda[3h_t^2 + 3(h_T^2 + h_B^2) + (h_E^2 + h_N^2)] \\ & - 4[3h_t^4 + 3(h_T^4 + h_B^4) + (h_E^4 + h_N^4)] \\ & - 3\lambda(g_1^2 + 3g_2^2) + \tfrac{3}{4} g_1^4 + \tfrac{3}{2} g_1^2 g_2^2 + \tfrac{9}{4} g_2^4 .\end{aligned} \tag{7}$$

Note that the Higgs coupling constant λ is no longer arbitrary but is constrained by the SUSY relation. As a renormalization group boundary condition at $\mu = M_{\mathrm{SUSY}}$, we impose the following SUSY relation on the Higgs coupling $\lambda(\mu)$ [21],

$$\lambda(M_{\mathrm{SUSY}}) = \tfrac{1}{4}[g_1^2(M_{\mathrm{SUSY}}) + g_2^2(M_{\mathrm{SUSY}})]\cos^2 2\beta, \tag{8}$$

where $\tan\beta$ is the ratio of the two vacuum expectation values (v_2/v_1). The initial values at M_Z for the gauge couplings are taken to be

$$g_1(M_Z) = 0.3578, \tag{9}$$

$$g_2(M_Z) = 0.6502, \tag{10}$$

$$\alpha_s(M_Z) = g_3^2(M_Z)/4\pi = 0.12. \tag{11}$$

Note that the uncertainties of our numerical results due to the experimental errors of $g_1(M_Z)$ and $g_2(M_Z)$ are negligible but is not so for the case of $\alpha_s(M_Z)$. Later, we will briefly discuss the uncertainty due to the error of $\alpha_s(M_Z)$. For the Yukawa couplings of the top quark and the fourth generation fermions, we impose the mass relation as a boundary condition:

$$m_i = h_i(m_i) v/\sqrt{2}, \tag{12}$$

where $v = \sqrt{v_1^2 + v_2^2}$, and $i = t, T, B, N$, and E. In order to calculate the Higgs boson mass m_H, we will use the relation [11]

$$m_H^2/M_Z^2 = 4\lambda(m_H)/[g_1^2(M_Z) + g_2^2(M_Z)]. \tag{13}$$

In Fig. 1, we show the lightest MSSM4 Higgs boson mass m_H as a function of the fourth generation mass m_4 for $m_t = 170$ GeV (a) for $M_{\mathrm{SUSY}} = 1$ TeV and (b) for $M_{\mathrm{SUSY}} = 10$ TeV (solid lines). We have also plotted in Fig. 1, for the sake of comparison, the corresponding lightest Higgs boson mass in the MSSM with three generations (MSSM3) (dotted lines). The upper and lower lines for both, MSSM4 and MSSM3, correspond to $|\cos 2\beta| = 1$ and 0, respectively. We see that, for small values of m_4, m_H is rather insensitive to m_4 and cannot be distinguished from the corresponding MSSM3 Higgs boson mass for fixed values of m_t, M_{SUSY}, and β. However, for large values of m_4, m_H rapidly increases as m_4 increases and there is a mass gap between the MSSM4 and the MSSM3. As one can see from Fig. 1, the difference of m_H between $|\cos 2\beta| = 1$ and 0 becomes smaller as m_4 increases and is significantly reduced as both m_4 and M_{SUSY} increase. We can see that, for $m_t = 170$ GeV and $M_{\mathrm{SUSY}} = 1(10)$ TeV, the upper bound on m_H varies from 120 (136) to 258 (355) GeV as m_4 is varied in the range 50–250 GeV. The lowest value of m_H (which occurs for $m_4 = 50$ GeV and $|\cos 2\beta| = 0$) is 92 (116) GeV.

Figure 2 shows m_H as a function of m_4 for several values of m_t with $|\cos 2\beta| = 1$ and $M_{\mathrm{SUSY}} = 1$ TeV. We regard the curves in this figure as the upper bounds on the lightest Higgs boson mass in the MSSM4 as a function of m_4 for each value of m_t at $M_{\mathrm{SUSY}} = 1$ TeV. For a given m_4, m_H increases as m_t increases. As one can also see from Fig. 2, the dependence of m_H on m_t becomes weaker as m_4 increases.

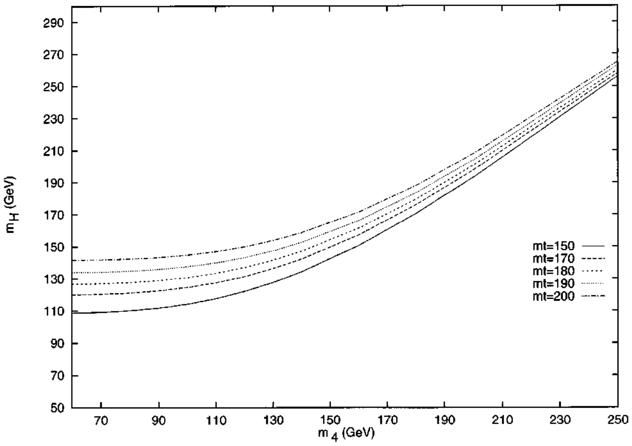

FIG. 2. Plots of m_H as a function of m_4 for several values of m_t with $|\cos 2\beta|=1$ and $M_{\text{SUSY}}=1$ TeV.

Now, we discuss the uncertainties contained in our calculation. Our numerical results have been obtained by integrating only the one-loop RGEs. For consistency, we also ignored threshold corrections at the SUSY scale whose effects are of the same magnitude as of the neglected two-loop contributions to the RGE. If we include two-loop contributions to the RGEs and maximal SUSY scale threshold corrections, m_H can be increased by about 10–25% as m_4 varies from 50 to 250 GeV for $m_t=170$ GeV and $M_{\text{SUSY}}=1$ TeV, while m_H in the MSSM3 by about 10%. Thus, for large values of m_4, we expect quite sizable contributions of those corrections to m_H. We will postpone the details of the two-loop analysis to a future work elsewhere. There is also uncertainty due to the experimental error of strong coupling constant. The shift of m_H due to $\Delta\alpha_s(M_Z)=\pm 0.005$ is about 5 GeV.

In conclusion, we have studied the upper bound on the lightest Higgs boson mass m_H in the MSSM with four generations by solving the one-loop RGEs. We have considered the fourth generation of quarks and leptons with the degenerate mass m_4. We have shown how the upper bound on m_H depends on m_4 as well as m_t. In the region of large m_4, the upper bound on m_H increases as m_4 increases, while it is rather insensitive to m_4 in the region of small m_4. We have also compared the bound with the one in the MSSM with three generations.

This work is supported by the Korea Science and Engineering Foundation through SNU CTP.

---

[1] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **74**, 2626 (1995); D0 Collaboration, S. Abachi *et al., ibid.* **74**, 2632 (1995).
[2] P. B. Renton, in 17th International Symposium on Lepton-Photon Interactions, Beijing, China, 1995 (unpublished).
[3] For a review see H. P. Nilles, Phys. Rep. **110**, 1 (1984).
[4] M. Sher, Phys. Rep. **179**, 273 (1989); Phys. Lett. B **317**, 159 (1993); **331**, 448 (1994); G. Altarelli and G. Isidori, *ibid.* **337**, 114 (1994); J. A. Casas, J. R. Espinosa, and M. Quirós, *ibid.* **342**, 171 (1995); J. R. Espinosa and M. Quirós, *ibid.* **353**, 257 (1995); and see also N. Cabibbo, L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B158**, 295 (1979).
[5] M. Lindner, Z. Phys. C **31**, 295 (1986); M. Sher, Phys. Rep. **179**, 273 (1989).
[6] H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); Phys. Rev. D **48**, 4280 (1993); Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991); Phys. Lett. B **262**, 54 (1991); J. Ellis, G. Ridolfi, and F. Zwirner, *ibid.* **257**, 83 (1991); *ibid.* **262**, 477 (1991); R. Barbieri, M. Frigeni, and F. Caravaglios, *ibid.* **258**, 167 (1991).
[7] R. Hempfling and A. H. Hoang, Phys. Lett. B **331**, 99 (1994); J. Kodaira, Y. Yasui, and K. Sasaki, Phys. Rev. D **50**, 7035 (1994); J. A. Casas, J. R. Espinosa, M. Quiros, and A. Riotto, Nucl. Phys. **B436**, 3 (1995); **B439**, 466(E) (1995); M. Carena, J. R. Espinosa, M. Quiros, and C. E. M. Wagner, Phys. Lett. B **355**, 209 (1995); M. Carena, M. Quiros, and C. E. M. Wagner, Nucl. Phys. **B461**, 407 (1996).
[8] K. S. Babu and E. Ma, Z. Phys. C **29**, 45 (1985).
[9] H. B. Nielsen, A. V. Novikov, V. A. Novikov, and M. S. Vysotsky, Phys. Lett. B **374**, 127 (1996).
[10] V. Novikov, in Microwave Background Anisotropies, Proceedings of the 31st Rencontres de Moriond, Les Arcs, France, 1996 (unpublished), Report No. hep-ph/9606318 (unpublished).
[11] See also Y. Okada *et al.* ([6]).
[12] K. Tabata, I. Umemura, and K. Yamamoto, Phys. Lett. B **129**, 80 (1983).
[13] H. E. Haber, in Electroweak Interactions and Unified Theories, Proceedings of the 30th Rencontres de Moriond, Meribel les Allues, France, 1995 (unpublished), p. 256, Report No. CERN-TH/95-178, hep-ph/9506426 (unpublished); in Proceedings of the International Europhysics Conference on High Energy Physics (HEP95), Brussels, Belgium, 1995 (unpublished), Report No. CERN-TH/96-08, hep-ph/9601331 (unpublished); M. Carena, H. E. Haber, and C. E. M. Wagner, Nucl. Phys. **B472**, 55 (1996); G. W. -S. Hou, Report No. NTUTH-96-03, hep-ph/9605203 (unpublished).
[14] J. Erler and P. Langacker, Phys. Rev. D **52**, 441 (1995); P. Langacker, Report No. NSF-ITP-95/40, hep-ph/9511207 (unpublished); K. Hagiwara, Report No. KEK-TH-461, hep-ph/9512425 (unpublished); K. Kang and S. K. Kang, Z. Phys. C **70**, 239 (1996); Z. Hioki, Report No. TOKUSHIMA 95-05, hep-ph/9511224 (unpublished); J. Ellis, G. L. Fogli, and E. Lisi, Z. Phys. C **69**, 627 (1996); P. H. Chankowski and S. Pokorski, Phys. Lett. B **356**, 307 (1995); S. Dittmaier, D. Schildknecht, and G. Weigleim, Report No. BI-TP 96/14, hep-ph/9605268 (unpublished).
[15] See also K. Kang and S. K. Kang, in *Quantum Infrared Physics*, Proceedings of the Workshop, Paris, France, 1994, edited by H. Fried and B. Muller (World Scientific, Singapore, 1995), Report No. hep-ph/9412368 (unpublished); in *Beyond the Standard Model IV,* Proceedings of the International Conference, Lake Tahoe, California, 1994, edited by J. Gunion *et al.* (World Scientific, Singapore, 1995), Report No. hep-ph/9503478 (unpublished), and references therein.
[16] LEP1.5 Collaboration, J. Nachtman, in Electroweak Interactions and Unified Theories, Proceedings of the 31st Rencontres

de Moriond, Les Arcs, France, 1996 (unpublished), Report No. hep-ex/9606015 (unpublished).

[17] J. W. Halley, E. A. Paschos, and H. Usler, Phys. Lett. **155B**, 107 (1985), and references therein; K. S. Babu and E. Ma, Z. Phys. C **29**, 41 (1985).

[18] For recent evaluation, see also Novikov ([10]).

[19] T. Inami, T. Kawakami, and C. S. Lim, Mod. Phys. Lett. A **10**, 1471 (1995); V. A. Novikov, L. Okun, A. Rozanov, and M. Vysotsky, *ibid.* **10**, 1915 (1995); A. Masiero, F. Feruglio, S. Rigolin, and R. Strocchi, Phys. Lett. B **355**, 329 (1995).

[20] T. P. Cheng, E. Eichten, and L. F. Li, Phys. Rev. D **9**, 2259 (1975); L. Maiani, G. Parisi, and R. Petronzio, Nucl. Phys. **B136**, 115 (1978); B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981); C. Hill, Phys. Rev. D **24**, 691 (1981); G. M. Asatryan, A. N. Ivannissyan, and S. G. Matinyan, Yad. Fiz. **53**, 592 (1991) [Sov. J. Nucl. Phys. **53**, 371 (1991)].

[21] H. Arason, D. Castano, B. Keszthelyi, S. Mikaelian, E. Piard, P. Ramond, and B. Wright, Phys. Rev. Lett. **67**, 2933 (1991).