Solutions to the R_b, R_c, and α_s puzzles by vector fermions

Chia-Hung V. Chang
Physics Department, National Tsing-Hua University, Hsinchu 30043, Taiwan, Republic of China

Darwin Chang
Physics Department, National Tsing-Hua University, Hsinchu 30043, Taiwan, Republic of China and Institute of Physics, Academia Sinica, Taipei, Republic of China

Wai-Yee Keung
Physics Department, University of Illinois at Chicago, Illinois 60607-7059

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We propose two minimal extensions of the standard model, both of which can accommodate the recent puzzling observations about the excess in R_b and the deficit in R_c. The discrepancy in the low energy and high energy determinations of α_s can be resolved in the second model. Each model requires three additional heavy vectorial fermions. The current phenomenological constraints and the new potential phenomena are also discussed. [S0556-2821(96)02823-8]

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I. INTRODUCTION

Recently it was reported [1,2] by the CERN e^+e^- collider LEP Collaborations that the measured rate of $Z\to b\bar{b}$ is greater than the prediction of the standard model, while that of $Z\to c\bar{c}$ is smaller. This is quite significant given the impressive confirmation of the standard model by other precision electroweak tests at the Z^0 resonance. Given $R_b\equiv\Gamma(Z\to b\bar{b})/\Gamma_{\rm had}$ and $R_c\equiv\Gamma(Z\to c\bar{c})/\Gamma_{\rm had}$, the discrepancies can be summarized as

	Measurement	SM	Pull
R_b	0.2219 ± 0.0017	0.2156	3.7
R_c	0.1543 ± 0.0074	0.1724	−2.5.

Here SM stands for the standard-model fit with $m_t=178$ GeV and $m_H=300$ GeV, and ''pull'' is the difference between measurement and fit in units of the measurement error.

At the same time, the α_s problem becomes more acute with improved precision data from Z decays. The strong coupling constant α_s extracted from high energy measurements at M_Z seems to be larger than that from low energy measurements, such as deep inelastic scattering and lattice calculations [1,3,4]. The $\alpha_s(M_Z)$ calculated from the total hadronic width in Z decays is 0.125 ± 0.005 [5,4]. On the other hand, low energy measurements all cluster around $\alpha_s(M_Z)\sim0.11$. It seems there is a substantial gap between the two. Although more data in the future might eliminate these discrepancies, it is possible that this ''R_b-R_c'' plus α_s crisis is indicating the same new physics beyond the standard model.

Several extensions [6–8] of the standard model have been proposed to address these puzzles. In these models, one-loop corrections to the $Zb\bar{b}$ vertex from the nonstandard sector will enhance the b quark partial width. With the hadronic total width also enhanced, the QCD corrections needed to fit the observed total width is reduced. Thus the observed data point to a smaller α_s than that in the standard model, as favored by low energy measurements. However, these attempts all fail to account for the large R_c deficit. In addition, the first two scenarios might be in potential conflict with top quark decay [9].

More recently, two papers [10,11] point to a new direction in the extensions of the standard model which may provide a simple solution to the above discrepancies. Both papers suggest to resolve the discrepancies by introducing new vectorial fermions that mix with b and/or c quarks. The mixing will reduce or enhance the couplings of the quarks to Z boson depending on the gauge quantum numbers of the new fermions. We shall call this class of solutions ''vectorial fermionic solutions'' to the puzzles. In Ref. [11], only a vectorial pair of singlet is introduced to reduce the partial width of $c\bar{c}$. This could solve the R_c puzzle while leaving the R_b puzzle only slightly ameliorated. On the other hand, in Ref. [10], a vectorial pair of singlet plus a vectorial pair of triplet are added to resolve both puzzles at the same time at tree level. As a price of solving both problems, Ma's model also reduces the prediction for the total hadronic width $\Gamma_{\rm had}$ and thus renders a surplus in the observed leptonic branching ratio $R_l\equiv\Gamma_{\rm had}/\Gamma_l$, which cannot be accommodated by assuming a smaller α_s. In this paper we propose and analyze two minimal extensions of the standard model, which are, nevertheless, sufficient to resolve the R_b and R_c puzzles and lower simultaneously the value of α_s extracted from Z decay. In the first minimal extension, only a vectorial triplet of fermions are needed; while in the second one, one needs a vectorial singlet plus a vectorial doublet of fermions. Unfortunately, the first model predicts an α_s that is below the low energy value if the current R_b,R_c data are used to fit the mixing. The second model, however, can give an α_s consistent with the low energy measurements.

We shall start by analyzing the fermion mixing in the general context and then demonstrate that our resulting models are indeed the simplest ones of the class.

II. VECTORIAL FERMION MODELS

In general, the coupling of Z meson with fermions can be written as

$$\frac{g}{\cos\theta_W} Z^{\mu}(g_L^f \overline{f_L}\gamma_\mu f_L + g_R^f \overline{f_R}\gamma_\mu f_R), \tag{1}$$

where

$$g_{L,R}^f = T_{fL,R}^3 - Q_f \sin^2\theta_W. \tag{2}$$

The coupling only depends on the weak isospin T^3 and electric charge Q of the fermion. Thus mixing with heavy fermions of different weak isospin T^3 could change the coupling of quarks with Z meson and the Z decay partial width. Take the partial width into $b\overline{b}$ as an example. Assume that there is a heavy fermion x of charge $-\frac{1}{3}$ and it mixes with quark b as well as d and s. Denote the mixing matrix among left-handed (right-handed) particles as U_L (U_R), which transforms mass eigenstates into gauge eigenstates. We shall specify the weak gauge eigenstates by fields with primes, while those without primes as the mass eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \\ x' \end{pmatrix}_{L,R} = \begin{pmatrix} U_{dd} & U_{ds} & U_{db} & U_{dx} \\ U_{sd} & U_{ss} & U_{sb} & U_{sx} \\ U_{bd} & U_{bs} & U_{bb} & U_{bx} \\ U_{xd} & U_{xs} & U_{xb} & U_{xx} \end{pmatrix}_{L,R} \begin{pmatrix} d \\ s \\ b \\ x \end{pmatrix}_{L,R}. \tag{3}$$

The coupling between mass eigenstate b_L and Z^0 would become

$$g_L^b = [T_{dL}^3 |U_{db}^L|^2 + T_{sL}^3 |U_{sb}^L|^2 + T_{bL}^3 |U_{bb}^L|^2 + T_{xL}^3 |U_{xb}^L|^2 - Q\sin^2\theta_W], \tag{4}$$

while g_R^b equals a similar expression with the subscript L replaced by R. Because the mixing matrix U is unitary and quarks d, b, and s share the same weak isospin T^3, g^b can be written as

$$g_{L,R}^b = T_{bL,R}^3 + (T_{xL,R}^3 - T_{bL,R}^3)|U_{xb}^{L,R}|^2 - Q\sin^2\theta_W. \tag{5}$$

The Z partial decay width into $b\overline{b}$ is proportional to $|g_L^b|^2 + |g_R^b|^2$.

$$[T_{bL}^3 + (T_{xL}^3 - T_{bL}^3)|U_{xb}^L|^2 - Q\sin^2\theta_W]^2 + [T_{bR}^3 + (T_{xR}^3 - T_{bR}^3)|U_{xb}^R|^2 - Q\sin^2\theta_W]^2. \tag{6}$$

It is different from that in the standard model. Whether the new fermion will enhance or reduce the partial width depends on its weak isospin T^3.

Now it is easy to see that we can reduce $\Gamma_{c\overline{c}}$ by adding a left-handed singlet of $T^3=0$ that mixes with c_L [10,11]. To increase $\Gamma_{b\overline{b}}$, a $T^3=-1$ left-handed fermion can be introduced to enhance $|g_L^b|^2$. A less obvious way is to mix b_R, which is of $T^3=0$, with a heavy right-handed doublet of $T^3=\frac{1}{2}$.

Next we shall show two minimal extensions of the standard model in which vectorial fermions with the above properties are introduced to resolve both the R_b and R_c puzzles simultaneously. These are the simplest models to accomplish that, with the smallest number of new particles, three species of vectorial fermions in both cases. We consider only adding vectorial fermions since anomalies are canceled automatically and these fermions could be heavy naturally.

In the first model, only one vectorial triplet is needed. The $T^3=0$ component will reduce $\Gamma_{c\overline{c}}$ and the $T^3=-1$ component will enhance $\Gamma_{b\overline{b}}$. The triplet Y can be written as

$$Y_{L,R} = \begin{pmatrix} y_1^{5/3} \\ y_2^{2/3} \\ y_3^{-1/3} \end{pmatrix}_{L,R},$$

with a gauge invariant mass term $M_Y \overline{Y_L} Y_R$. The mixing is induced by Yukawa couplings between the triplet Y_R and left-handed quark doublets:

$$\begin{aligned} &\xi_3[\overline{y_{1R}} t_L' \phi^+ + \overline{y_{2R}}(t_L'\phi^0 + b_L'\phi^+)/\sqrt{2} + \overline{y_{3R}} b_L' \phi^0], \\ &\xi_2[\overline{y_{1R}} c_L' \phi^+ + \overline{y_{2R}}(c_L'\phi^0 + s_L'\phi^+)/\sqrt{2} + \overline{y_{3R}} s_L' \phi^0], \\ &\xi_1[\overline{y_{1R}} u_L' \phi^+ + \overline{y_{2R}}(u_L'\phi^0 + d_L'\phi^+)/\sqrt{2} + \overline{y_{3R}} d_L' \phi^0]. \end{aligned} \tag{7}$$

In addition, we have the ordinary Yukawa couplings in the standard model.

y_3 mixes with the down quarks d, s, and b. We will use the biunitary transformation to diagonalize the 3×3 mass matrix between d, s, and b. The mass matrix between $D_L' \equiv (d_L', s_L', b_L', y_{3L}')$ and $D_R' \equiv (d_R', s_R', b_R', y_{3R}')$ then becomes

$$(\overline{d_L'}, \overline{s_L'}, \overline{b_L'}, \overline{y_{3L}'}) \begin{pmatrix} m_d & 0 & 0 & \xi_1 v \\ 0 & m_s & 0 & \xi_2 v \\ 0 & 0 & m_b & \xi_3 v \\ 0 & 0 & 0 & M_Y \end{pmatrix} \begin{pmatrix} d_R' \\ s_R' \\ b_R' \\ y_{3R}' \end{pmatrix} \equiv \overline{D_L'} M_d D_R'. \tag{8}$$

M_d can be written as

$$M_d = \begin{pmatrix} \widetilde{M}_d & J \\ 0 & M_Y \end{pmatrix} = U_L^d \mathcal{D}_d U_R^{d\dagger}, \tag{9}$$

with $\widetilde{M}_d$ as a 3×3 matrix, which is diagonal here and J is a 3×1 column. It is *natural* to assume that the gauge invariant M_Y is much larger than all the other elements of the matrix. The diagonalization then takes a simple form. The mixing matrix U_L (U_R) is the matrix that diagonalizes $M_d M_d^\dagger$ ($M_d^\dagger M_d$) into $\mathcal{D}_d$. $M_d^\dagger M_d$ has only one large element at the lower right corner with all the other elements suppressed by $(m/M_Y)^2$. Thus the mixing of y_{3R} with d_R, s_R, b_R, U_{yi}^R $(i=d,s,b)$, is also suppressed by $(\xi v/M_Y)^2$ and negligible. This is a result of the fact that we cannot construct mixing Yukawa couplings among the triplet y_{3L}, the singlet q_R, and the doublet Higgs boson. The mixing between y_{3L} and b_L, s_L, and d_L is more important. Write U_L as

$$U_L=\begin{pmatrix} K & R \\ S & T \end{pmatrix}. \tag{10}$$

K, R, and S are, respectively, a 3×3 matrix, a 1×3 column, and a 3×1 row, and T is a number. The various elements can be solved in the large M_Y approximation [12]. In this approximation, T is equal to one. K equals the unitary matrix that diagonalizes $\widetilde{M}_d\widetilde{M}_d^\dagger$ which is just unity matrix in this case. The column R and row S also can be calculated

$$R=\frac{1}{M_Y}J, \quad S=-\frac{1}{M_Y}J^\dagger K=-\frac{1}{M_Y}J^\dagger. \tag{11}$$

The mixing of d_{iL} with y_{3L}, i.e., R_i, S_i is approximately $\xi_i v/M_Y$. As a result, $\Gamma_{b\bar{b}}$ is proportional to

$$(-\tfrac{1}{2}-\tfrac{1}{2}|S_3|^2+\tfrac{1}{3}\sin^2\theta_W)^2+(\tfrac{1}{3}\sin^2\theta_W)^2, \tag{12}$$

with

$$S_3=-\frac{\xi_3 v}{M_Y}. \tag{13}$$

To fit the observed R_b, we need

$$|S_3|^2=\left(\frac{\xi_3 v}{M_Y}\right)^2=0.0127\pm 0.0034. \tag{14}$$

The charge $\frac{2}{3}$ quarks will mix with y_2 with the mass matrix

$$M_u=\begin{pmatrix} \widetilde{M}_u & \dfrac{J}{\sqrt{2}} \\ 0 & M_Y \end{pmatrix}. \tag{15}$$

Note that J is identical to the same column in M_d. However $\widetilde{M}_u$ is no longer diagonal in this basis. $\widetilde{M}_u\widetilde{M}_u^\dagger$ is diagonalized by a 3×3 matrix V_0. Denote the unitary matrix that diagonalizes $M_u M_u^\dagger$ as U_L':

$$U_L'=\begin{pmatrix} K' & R' \\ S' & T' \end{pmatrix}. \tag{16}$$

In the heavy M_Y limit,

$$K'\approx V_0^\dagger, \tag{17}$$

and it can be approximately identified as the Kobayashi-Maskawa matrix. Also,

$$S'=-\frac{1}{\sqrt{2}M_Y}J^\dagger V_0^\dagger. \tag{18}$$

The partial width $\Gamma_{c\bar{c}}$ is proportional to

$$(\tfrac{1}{2}-\tfrac{1}{2}|S_2'|^2-\tfrac{2}{3}\sin^2\theta_W)^2+(-\tfrac{2}{3}\sin^2\theta)^2. \tag{19}$$

It is smaller than the corresponding value when there is no mixing. To fit the data, we need

$$|S_2'|^2=0.045\pm 0.019. \tag{20}$$

Note that S' is related to S through the mixing matrix V_0. There is no separate parameter for the charge $\frac{2}{3}$ quarks. The mixing is totally fixed by three parameters $\xi_{1,2,3}$ and the heavy fermion mass M_Y.

In the second model, we shall introduce a vectorial doublet and a vectorial singlet. The singlet, call it x, has charge $\frac{2}{3}$ and will reduce R_c just like y_2 in the first model. Choose x_R to be the only right-handed fermion with the gauge invariant mass term $M_x\overline{x_L}x_R$ with x_L. The mass mixing is induced by Yukawa couplings $\xi_i'\overline{q_{Li}}\widetilde{\phi}x_R$ for $i=1$, 2, and 3. The analysis is the same as y_2 in the first model. To fit the data of R_c, we need

$$|S_2'|^2=\left(\frac{\xi_2' v}{M_x}\right)^2=0.045\pm 0.019. \tag{21}$$

The doublet will have the weak hypercharge $-\frac{5}{3}$,

$$\Psi_{L,R}=\begin{pmatrix} \Psi_1^{-1/3} \\ \Psi_2^{-4/3} \end{pmatrix}_{L,R},$$

with a gauge invariant mass term $M_\Psi\overline{\Psi_L}\Psi_R$. The Yukawa coupling between Ψ and ordinary quarks are

$$\eta_3\overline{\Psi_L}\widetilde{\phi}b_R+\eta_2\overline{\Psi_L}\widetilde{\phi}s_R+\eta_1\overline{\Psi_L}\widetilde{\phi}d_R. \tag{22}$$

The coefficients η_i for the doublet Ψ need not be the same as those ξ_i' for the singlet x. Therefore, more parameters are involved in the second model. The mass matrix is

$$(\overline{d_L'},\overline{s_L'},\overline{b_L'},\overline{\Psi_{1L}'})\begin{pmatrix} m_d & 0 & 0 & 0 \\ 0 & m_s & 0 & 0 \\ 0 & 0 & m_b & 0 \\ \eta_1 v & \eta_2 v & \eta_3 v & M_\Psi \end{pmatrix}\begin{pmatrix} d_R' \\ s_R' \\ b_R' \\ \Psi_{1R}' \end{pmatrix}$$

$$\equiv\overline{D_L'}M_d D_R'. \tag{23}$$

Contrary to the previous case, the mixing between Ψ_L with d_L, b_L, and s_L is suppressed by $(m/M_\Psi)^2$. The reason is that we cannot construct mixing Yukawa couplings among the three doublets Ψ_{1R}', q_L', and Higgs boson. Now the mixing of Ψ_R with d_R, b_R, and s_R is specified by a row matrix S_R, which is defined similarly by Eq. (10) for the unitary matrix U_R. The entries in S_R are of the order $\eta v/M_\Psi$. The decay width $\Gamma_{b\bar{b}}$ is proportional to

$$\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)^2+\left[\frac{1}{2}\left(\frac{\eta_3 v}{M_\Psi}\right)^2+\frac{1}{3}\sin^2\theta_W\right]^2. \tag{24}$$

To fit the data, we need

$$S_{R3}^2=\left(\frac{\eta_3 v}{M_\Psi}\right)^2=0.059\pm 0.016. \tag{25}$$

Generally, by assuming no nonstandard Higgs boson in the theory, if the vectorial fermions are triplets or singlets, the effects on g_L will dominate because in such case U_R is much smaller than U_L. The singlet with the b-quark charge will only reduce g_L^b which is in the wrong direction, while the singlet with the c-quark charge will reduce g_L^c as data require. In the case of a triplet, it can either increase or reduce both g_L^b or g_L^c depending on the hypercharge.

On the other hand, if the new vectorial fermions are a doublet, the effects on g_R will dominate while those on g_L are largely unchanged. To increase g_R^b we need the new vectorial ''down-type'' quark to have $T^3=+\frac{1}{2}$. To reduce g_R^c we also need the new vectorial ''up-type'' quark to have $T^3=+\frac{1}{2}$.

From these arguments, it is straightforward to show that the two models we have are the ones with a minimal number (three) of new vectorial fermions. If one allows four new vectorial fermions, there are also two interesting models that can be considered. One of them is adopted by Ma in Ref. [10] and the other model uses two vectorial doublets: one doublet with $Y=-\frac{5}{3}$ to increase g_L^b and another doublet with $Y=\frac{1}{3}$ to reduce g_L^c. We shall not discuss these nonminimal models in details.

In our models, the strong coupling constant extracted from R_l is different from the standard model. While $\Gamma_{b\bar{b}}$ is enhanced and $\Gamma_{c\bar{c}}$ reduced, $\Gamma_{u\bar{u}}$, $\Gamma_{d\bar{d}}$, and $\Gamma_{s\bar{s}}$ change as well. For simplicity we ignore the changes in $\Gamma_{u\bar{u}}$ and $\Gamma_{d\bar{d}}$. Their mixing with the vectorial fermions is constrained by the kaon flavor-changing neutral current (FCNC) limit as discussed later. However the enhancement in $\Gamma_{s\bar{s}}$ could be substantial. Overall, the $\Gamma_{\rm had}$ without QCD corrections could be enhanced. The $\alpha_s(M_Z)$ extracted from R_l would be smaller than the standard model value since a smaller α_s gives smaller QCD enhancement corrections [14]. Note that R_b and R_c are insensitive to α_s. In the first model, the change in $\Gamma_{s\bar{s}}$ is related to the change in $\Gamma_{c\bar{c}}$, both determined by the parameter ξ_2. Unfortunately, this model cannot accommodate simultaneously the measured R_c, R_b, and α_s. With ξ_2 and ξ_3 determined by the R_b and R_c data, the α_s extracted from R_l would be reduced compared to the standard model result, which is consistent with the tendency of low energy measurements, but it is numerically too small. However, with the experiment data still changing, this model may be useful in the future. On the other hand, in the second model, the change in $\Gamma_{s\bar{s}}$ is determined by a separate parameter η_2. Thus the parameters can be chosen so that all the puzzles are resolved. To extract an α_s of 0.11, as favored by low energy measurements, η_2 should satisfy

$$\left(\frac{\eta_2 v}{M_\Psi}\right)^2 = S_{R2}^2 \sim 0.14 \pm 0.08, \tag{26}$$

with the uncertainty coming from R_b and R_c.

In contrast, Ma's model [10] omits the mixing between the heavy fermion and the s quark. Thus $\Gamma_{\rm had}$ is reduced since the absolute deviation of R_c is larger than that of R_b. Extracted from a smaller prediction for R_l, the strong coupling constant [15] becomes 0.18 in that model, even higher than the original high value of 0.125. This led Ma in his paper [10] to assign the heavy fermion a relatively small mass $M_x<72$ GeV so as to open a new channel for the Z boson decaying into this heavy fermion. In our second model, the α_s puzzle is resolved because of the enhancement in R_s. Experimentally, it is challenging to measure this R_s effect. If this can be done, it will be the most direct test of our model.

III. CONSTRAINTS

In these vectorial fermion models, tree-level flavor-changing neutral currents (FCNC) will, in general, arise since quarks mix with fermions of different weak isospin. Next we shall analyze the FCNC constraints, especially from the kaon decays.

Because of the Glashow-Iliopoulos-Maiani (GIM) mechanism, there will be no FCNC if the heavy vectorial fermions have the same weak isospin T_3 as the quarks they mix with. In the first model, the only component in the neutral current that will generate tree-level FCNC in the kaon decay is $-\frac{1}{2}\overline{y'_{3L}}\gamma^\mu y'_{3L}$. It will give rise to a FCNC vertex involving mass eigenstates d and s:

$$-\tfrac{1}{2}(U_L)_{41}^*(U_L)_{42}\overline{d_L}\gamma^\mu s_L, \tag{27}$$

with

$$(U_L)_{41}=S_1\sim\frac{\xi_1 v}{M_Y}, \quad (U_L)_{42}=S_2\sim\frac{\xi_2 v}{M_Y}\sim S_2'. \tag{28}$$

Here $\xi_2 v/M$ is fixed by R_c to be about 0.2 from Eq. (20). Thus the coefficient of the FCNC vertex,

$$-\frac{g}{2\cos\theta_W}\overline{d_L}\gamma^\mu s_L, \tag{29}$$

is of the order $0.2\times\xi_1 v/M_Y$. The kaon decay $K_L\to\mu^+\mu^-$ restricts this coefficient to be $<3.1\times10^{-5}$ [13]. Take $M_Y\sim200$ GeV as an illustration. The bound for $\xi_1 v$ is $\xi_1 v<32$ MeV. Given the d quark mass of about 10 MeV, the constraint is still quite natural. If the vectorial fermion is heavier, the constraint will be even looser. In the second model, there are more parameters involved. The kaon FCNC constraints will now impose a limit on $\eta_1\eta_2$ from the right-handed current. But now η_2 is no longer fixed by R_c fitting, which is related to ξ_2' instead.

In our models, the couplings of quarks to the charged W gauge boson are written in the usual fashion,

$$\mathcal{L}_W=\frac{g}{\sqrt{2}}(\overline{u_L},\overline{c_L},\overline{t_L})V_{KM}\begin{pmatrix} d_L\\ s_L\\ b_L\end{pmatrix}W^+ + \text{H.c.}, \tag{30}$$

where the Kobayashi-Maskawa matrix V_{KM} is a 3×3 submatrix of the 4×4 matrix:

$$V_4=(U_L')^\dagger\mathcal{I}U_L \quad \text{with } \mathcal{I}=\text{diag}(1,1,1,a). \tag{31}$$

The Clebsch-Gordan coefficient a is equal to $\sqrt{2}$ in the first model and 0 in the second model. In general, neither the full 4×4 matrix $V_4=(U_L')^\dagger\mathcal{I}U_L$ nor its 3×3 submatrix V_{KM} are unitary. However, the unitarity of U_L', U_L, combined with the current data of KM matrix elements, imposes limits on the mixings with vectorial fermions. To analyze these limits, one has to go beyond the leading order of $1/M_Y$ (or $1/M_x$) in the diagonalization of mass matrices. We have worked out V_{KM} to order $(1/M_Y)^2$ by brute force diagonalization. To simplify the presentation, we shall formulate the result in the following way which is more intuitive. Since the Kobayashi-

Maskawa (KM) matrix data is most accurate for the first two generations of quarks, we omit b and t in this discussion (though they can be included without affecting the following conclusions). It is further assumed that c_L mixes with u_L by the angle θ_{0C} of the matrix V_0 which diagonalizes $\widetilde{M}_u\widetilde{M}_u^\dagger$ [see the notation in Eqs. (15)–(17)] before mixing with the vectorial fermion. To zeroth order in $1/M_Y$, θ_{0C} is the same as the Cabibbo angle θ_C. The mixing of the first generation quarks with vectorial fermions is small as discussed above and will be ignored. The mixing between the combination $c_L\cos\theta_{0C}+u_L\sin\theta_{0C}$ and the vectorial fermion is denoted as θ' and the mixing of s_L with its corresponding vectorial fermion θ. Note that θ and θ' are related to S_2 and S_2', respectively, in the leading order by Eq. (11) and Eq. (18):

$$\theta=S_2, \quad \theta'=S_2'/\cos\theta_{0c}. \tag{32}$$

Both angles are of order $1/M_Y$. It can be shown that the KM elements V_{ud}, V_{us} can be written as

$$V_{ud}=\cos\theta_{0C}, \quad V_{us}=\sin\theta_{0C}(\cos\theta\cos\theta'+a\sin\theta\sin\theta'). \tag{33}$$

Thus we have, to order $(1/M_Y)^2$,

$$V_{ud}^2+V_{us}^2\approx 1+\sin^2\theta_C(-\theta^2-\theta'^2+2a\theta\theta'). \tag{34}$$

This result is true for any value of a. Note that if a were equal to one, then V_4 would become unitary and therefore $V_{ud}^2+V_{us}^2$ would have to be less than or equal to 1, as satisfied by the above equation. However for a larger than one, $V_{ud}^2+V_{us}^2$ in general can be larger than one. The current upper and lower bounds at 90% C.L. for $V_{ud}^2+V_{us}^2-1$ are [16]

$$-0.0050<V_{ud}^2+V_{us}^2-1<0.002. \tag{35}$$

In our first model, $\theta=\sqrt{2}\theta'$. Therefore,

$$V_{ud}^2+V_{us}^2\approx 1+\theta'^2\sin^2\theta_C. \tag{36}$$

The above bound requires $\theta'^2<0.04$, which is marginally enough for the deficit of R_c in the first model. Furthermore, one can expect that when the t quark is included, the constraint will be loosened because the factor 1 on the right-handed side of Eq. (36) would be reduced. In the second model, the mixing angle θ of s_L with the left–handed vectorial fermion Ψ_{1L} is suppressed by $1/M_\Psi$ compared to the mixing of s_R. Thus, the angle θ is practically zero. Since the coefficient a is also zero in this case, we get

$$V_{ud}^2+V_{us}^2-1\approx -\theta'^2\sin^2\theta_C. \tag{37}$$

We have

$$\theta'^2\sin^2\theta_c<0.005, \quad \text{or} \quad S_2'^2<0.1. \tag{38}$$

The value of $S_2'^2$ determined by R_c from Eq. (21) is clearly allowed by the unitarity bound in the second model.

The forward and backward asymmetries $A_{\rm FB}^b$ and $A_{\rm FB}^c$ also will be affected by the mixing. The prediction in the first model agrees well with the experimental measurements, as shown by Ma [10]. In the second model, the asymmetry $A_{\rm FB}^b$ is different from the first one. Using a standard-model fit result for $A_{\rm FB}^b$ equal to 0.1041 [3], we calculate the prediction for $A_{\rm FB}^b$ in the second model to be 0.0981 ± 0.002, which agrees with the observed value 0.0997 ± 0.0031 [3]. The uncertainty in the prediction comes from the uncertainty in S_{R3}^2.

The oblique radiative corrections are not significantly affected by the mixing. The vector nature of these new particles allows them to decouple at the heavy mass limit. As long as their masses are above the electroweak scale and the mixings are small, the changes will be smaller than the experimental uncertainties.

One may wonder how these new vectorial fermions can be accommodated in a grand unified theory. The vectorial triplet Y can be found is (3,1,15) of $SU(2)_L\times SU(2)_R\times SU(4)$ which in turn can be found in 210 multiplet of $SO(10)$. The vectorial doublet Ψ is a bit harder to accommodate. It can be found in $(2,1,20)+(2,1,\overline{20})$ $\underline{SU(2)_L}\times SU(2)_R\times SU(4)$ which in turns can be found in $\mathbf{144}+\overline{\mathbf{144}}$ of $SO(10)$ or 650 of E_6.

Note added in proof: The updated world average value of $R_b=0.2178\pm0.0011$ was recently reported [17] at the 1996 DPF Meeting. This experimental value is 1.8σ above the standard-model prediction. The reduced discrepancy decreases the mixing angles in Eqs. (14) and (25) to $|S_3|^2=0.0047\pm0.0023$ or $|S_{R3}|^2=0.023\pm0.011$, respectively. The new value of R_c is now within 1σ of the standard model prediction. Thus S_2' or S_{R2} are consistent with zero. It is interesting to note that the first model with the vectorial fermion triplet can now also accommodate the measured R_b and α_s as the constraint of R_c disappears.

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