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Supersymmetric framework for a dynamical fermion mass hierarchy

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We propose a new framework for constructing supersymmetric theories of flavor, in which flavor symmetry breaking is triggered by the dynamical breakdown of supersymmetry at low energies. All mass scales in our scheme are generated from the supersymmetry-breaking scale $\Lambda_{SSB} \approx 10^7$ GeV through radiative corrections. We assume a spontaneously broken flavor symmetry and the Froggatt-Nielsen mechanism for generating the fermion Yukawa couplings. Supersymmetry breaking radiatively induces a vacuum expectation value for a scalar field, which generates invariant masses for the Froggatt-Nielsen fields at $M_F \approx 10^4$ GeV. ''Flavon'' fields φ, which spontaneously break the flavor symmetry, naturally acquire negative squared masses due to two-loop diagrams involving the Froggatt-Nielsen fields, and acquire vacuum expectation values of order $\langle \varphi \rangle \approx M_F/16\pi^2$. The fermion mass hierarchy arises in our framework as a power series in the ratio $\langle \varphi \rangle / M_F \approx 1/16\pi^2$. [S0556-2821(96)03323-1]

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I. INTRODUCTION

Two outstanding problems of particle physics both involve the origin of symmetry breaking. How does the electroweak gauge symmetry break, allowing the W and Z bosons to acquire mass? And secondly, what breaks the flavor symmetry of the standard model gauge interactions, allowing the quark and leptons to also become massive? The mechanisms for these symmetry breakings must involve new particles and interactions. Furthermore, this new physics must involve new mass scales: the physics of electroweak symmetry breaking (EWSB) must provide an origin for the weak scale M_Z, and the physics of flavor symmetry breaking (FSB) must involve a mass scale M_F.

In the standard model, the interactions of the Higgs doublet H generate both EWSB and FSB. Although the Higgs sector is extremely economical it provides no understanding for the small size of the weak scale, $\langle H \rangle / M_{\rm Pl}$, nor for the small size of FSB, $m_{q,l}/M_Z$. Indeed, the Yukawa couplings of the standard model are arbitrary, explicit, FSB parameters. An understanding of fermion masses would result if these small dimensionless parameters were given in terms of a small ratio $\langle \varphi \rangle / M_F$ [1], where $\langle \varphi \rangle$ is the vacuum expectation value (VEV) of a flavon field which spontaneously breaks a flavor group G_f. However, such a scheme involves three mass scales: $\langle H \rangle$, $\langle \varphi \rangle$, and M_F.

In theories with weak scale supersymmetry, the weak scale $\langle H \rangle$ is determined to be comparable to the superpartner masses $\tilde{m}$ which are derived from two other scales: the primordial supersymmetry-breaking scale Λ_{SSB} and the messenger scale $M_{\rm mess}$. Specific models show that it is possible to generate supersymmetry breaking, and therefore Λ_{SSB}, by dimensional transmutation from nonperturbative dynamics [2]. The messenger scale describes the softness of the superpartner masses $\tilde{m}$ which rapidly vanish at scales above $M_{\rm mess}$.

Hence in supersymmetric theories there are generally four mass scales: two to describe supersymmetry breaking Λ_{SSB} and $M_{\rm mess}$, which lead to EWSB, and two to describe flavor physics and FSB, M_F and $\langle \varphi \rangle$. In supergravity theories, supersymmetry breaking is transmitted to superpartners via supergravitational interactions, so that $M_{\rm mess} = M_{\rm Pl}$ and Λ_{SSB} is determined to be 10^{11} GeV. If $\Lambda_{SSB} < 10^{11}$ GeV, sufficient supersymmetry breaking can be transmitted to the superpartners by gauge interactions, and it is this case which we study in this paper [3,4]. In these theories $M_{\rm mess} \approx 1/(16\pi^2)\Lambda_{SSB}$ can arise in perturbation theory [5]. The messenger sector contains a set of vectorlike generations, X and $\bar{X}$, which acquire both supersymmetry preserving and supersymmetry breaking masses: $M_{\rm mess}[X\bar{X}]_F + M^2_{\rm mess}[X\bar{X}]_A$. On integrating these heavy vector generations out of the theory, the standard model gauge interactions transmit the supersymmetry breaking to the superpartners, giving $\tilde{m} \approx (1/16\pi^2)^2 \Lambda_{SSB}$. Furthermore, renormalization group scalings induced by the large top quark Yukawa coupling, λ_t, induce a negative shift in the Higgs mass squared: $\Delta m_H^2/m_H^2 \approx -3/4\pi^2 \ln(M_{\rm mess}/300\ \text{GeV})(m^2_{\tilde{t}}/m^2_H)$. Since $m_{\tilde{t}}/m_H \approx \alpha_3/\alpha_2$, this triggers EWSB. Thus $M_{\rm mess}$ and $\langle H \rangle$ are understood as arising from Λ_{SSB} by a successive cascade of perturbative loop factors:

$$\Lambda_{SSB} \to M_{\rm mess} \to \tilde{m}, \langle H \rangle. \tag{1.1}$$

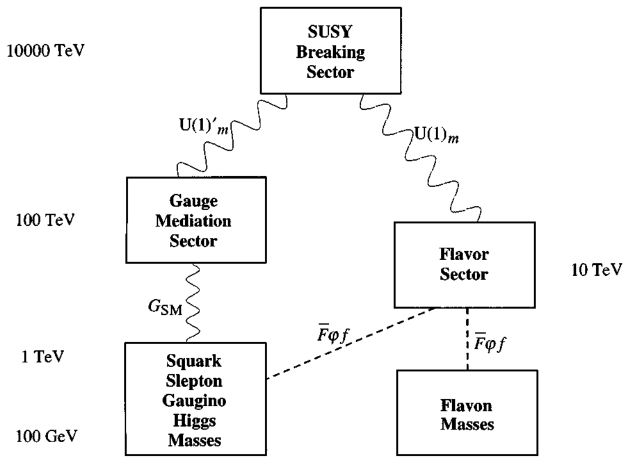

FIG. 1. Schematic structure of the model. The wavy lines indicate radiative corrections due to gauge interactions, while the dashed lines are due to superpotential interactions.

In this paper we study whether the scales of FSB can be similarly derived from $\Lambda_{\rm SSB}$ by a succession of perturbative loops:

$$\Lambda_{\rm SSB}\rightarrow M_F \rightarrow \langle \varphi \rangle. \qquad (1.2)$$

The most successful scheme for generating fermion mass hierarchies from flavor symmetry breaking VEVs, $\langle \varphi \rangle$, involves the mixing of heavy vectorlike generations, F and $\bar{F}$, with the light generations, f: $[M_F \bar{F}F+\bar{F}\varphi f]_F$. The flavor symmetry group, G_f, prevents direct Yukawa couplings for the light quarks and leptons $[ffH]_F$, but these are generated from f-F mixing from the allowed couplings $[\bar{F}fH+\bar{F}FH]_F$. It is intriguing that the cascade of (1.1) for EWSB and (1.2) for FSB both require vectorlike generations at the intermediate stage. We will argue, however, that F and X cannot be identical. In particular, only X has a large supersymmetry breaking mass, $[\bar{X}X]_A$, and only F has direct Yukawa couplings with ordinary matter $[\bar{F}\varphi f]_F$. These distinctions, which arise because F transforms nontrivially under G_f while X is trivial, result in an important difference between the last cascade of (1.1) and (1.2). In the case (1.1), the last cascade is induced by standard model gauge interactions, and leads to positive squared masses for the scalar superpartners. On the other hand, the last cascade of (1.2) is induced by the Yukawa couplings $[\bar{F}\varphi f]_F$ and produces negative squared masses for φ, triggering FSB. In our scheme, both mass scales of the EWSB sector, $M_{\rm mess}$ and $\langle H \rangle$, and both mass scales of the FSB sector, M_F and $\langle \varphi \rangle$, are generated via perturbative loops from the single dynamical scale $\Lambda_{\rm SSB}$, as illustrated in Fig. 1. We will see later that $\langle \varphi \rangle / M_F \approx c/(16\pi^2)$, where c represents a product of several coupling constants, so that c may easily vary between 1/10 and 10. This is sufficient for constructing viable models of the fermion Yukawa matrices, like those in Refs. [6,7].

It is well known that theories of weak scale supersymmetry also need mass terms which couple the two Higgs doublets: $\mu[H_u H_d]_F$ and $m_3^2[H_u H_d]_A$. In supergravity theories, μ plausibly arises from higher order D terms, giving $\mu = \Lambda_{\rm SSB}^2/M_{\rm Pl}$, and a nonzero μ leads automatically to a nonzero m_3^2. With gauge mediated supersymmetry breaking, $\Lambda_{\rm SSB}$ is too small to allow such an origin for μ: the origin of μ and m_3^2 is problematic. Our scheme allows a simple origin for both μ and m_3^2, as we will see in Section III.

The main accomplishment of this paper is the generation of flavor mass scales M_F and $\langle \varphi \rangle$ in a way which is analogous, but not identical, to the generation of $M_{\rm mess}$ and $\langle H \rangle$. In Sec. II we survey the possible origins for fermion mass hierarchies in supersymmetric theories, and find that several alternative options are not very promising. In Sec. III we give the structure of the messenger sector at $M_{\rm mess}$ and of the flavor sector at M_F. We discuss the differences between these sectors, showing that they cannot be identical. Keeping the group structure for G_f general, we show explicitly how radiative corrections trigger the FSB VEV $\langle \varphi \rangle$. We also study the EWSB sector in this framework.

The flavor physics scales of our framework are sufficiently low, of order 10 TeV for M_F and 100s of GeV for $\langle \varphi \rangle$, to be both dangerous and interesting from the viewpoint of flavor-changing and CP-violating processes. In Sec. IV we study amplitudes for these processes induced by integrating out the heavy vector generations F and the flavon fields φ. These rare processes provide constraints on our framework which are very different from the constraints they impose on supergravity theories [6]. Hence the model building choices for the group G_f, and for the representations of φ, F, and f, are governed by constraints which are summarized in the conclusions, and which differ greatly from the supergravity case.

II. FERMION MASS HIERARCHY IN SUPERSYMMETRIC MODELS

The hierarchy in fermion Yukawa couplings is one of the major puzzles in the standard model. The only known fermion with a Yukawa coupling of order one is the top quark, while all other fermions have Yukawa couplings that are significantly smaller.[1] The existence of small parameters in the standard model Lagrangian is natural in the sense of 't Hooft: When all the Yukawa couplings are set to zero, the standard model is invariant under a global $U(3)^5$ flavor symmetry. Thus, the symmetry of the theory is enhanced as the Yukawa couplings are reduced. While this explains why small Yukawa couplings are technically natural, it sheds no light on how such small parameters arise. Thus, we would like to understand how a low-energy effective theory containing small Yukawa couplings can arise when the corresponding high-energy theory involves no small parameters at all.

One way of framing this problem is to assume that the Yukawa couplings of the light fermions are forbidden at high energies by a flavor symmetry G_f, which is a subgroup of $U(3)^5$. The top quark Yukawa coupling may be invariant under the flavor symmetry. Then, the problem at hand is to understand why G_f is broken only by a small amount. Since the flavor scale is generally much larger than the weak scale, it is natural to work in the supersymmetric context. Then the

[1]If $\tan\beta$ is large, the bottom quark and tau lepton may also have order 1 Yukawa couplings.

hierarchy between the weak scale, the flavor scale, and any other high scales in the problem will be stable against radiative corrections.

Supersymmetry, however, makes the task of generating small couplings a challenging one. If small Yukawa couplings are not present in the original superpotential, then the supersymmetric nonrenormalization theorem tells us that such couplings will *never* be generated at any order in perturbation theory. Therefore, the mechanism of flavor symmetry breaking must be linked either to supersymmetric nonperturbative effects, or to supersymmetry breaking. Three popular schemes have been proposed for generating small Yukawa couplings in supersymmetric models: string compactification, radiative generation, and the Froggatt-Nielsen mechanism. Let us review these possibilities and consider the limitations of each:

I. String compactification. It is possible that the small Yukawa couplings are simply present as a boundary condition due to physics at the string scale. All coupling constants in string theory are supposed to be proportional to a single string coupling constant, which is of the same order as the gauge coupling constants, i.e., order 1. However, couplings in the superpotential depend on the compactification. In orbifold models, if chiral fields belong to twisted sectors with different fixed points, their superpotential couplings are suppressed by e^{-R^2} in the limit where the size of the compactified manifold R is large [8]. In this case, small numbers arise as a result of the compactification. It has been pointed out that the radius R does not need to be much larger than the string scale [9]. One possible problem with this scenario is that different generations have different modular weights, and the scalar masses are therefore nonuniversal. This may lead to dangerous flavor-changing neutral current effects [10].

II. Radiative generation. If small Yukawa couplings are not already present at the string scale, one may try to generate them through radiative corrections that involve the soft supersymmetry-breaking operators. In scenarios of this type, the flavor symmetry G_f is broken by the nonvanishing entries of the scalar mass matrices, while the fermion Yukawa matrices retain their flavor-symmetric form at the tree level. This can be a consequence of the spontaneous breakdown of the flavor group. Yukawa couplings are then generated at the higher loop level when the superpartners are integrated out. The limitation of this approach is that it is very difficult to generate small Yukawa couplings for both the first and second generation fermions, assuming the minimal particle content of the supersymmetric standard model [11,12]. While the small first-generation Yukawa couplings may be understood as radiative effects, those of the second generation must be generated by a separate mechanism. In addition, the models that have been proposed require a special mechanism to ensure separate muon number conservation in the lepton sector to evade the tight constraints from $\mu \to e\gamma$.

III. Froggatt-Nielsen mechanism. The Froggatt-Nielsen mechanism [1] is perhaps the most popular mechanism for generating small Yukawa couplings. Fields of the first and second generations have no direct Yukawa couplings to the Higgs bosons, as a consequence of a flavor symmetry. On the other hand, heavy vectorlike fields couple to the Higgs fields with $O(1)$ strength. When the flavor symmetry breaks spontaneously, a small mixing is induced between the light generations and the vectorlike fields, as described in the Introduction. When the vectorlike fields are integrated out, small Yukawa couplings are generated in the low-energy effective theory, having the form

$$\left(\frac{\langle\varphi\rangle}{M_F}\right)^n ffH. \tag{2.1}$$

The smallness of the light fermion Yukawa couplings is a consequence of a hierarchy between two scales $\langle\varphi\rangle/M_F$, where M_F is the mass scale of heavy vectorlike fields, and $\langle\varphi\rangle$ is the vacuum expectation value of a flavon field that spontaneously breaks G_f. This is the mechanism of generating small Yukawa couplings that we will adopt in this paper.

There are two important questions associated with supersymmetric models that involve the Froggatt-Nielsen mechanism. First, one may worry that the scalar mass matrices may not be sufficiently degenerate to suppress flavor-changing processes, since the three generations couple differently to the heavy states. However, the flavor symmetry restricts the form of both the Yukawa and scalar mass matrices, and this can be sufficient to prevent any flavor-changing problems. The flavor symmetry can either enforce a sufficient degeneracy among the scalar states, or align the Yukawa and scalar mass matrices so that flavor changing neutral current processes are adequately suppressed [13,16]. In many models based on the Froggatt-Nielsen mechanism, one typically needs $\langle\varphi\rangle/M_F \lesssim 0.01$–$0.05$; powers of this ratio appear in the operators that generate the light fermion Yukawa couplings. Thus, we are lead to the second question, which is more fundamental: from where does the hierarchy $\langle\varphi\rangle/M_F \ll 1$ originate? This is the main issue of the paper.

Nonrenormalization theorems tell us that it is impossible to generate the $\langle\varphi\rangle/M_F$ hierarchy perturbatively in models with unbroken supersymmetry, unless the scales are put into the superpotential by hand. This is exactly what we would like to avoid. Therefore, the only logical possibilities are that the origin of scales is either triggered by (a) nonperturbative effects or by (b) supersymmetry breaking (or both). Let us consider each of these possibilities:

(a) *Nonperturbative effects*. One can imagine that the Froggatt-Nielsen mass scale M_F and the scale $\langle\varphi\rangle$ are generated by separate gauge groups that each become strong at scales much lower than the Planck scale. The scale parameter of each gauge group is given by $\Lambda_i = M_* e^{-8\pi^2 b_0^i/g_i^2}$, where g_i are the gauge coupling constants at $M_* = M_{\rm Pl}/\sqrt{8\pi}$ and b_0^i are their β function coefficients. When the model is incorporated into supergravity, a condensation due to strong gauge dynamics is likely to break supersymmetry. Even when the strongly interacting groups do not couple directly to the fields in the minimal supersymmetric standard model (MSSM), their scales have to be smaller than $\Lambda_i \lesssim (m_W M_*^2)^{1/3} \simeq 10^{13}$ GeV. Thus, a small ratio in scales $\Lambda_1/\Lambda_2 \sim 0.01$–$0.05$ requires a special arrangement in the particle content and a mild fine-tuning in the initial gauge couplings of both gauge groups. The tuning becomes more and more severe as one considers lower scales for the Λ_i.

(b) *Supersymmetry breaking.* The other possibility is to use soft supersymmetry breaking parameters to generate the scales $\langle\varphi\rangle$ and M_F. We discuss two cases separately. The first possibility is that supersymmetry breaking occurs in the hidden sector, and all soft supersymmetry breaking terms are generated at the Planck scale, at the same order of magnitude. The other possibility is that supersymmetry breaking occurs at low energy $\sim 10^7$ GeV, and is transmitted to the MSSM fields via renormalizable interactions.

The hidden sector case suffers from naturalness problems similar to those that we encountered earlier. The basic difficulty is that there is only one scale in the problem from which we would like to generate two scales. If all mass scales are generated by supersymmetry breaking with a generic superpotential, they will all be of order the weak scale, with no hierarchy among them. Therefore, one needs to rely on flat directions of the superpotential to generate scales much higher than the weak scale. One may try to be economical by identifying M_F with M_* (or $M_{\rm string}$) and then hope to generate $\langle\varphi\rangle\sim 10^{-2}M_*$. A difficulty with this idea is that one needs to forbid higher dimension operators in the superpotential of the form φ^{n+3}/M_*^n, up to $n \gtrsim 7$ in order that $\langle\varphi\rangle$ not be pushed down to lower scales. If the flavor symmetry is Z_N, then one needs a relatively large $N\gtrsim 10$. If the flavor symmetry is continuous, it needs to be gauged or it will be violated by Planck scale effects. If it is a gauged U(1) symmetry, we always require fields with positive and negative charges to cancel the anomaly. As a result, it is likely that there will be higher dimension operators allowed by the gauge symmetry that reduce $\langle\varphi\rangle$. This statement trivially extends to non-Abelian gauge symmetries as well. Therefore, a gauged flavor symmetry had better be anomalous, with the anomaly cancelled by the Green-Schwarz mechanism. This possibility has been studied by many authors, who have focused on obtaining the correct $\sin^2\theta_W=3/8$ from the anomaly cancellation condition [14]. In general, however, this scenario leads to nondegenerate scalar masses because the U(1) charge assignments are generation dependent [15]. If there is quark-squark alignment [16], then scalar nondegeneracy may not lead to dangerous flavor changing effects. However, we are not aware of any models in which alignment is achieved using the same U(1) whose anomalies are canceled by the Green-Schwartz mechanism. It remains to be seen whether a model of this type can be constructed.

The obvious way to avoid the problems with higher dimension operators is to lower the scales of M_F and $\langle\varphi\rangle$. The potential along a flat direction is given by $m^2(\varphi)|\varphi|^2$ where $m^2(\varphi)$ is the effective supersymmetry breaking squared mass which satisfies the renormalization group equation. The minimum of the potential is generated around the scale where $m^2(\varphi)$ crosses zero. In this way, one can easily obtain the invariant mass M_F of the vectorlike Froggatt-Nielsen fields such that $M_F \ll M_{\rm Pl}$; then higher dimension operators become completely irrelevant. Once the scale M_F is generated, the vectorlike fields decouple from the renormalization group equation of the flavon mass squared. Therefore the running of the flavon mass squared is slowed, and it crosses zero at a much lower scale, which is likely to be much less than one hundreth of M_F. To obtain $\langle\varphi\rangle/M_F\sim 0.01$ requires the model to be very carefully arranged.

Models with low-energy supersymmetry breaking do not suffer from the difficulties discussed above. While supersymmetry breaking in the hidden sector presented us with only one scale from which we needed to generate two, low-energy supersymmetry breaking mediated by renormalizable interactions tends to produce a multitude of scales. Since supersymmetry breaking is mediated to the MSSM fields, or to any other fields in the theory, via renormalizable interactions, the effects are not always transmitted at the same order in perturbation theory. Thus, it is natural to obtain many different mass scales separated from each other by powers of $1/(16\pi^2)$.

This observation suggests an intriguing scenario: Supersymmetry is broken at a scale $\Lambda_{\rm SSB}\sim 10^7$ GeV. This scale is determined by dimensional transmutation, and is not directly input into the theory. Supersymmetry breaking is mediated via renormalizable interactions to all other fields in the theory. This occurs at varying order in perturbation theory, producing a hierarchy of scales, of the form $(1/16\pi^2)^n\Lambda_{\rm SSB}^2$. Thus, the small flavor symmetry breaking parameters described earlier are identified with the ratio of some of these scales. Notice that the range of $\langle\varphi\rangle/M_F$ that is required by the Froggatt-Nielsen mechanism, $\simeq 0.01$–0.05, corresponds to a loop factor ($1/4\pi$ or $1/16\pi^2$) times an order one coefficient. In this framework, all coupling constants in the superpotential can be of order 1, and all mass scales generated from a single scale, the scale at which supersymmetry is broken.

In the rest of the paper we show how our framework may be implemented. We focus on the generic structure of our framework and demonstrate that it is phenomenologically viable. We do not go into a detailed discussion of particular flavor symmetries. Our framework is compatible with a variety of explicit flavor models that are described in the literature, including $U(1)^3$ [16,6], $\Delta(75)$ [17], and $(S_3)^3$ [18,7,19].

III. FRAMEWORK

The overall structure of our model is summarized schematically in Fig. 1. Supersymmetry is broken at the scale $\Lambda_{\rm SSB}$, and communicated to both the gauge-mediation (GM) and flavor sectors via two-loop diagrams. We assume that different U(1) gauge interactions act as the messengers of supersymmetry breaking to each of these sectors. Thus, the GM and flavor mass scales are given by $\Lambda_{\rm GM}\approx g_m'^2\Lambda_{\rm SSB}/16\pi^2$ and $\Lambda_{\rm flav}\approx g_m^2\Lambda_{\rm SSB}/16\pi^2$, where g_m' and g_m are the messenger U(1) gauge couplings.[2] The ordinary superparticles $\tilde{f}$ and the flavor symmetry breaking fields φ communicate with the supersymmetry breaking sector through four-loop diagrams, and develop masses of order $g_m'^2\Lambda_{\rm SSB}/(16\pi^2)^2$ and $g_m^2\Lambda_{\rm SSB}/(16\pi^2)^2$, respectively. The ordinary Higgs field H develop masses comparable to those of $\tilde{f}$ for the choice $g_m'>g_m$ that we assume below.

A. Constraints on the flavor sector

The requirement that we generate TeV scale masses for the ordinary superparticles fixes $\Lambda_{\rm GM}$ at approximately 100 TeV. This is consistent with a supersymmetry breaking scale

[2] $\Lambda_{\rm GM}$ and $\Lambda_{\rm flav}$ also depend on couplings in the supersymmetry-breaking sector which we have omitted for simplicity.

$\Lambda_{\rm SSB}\approx 10\,000$ TeV. On the other hand, we choose the flavor scale to be somewhat lower, $\Lambda_{\rm flav}\approx 10$ TeV, so that we do not generate negative squared masses for the Higgs fields that are too large (see Sec. III E). In addition, this choice reduces flavor changing neutral current effects that originate from the supersymmetry-breaking masses of the Froggatt-Nielsen (FN) fields (see Sec. III G). The difference between the GM and flavor scales can be obtained by choosing the messenger U(1) gauge couplings such that $g_m/g'_m\approx 1/3$, which does not constitute a significant fine-tuning.[3] With the flavor scale at 10 TeV, the flavon fields φ develop masses of order a few hundred GeV. We will show in Sec. IV that the possibility of flavon fields with masses in the few hundred GeV range is not excluded by the current phenomenological constraints.

In order to generate the flavor scale in the way suggested above, we must first address some immediate phenomenological difficulties. Let us assume that there is a field a whose vacuum expectation value (VEV) generates the mass of the FN fields. Consider the superpotential couplings

$$W=\alpha a F\overline{F}+\beta\overline{F}\varphi f, \tag{3.1}$$

where F and $\overline{F}$ are FN fields, φ is a flavon, and f is an ordinary matter field. The first term determines the FN mass scale $M_F=\alpha\langle a\rangle$, while the second term generates the desired mixing between the ordinary fields and the heavy FN fields beneath the flavor-symmetry-breaking scale. Since we assume that the VEV of a is a consequence of supersymmetry breaking, we expect that the auxiliary component of a will also be nonvanishing, and in general,

$$\langle F_a\rangle\approx\langle a\rangle^2. \tag{3.2}$$

Now consider the scalar mass squared matrix for the ordinary and the FN fields. If a has a nonvanishing F component, then there will be a scalar B-term of the form $F\overline{F}$ of order $\alpha\langle F_a\rangle\approx M_F^2$ From Eq. (3.1), the scalar mass squared matrix is then given by

$$\begin{pmatrix} M_F^2 & \alpha\langle F_a\rangle & 0 \\ \alpha\langle F_a\rangle & M_F^2 & \beta M_F\langle\varphi\rangle \\ 0 & \beta M_F\langle\varphi\rangle & \beta^2\langle\varphi\rangle^2 \end{pmatrix} \tag{3.3}$$

in the basis $(\overline{F}^*,F,f)$. In the case where $\langle F_a\rangle=0$, the matrix (3.3) has one zero eigenvalue (corresponding to the physical squarks or sleptons), and two eigenvalues of order M_F^2. When F_a is nonvanishing, the zero eigenvalue is shifted to

$$-\beta^2\left(\frac{\langle\varphi\rangle}{M_F}\right)^2\langle F_a\rangle^2/M_F^2. \tag{3.4}$$

The fact that the lightest eigenvalue is negative is not a problem by itself, since there are larger positive contributions to the squared masses from the gauge-mediation diagrams in the gauge mediation sector of the model, as we will see later. However, the contribution in Eq. (3.4) is *flavor dependent*, and can lead to large flavor changing neutral current effects. The simplest way of avoiding these difficulties is to construct models in which F_a is naturally much smaller than $\langle a\rangle^2$, so that the effect of Eq. (3.4) is phenomenologically irrelevant.[4]

Thus, we choose to build a model in which the messenger sector for generating the FN mass scale has some mechanism of protecting the a field from acquiring an F component, at least at tree level. The situation is quite different in the gauge mediation sector, where the field analogous to a must acquire both scalar and F-component VEVs to produce the correct nonsupersymmetric spectrum of vectorlike states [5]. Thus, in addition to a differing sequence of mass scales, the two branches of Fig. 1 are distinguished by the properties of the field that couples to the vectorlike multiplets; thus, the GM and flavor sectors cannot be identified. We will see this explicitly in the model that we present below.

B. The supersymmetry breaking sector

We assume that supersymmetry is broken in a sector of the model that is nearly isolated from all other sectors. The only communication between fields in the supersymmetry breaking sector and the remaining fields in the theory is through the two messenger U(1) gauge interactions. Fields $\xi_\pm$ that carry either of the messenger charges can communicate with the supersymmetry breaking sector through two-loop diagrams. When supersymmetry is broken, the $\xi_\pm$ fields can acquire supersymmetry breaking masses. In some models of dynamical supersymmetry breaking, like the SU(6) $\times$U(1) model discussed in Refs. [5,20], it is known that the ξ fields can acquire negative squared masses after supersymmetry is broken. Although our framework involves two messenger U(1) gauge groups, we assume that the same is possible here. We view this as a mild restriction on the types of model that can serve as an adequate supersymmetry breaking sector. Note that there are many models of dynamical supersymmetry breaking that contain two nonanomalous U(1) factors that can be gauged. Examples include the SU(9) model

[3]While we have introduced two messenger U(1) gauge groups to obtain different flavor and GM mass scales, the gauge structure of our model has an additional benefit. If there were only one U(1) gauge interaction coupling to the two otherwise disconnected sectors of the model, we would be left with an extra Nambu-Goldstone boson after symmetry breakdown. While it is not clear whether such a massless scalar boson would do any harm phenomenologically, we have chosen to avoid this situation completely: the Nambu-Goldstone boson is absorbed by the additional U(1) gauge field.

[4]If the squarks have masses around 1 TeV, then flavor-changing neutral current (FCNC) effects in the quark sector due to Eq. (3.4) may not necessarily be fatal. For example, in an explicit model of flavor with $\langle\varphi\rangle/M_F\sim 1\times 10^{-2}$, the (1,2) elements of the squark mass squared matrices will be of the order $\widetilde{m}_{12}^2/\widetilde{m}^2\approx 0.01$, assuming that $F_a\approx\langle a\rangle^2=$ (10 TeV)^2. This is in borderline agreement with the current experimental bounds. The real problem arises is the lepton sector, where the right-handed sleptons are a factor of 7 lighter than the squarks. Lepton flavor violation will be present at an unacceptable level unless a separate FN sector is constructed for the leptons that preserves electron or muon number. The solution presented in the text does not place additional restrictions on the flavor structure of the model.

with an antisymmetric tensor and five antifundamentals [2], and the SU(2) model with four doublets and six singlets [21]. Both of these models possess large global symmetries [SP(4) or SU(4)] that contain a nonanomalous U(1)×U(1) subgroup.[5] It is reasonable to assume that in some of these models, there are regions of parameter space in which it is possible to generate negative squared masses for fields carrying either of the messenger U(1) charges. This point will be assumed in the next two subsections.

C. Gauge mediation sector

We first consider the sector of the theory that generates the gauge mediation mass scale, following the work of Dine, Nelson, Nir, and Shirman [20]. The superpotential for this sector is given by

$$W = -h_1'\xi_+'\xi_-'S_1' + \frac{\lambda_1'}{3}S_1'^3 + \alpha_1' S_1' X\overline{X}, \tag{3.5}$$

where X and $\overline{X}$ are vectorlike fields that carry standard model quantum numbers, and S_1' is a gauge singlet. The ξ' fields are charged under the messenger U(1) gauge group that connects the superpotential above to the supersymmetry breaking sector of the theory. To prevent the fields in Eq. (3.5) from coupling to fields in the flavor sector, we will impose a Z_3 "sector symmetry" under which all the GM sector fields transform by the phase $e^{2i\pi/3}$.[6] Then (3.5) includes the most general renormalizable interactions consistent with the symmetries of the theory. The two-loop diagrams described in Sec. III B contribute to the ξ_+' and ξ_-' masses after supersymmetry is broken. Thus, in addition to the superpotential given above, we assume there are soft supersymmetry-breaking masses

$$V = -m'^2|\xi_+'|^2 - m'^2|\xi_-'|^2. \tag{3.6}$$

With $\Lambda_{\rm SSB} \approx 10\,000$ TeV, we expect the $\xi_\pm'$ masses m' to be of order $(\alpha_m'/4\pi)\Lambda_{\rm SSB} \approx 100$ TeV, where $\sqrt{4\pi\alpha} = g_m'$ is the relevant messenger U(1) gauge coupling. The negative squared masses for the $\xi_\pm'$ generate nonvanishing VEVs for both the scalar and F components of S_1', of order m' and m'^2 respectively. This leads to a nonsupersymmetric spectrum for the fields X and $\overline{X}$, which can communicate with the ordinary fields $\tilde{f}$ via standard model gauge interactions.

If the X and $\overline{X}$ form a **5** and $\overline{\mathbf{5}}$ of SU(5), then the ordinary gaugino and squark masses are given by [5]

$$m_i = \frac{g_i^2}{16\pi^2}\frac{\langle F_{S_1'}\rangle}{\langle S_1'\rangle}, \tag{3.7}$$

$$\tilde{m}^2 = \sum_i 2C_F^{(i)}\left(\frac{g_i^2}{16\pi^2}\right)^2 \frac{\langle F_{S_1'}\rangle^2}{\langle S_1'\rangle^2}, \tag{3.8}$$

where the g_i are standard model gauge couplings, and the Casimir C_F is 3/4 for SU(2) doublets, 4/3 for SU(3) triplets, and $(3/5)Y^2$ for fields with ordinary hypercharge Y. For our choice $\Lambda_{\rm GM} = \langle F_{S_1'}\rangle/\langle S_1'\rangle = 100$ TeV, we obtain the squark and slepton masses (at 100 TeV),

$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$
1140	1100	1100	350	150

and the gaugino masses (at 1 TeV)

$\tilde{B}$	$\tilde{W}$	$\tilde{g}$
130	270	930,

in GeV. Notice that gauge-mediation renders the squarks and gluinos heaviest, with masses around 1 TeV.

It is important to point out that there are a number of problems with the messenger superpotential in Eq. (3.5).[7] First, the coupling h_1' must satisfy $h_1'^3 < 2\lambda_1' g_m'^2$,[8] so that the D-term vanishes at the minimum of the above potential. Second, the coupling h_1' must be also smaller than λ_1' so that the desired vacuum with $\langle S\rangle \neq 0$ is a local minimum. This is in disagreement with the claim made in Ref. [20]. The region $\alpha_1'\lambda_1'/(\lambda_1'+\alpha_1') < h_1' < \lambda_1'$ is also problematic, because $\alpha_1'\langle F_{S_1'}\rangle$ will be larger than $(\alpha_1'\langle S_1'\rangle)^2$, and the scalar mass squared matrix for the X and $\overline{X}$ fields will have a negative eigenvalue. This would imply that the standard model gauge group is broken at the 100 TeV scale. Therefore, we need $h_1' < \alpha_1'\lambda_1'/(\lambda_1'+\alpha_1')$. Even in this region, there is still a global minimum of the potential where the standard model gauge group is broken. To see this, notice that in the supersymmetric limit there is a flat direction in which $S_1'=0$, $\xi_+'=\xi_-'$, $X=\overline{X}$, and $h_1'\xi_+'\xi_-' = \alpha_1' X\overline{X}$. Along this direction, the potential becomes increasingly negative until the negative mass squared for the ξ' fields $-m'^2$ disappears beyond $\xi_\pm' \approx \Lambda_{\rm SSB}$. For larger values of ξ', the potential becomes flat with $V \simeq -m'^2\Lambda_{\rm SSB}^2$, which is much lower than the local minimum, where $V \simeq -m'^4$. In order to avoid this problem, it seems that the gauge mediation sector must be modified. One obvious solution is to introduce another singlet field S_2' with the the additional superpotential couplings

$$\Delta W = -h_2'\xi_+'\xi_-'S_2' + \frac{\lambda_2'}{3}S_2'^3 + \alpha_2' S_2' X\overline{X} + (S_1', S_2' \text{ couplings}). \tag{3.9}$$

Unless $h_2'/\alpha_2' = h_1'/\alpha_1'$, there is no longer a flat direction. The desired vacuum can be a global minimum in a certain range of parameters as discussed above. The precise conditions on the parameters are complicated and not worth presenting here. It should be stressed that the details of the gauge mediation sector are irrelevant to the flavor sector presented in

[5]In addition, Fayet-Illiopoulos D terms are not generated in these models because of unbroken discrete symmetries [20].

[6]Without this symmetry, the singlet S_1' above could also couple to the FN fields. Then we would not be able to avoid the phenomenological disasters described in Sec. III A.

[7]Shortly after our original preprint similar observations were made independently in Ref. [22].

[8]We thank the authors of Ref. [22] for pointing out that our original bound $h_1' < g$ was not necessary.

the next subsection. Thus, any workable alternative to Eq. (3.5) can be adopted without altering the conclusions of this paper.

D. The flavor sector

As we described in Sec. III A, the field that determines the FN mass scale, a, must have no F component at tree level. If a self-coupling term λa^3 were present in the superpotential, then we would expect the a field to acquire an F component $F_a = 3\lambda a^2 \neq 0$ when a acquires a VEV. Therefore, we will begin by imposing a Z_2 symmetry under which the a field is odd. In the superpotential that we present below, this will be sufficient to prevent F_a from acquiring a VEV at lowest order in perturbation theory.

Since we would like to generate $\langle a \rangle$ from supersymmetry breaking, the field a must couple, at least indirectly, to fields that are charged under the messenger U(1). The simplest renormalizable superpotential that generates a VEV for a is

$$W = h_1 \xi_+ \xi_- S_1 - g_1 S_1 a^2 + \frac{\lambda_1}{3} S_1^3, \tag{3.10}$$

where the $\xi_\pm$ and S_1 fields are even under the Z_2, and we allow no dimensionful couplings. As before, we assume that the ξ fields develop negative squared masses when supersymmetry is broken:

$$V = -m^2 |\xi_+|^2 - m^2 |\xi_-|^2, \tag{3.11}$$

where m is of order $(\alpha_m/4\pi)\Lambda_{\rm SSB} \approx 10$ TeV. Thus, the $\xi_\pm$ fields acquire VEVs $\langle \xi_+ \rangle = \langle \xi_- \rangle = \langle \xi \rangle$ which are of order m. Given these VEVs, the S_1 field develops a mass $h_1 \langle \xi \rangle$. As long as $\lambda_1 < h_1$, S_1 remains at the origin, while $F_{S_1} = h_1 \langle \xi \rangle^2 \neq 0$. Since $F_{S_1} \neq 0$, the scalar (a, a^*) mass squared matrix develops a negative eigenvalue, and a then obtains a VEV. However, S_1 does not have a VEV in this parameter range, so F_a exactly vanishes, as desired.

Unfortunately, this simple superpotential has the same problem that we encountered with the first superpotential presented in Sec. III C: there is a flat direction where $h_1 \xi_+ \xi_- = g_1 a^2$. As a consequence of the negative squared masses of the $\xi_\pm$ fields, the potential has a running-away behavior along the flat direction, and thus we expect that $\langle \xi \rangle$ will be at least of order $\Lambda_{\rm SSB}$. To eliminate the unwanted flat direction, we introduce a new field, S_2, with the following superpotential interactions:

$$\Delta W = h_2 \xi_+ \xi_- S_2 - g_2 S_2 a^2 + \frac{\lambda_2}{3} S_2^3 + (S_1, S_2 \text{ couplings}). \tag{3.12}$$

Again, all the fields above are even under the Z_2 symmetry, except for a. The coupling $\alpha a F \bar{F}$ generates the desired masses of the FN fields, without the bilinear supersymmetry breaking mass terms. Notice that the superpotential in the flavor sector is actually identical to the one in the gauge mediation sector providing we make the identification $X\bar{X} \leftrightarrow a^2$. The only difference between these sectors is the allowed range of the superpotential couplings. In the flavor sector we take the λ_i to be smaller than the h_i, so that S_1 and S_2 will not develop VEVs. In the gauge-mediation sector, we take the λ_i' to be larger than the h_i' so that S_1' and S_2' fields do acquire VEVs, while X and $\bar{X}$ do not.

To study the scalar potential of the theory, it is convenient to work with the redefined superpotential couplings

$$W = h \xi_+ \xi_- \chi + g \xi_+ \xi_- \eta - \lambda a^2 \eta + + \frac{k_1}{3} \chi^3 + k_2 \chi^2 \eta + k_3 \chi \eta^2 + \frac{k_4}{3} \eta^3, \tag{3.13}$$

where χ and η are linear combinations of S_1 and S_2. h, g, and k_i $(i=1,2,3,4)$ are coupling constants of the redefined fields. Using suitable phase rotations of fields, we can take h, g and λ real and positive without a loss of generality. The scalar potential of the theory is given by

$$\begin{aligned} V = &|h \xi_+ \xi_- + k_1 \chi^2 + 2k_2 \chi \eta + k_3 \eta^2|^2 + |\xi_+ (h\chi + g\eta)|^2 \\ &+ |\xi_- (h\chi + g\eta)|^2 + |g \xi_+ \xi_- - \lambda a^2 + k_2 \chi^2 \\ &+ 2k_3 \chi \eta + k_4 \eta^2|^2 + |2\lambda a \eta|^2 + \frac{g_m^2}{4} (|\xi_+|^2 - |\xi_-|^2)^2 \\ &- m^2 |\xi_+|^2 - m^2 |\xi_-|^2. \end{aligned} \tag{3.14}$$

For simplicity, we assume $g_m > h$. Then the potential is minimized when the messenger U(1) D-term contribution to the potential is minimized: $\langle \xi_+ \rangle = \langle \xi_- \rangle$.[9] In some region of parameter space, the rest of the potential is minimized when $\langle \eta \rangle = \langle \chi \rangle = 0$, and

$$\langle a \rangle = \frac{1}{h} \sqrt{\frac{g}{\lambda}} m \ \text{ and } \langle \xi_\pm \rangle = \frac{1}{h} m. \tag{3.15}$$

It is straightforward to check that the F components of a, ξ_+, ξ_-, and η vanish at this minimum; χ develops an F component $F_\chi = m^2/h$. Of course, there are corrections to these results that are suppressed by loop factors. For example, we expect supersymmetry breaking to generate soft trilinear terms in addition to the squared masses of the $\xi_\pm$ fields. Since these terms appear at the same order in the loop expansion as the $\xi_\pm$ masses, they will have coefficients $A \approx \Lambda_{\rm SSB}/(16\pi^2)^2$, which implies that $A/m \approx 1/(16\pi^2)$. In this case, we find that a nonvanishing F component of a is indeed generated. However, it is more than adequately suppressed: $\langle F_a \rangle \approx \langle a \rangle^2/(16\pi^2)$.

Given these VEVs, we can now study the effects of supersymmetry breaking on the scalar and fermion spectra of our model. Only the fields that mix with a are relevant to the generation of flavon masses, as we will see below. It is straightforward to show that the scalar and fermion masses matrices are two-by-two block diagonal in the basis $(\xi, a, \chi, \eta, \xi')$, where $\xi = (\xi_+ + \xi_-)/\sqrt{2}$ and $\xi' = (\xi_+ - \xi_-)/\sqrt{2}$. Since ξ is the only field that mixes with a, we focus on the (ξ, a) submatrices. In this basis, the mass squared matrices for the real and imaginary scalar components are given by

[9] If $g_m < h$, the potential prefers $\langle \xi_+ \rangle \neq \langle \xi_- \rangle$ and develops a different minimum.

$$M_r^2=\begin{bmatrix} 2h^2+2g^2 & -2g\sqrt{2\lambda g} \\ -2g\sqrt{2\lambda g} & 4g\lambda \end{bmatrix}\frac{m^2}{h^2} \tag{3.16}$$

and

$$M_i^2=\begin{bmatrix} 2g^2 & -2g\sqrt{2\lambda g} \\ -2g\sqrt{2\lambda g} & 4g\lambda \end{bmatrix}\frac{m^2}{h^2}, \tag{3.17}$$

while the squared mass matrix for the fermionic components is

$$M_f^2=\begin{bmatrix} 2h^2+2g^2 & -2g\sqrt{2\lambda g} \\ -2g\sqrt{2\lambda g} & 4g\lambda \end{bmatrix}\frac{m^2}{h^2}. \tag{3.18}$$

Notice that M_r^2 and M_f^2 are identical, while M_i^2 differs in its (1,1) component. As far as we are interested in the effects of supersymmetry breaking in the ξ-a spectrum alone, we may evaluate the deviation of a given Feynman diagram about its supersymmetric limit by replacing the propagator for the imaginary components of ξ and a by the difference

$$i(p^2-M_i^2)^{-1}-i(p^2-M_f^2)^{-1}. \tag{3.19}$$

The potential (3.14), combined with a term $\alpha a\overline{F}F$, generates the supersymmetric mass of FN fields while avoiding supersymmetry breaking bilinear term. This is a consequence of the Z_2 symmetry that we imposed at the start. For simplicity, we assume that the FN fields F and ordinary matter fields f transform in the same way under this Z_2. The couplings $a\overline{F}F$ and $\overline{F}\varphi f$ then imply that the flavons φ are necessarily odd under this Z_2.

Now we are in a position to show that supersymmetry breaking generates a negative mass squared for the flavon fields φ. To communicate flavor symmetry breaking to the ordinary fields, we assume the superpotential couplings

$$W_\varphi=\beta\overline{F}\varphi f+\gamma\varphi^2\varphi'+\delta\varphi'^3/3, \tag{3.20}$$

where β, γ, and δ are coupling constants. The fields φ' are even under the Z_2, and hence do not have an $\overline{F}\varphi' f$ coupling. The interactions in Eq. (3.13) and Eq. (3.20) allow us to write down the two-loop diagrams presented in Fig. 2. Only the diagrams that involve the imaginary part of a are shown. The cross on the a line indicates we are using the (2,2) element of the difference propagator in Eq. (3.19). The details of this calculation are presented in the Appendix. The result is

$$m_\varphi^2\approx-\frac{N_c}{(16\pi^2)^2}\frac{\alpha^2\beta^2g^3\lambda}{h^4}m^2, \tag{3.21}$$

where N_c is the number of colors, and we have assumed that all superpotential couplings are of order unity. This generates the vacuum expectation values

$$\langle\varphi\rangle^2=\frac{\delta^2}{8\gamma^3(\delta-\gamma)}(-m_\varphi^2), \quad \langle\varphi'\rangle^2=\frac{\delta-2\gamma}{8\gamma^2(\delta-\gamma)}(-m_\varphi^2), \tag{3.22}$$

if $\delta>2\gamma$, or

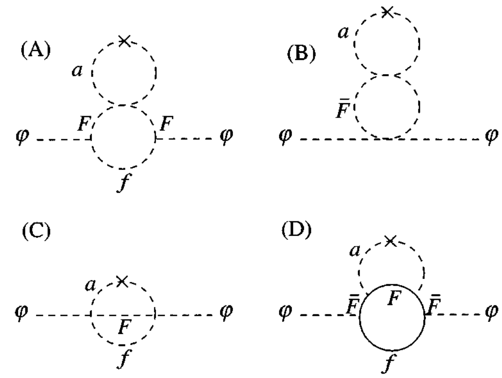

FIG. 2. Two-loop Feynman diagrams that generate the negative squared flavon masses. The cross on the a propagator indicates the supersymmetry-breaking effect given in Eq. (3.19). See text for more details.

$$\langle\varphi\rangle^2=\frac{1}{2\gamma^2}(-m_\varphi^2), \quad \langle\varphi'\rangle^2=0, \tag{3.23}$$

if $\delta<2\gamma$. For the first choice of parameters, φ' acquires a VEV. We will see in Sec. IV A that this may be desirable in some cases to avoid model-dependent pseudo-Nambu-Goldstone bosons that are too light. With the second choice of parameters, φ' does not acquire a VEV, and F_φ remains vanishing. This choice will be useful when we need to suppress the flavor-dependent trilinear soft supersymmetry breaking terms generated when $F_\varphi\neq 0$; these will be discussed in Sec. III G. The exact pattern of φ' VEVs and F components is a highly model-dependent issue. Equations (3.22) and (3.23) allow us to leave our options open; it is quite likely that the relative size of γ and δ differs from one flavon to the other.

Equations (3.22) and (3.23) are the origin of the hierarchical pattern of flavor symmetry breaking in our model. Notice that Eqs. (3.21), (3.22), and (3.23) depend on the product of many different coupling constants, so that the precise ratio $\langle\varphi\rangle/m$ can vary significantly from one flavon to the other, even when the individual couplings are not far from unity. Thus, it is possible that the flavons coupling to leptons may have vacuum expectation values that are systematically smaller than those coupling to quarks, as a consequence of the different renormalization group running of the corresponding superpotential couplings. This may partially alleviate the more severe flavor-changing problem in the lepton sector. The typical parameter range we have in mind is $m_\varphi^2\sim(400\text{ GeV})^2$ in the quark sector and $\sim(100\text{ GeV})^2$ in the lepton sector; with the coupling constants γ,δ varying between 1/3 to unity, we obtain $\langle\varphi\rangle\sim 0.4$–1 TeV and $\langle\varphi\rangle\sim 100$–300 GeV, respectively.

To properly implement the FN mechanism, we must also include the Yukawa couplings of the ordinary Higgs fields H

$$W_H=\zeta\overline{F}Hf+\kappa\overline{F}FH+h_t ffH, \tag{3.24}$$

where ζ and κ are coupling constants. The term proportional to h_t follows from our assumption that the top quark Yukawa coupling is invariant under the flavor symmetries of the theory. There are several ways in which we may extend the

Z_2 symmetry to the H, F, and f fields that are consistent with the interactions in (3.24). The simplest choice is

$\overline{F}$	F	f	φ	φ'	H
$-$	$+$	$+$	$-$	$+$	$+$.

It is interesting to note that the form of our superpotential interactions may be guaranteed by other discrete symmetries. One rather nice possibility is a Z_4 symmetry, where the fields have the charges:

$\overline{F}$	F	f	φ	φ'	H
i	i	i	$-$	$+$	$-$.

In this case, the a VEV breaks this Z_4 down to a Z_2 which is precisely the matter parity (or R parity) needed to forbid dangerous baryon- or lepton-number-violating interactions at the renormalizable level.

We will proceed with our discussion assuming that the form of our superpotential interactions are restricted by the Z_2 symmetry introduced earlier. All that remains is to specify the sector of the theory that is responsible for electroweak symmetry breaking (EWSB). This is considered in the next section.

E. Electroweak symmetry breaking

Perhaps the simplest solution for EWSB is achieved by assuming the couplings

$$\Delta W_1 = \beta_{\varphi'} \varphi'_\mu F F, \tag{3.25}$$

$$\Delta W_2 = \lambda_5 \varphi'_\mu H_u H_d - \frac{\lambda_6}{3} \varphi'^3_\mu . \tag{3.26}$$

The FN field F in ΔW_1 is neutral under standard model gauge interactions. The field φ'_μ is even under the Z_2 symmetry like the φ' fields introduced earlier, though we assume that it has no couplings to the other flavons φ to simplify the discussion. Then, Eq. (3.26) is the usual superpotential of the next-to-minimal supersymmetric standard model, except we assume that φ'_μ and H have nontrivial flavor transformation properties. This must be the case if we are to prevent couplings χHH or ηHH. The Higgs fields H_u and H_d acquire positive squared masses at the scale $\Lambda_{\rm GM}$ from gauge mediation diagrams, so that $m_H^2(\Lambda_{\rm GM}) \approx 350$ GeV. The contribution to the Higgs masses from the flavor sector is negligible given our choice of scales, $\Lambda_{\rm flav} \sim \Lambda_{\rm GM}/10$. If we had chosen them to be comparable, we would have had an additional negative contribution to the $m_H^2 \simeq -(3\ {\rm TeV}^2)^2$ which is too large for correct electroweak symmetry breaking. As the Higgs masses are run to lower energies, $m_{H_u}^2$ becomes negative due to the effect of the top quark Yukawa coupling and the squark squared masses. However, the heaviness of the squarks in models with gauge-mediated SUSY breaking forces $m_{H_u}^2$ to become negative much more rapidly than it does in the MSSM, so that $m_{H_u}^2 \approx -(500\ {\rm GeV})^2$ at the weak scale. If φ'_μ has no soft supersymmetry breaking mass, then the simple extension of the Higgs sector in Eq. (3.26) does not work phenomenologically: there are always scalar and pseudoscalar states that are light enough to be produced in Z decay. This problem led the authors of Refs. [5,23,20] to consider much more complicated Higgs sectors. In our framework, the situation is somewhat better. The coupling of φ'_μ to the flavor sector of the model through Eq. (3.25) leads to a two-loop negative mass squared $m^2_{\varphi'_\mu} \approx -(100\ {\rm GeV})^2$, like the flavon fields φ. The negative mass squared and the $\lambda_6 \varphi'^3_\mu/3$ term force both $\langle \varphi'_\mu \rangle$ and $\langle F_{\varphi'_\mu} \rangle = \lambda_6 \varphi'^2_\mu$ to become nonvanishing, and hence generate μ and m_3^2 parameters. We studied the potential following from Eq. (3.26) numerically, and obtained local minima in which all the physical scalar and pseudoscalar states are sufficiently heavy.[10] However, this required a fine-tuning of the couplings λ_5 and λ_6. Thus, the longstanding problem of generating the μ and m_3^2 parameters naturally in models with gauge-mediated supersymmetry breaking is not immediately resolved in our framework. We considered other EWSB superpotentials, e.g., those allowing φ'_μ to have couplings to the other flavons of the form $\varphi^2 \varphi'_\mu$ but our conclusions remain unchanged. Regardless of the details of the superpotential, we always found that a fine-tuning of parameters was necessary to compensate for the very large negative value of $m_{H_u}^2$.

Of course, it is possible that there is some explicit model of flavor compatible with our framework, that provides additional contributions to μ, m_H^2, and m_3^2. For example, in some model there may be FN fields with even matter parity that mix with Higgs fields. Such a mixing induces a positive mass squared to the Higgs bosons. Or, there may be higher dimension operators of the form $a(\langle \varphi \rangle / M_F) H_u H_d$, which generates μ and m_3^2 at the desired orders of magnitude. Since this is a model-dependent question, we will not pursue this issue further here.

F. *R* axions

The absence of dimensionful parameters in the theory we have presented leads to an effective global R-symmetry under which all fields transform with charge 2/3. Since our superpotential is partitioned into "sectors," which are relatively isolated from each other, one may worry that there are separate and potentially dangerous R axions associated with each. In this section, we show that all the model-independent R axions that are present in our framework are phenomenologically harmless. There could be additional light scalar bosons that arise as a consequence of accidental global symmetries in specific flavor models; we discuss how these may be avoided in Sec. IV A.

There are four approximate R symmetries in our framework, corresponding to each of the nearly decoupled sectors in which spontaneous symmetry breaking occurs: the supersymmetry breaking sector at the scale $\Lambda_{\rm SSB}$, the gauge mediation sector at $\Lambda_{\rm GM}$, the flavor sector at $\Lambda_{\rm flav}$, and the EWSB-FSB sector at a few hundred GeV. The lines in Fig. 1 that connect the different sectors explicitly break the inde-

[10]A soft trilinear coupling $A\varphi'_\mu H_u H_d$ is generated at one loop, which pushes the lightest pseudoscalar mass above 10 GeV. This is sufficient to evade the bounds from astrophysics and cosmology, quarkonium decay, and beam dump experiments.

pendent $U(1)_R$ symmetries, leaving one unbroken linear combination acting on all the sectors. This corresponds to the non-anomalous R-symmetry in the supersymmetry breaking sector, with all fields in the other sectors transforming with charge 2/3. The remaining three linear combinations are explicitly broken by the messenger U(1) gauge interactions, the indirect coupling between the a field and the flavons through loops of FN fields in the flavor sector, and the indirect coupling between the Higgs fields and S through loops of the X field, in the gauge mediation sector. Once the $U(1)_R$ is spontaneously broken in the supersymmetry breaking sector, the low-energy effective theory beneath $\Lambda_{\rm SSB}$ contains explicit $U(1)_R$ breaking parameters. Below we estimate the masses for the one true R-axion and the three ''would-be'' R-axions separately.

The only true R-axion is the first linear combination described above. Since its decay constant is high, $F\sim 10^7$ GeV, all direct search experiments are irrelevant. The only potential problem is its possible contribution to the cooling of red giant stars. However, we expect this R axion to obtain a mass of order 100 MeV via the same mechanism which cancels the cosmological constant [24], and thus it is astrophysically harmless. The cosmological implications of this R axion are less clear. Since this issue is not specific to our framework, we will not consider it further.

The would-be R axion in the gauge mediation sector obtains a mass of order 10 TeV in the following manner. Since global R symmetry is spontaneously broken in the dynamical supersymmetry breaking sector, the messenger U(1) gaugino acquires a Majorana mass at the one-loop level. We explicitly checked this point in the case of the SU(6)×U(1) model. Then a trilinear coupling $-Ah'\xi'_+\xi'_-S'$ is generated at the two-loop order with $A\sim\Lambda_{\rm SSB}/(16\pi)^2\sim\Lambda_{\rm GM}/(16\pi)$. This coupling explicitly breaks the global R symmetry in the gauge mediation sector, and the R axion aquires a mass of order $m^2_{A^0}\sim\langle Ah'\xi'_+\xi'_-\rangle/\Lambda^2_{\rm GM}\sim(10\ {\rm TeV})^2$ according to Dashen's formula. It is completely harmless given this large mass.

Similarly, the would-be R axion in the flavor sector obtains a mass via the analogous trilinear coupling $-Ah\xi_+\xi_-\chi$. While its mass is much smaller, $\sim$100 GeV, it is still heavy enough to avoid all existing phenomenological constraints. Recall χ does not acquire a VEV in the absence of the trilinear coupling, and hence $\langle\chi\rangle\sim A\sim\Lambda_{\rm flav}/16\pi^2$ as discussed in Sec. III D. Therefore the operator which explicitly breaks R symmetry is suppressed. Dashen's formula gives $m^2_{A^0}\sim\langle A\xi_+\xi_-\chi\rangle/\Lambda^2_{\rm flav}\sim(100\ {\rm GeV})^2$.

Finally, there is a separate would-be R axion in the flavon superpotential that gains a mass of order 10 GeV as a consequence of the soft trilinear flavon interactions. Trilinear couplings of the form $A\varphi^2\varphi'$ are generated at order $A\sim 1$ GeV through one-loop diagrams involving the FN fields and an insertion of their supersymmetry breaking bilinear mass term $\propto\langle F_a\rangle$. As long as (at least) one of φ' obtains a VEV comparable to φ, the R axion obtains a mass of order $m^2_{A^0}\sim\langle A\varphi^2\varphi'\rangle/\langle\varphi\rangle^2\sim(10\ {\rm GeV})^2$. In Sec. III D we showed that the relative size of the two couplings γ and δ determines whether or not φ' acquires a VEV. If $\langle\varphi'\rangle=0$ at lowest order for all φ', then $\langle\varphi'\rangle$ is induced only through trilinear couplings, and the R axion mass goes down to the 1 GeV level. In this case, the quarkonium decay $\Upsilon\to A^0\gamma$ excludes the model. On the other hand, there are no constraints from flavor-changing processes like $K^0\to$ virtual $A^0\to\overline{K^0}$ because this R axion couples to the overall R charge 2/3 of each ordinary matter fields and its coupling is therefore flavor blind. Note also that the coupling of A^0 is axial and hence proportional to the fermion masses; this makes it impossible to find A^0 as an s-channel resonance at e^+e^- experiments. The beam dump experiments do not constrain axionlike fields above the GeV range. Known astrophysical sources do not produce particles above the GeV range either. Cosmology is also not likely to constrain such a particle because it decays relatively quickly as $A^0\to b\bar{b}$ with a width much larger than a keV. We are not aware of any experimental constraints which exclude the existence of a scalar boson in the 10 GeV range which couples universally to all fermion axial currents.

G. The flavor changing problem

Now that we have outlined the important features of our model, we return to the issue of flavor changing neutral currents, and how they constrain our choice of scales. In our framework, the ordinary squarks and sleptons receive four contributions to their squared masses:

(i) A positive, flavor-blind contribution from gauge-mediation diagrams, of order $\Lambda^2_{\rm GM}/(16\pi^2)^2$.

(ii) A negative, flavor-symmetric contribution from two-loop diagrams like those in Fig. 2, except with the φ and f lines interchanged, of order $-\Lambda^2_{\rm flav}/(16\pi^2)^2$. The term ''flavor-symmetric'' refers to operators which respect the flavor symmetry of the model, without necessarily being flavor blind. This can be the case, for instance, if the flavor symmetry is Abelian.

(iii) A positive, flavor-dependent contribution due to the supersymmetry breaking scalar masses of the FN fields. Note that the one-loop subdiagram in Fig. 2 will give the scalar F and $\bar{F}$ fields supersymmetry breaking squared masses, of order $\Lambda^2_{\rm flav}/(16\pi^2)$. This alters the (1,1) and (2,2) entries of the scalar mass matrix in Eq. (3.3), leading to a flavor-dependent shift in the lightest eigenvalue of order $+\Lambda^2_{\rm flav}/(16\pi^2)(\langle\varphi\rangle/M_F)^2$.

(iv) A negative, flavor-dependent contribution as in (3.7), due to the small nonvanishing F component of the field a. With $\langle F_a\rangle\approx\langle a\rangle^2/(16\pi^2)$, this effect is of order $-\Lambda^2_{\rm flav}/(16\pi^2)^2(\langle\varphi\rangle/M_F)^2$.

The first contribution was estimated in Sec. III C, assuming $\Lambda_{\rm GM}=\langle F_S\rangle/\langle S\rangle=100$ TeV. With $\Lambda_{\rm flav}=10$ TeV, we may estimate the remaining contributions:

$$\text{(ii)}\quad -(100\ \ {\rm GeV})^2,$$

$$\text{(iii)}\quad +(1000\ \ {\rm GeV})^2(\langle\varphi\rangle/M_F)^2,$$

$$\text{(iv)}\quad -(100\ \ {\rm GeV})^2(\langle\varphi\rangle/M_F)^2.$$

Notice that contribution (ii) is much smaller than contribution (i), given the choice $\Lambda_{\rm GM}=10\Lambda_{\rm flav}$. Thus, the flavor-blind component of the squark and slepton masses is exactly what we would expect in the kind of scenario proposed by

Dine and Nelson. If the flavor symmetry does not guarantee degeneracy of the squarks (or sleptons) of the first two generations, then the flavor-symmetric contributions (ii) can lead to flavor changing neutral current effects. In the quark sector, the constraints on K-$\bar{K}$ mixing are satisfied rather easily for 1 TeV squarks, so there is no restriction on the flavor structure of the model. However, the lightness of the sleptons makes the situation in the lepton sector more dangerous; the flavor-symmetric contributions (ii), may favor some flavor models over others. Note that in models with a non-Abelian flavor symmetry in which the first two generations transform as a doublet, there will be no constraint on the contribution (ii). The flavor-dependent contribution (iii) dominates over (iv), since the latter is suppressed by the the smallness of $\langle F_a \rangle$ in the FN sector of our model. With $\Lambda_{\rm flav}=10$ TeV, (iii) is marginally consistent with the bounds from flavor changing processes, assuming that the $\langle \varphi \rangle / \Lambda_{\rm flav}$ are of order 10^{-1} in the quark sector, and a few $\times 10^{-2}$ in the lepton sector. Since (iii) scales as $\Lambda^2_{\rm flav}$, we would not be able to construct a viable model had we chosen $\Lambda_{\rm flav}$ to be much larger than 10 TeV. On the other hand, we will see in Sec. IV that the exchange of the relatively light flavon fields are also marginally consistent with the bounds on FCNC processes, for $\Lambda_{\rm flav}\approx 10$ TeV. Thus, lowering $\Lambda_{\rm flav}$ significantly is also phenomenologically unacceptable. Our choice for $\Lambda_{\rm flav}$ is a reasonable compromise, given the constraints on flavor changing processes detailed in Sec. IV.

The constraints on the left-right mass matrices, on the other hand, are very weak in our framework. The left-right masses originate from the effective Yukawa couplings,

$$W_{\rm eff}=\left(\frac{\varphi}{\Lambda_{\rm flav}}\right)^n QdH_d + \text{ up-quark, leptons,} \tag{3.27}$$

when one φ field is set to its F component, while the remaining φ fields and the Higgs field H are all set to their VEVs. Thus, the left-right mass terms are of the order

$$m^2_{\rm LR}\simeq m_f\left(\frac{\langle F_\varphi \rangle}{\langle \varphi \rangle}\right), \tag{3.28}$$

where m_f stands for the mass of a light quark or lepton. Therefore, the left-right mass terms are always proportional to the corresponding fermion masses, which is not necessarily true in the case of supergravity. In the quark sector, all squarks are at 1 TeV while the effective A parameters are about 400 GeV or less. This is phenomenologically safe by itself. In the lepton sector, Eq. (3.28) may lead to a large mixing between smuons and selectrons, and hence an unacceptably large $\mu \to e\gamma$ decay rate. Fortunately, this can be avoided in a number of ways. For instance, if $\delta < 2\gamma$ (see discussions in Sec. III D), φ' does not acquire a VEV and hence F_φ vanishes identically. A trilinear coupling among flavons induce $\langle \varphi' \rangle$ only at a higher order in $1/16\pi^2$. In this case there is no further restriction on the flavor model. On the other hand, an alignment or non-Abelian flavor symmetry can suppress the off-digonal entry in the left-right mass matrix in the basis where fermion masses are diagonal.

Finally, one may worry that operators involving the F component of the a field may contribute to left-right mass mixing at a more dangerous level than the operators involving F_φ. The effective Yukawa operators of the form

$$W_{\rm eff}=\left(\frac{\varphi}{M_F}\right)^n ffH \tag{3.29}$$

generate trilinear couplings in the following way. Since $M_F=\alpha\langle a \rangle$, they should be written more correctly as

$$W_{\rm eff}=\left(\frac{\varphi}{\alpha a}\right)^n ffH, \tag{3.30}$$

and an expansion of the chiral superfield a around its VEV as $a=a+\theta^2 F_a$ generates trilinear couplings

$$V_{\rm eff}=\left(\frac{\varphi}{\alpha\langle a \rangle}\right)^n n\left(\frac{\langle F_a \rangle}{\langle a \rangle}\right)\tilde{f}\tilde{f}H. \tag{3.31}$$

Recall $\langle F_a \rangle / \langle a \rangle \sim \langle a \rangle / 16\pi^2 \sim 100$ GeV. If the power n is different for different generations, the left-right mass terms generated from this operator cannot be simultaneously diagonalized with the fermion masses. This is not a problem in the quark sector, but could be serious in the lepton sector. One way to avoid this problem is to have an alignment. Another way is to obtain the Yukawa couplings of the first two generations in any given Yukawa matrix from a set of operators in the high-energy theory that all involve the same powers of $\langle \varphi \rangle / M_F$; then these contributions to $m^2_{\rm LR}$ will be diagonal in the fermion mass basis, and will not give additional constraints.

IV. PHENOMENOLOGY OF A LOW FLAVOR SCALE

In the previous section, we found that the flavor-blind, gauge-mediated contribution to the scalar masses dominates over any flavor-dependent splittings induced by the mixing of the light families with the FN fields. Thus, the super-Glashow-Iliopoulos-Maiani (GIM) mechanism is effective and the usual supersymmetric flavor problem (i.e., the large FCNCs effects generated by the exchange of sfermions in loops) is greatly reduced. As a consequence, the usual constraints placed on the flavor structure of the model are significantly weakened in our framework. This is in sharp contrast to the case of gravity-mediated supersymmetry breaking, where the flavor symmetry must either guarantee a high degree of sfermion degeneracy or an alignment between fermion and sfermion mass matrices to avoid the SUSY flavor problem.

However, with the FN scale at ~ 10 TeV and flavon VEVs and masses in the few hundred GeV range, we must consider new contributions to FCNCs originating from the exchange of the physical states of the flavor sector. In this section, we describe the phenomenological constraints that FCNC processes impose on theories with low flavor scales. Most of the new flavor-violating effects are described by four-fermion operators, with coefficients C/M^2. In Table I, we present bounds on the coefficients C assuming $M=M_F=10$ TeV, for a number of four-fermion operators that contribute to rare processes. We then estimate the coefficients C for the flavor-violating operators that may arise in our framework, and determine in what ways the bounds in Table I constrain the flavor structure of the model.

TABLE I. Constraints on the four fermion operators $(C/M^2)O$ from various rare processes, with $M=$ 10 TeV. A,B are color indicies. Other relevant operators can be obtained from the above by charge conjugation. This table then exhausts all possible four fermion operators contributing to these flavor changing processes. L and R are chiral projection operators for left-handed and right-handed fields, respectively. We have used $(m_s+m_d)=$ 160 MeV. The bounds on Δm_i scale as $(160\ \text{MeV})^2/f_i^2 B_i$, where $i=K,D,B$ and f_i, B_i are the relevant decay and bag constants respectively.

Process	O	$C<$
	$(\overline{d^A}\gamma^\mu L s_A)(\overline{d^B}\gamma_\mu L s_B)$	4×10^{-5}
	$(\overline{d^A}L s_A)(\overline{d^B}L s_B)$	6×10^{-6}
Δm_K	$(\overline{d^A}L s_B)(\overline{d^B}L s_A)$	3×10^{-5}
	$(\overline{d^A}L s_A)(\overline{d^B}R s_B)$	5×10^{-6}
	$(\overline{d^A}L s_B)(\overline{d^B}R s_A)$	2×10^{-5}
	analogous to above	4×10^{-4}
	‘‘	6×10^{-4}
$\Delta m_D, \Delta m_B$	’’	3×10^{-3}
	‘‘	5×10^{-4}
	’’	4×10^{-3}
	$(\overline{\mu}\gamma^\mu L e)(\overline{e}\gamma_\mu L e)$	2×10^{-3}
$\mu\to 3e$	$(\overline{e}L\mu)(\overline{e}L e)$	9×10^{-3}
	$(\overline{e}L\mu)(\overline{e}R e)$	7×10^{-3}
$K_L\to\mu^+\mu^-$	$(\overline{d}\gamma^\mu L s)(\overline{\mu}\gamma_\mu L\mu)$	4×10^{-2}
	$(\overline{d}L s)(\overline{\mu}L\mu),(\overline{d}L s)(\overline{\mu}R\mu)$	4×10^{-3}
$K_L\to\mu e$	$(\overline{d}\gamma^\mu L s)(\overline{\mu}\gamma_\mu L e)$	4×10^{-3}
	$(\overline{d}L s)(\overline{\mu}L e),(\overline{d}L s)(\overline{\mu}R e)$	3×10^{-4}
$K_L\to e^+e^-$	$(\overline{d}\gamma^\mu L s)(\overline{e}\gamma_\mu L e)$	6×10^{-1}
	$(\overline{d}L s)(\overline{e}L e),(\overline{d}L s)(\overline{e}R e)$	3×10^{-4}

There are four logical possibilities as flavor groups: combinations of local or global, and discrete or continuous. In all cases, flavor-violating operators may be generated at two distinct scales: at $\sim$10 TeV, where the FN fields F and $\overline{F}$ are integrated out, and at a few hundred GeV where the flavon fields φ are integrated out. If there exist yet lighter degrees of freedom such as pseudo Nambu-Goldstone bosons, they may induce further flavor-changing operators. We will discuss general constraints which apply to all cases in Sec. IV A. By itself, this discussion will present all the constraints relevant to discrete flavor symmetries, except those relating to a possible domain wall problem; we do not have anything to add on this point.[11] On the other hand, a broken continous flavor symmetry produces additional degrees of freedom (flavor gauge bosons in the local case and ‘‘familons’’ in the global case) which induce new flavor-changing phenomena.

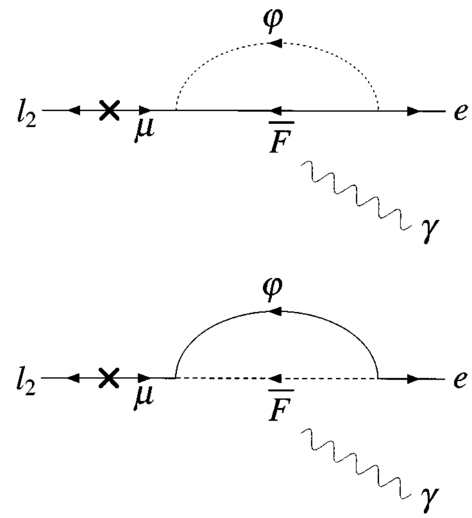

FIG. 3. Feynman diagrams contributing to $\mu\to e\gamma$.

These two cases are discussed separately in Secs. IV B and IV C. Most of the constraints are new as far as we know, though some were briefly discussed in [6].

A. General constraints from FCNC

The existence of FN fields at a 10 TeV scale and flavons at a few 100 GeV scale can induce new flavor-changing processes. The phenomenological constraints are identical for discrete or continuous flavor symmetries, except those induced by flavor gauge bosons or familons. They will be discussed separately in the next subsections. We discuss constraints common to all possible flavor groups in this subsection.

1. Flavor-violating operators induced at FN scale

The $\overline{F}\varphi f$ couplings lead to diagrams with f fields on the external lines, and the FN and flavon fields in loops. Flavor-violating operators, like those listed in Table I, are induced in the low-energy theory when the FN and φ fields are integrated out. All superpotential couplings which we generically refer to as h are of order 1, so these operators are only suppressed by loop factors and the mass of the FN fields $M_F \simeq 10$ TeV. In our numerical estimates below, we set all the $\overline{F}\varphi f$ Yukawa couplings to 1, and assume that the multiplicity of the particles running around internal loops is also 1. We consider flavor violating processes in the lepton, quark and mixed lepton-quark sectors separately.

(i) Lepton sector

$\mu\to e\gamma$: The amplitude for $\mu\to e\gamma$ comes from the diagrams of Fig. 3. Crucially, this amplitude, like all magnetic transitions, vanishes in the supersymmetric limit. For $M_F=$ 10 TeV, we find

$$B(\mu\to e\gamma)\sim 10^{-10}h^4[f(m_{\varphi_R}^2/M_F^2)+f(m_{\varphi_I}^2/M_F^2) -2f(m_{\varphi_f}^2/M_s^2)]^2, \qquad (4.1)$$

where the $m_{\varphi_{R,I}}$ are the masses of the real and imaginary components of the scalar part of φ, m_{φ_f} is the mass of the

[11]It may be worth recalling that a global discrete symmetry could be anomalous. Then the domain walls dissolve due to instanton effects and do not cause cosmological embarassments [25]. On the other hand, a global discrete symmetry is probably spoiled by quantum gravitational effects. Still, it could well arise as an accidental low-energy symmetry especially when one considers a low flavor scale as in our framework. Nonrenormalizable operators suppressed by $1/M_{\rm Pl}$ may also solve the domain wall problem even if the discrete symmetry is anomaly free [5].

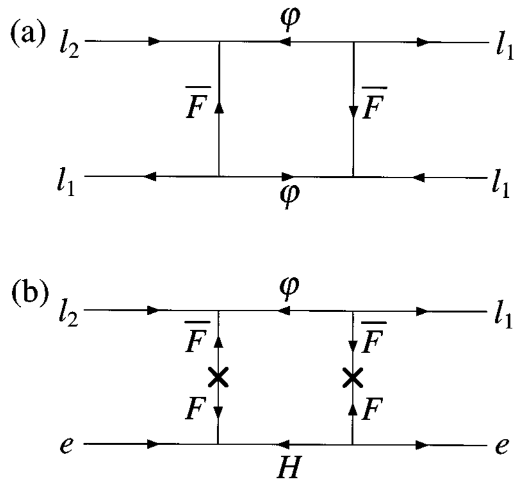

FIG. 4. Superdiagrams contributing to $\mu \rightarrow eee$.

fermionic part of φ, M_F is the mass of the FN fermion, M_s the mass of the FN scalar, and

$$f(x)=\frac{2+3x-6x^2+x^3+6x\ln(x)}{12(x-1)^4}. \tag{4.2}$$

Since $m_\varphi / M_F \sim 10^{-2}$, the quantity in square brackets in Eq. (4.1) is highly suppressed; we find that it is always numerically much smaller than ~ 0.001. Thus, $B(\mu \rightarrow e\gamma) \lesssim 10^{-16}$, well beneath the current bound $B(\mu \rightarrow e\gamma) < 5\times 10^{-11}$. We conclude that the $\mu \rightarrow e\gamma$ operator is not dangerous, even if it is allowed in the flavor-symmetric limit.

$\mu \rightarrow 3e$: If allowed by the flavor symmetry, box diagrams with internal FN and flavon fields can generate four-fermion operators that contribute to $\mu \rightarrow 3e$. Even if these operators are forbidden in the flavor symmetric limit, they may be generated after we rotate a flavor-symmetric operator to the mass eigenstate basis. The diagram of Fig. 4(a) (the flavor symmetric version of which necessarily exist) generates the interaction

$$\frac{h^4}{32\pi^2 M^2}(\overline{\mu}\gamma^\mu L e)(\overline{e}\gamma_\mu L e). \tag{4.3}$$

Even if this operator is allowed in the flavor symmetric limit, the coefficient $C \sim 3\times 10^{-3}$ is in borderline agreement with the constraint given in Table I.

The diagram of Fig. 4(b), on the other hand, does not necessarily exist. It can only be generated if there is a direct Yukawa coupling between the Higgs, FN, and lepton fields of the first two generations. This box diagram diverges in the infrared, and is cut off by the mass m_φ. From this diagram we generate the operator

$$\frac{h^4}{8\pi^2 M^2}[1-\ln(M/m_\varphi)](\overline{e}L\mu)(\overline{e}Re). \tag{4.4}$$

With $m_\varphi / M_F \sim 10^{-2}$, the coefficient of this operator is $C \sim 4\times 10^{-2}$, which is bigger than the bound in Table I by a factor of ~ 7. This is not necessarily a disaster, but it is certainly safer to forbid the diagram 4(b) in the flavor symmetric limit. If the above operator is generated through mixing with an angle $\sim \sqrt{m_e/m_\mu} \sim 0.1$ it is no longer dangerous phenomenologically.

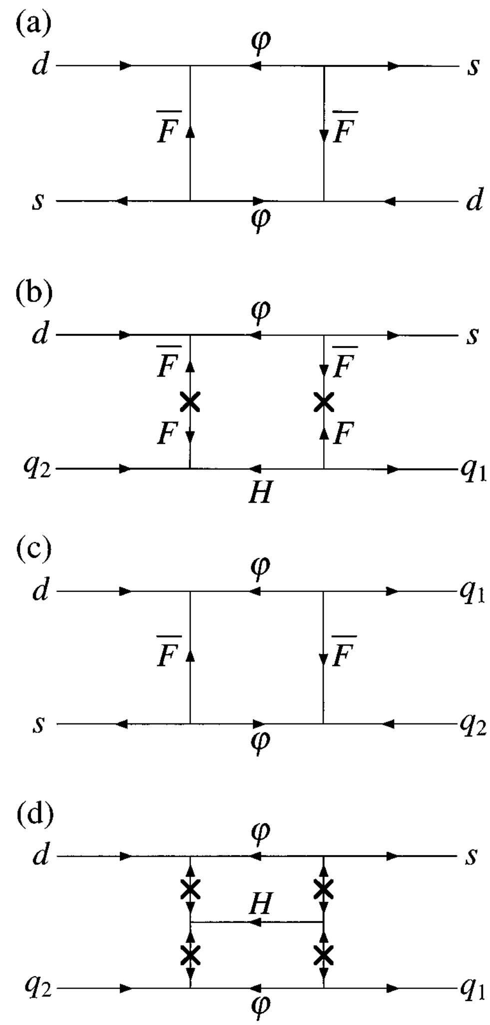

FIG. 5. Superdiagrams contributing to neutral meson mixing.

(ii) Quark sector

$b \rightarrow s\gamma$: The rate is completely negligible, for the same reasons given in our discussion of $\mu \rightarrow e\gamma$.

Δm_K: The diagram of Fig. 5(a) generates the interaction

$$\frac{h^4}{32\pi^2 M^2}(\overline{d^A}\gamma^\mu L s_A)(\overline{d^B}\gamma_\mu L s_B). \tag{4.5}$$

The coefficient $C \sim 3\times 10^{-3}$ is ~ 75 times bigger than the bound in Table I. Thus, this diagram must be forbidden in the flavor symmetric limit. With a mixing suppression of $\sim \lambda^2$, the coefficient is only 3 times bigger than the bound. However, it is easy to compensate for this factor by choosing all the Yukawa couplings in the diagram to be 1/2 instead of 1.

The diagram in Fig. 5(b) only exists if there are flavons coupling to both left- and right-handed superfields. If such couplings exist, we generate the operator

$$\frac{h^4}{16\pi^2 M^2}(\overline{d^A}L s_B)(\overline{d^B}L s_A), \tag{4.6}$$

after appropriate Fierz transformations. The coefficient $C\sim 6\times 10^{-3}$ is ~ 200 times bigger than the bound in Table I, and we conclude that this diagram must be forbidden in the flavor symmetric limit. If we include a mixing suppression $\sim\lambda^2$, the coefficient is still too large by a factor of 10. In this case, however, we can reduce the amplitude sufficiently by choosing all the Yukawa couplings in the diagram to be $h\sim 1/2$ instead of 1.

The diagram in Fig. 5(c) is by far the most dangerous one we will encounter. It only exists if there is a direct Yukawa coupling between down or strange, FN and Higgs fields. As in the case of $\mu\to 3e$, this box diagram is enhanced by an infrared divergence. We obtain the operator

$$\frac{h^4}{8\pi^2 M^2}[1-\ln(M_F/m_\varphi)](\overline{d^A}Ls_A)(\overline{d^B}Rs_B). \tag{4.7}$$

If we take $m_\varphi/M_F\sim\lambda^2$, the coefficient $C\sim 2\times 10^{-2}$ is ~ 3000 times larger than the corresponding bound in Table I; even with λ^2 suppression, it is still ~ 100 times too big. Notice that we cannot reduce the magnitude of this operator by choosing smaller couplings in the box diagram. The product of the two couplings on the left-hand side of the diagram with $\langle\varphi\rangle/M_F$ gives us an element of a quark Yukawa matrix; thus, we can only reduce the couplings in the box diagram if we increase $\langle\varphi\rangle/M_F$. Note also that this diagram is particularly worrisome in a theory with large $\tan\beta$. Recall that the negative squared masses for the flavon fields were naturally of order $[1/(16\pi^2)^2]M_F^2$, and thus $\langle\varphi\rangle/M_F$ fell in the range $\lambda^2-\lambda^3$. However, for large $\tan\beta$, the strange Yukawa coupling is itself of order $\lambda^2-\lambda^3$, and we therefore require a direct coupling between the strange quark, Higgs boson, and a FN field. This is precisely the situation that gives us the disastrous contribution to Δm_K in Eq. (4.7).

Finally, even when there is no direct Yukawa coupling between down or strange quark, Higgs boson, and FN fields, we can have a two-loop contribution to K-$\bar{K}$ mixing as shown in Fig. 5(d). We generate exactly the same effective operator as in Eq. (4.7), and we find for $m_\varphi\sim\lambda^2 M_F$ a coefficient $C\sim 3\times 10^{-5}$, which is 5 times bigger than the bound. This can easily be avoided either by choosing slightly smaller couplings in the diagram, or by forbidding the operator in the flavor symmetric limit.

It is important to point out that our conclusions remain unchanged if we include CP violating phases of order 1 in the theory. In this case, the constraint from ϵ_K is ~ 10 times stronger than the one from Δm_K that we have presented here. However, we have already concluded from the FCNC considerations that the corresponding $\Delta S=2$ operators must be forbidden in the flavor symmetry limit. In a model where this is the case, CP violation does not provide us with any additional generic constraints.

$\Delta m_D, \Delta m_B$: The only significant constraint from either of these processes comes from the analogue of the dangerous diagram 5(c) above. We conclude that this diagram should be forbidden in the flavor symmetric limit; if it is induced through mixing from a flavor symmetric operator, then its effects are on the borderline from experimental constraints.

(iii) Mixed quark-lepton sector

If some flavons couple to both the quark and lepton sectors, they can give a significant contribution to $K_L\to\ell^+\ell^-$ via box diagrams analogous to those in Figs. 5(a)–5(d). We have already learned that we must forbid direct couplings between the down or strange quark, Higgs boson, and FN fields, so we do not consider the analogue of Fig. 5(c): this diagram would give too large an amplitude for $K\to\mu e, ee$ by a factor ~ 200 if it were generated in the flavor symmetric limit. All the remaining diagrams are on the borderline even if they are present in the flavor symmetric limit.

TABLE II. Constraints on Θ_{ij} from exchange of physical flavons. The mass and VEV of the flavons coupling to quarks alone are taken to be at 400 GeV, all others at 100 GeV.

Process	Constraint
$\mu\to 3e$	$\Theta_{\mu e}<130$
Δm_K	$\Theta_{ds}<0.2$
Δm_D	$\Theta_{uc}<0.3$
Δm_B	$\Theta_{bd}<0.09$
$K_L\to\mu^+\mu^-$	$\Theta_{ds}<0.2$

2. Flavor violating operators induced at the flavor symmetry breaking scale

These operators arise from tree-level exchange of physical flavons. Recall that we generate higher dimension operators of the form

$$\left(\frac{\varphi}{M_F}\right)^n f_i f_j^c H \tag{4.8}$$

after integrating out the FN fields. Here the f_i are ordinary fields of the first two generations. When the φ fields acquire VEVs, the operator in (4.8) gives us an element of the corresponding Yukawa coupling matrix. If we now set all but one of the flavon fields and the Higgs field to their VEVs, we generate a Yukawa coupling between the light fermions and flavons:

$$\left(\frac{\langle\varphi\rangle}{M_F}\right)^{n-1}\frac{\langle H\rangle}{M}\varphi f_i f_j^c\sim\frac{m_{ij}}{\langle\varphi\rangle}f_i f_j^c\varphi. \tag{4.9}$$

We do not show an exact equality in Eq. (4.9) since, in general, the mass matrix element m_{ij} receives contributions from several different operators of the form (4.8), with different order 1 coefficients. Of course, it may be the case that in specific models, the flavon couplings will become flavor diagonal when we rotate to the fermion mass basis. Generically, however, this will not be the case, and we will obtain flavor-violating four-fermion operators when we integrate out the physical flavons at tree level. Let us write $m_{ij}=\max(m_i,m_j)\Theta_{ij}$. Then, fixing the mass of the flavons coupling only to the quark sector at $\sim\lambda^2\times 10$ TeV ~ 400 GeV and the mass of the flavons coupling to the leptons or leptons and quarks at ~ 100 GeV,[12] we present the strongest constraints on the magnitudes of the Θ_{ij} in Table II. In all cases, the Θ_{ij} can be as large or larger than the corresponding CKM matrix element, and thus physical flavon exchange does not give us significant constraints.

[12]See Section III D for these estimates.

3. Pseudo Nambu-Goldstone bosons

The above analysis assumes that all the physical flavons get masses of order the VEV of the flavon field. However, given many flavons and the restriction to renormalizable superpotentials, it is often the case that there are approximate, accidental continuous symmetries of the tree-level potential, producing pseudo-Nambu-Goldstone bosons (PNGB) which pick up a mass only at loop level $\sim\langle\varphi\rangle/4\pi$ which may be as small as 10 GeV. These PNGBs can have flavor-violating couplings, and (especially for the PNGBs coupling to the first two generations) mediate disastrously large FCNC. Thus, a specific model must ensure that most of the PNGBs receive a mass directly at tree level. This puts a constraint on the flavor symmetry and flavon particle content. The absence of accidental global symmetries must be checked in each explicit model of flavor.

Even when there is no accidental global symmetry which results in PNGB with flavor-dependent coupling, there is always one model-independent PNGB. Since the superpotential is purely trilinear, it necessarily has a tree level R symmetry under which all fields have R charge 2/3 and the negative squared masses for the flavon fields do not break this R symmetry. We discussed this particular ''model-independent R axion'' in Sec. III F and showed it does not have flavor-changing interaction and is phenomenologically harmless as long as it is heavier than 10 GeV.

4. Summary of general constraints

Let us summarize the major constraints that emerged from our analysis. We have found that the flavor symmetry must forbid all K-$\bar{K}$ mixing operators in the flavor symmetric limit. Furthermore, there can be no direct Yukawa coupling of the FfH type between down or strange quark, Higgs boson and FN fields; this in particular causes great difficulty for a scenario with large $\tan\beta$. Once these constraints are satisfied, CP violation does not put any further significant constraints even with order 1 phases. Notice that the constraints on the flavor group we have found are quite different from the usual ones needed to guarantee sfermion degeneracy. For instance, an SU(2) flavor symmetry with the first two generation fields in a doublet is sufficient to guarantee sfermion degeneracy in the flavor symmetric limit; however the dangerous operator $(\epsilon^{jk}\bar{d}_i R q_k)(\epsilon^{kl}\overline{q_k} R d_l)$ [with i,j,k,l flavor SU(2) indices] gives large K-$\bar{K}$ mixing while being completely flavor symmetric. Similarly, U(1)'s (or discrete subgroups) can be used to forbid all K-$\bar{K}$ mixing operators in the flavor symmetric limit but cannot in themselves guarantee sfermion degeneracy. Also, a specific model must sufficiently break any accidental global symmetries which give rise to light PNGBs. These new constraints differ from the ones we usually encounter in supergravity scenarios, and suggest new avenues for flavor model building.

B. Continuous, global flavor symmetries

In addition to the constraints presented in the previous subsection, continuous flavor symmetries lead to other, often problematic, contributions to flavor changing processes. In the case of global flavor symmetries, we must contend with the Nambu-Goldstone bosons (''familons'') that arise when the flavor group is spontaneously broken. If some NG bosons have flavor-violating couplings (which is certainly the case if the group has a non-Abelian component), FCNC constraints, from processes like $K\to\pi+$ familon and $\mu\to e+$ familon, push the flavor symmetry breaking scale above $\sim 10^{11}$ GeV. However, if the flavor group has only U(1) factors, the familons may have purely diagonal couplings in the flavor basis. If the first two generation fields have different charges under some U(1) factor, the alignment between flavor and mass eigenstate bases must be very precise (~ 1 TeV/10^{11} GeV$=10^{-8}$) to avoid FCNC constraints, which implies that a mechanism for perfect alignment is required. For instance, we can imagine that all left-handed fields in the theory have the same charge, whereas the right-handed fields have different charges. Since the only absolute requirement we have is for the existence of nontrivial rotations on the left-handed fields to generate the CKM matrix, the flavor and mass bases may be exactly aligned for the right-handed fields. Then, the rotation in going to the mass eigenstate basis for the left-handed fields does not induce any off-diagonal familon coupling (since the left handed charges are generation blind), while there is no rotation on, and hence no off-diagonal familon coupling to the right handed fields. While this sort of idea is not excluded, it is clear that continuous global flavor groups are strongly constrained by the requirement of purely diagonal familon couplings.

C. Continuous, local flavor symmetries

In the case of a gauged flavor symmetry, a new contribution to flavor violating processes comes from the exchange of massive flavor gauge bosons at the ~ 100 GeV scale. This source of FCNCs will place significant restrictions on the form of the flavor group and symmetry breaking sector, as we will see below. In addition, when a gauged flavor symmetry is spontaneously broken, degeneracy between the first and second generation sfermions may be spoiled by flavor-dependent D terms in the scalar potential [26]. We consider both issues below.

1. Flavor gauge boson exchange

Let us examine the effects of gauge boson exchange in the down quark sector. Suppose that the left-handed Weyl spinor quarks q_L (grouped into a three-vector in generation space) transform under a representation T_L^a of the flavor group while the right-handed down quarks d_R transform under $T_{d_R}^a$. In the flavor basis (denoted by primes), the flavor current is given by

$$J^a{}_\mu = \overline{d'_L}\overline{\gamma_\mu}T_L^a d'_L + \overline{d'_R}\overline{\gamma_\mu}T_{d_R}^a d'_R. \tag{4.10}$$

After integrating out the massive flavor gauge bosons, we generate the following four-fermion operators:

$$\sum_a \frac{g^2}{M_a^2} J^{a\mu}J^a_\mu = \sum_a \frac{g^2}{M_a^2}(\overline{d'_L}\overline{\gamma_\mu}T'^a_L d'_L + \overline{d'_R}\overline{\gamma_\mu}T'^a_{d_R}d'_R)^2. \tag{4.11}$$

In the second expression we have rotated to the mass basis $q=U_q q'$, $d^c=U_{d^c}d^{c\prime}$, and the $T^{a\prime}$ are the flavor group generators in the mass basis, $T^{a\prime}=UT^aU^\dagger$. Suppose that the pattern of flavor symmetry breaking is such that the M_a are different from each other. If the flavor group is Abelian and there is precise alignment between flavor and mass eigenstates, all the operators above are flavor diagonal. However, if there is no alignment, the M_a must be pushed above $\sim$1000 TeV to avoid FCNC constraints (in particular, from Δm_K). Thus, for an Abelian flavor symmetry broken below the TeV scale, the alignment between mass and flavor bases must be precise better than $\sim$1 TeV/1000 TeV$=10^{-3}$. For non-Abelian flavor groups, a given $T^{a\prime}$ will have off-diagonal flavor-violating elements. When the M_a are all different, there is no hope that summing over a will yield a flavor conserving result, once again forcing the M_a to above 1000 TeV. Thus, if the flavor group is non-Abelian and broken at the TeV scale, some mechanism must guarantee that the flavor gauge bosons (especially coupling to the first two generations) have identical mass at least at tree level. For this to happen it seems neccessary to have some accidental ''custodial'' symmetry analogous to the one which forces $\rho=1$ at tree level in the standard model. We have not succeeded in finding a model of flavor which simultaneously guarantees sufficient flavor gauge boson degeneracy and produces the Cabibbo angle, but this remains an interesting direction to explore.

2. D term splitting

In Ref. [26], it was shown that flavor D terms can split the first two generation sfermion masses by an amount independent of the flavor gauge couplings, if flavor symmetry breaking occurs in the supersymmetric limit. In this case, the splittings cannot be made small by reducing the flavor gauge coupling. This is also the case in supergravity scenarios that generate $\langle\varphi\rangle/M_F$ along flat directions after supersymmetry is broken. In our framework, however, the situation is different. The flavor sector potential has a stable minimum as the flavor gauge coupling is taken to zero. Thus, the flavor D terms can be made arbitrarily small.

As an example, consider an SU(2) theory with a doublet flavon φ_a and a triplet field Σ^{ab}, with superpotential $W=\lambda\varphi_a\varphi_b\Sigma^{ab}$. Suppose that only the doublets talk to the FN fields and hence get a negative mass squared at two loops. The crucial point is that, even if the gauge coupling is put to zero, the potential has a stable minimum; in the $g\rightarrow 0$ limit the superpotential part of the potential is minimized by putting $\Sigma^{ab}=0$, and we have

$$V=|\lambda|^2(\varphi^\dagger\varphi)^2-m^2\varphi^\dagger\varphi \tag{4.12}$$

and without loss of generality we can choose $\langle\varphi\rangle=(m/\sqrt{2}|\lambda|,0)$. For sufficiently small g, the D term contribution is a small perturbation to the above potential. In particular, the VEV of the flavon D term is $\sim m^2$, and so the induced D term splitting between first two generation sfermions is $\sim g^2m^2$. Now, for squark masses of $\sim$1 TeV, K-$\bar{K}$ mixing constrains $\theta_c(\widetilde{m}_d^2-\widetilde{m}_s^2)/\widetilde{m}^2<0.01$. Thus, for $m\sim 500$ GeV we must have $g^2<1/5$, making this gauge coupling moderately larger than e. Given that many fields transform under this flavor SU(2), it could be that the SU(2) coupling is non-asymptotically free, in which case a small value for the coupling could be naturally explained.

In the lepton sector, it is difficult to ensure that the D terms do not spoil slepton degeneracy without running into trouble with a very light flavor gauge boson coupling to leptons. The reason is that the D terms splitting has the form $\delta m_{\tilde{l}}^2\sim g^2\varphi^\dagger T^a\varphi\sim M^2$ where M is the mass of the flavor gauge boson. On the other hand, for slepton masses of $m_{\tilde{l}}\sim$ 100 GeV and slepton mixing of $\sim\sqrt{m_e/m_\mu}$, the constraint from $\mu\rightarrow e\gamma$ demands that $\delta m_{\tilde{l}}^2/m_{\tilde{l}}^2<.01$, putting the flavor gauge boson at a mass less than about 30 GeV. Thus, in order to have gauged flavor symmetries in the lepton sector, we must have some mechanism either to cancel the unwanted D term splitting between selectron and smuon masses or to guarantee the absence of slepton mixing in the first two generations.

V. CONCLUSIONS

Understanding the origins of supersymmetry breaking (SSB) and flavor symmetry breaking (FSB) are two of the greatest challenges for theories with weak-scale supersymmetry. Much of the structure of the theory is dictated by the mechanisms and scales for these symmetry breakings. In particular, the degree to which the soft supersymmetry breaking operators contain information about flavor depends on the relative size of the flavor scale, M_F, and the messenger scale for supersymmetry breaking $M_{\rm mess}$. In supergravity theories with $M_F<M_{\rm mess}\approx M_{\rm Pl}$, the interactions of the flavor scale leave an imprint on the soft operators; while in theories of gauge-mediated supersymmetry breaking, with $M_{\rm mess}<M_F$, they do not.

In this paper we have studied a framework in which $M_{\rm mess}$ and M_F are comparable because they have a common origin — the scale of dynamical supersymmetry breaking, $\Lambda_{\rm SSB}$. This scheme allows a unified view of FSB and EWSB — indeed the FSB VEVs $\langle\varphi\rangle$ are comparable to the EWSB VEVs $\langle H\rangle$. This unification is illustrated in Fig. 1, which shows the Froggatt-Nielsen sector as being the flavor analogue of the gauge mediation messenger sector. Indeed these sectors bear more than just a passing resemblance: both contain heavy vector generations of matter. Fundamentally, they are distinguished only by whether these heavy vector generations have large supersymmetry breaking contributions to their masses. We have given explicit models for the messenger sector, (3.5) + (3.9), and for the Froggatt-Nielsen sector (3.10) + (3.12). These models are both variants of a basic model which has two phases, with the vacuum choice dependent on the values of the dimensionless couplings. The sectors are chosen to be in opposite phases, so that the heavy vector generations feel supersymmetry breaking strongly in the gauge mediation sector but only mildly in the flavor sector. These models, while certainly not unique, illustrate our scheme and explicitly show how the flavon fields acquire negative squared masses triggering FSB, in a way which is analogous but not identical to the triggering of EWSB.

The interactions which feed supersymmetry breaking to the superpartners are phenomenologically very important. Supergravity mediation has been the most studied case, and mediation by the known gauge interactions is the only other case that has received significant attention. Our scheme does have mediation via the known gauge interactions, but in addition there is mediation via the superpotential interactions of the Froggatt-Nielsen sector. This provides a new origin for contributions to the soft operators; in particular it provides the dominant contribution to the soft trilinear A terms, and important flavor-dependent contributions to the scalar mass matrices.

We believe that the unified scheme for FSB and EWSB introduced in this paper, and summarized in Fig. 1, provides significant motivation for studying a new class of models for flavor. This new class of models, although based on the old ideas of Froggatt and Nielsen, are substantially different from the theories which have been constructed up to now. This is partly because the messenger sector provides a dominant, flavor-independent, contribution to the squark and slepton masses, and partly because flavon VEVs of order the weak scale introduce several new flavor-changing constraints.

The construction of any Froggatt-Nielsen model requires a choice for the flavor group, G_f, and for the G_f transformations of the flavons φ, the heavy vector generations F and the light generations f. The class of models which is consistent with the framework of this paper has these choices severely restricted by the following six constraints.

(1) G_f is preferred to be discrete — a continuous global G_f gives unacceptable familons, while a continuous gauged G_f has additional flavor dependent scalar mass contributions from D^2 terms.

(2) The size of the flavon VEVs is given by $\langle\varphi\rangle/M_F \approx 1/16\pi^2 \approx \lambda^2$ or λ^3, where $\lambda=0.22$. This hierarchy is determined by the order in perturbation theory at which the flavon VEVs are generated, and the requirement that all dimensionless couplings be of order unity. The top quark Yukawa interaction must be G_f allowed.

(3) Sufficient trilinear flavon interactions must be G_f allowed so that there are no accidental U(1) flavor groups which are spontaneously broken. Since there must be several different flavon fields, this is a very powerful constraint on the theory.

(4) Box diagrams involving internal heavy vector generations and flavons must not generate K_L-K_S mixing in the G_f symmetric limit. For example, this means that the 12 or 21 entries of the down Yukawa matrix may not be generated at linear order in φ/M_F. This excludes the case of very large $\tan\beta$ where the b Yukawa coupling is of order unity, since in that case these entries are expected to be linear in φ in order to generate the Cabibbo angle. The only way to avoid this conclusion is if the Cabibbo angle is generated from the up sector, in which case D-$\bar{D}$ mixing is predicted at the level of the present experimental limit.

(5) From the previous point it follows that the b and τ Yukawa couplings should arise at linear order in φ. This means that $\tan\beta$ is expected to be low, less than about 3. The mixing V_{cb} should arise at order φ in the up sector, or φ^2 in the down sector. Entries of the light 2×2 block of the Yukawa matrices should be at most of order φ^2.

(6) To prevent dangerous nonuniversal A terms, all nonzero entries of the light 2×2 block of the down and lepton Yukawa matrices must be of order φ^2.

Each of the above constraints is very significant, and none of them applies to Froggatt-Nielsen models with a high flavor scale and supergravity mediated supersymmetry breaking. Furthermore, these constraints are completely independent of the particular models chosen to give masses to the heavy vector generations in the gauge mediation and Froggatt-Nielsen sectors; there may be additional model-dependent constraints. For example, the explicit flavor sector of Sec. III which led to masses for the heavy vector generations of the Froggatt-Nielsen sector was based on a Z_2 symmetry. The flavons separated into two categories: φ which have $\bar{F}\varphi f$ type couplings and φ', which do not. The trilinear interactions necessary to prevent accidental flavor $U(1)$s then take the form $\varphi'^3+\varphi'\varphi^2$. Finally there may be further constraints which arise from the mechanism used to generate effective μ and m_3^2 terms. In Sec. III E this was accomplished by an interaction of the form $\varphi_\mu H_u H_d$, implying that at least one of the Higgs fields must transform nontrivially under the flavor group.

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APPENDIX A: TWO-LOOP INTEGRALS

In this Appendix, we evaluate the diagrams in Fig. 2 using the difference of propagators given in (3.19):

$$i\left[p^2-\begin{pmatrix}2g^2 & -2g\sqrt{2\lambda g}\\ -2g\sqrt{2\lambda g} & 4g\lambda\end{pmatrix}\frac{m^2}{h^2}\right]^{-1} - i\left[p^2-\begin{pmatrix}2(g^2+h^2) & -2g\sqrt{2\lambda g}\\ -2g\sqrt{2\lambda g} & 4g\lambda\end{pmatrix}\frac{m^2}{h^2}\right]^{-1}. \quad \text{(A1)}$$

Since these diagrams only involve modification of the a propagator, we need the (2,2) component of the expression above:

$$\frac{-i(16\lambda g^3 m^6)}{p^2[h^2p^2-2gm^2(g+2\lambda)][h^2p^4-2m^2(g^2+h^2+2g\lambda)p^2+8g\lambda m^4]}. \quad \text{(A2)}$$

The sum of the diagrams in Fig. 2 then gives us

$$\int\frac{d^4p}{(2\pi)^4}\frac{-i(8\alpha^2\beta^2g^3\lambda m^6)}{[h^2p^2-2gm^2(g+2\lambda)][h^2p^4-2m^2(g^2+h^2+2g\lambda)p^2+8g\lambda m^4]}\int\frac{d^4l}{(2\pi)^4}\frac{l^2+M^2}{l^2(l^2-M^2)^2[(p-l)^2-M^2]}, \quad \text{(A3)}$$

where $M=\alpha\langle a\rangle$ is the Froggatt-Nielsen mass scale. Note that diagrams (B), (C), and (D) in Fig. 2 are individually ultraviolet divergent, but their sum is finite. The d^4l integral in Eq. (A3) can be done analytically by conventional methods. To evaluate the remaining p integral, we first go to Euclidean space, and do the trivial angular integration. What remains is a one-dimensional integral in the Euclidean radial coordinate p_E:

$$\frac{i}{(16\pi^2)^2}(16\alpha^2\beta^2g^3\lambda m^6) \times \int_0^\infty dp_E p_E^3 F(p_E^2/M^2) D_1(p_E^2) D_2(p_E^2), \tag{A4}$$

where

$$D_1(p_E^2)=\frac{1}{M^2[h^2p_E^2+2gm^2(g+2\lambda)]}, \tag{A5}$$

$$D_2(p_E^2)=\frac{1}{h^2p_E^4+2m^2(g^2+h^2+2g\lambda)p_E^2+8g\lambda m^4}, \tag{A6}$$

and

$$F(x)=\frac{-4-2x}{\sqrt{x(4+x)}}\tanh^{-1}\sqrt{\frac{x}{4+x}}+\frac{x+1}{x}\ln(1+x). \tag{A7}$$

The function $F(x)$, plotted in Fig. 6, is positive definite. We have chosen g and λ to be real and positive without loss of generality, by making suitable field phase rotations. Therefore the entire integrand of Eq. (A4) is also positive definite. If we use a different phase convention, the VEVs and mass spectrum change accordingly but the final sign of the mass squared remains the same. We conclude that the mass squared generated in the flavon potential is negative. The dp_E integral can be evaluated numerically, and the results are consistent with the order of magnitude estimate given in the text.

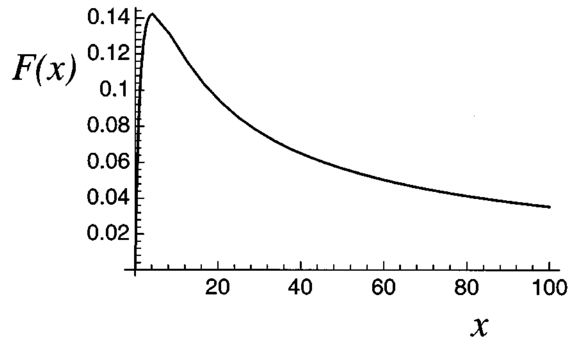

FIG. 6. The function $F(x)$.

We have shown that the two-loop diagrams in Fig. 2 generate finite negative definite squared masses for the flavons. One also obtains apparently ultraviolet-divergent contributions at third or higher loop orders in perturbation theory. However, the ultraviolet divergence is cut off at $\Lambda_{\rm SSB}$ because the supersymmetry is restored above this scale, and one needs to take into account that the supersymmetry breaking mass parameter m^2 vanishes above $\Lambda_{\rm SSB}$. Therefore higher order divergent contributions are suppressed compared to the two-loop finite ones by a factor of

$$\sim\frac{1}{16\pi^2}\ln\frac{\Lambda_{\rm SSB}^2}{m^2}\simeq 0.09, \tag{A8}$$

and hence can be neglected.

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