Projection scheme for handling large-number cancellation related to gauge invariance

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A scheme, the so-called ''projection,'' for handling singularities in processes such as $e^+e^-\to t\overline{b}e^-\overline{\nu}$ (or $e^+e^-\to u\overline{d}e^-\overline{\nu}$) is proposed. In the scheme, with the help of gauge invariance, the large power quantities $(s/m_e^2)^n$ $(n\geqslant 1; s\to\infty)$ are removed from the calculation totally, while in the usual schemes the large quantities appear and only will be canceled at last. The advantages of the scheme in numerical calculations are obvious; thus, we focus our discussions mainly on the advantages of the scheme in the special case where the absorptive part for some propagators relevant to the process could not be ignored, and a not satisfactory but widely adopted approximation is made; i.e., a finite constant ''width'' is introduced to approximate the absorptive part of the propagators phenomenologically even though QED gauge invariance is violated. [S0556-2821(96)04323-8]

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I. INTRODUCTION

Since the top quark mass has been measured at Fermilab [1,2] and the Higgs boson mass seems to possess some constraints [3], at present there are better grounds to precisely testify do the validity of the standard model. The CERN e^+e^- collider LEP II of 200 GeV and the Next Linear Collider (NLC) of about 500 GeV probably can do the job further in the near future.

Of the possible reactions, $e^+e^-\to t\overline{b}e^-\overline{\nu}$ is an interesting one. It is asserted that of the Feynman diagrams, the four associated with a t-channel photon exchange shown in Fig. 1, being gauge-invariant themselves, are dominant over others as $\sqrt{s}>250$ GeV [4]. The propagator of the photon is proportional to $1/k^2$ where $k^2\equiv(p_1-p)^2$ and p_1 and p are the momenta of the outgoing and incoming electrons. The kinematics tells us

$$k^2=(p_1-p)^2=2m_e^2-2E_1E+2|\vec{p}_1||\vec{p}|\cos\theta_e \sim\left(2-\frac{|\vec{p}_1|}{|\vec{p}|}-\frac{|\vec{p}|}{|\vec{p}_1|}\right)m_e^2-2|\vec{p}_1||\vec{p}|(1-\cos\theta_e), \quad (1)$$

as θ_e, the angle between $\vec{p}_1$ and $\vec{p}$ approaches to zero, $k^2\to 0$, if $m_e\to 0$, i.e., the photon approaches to mass shell; i.e., the propagator becomes singular. This singularity is essential, because even after the final-state phase-space integration it still survives. One can divide the integration over θ_e into two parts:

$$\int_0^{(\theta_e)_{\rm cut}}\sin\theta_e d\theta_e f(\theta_e,m_e)+\int_{(\theta_e)_{\rm cut}}^{\infty}\sin\theta_e d\theta_e f(\theta_e,m_e), \quad (2)$$

where $(\theta_e)_{\rm cut}$ is a small angle, and may be related to experimental measurements as one chooses. For the second integral, regular m_e can be safely set to approximately zero, whereas in general up to the first order of $(\theta_e)_{\rm cut}$, the first integration may be written as

$$\left[a\left(\frac{s}{m_e^2}\right)^n+b\left(\ln\frac{s}{m_e^2}\right)+\cdots\right]\cdot\Delta\theta_e, \quad (3)$$

where $n\geqslant 1$ and $\Delta\theta_e$ is a small angle equivalent to $(\theta_e)_{\rm cut}$.

Since the small-angle electron that finally scatters into the beam tube (i.e., θ_e being very small), cannot be detected, the phase-space integration for final state should always start from the small angle instead of zero if the electron is ''exclusively'' measured, so the $(\theta_e)_{\rm cut}$ in Eq. (2) has physical meaning. However, if the electron in the reaction $e^+e^-\to t\overline{b}e^-\overline{\nu}$ is detected inclusively, the small θ_e contribution cannot be negligible.

Note that in Eq. (3) the power terms $(s/m_e^2)^n\Delta\theta$ being very large even for very small $\Delta\theta$ and the logarithmic terms $\ln(s/m_e^2)\Delta\theta$ being of much milder divergent behavior at $\theta\sim 0$, all of the terms may be suppressed if there is some symmetry, e.g., the gauge symmetry for the concerned process. Many authors have investigated this process. Raidal *et al.* [5], Panella *et al.* [6] obtained quite different conclusions subsequently; then Boos *et al.* [4] pointed out that due to an extravagant destructive interference among the four diagrams, the unexpected large contributions disappear after summing up the contributions with the large number cancellation.

It is proved that due to the gauge invariance contributing to the process the troublesome power terms $(s/m_e^2)^n$ do not exist at the final cross section but only the well-known logarithmic term as $\ln(s/m_e^2)$, that also forms the basis of the Weiszäker-Williams approximation [7]. In principle, a straightforward calculation can give the correct final result as done in Ref. [4] as long as the numerical calculation keeps all the large numbers accurate enough. Even though the troublesome power terms $(s/m_e^2)^n$, generating large quantities, would kill the large quantities, and much smaller ones are retained due to the destructive interference nature. In practice, such cancellations may cause 6 to 8 magnitude orders reduction (see Ref. [4]), so it very often causes a problem, i.e., it can lead to a totally wrong result for numerical computation; at least, it becomes very difficult to estimate the errors of the computation.

Formulating the above statement, one can write the amplitude contributed by each individual of the four diagrams as

$$M_i = a_i + b_i \cos\theta, \tag{4}$$

where θ is the angle between the three-momenta of the incoming and outgoing electrons. The troublesome photon propagator contributes a factor $1/[\alpha+\beta\ \sin^2(\theta/2)]^2$ to the differential cross section. α is proportional to m_e^2 and β is related to the energy, so as $\theta \to 0$ and $m_e \to 0$, this is a singular term. The final-state phase-space integration includes a part over the solid angle $d\ \sin^2(\theta/2)$, therefore it alleviates the singular degree. When we take the integration of $|\Sigma M_i|^2$ over $d\ \sin^2(\theta/2)$, we have

$$\begin{aligned}
&\int d\ \sin^2\left(\frac{\theta}{2}\right)\left|\sum_{i=1}^{4}(a_i+b_i\cos\theta)\right|^2 \frac{1}{(\alpha+\beta\ \sin^2(\theta/2))^2}\\
&=\int d\ \sin^2\left(\frac{\theta}{2}\right)\sum_{i,j}[a_i^*a_j+(a_i^*b_j+a_jb_i^*)\cos\theta\\
&\quad +b_i^*b_j\cos^2\theta]\frac{1}{(\alpha+\beta\sin^2(\theta/2))^2}\\
&=\int d\ \sin^2\left(\frac{\theta}{2}\right)\left[\left|\sum_i (a_i+b_i)\right|^2\right.\\
&\quad -\sum_{i,j}(4b_i^*b_j+2(a_i^*b_j+a_jb_i^*))\sin^2\left(\frac{\theta}{2}\right)\\
&\quad \left.+4\sum_{i,j} b_i^*b_j\sin^4\left(\frac{\theta}{2}\right)\right]\frac{1}{(\alpha+\beta\ \sin^2(\theta/2))^2}.
\end{aligned}\tag{5}$$

It is easily rewritten as

$$\begin{aligned}
&\int_0^1 dx\frac{(A+Bx+Cx^2)}{(\alpha+\beta x)^2}\\
&=\int_0^1 dx\left[\frac{C}{\beta^2}(\alpha+\beta x)^2+\frac{1}{\beta}\left(\frac{-2\alpha C}{\beta}+B\right)(\alpha+\beta x)\right.\\
&\quad \left.+\left(A+\frac{\alpha^2C}{\beta^2}-\frac{\alpha B}{\beta}\right)\right]\frac{1}{(\alpha+\beta x)^2},
\end{aligned}\tag{6}$$

where $x=\sin^2\theta/2$.

It is easy to see that the first term is completely benign as $m_e \to 0$, and the second term gives a logarithmic term $\ln m_e^2$ while the last one produces an extravagantly singular term proportional to $1/m_e^2$ at $\theta \to 0$. Therefore as we discussed above, the gauge invariance demands vanishing of the last term; namely one should expect

$$A+\frac{\alpha^2 C}{\beta^2}-\frac{\alpha B}{\beta}=0, \tag{7}$$

where

$$A=\left|\sum_i (a_i+b_i)\right|^2, \tag{8}$$

$$B=-\sum_{i,j}(4b_i^*b_j+2a_i^*b_j+2a_jb_i^*), \tag{9}$$

$$C=4\sum_{i,j} b_i^*b_j. \tag{10}$$

Furthermore, a very interesting and important issue has been addressed recently; i.e., some authors [8–10,12] pointed out that to get rid of the singularity at the W-boson propagator $1/(q^2-M_W^2)$ for higher energies, a regular Breit-Wigner form $1/(q^2-M_W^2+i\Gamma_W M_W)$ where Γ_W is the measured decay width of the W boson is introduced, whereas the gauge invariance of QED is violated. The power divergent terms may appear again. It is well known that the QED gauge invariance is a fundamental principle and cannot be upset at any case. Here the apparent violation is artificial or due to an inappropriate approximation and misapplication of the Breit-Wigner form. They suggested many approaches to restore the gauge invariance. However, since most of the treatments possess certain arbitrariness depending on the

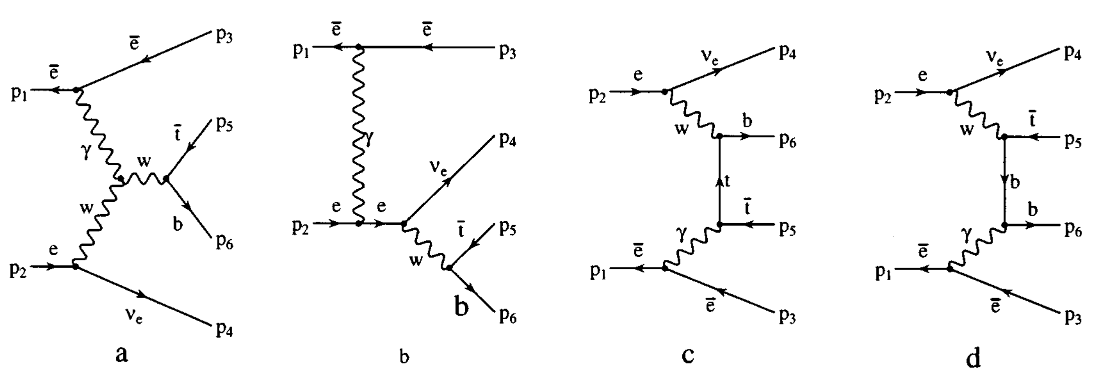

FIG. 1. (a) through (d), the Feynman diagrams where a $t-$channel photon propagator is involved.

TABLE I. The cross sections of $e^+e^- \to u\overline{d}e^-\overline{\nu_e}$ obtained in the projection scheme (the definition of the W width appearing in the table may be found in Ref. [12].

σ (pb)	0.08977 ($\pm$0.000200)	with fixed W width
σ (pb)	0.08983 ($\pm$0.000200)	with running W width

way of restoring the gauge invariance as long as it is not based on a solid stone of the quantum field theory, even if the gauge invariance is respected, some ''unphysical'' contributions emerge in the final results. Aeppli [10] and Papavassiliou, and Pilaftsis [11] provide very elegant ways of setting more solid foundations to deal with the width of the W propagator for the processes; namely loop contributions for its self-energy and vertex are considered, and thus the effective width Γ in the W propagator is a function of momentum. But this procedure involves complicated loop calculations so the intuitive meaning of the width is lost, and is the convenience for cross section evaluation. Argyres *et al.* [12] also suggested that by taking into account the absorptive part of the triangular loop correction to the $WW\gamma$ vertex, one can regain the gauge invariance. The method they provided is practically efficient for real processes and the results obtained in various schemes that restore gauge invariance coincide with each other (see Table I of Ref. [12]).

Alternatively, we propose a different scheme to approach the problem, namely ''project out'' the large component from each piece of the amplitude (corresponds to each diagram), by means of choosing a special gauge based on the gauge invariance of the processes, thus the power terms, i.e., the large numbers, do not appear in the calculations completely. Furthermore, even if the gauge invariance is artificially violated, in this scheme, the additional contribution related to the violation is also suppressed. For instance, even though the naive Breit-Wigner formulation of the propagator for describing the unstable nature of the particle is adopted, the large terms are eliminated and only the terms $\Gamma_W/M_W \ln(s/m_e^2)\Delta\theta$ survive. A more precise discussion and comparisons of the results obtained in this scheme with the others, especially that of the authors of Ref. [12] will be given below. Reasonable consistency with other schemes is found.

II. THE PROJECTION SCHEME

(i) The scheme.

The amplitude corresponding to the Feynman diagrams shown in Fig. 1(a) through (d) characterized by possessing a common electron line, can be written as

$$M=\sum_{i=1}^{4} M_i = l^\mu T_\mu \frac{-i}{k^2} = \overline{u_e}(p_1,s_1)\gamma^\mu u_e(p,s)\frac{-i}{k^2} \times (T_1+T_2+T_3+T_4)_\mu , \tag{11}$$

where l_μ is the lepton current, u_e's are the incoming and outgoing electrons and T_i's are the effective currents determined by the weak interaction and $k^2=(p_1-p)^2$ is the squared momentum carried by the photon. Due to the gauge invariance for the four diagrams themselves, we have

$$k\cdot T \equiv k^\mu \cdot \sum_{i=1}^{4} T_{i\mu}=0. \tag{12}$$

As aforementioned, through a straightforward calculation one may show that each of T_i's, $i=1,\ldots,4$ itself contributes power terms s/m_e^2 to the cross section at the vicinity of $\theta_e=0$. The ''total cross section'' σ (here only the four Feynman diagrams in Fig. 1 are included, and it is introduced only for the discussion's convenience),

$$\sigma=\frac{1}{4s^2}\int\prod_{j=1}^{4}\frac{d^3p_j}{(2\pi)^3 2E_j}\sum_{\text{all spins}}|M|^2 = \frac{1}{4s^2}\int\prod_{j=1}^{4}\frac{d^3p_j}{(2\pi)^3 2E_j}\sum_{\text{all spins}}\left|\sum_{i=1}^{4} l^\mu T_{i\mu}\right|^2\frac{1}{k^4}, \tag{13}$$

where p_j's are the momenta of the outgoing $t,\overline{b},e^-$, and $\overline{\nu}.$ Note once more it has been proved [4] that the final-state integration only results in a $\ln(s/m_e^2)$ term, but not any power term $(s/m_e^2)^n$ $(n\geqslant 1)$. The disappearance of the troublesome power terms finally is due to the gauge invariance of QED.

Nevertheless, there is still the problem in the numerical calculation that the ''cross section'' involves subtraction among large quantities with a very small quantity remaining as shown in Fig. (3) of Ref. [4]. To solve the problem, we propose the so-called project scheme by choosing a very special gauge. One can add an arbitrary term proportional to k^μ to the lepton current l^μ such as

$$l'_\mu = l_\mu - ck_\mu . \tag{14}$$

Due to the gauge invariance, here c may be any variable or constant. The idea of the projection scheme is to subtract a suitable quantity from every amplitude (corresponding to each Feynman diagram) by choosing a proper gauge (here the quantity c) , so as to project out a fraction that results in the large power divergent term in the final cross section. Indeed the idea may be carried through successfully as follows. To make each component of the four-vector l'_μ minimal, we choose a condition

$$\frac{\delta}{\delta c}\max(|l'_0|,|l'_1|,|l'_2|,|l'_3|)=0. \tag{15}$$

To be symmetric, alternatively we adopt the following condition instead:

$$\frac{\delta}{\delta c}(l'^*_0 l'_0 + l'^*_1 l'_1 + l'^*_2 l'_2 + l'^*_3 l'_3)=0. \tag{16}$$

Note here that the summation $\Sigma l'^*_i l'_i$ is defined in ''Euclidean space measure'' but not in a Minkovsky one, thus it minimizes the squared radius of the Euclidean four-sphere. Then we obtain

$$c=\frac{k_0 l_0 + k_1 l_1 + k_2 l_2 + k_3 l_3}{k_0^2+k_1^2+k_2^2+k_3^2}, \tag{17}$$

under the condition. Because $k^\mu l_\mu = 0$ (with the metric $(1,-1,-1,-1)$ as convention), we have

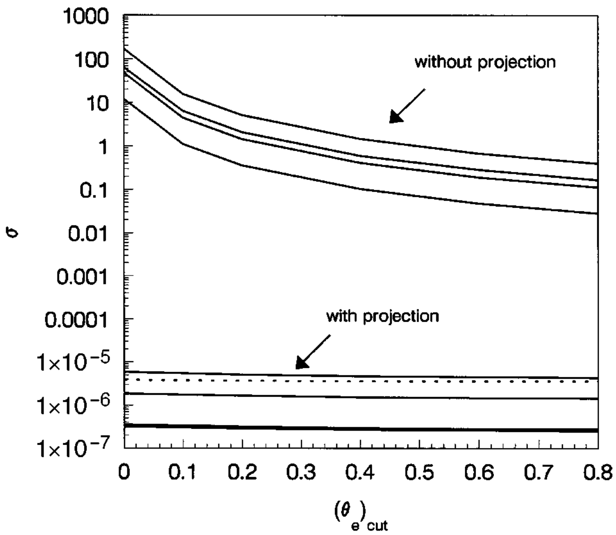

FIG. 2. The dependence of the cross sections on the angle cut $(\theta_e)_{\rm cut}$. The upper four curves show a rapid rise near $(\theta_e)_{\rm cut}\to 0$, which correspond to the cross sections of the individual diagrams of Fig. 1 calculated in the regular scheme. The lower four solid lines also correspond to the individual diagrams of Fig. 1, but are calculated with the projection scheme. The dashed line is the total cross section. To compare with previous calculations, we take $m_t=140$ GeV and $\sqrt{s}=190$ GeV.

$$c=\frac{2k_0l_0}{||k||^2},$$

where $||k||^2\equiv k_0^2+k_1^2+k_2^2+k_3^2$. Thus with the gauge c, the lepton current may be replaced by

$$l'_\mu=l_\mu-\frac{2k_0l_0}{||k||^2}k_\mu, \tag{18}$$

which indeed projects out the large term from l_μ. Due to the smallness of m_e in the process, one can expand k_0l_0 in m_e and only keep terms up to m_e^3 in the series.

To see the results, by comparing with Eq. (4) under the limit of $m_e^2\to 0$, i.e., α in Eq. (5) is zero, if one calculates the amplitude for each diagram in terms of the projection scheme, one will find that each amplitude is proportional to $1/\sin(\theta/2)$ instead of $1/\sin^2(\theta/2)$ in the usual scheme, thus the singularity becomes mild and after the integration over final phase space, only the logarithm term remains, even for the contribution from each diagram individually.

To show the advantages more precisely, we recalculate the process numerically. The numerical results are shown in Fig. 2. We plot the dependence of the cross sections on the $(\theta_e)_{\rm cut}$, which is the lower limit of the angle integration. In order to compare with the results of Ref. [4], we recalculate the contributions from the four diagrams of Fig. 1 (without the interference among them) in terms of the usual way, where we deliberately choose $m_t=140$ GeV and $\sqrt{s}=190$ GeV precisely as given in Ref. [4]. The individual curves are the upper four in the figure and they are exactly consistent with those of Ref. [4]. It is noticed that the total cross section is lower than them by 8 magnitude orders as pointed out by Boos *et al.* The lower four curves correspond to the contributions from the four diagrams individually also, but are calculated in the projection scheme. The total cross section exactly coincides with that obtained in the usual scheme where the high accuracy computation technique is employed. It is interesting to note that the ''individual'' curves obtained by the projection scheme have the same order as the total cross section.

(ii) Under the approximation of a finite W-boson width Γ_W.

As the concerned energy is relatively low as long as all intermediate W bosons are far away from its mass shell, the propagator can be written as

$$\frac{-i}{q^2-M_W^2+i\epsilon}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_W^2}\right).$$

As showed by Kurihara [8], with propagators corresponding to zero width, it is easy to check $k^\mu T_\mu=0$, i.e., the gauge invariance holds. However, when the other s-channel diagrams for the process $e^+e^-\to t\bar{b}e^-\bar{\nu}$ are concerned and as the energy is increasing ($\sqrt{s}\geqslant m_t+m_b+M_W$), q^2 of the process may cross the mass shell of W boson and the singularity of the propagator would result in a new singularity. In fact this divergence is caused by an unsuitable approximation. Since the intermediate boson is not a stable particle, the propagator should be modified, for instance,

$$\frac{-i}{q^2-M_W^2+i\Gamma_W M_W}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{m_W^2}\right).$$

However, it still is a problem deciding what Γ_W in the propagator is. Generally it is an ''effective'' width, corresponding to the absorptive part of the self-energy of the particle; therefore, only the s-channel W-boson, being timelike, can be nonzero, whereas the t channel is spacelike, so will always be zero. Therefore Γ_W in the propagator should be a function of momentum behaving as

$$\Gamma_W(q^2)=\Gamma_1(q^2)\,\theta(q^2-\Lambda_1^2)+\Gamma_2(q^2)\,\theta(q^2-\Lambda_2^2)+\cdots, \tag{19}$$

where Λ_i^2 is the threshold of a corresponding channel (i). In practice, the Breit-Wigner formulation is adopted widely [13], i.e., one has

$$\Gamma_W(q^2)=\begin{cases}\overline{\Gamma_W}, & q^2>0 \quad (s \text{ channel}),\\ 0 & q^2<0, \quad (t \text{ channel}),\end{cases} \tag{20}$$

where $\overline{\Gamma_W}$, being constant, is taken as the measured width of the real W boson. This brings in an inconsistency, i.e., the gauge invariance of QED is violated artificially. To amend the fake violation, many authors proposed various methods [8,9], and Aeppli [10] summarized them and indicated that all schemes may possess some unphysical additions artificially imposed to the results.

In our scheme for taking the special gauge [Eq. (17)], thanks to the projection in the gauge, the large terms $(s/m_e^2)^n\Delta\theta$ do not occur at all, whereas in usual schemes they appear in the intermediate stage of the calculation. When gauge invariance is violated, such dangerous power terms still do not appear; even when the gauge invariance is violated, in the final result only the term proportional to $\ln(s/m_e^2)$ appears in terms of the projection scheme. In summary, when the gauge invariance is artificially violated, in this scheme there could be only additional terms,

$$a\left(\frac{\Gamma_W}{M_W}\right)^m\left(\frac{s}{m_e^2}\right)^n\times\Delta\theta+b\left(\frac{\Gamma_W}{M_W}\right)^{m'}\ln\frac{s}{m_e^2}\Delta\theta, \tag{21}$$

which vanish as $\Gamma_W\to 0$. In the Breit-Wigner formulation, both a and b terms in Eq. (21) are nonzero and any procedure to restore the gauge invariance would make the power term disappear, i.e., impose a to be zero, and bring in some change to the logarithmic term. The change, in fact, must involve unphysical components due to the violating of the gauge invariance.

For an explicit comparison with the literature [12], we have calculated the cross section of $e^+e^-\to ude^-\overline{\nu_e}$ for $\sqrt{s}=175$ GeV. As the collision energy is so high, we have adopted a little larger fine-structure constant:

$$\alpha_e(s=175^2\ \text{GeV}^2)=1/125.0$$

in our computations. The numerical values for the cross section we obtained are listed in Table I. When calculating the values in the table, the parameters

$$M_W=80.22\ \text{GeV},$$

$$m_e=0.511\times 10^{-3}\ \text{GeV},$$

$$\alpha_e(0)=1/127.034,$$

$$\sin\theta_W=0.232,$$

$$50<\sqrt{P_+^2}<110\ \text{GeV},$$

are taken, along with the definition $P_+=P_l+P_\nu$. The coupling constant α_e is mainly based on the formulas [15]. In the calculations $m_t=176$ GeV is used.

To compare with the results of Ref. [12], their calculation for the cross section is 0.08887(8)pb for fixed width $(\theta_{\min}=0)$, while from Table I one may see ours is about 1% larger only. The result with running width is only 0.07% larger than that for the fixed width. We should note here that a comparatively larger value for α_e is adopted in our calculation, hence a slightly larger number should be expected.

III. DISCUSSIONS AND CONCLUSION

To solve the problem of large number cancellations around the singularity such as that in the forward direction for the process $e^+e^-\to tbe^-\overline{\nu}$, we propose a ‘‘projection’’ scheme so as to *a priori* project out the large quantity in each amplitude where a t-channel photon-propagator is involved.

Figure 2 shows that in usual schemes, the curves corresponding to the contribution of the individual diagrams of Fig. 1 rise very fast as $(\theta_e)_{\text{cut}}$ approaches zero, but the total cross section does not. It is a result of the gauge invariance as discussed above. In contrast, in the new scheme (a special gauge is applied) the contribution presents a smooth behavior at zero-$(\theta_e)_{\text{cut}}$.

Indeed, this intriguing problem was conceived by some authors a long time ago and they employed a special scheme [14]. The squared amplitude is

$$\frac{1}{4}\sum |T|^2=L_0^{\mu\nu}(p,p_1)H_{\mu\nu}(p_t,p_b,q), \tag{22}$$

where $L_0^{\mu\nu}$ is the lepton current part, $q=p-p_1$ and $q^\mu H_{\mu\nu}=0$ by gauge invariance. In general, $L_0^{\mu\nu}$ may be written as

$$L_0^{\mu\nu}=(\epsilon_1^\mu\epsilon_1^\nu+q^2g^{\mu\nu})L(q^2), \tag{23}$$

where $L(q^2)$ is a scalar function; the polarization may be written as

$$\epsilon_1^\mu=p^\mu+p_1^\mu+Z_1q^\mu,$$

where Z_1 stands for an arbitrary parameter. The authors proved that if a special choice,

$$Z_1=-(p^0+p_1^0)/q^0,$$

is taken, and further to demand $\epsilon_1^0=0$ and

$$\epsilon^2=-|\vec{\epsilon}_1|^2=4m_e^2+(Z_1^2-1)q^2+0(m_e^2),$$

the power terms $(s/m_e^2)^n$ can be effectively eliminated. Our projection scheme is in a way parallel to their treatment. Our scheme systematically handles the power singularity at the collinear limit. We adopt the projection at the amplitude level while the authors of Ref. [14] dealt with it at the amplitude-square level.

As pointed out above, so far there is no very satisfactory (simple, intuitive, and not breaking the existent symmetries etc.) way to dictate the absorptive part of the propagator when the ‘‘finite width’’ effects cannot be ignored. Usually when the finite width is introduced phenomenologically the gauge invariance is artificially violated. Large power singular terms generally emerge. Therefore one would try some methods to restore the gauge invariance, but so far most of the treatments (there are a few exceptions, e.g., Ref. [12]) ‘‘planting in’’ gauge invariance by hand, may get rid of the unphysical power singular terms, but at the same time would bring in other new unphysical and undesired changes. In our scheme, the unphysical power singularity is eliminated from the very beginning, and is therefore even with the artificial gauge invariance violation. Even though an unphysical logarithmic term due to the violation of the gauge invariance indeed emerges and is added to the final result, compared to other schemes this additional unphysical contribution is much suppressed and the influence to the total cross section is within 1%, a tolerable error at least for the tree level. It is because for all known unstable particles that we have so far treated, we always have the width much smaller than the mass, e.g., for W boson we have $\Gamma_W\ll M_W$, the extra term, behaving as Eq. (21), does not make a substantial contribution at the highest energy in the foreseen future.

For $e^+e^-\to tbe^-\overline{\nu}$, since in the four t-channel photon exchange diagrams of Fig. 1, the W boson cannot go onto its mass shell, whether at s or t channels, so the finite Γ_W should not give rise to any substantial change in that case. Our results with the W propagator having a finite width only at the timelike region are numerically consistent with that of null Γ_W, and the error is within 1%, so it confirms our aforementioned discussions that the gauge-invariance violation can only cause a term proportional to $(\Gamma_W/M_W)^2 \ln s/m_e^2$. In the process $e^+e^-\to ude^-\overline{\nu}$, the effects of a finite width Γ_W become important even for the four photon-exchange diagrams in Fig. 1. With our scheme, the troublesome unphysical power term does not appear either, and the unphysical term corresponding to artificial gauge invariance is also suppressed to $(\Gamma_W/M_W)^2$ order, so they are negligible up to a sufficient accuracy, for example as $\sqrt{s}=200$ GeV, $(\Gamma_W/M_W)^2 \ln s/m_e^2 \sim 0.016$, and $0.016\Delta\theta$ must be much less than 1. The results, shown in Table I, indicate that the effect of violating gauge invariance caused by the running W width does not affect the final conclusion within a range of 1%. In contrast, without the projection, the power term caused by the artificial gauge invariance violation is too large to tolerable, in fact, it blows up the numerical results (see Table I of Ref. [12]).

It is certain that the scheme of Ref. [12] is more solid from a theoretical point of view that is based on more solid ground, such as quantum field theory where through loops one can connect the vertex to the self-energy diagrams to restore the gauge invariance when finite width effects are concerned. Even though this is the case, we still should note that if one restricts oneself to work in an exact perturbative theory, surely the gauge invariance will be kept order by order; however, the W propagator cannot be simply written in the compact form $-i/(q^2-m_W^2+i\Gamma_W M_W)$, which is a result of resummation of chain diagrams. Thus it is not easy to mend the singularity problem at a given order. In fact, what we need is to eliminate the dangerous power divergence caused by the artificial violation of gauge invariance so as to reach a reliable result to the desired accuracy. To serve is goal, one either restores the gauge invariance to automatically remove the dangerous power divergence or gets rid of the troublesome term directly as we do in this work. For restoration of the gauge invariance, an appropriate scheme is to include all loop corrections of the self-energy and vertex whose absorptive parts would consistently result in the imaginary part in the full propagator of W bosons, Z bosons, and fermions and retains the gauge invariance [11,12]. In Ref. [12], the authors proved that the absorptive part of the triangular loop compensates the unbalance at s and t channels due to introducing finite width to the unstable W boson, so the gauge invariance is regained in the calculations. The authors showed that deviations for various schemes that are adopted to retain gauge invariance and eliminate the power divergence s/m_e^2 are reasonably small. In our scheme, we simply avoid the trouble of power divergence and suppress the gauge invariance violation effects. Indeed, in principle and in practice our scheme may let the calculations escape from the problems due to violation of the gauge invariance caused by phenomenologically introducing a finite width in the propagator(s).

The advantages of the scheme are obvious. Many large number cancellations due to internal gauge invariance in the concerned process are avoided. Those advantages are crucial sometimes for numerical calculations. Futhermore, a simple but rough numerical computation indicates that the final results for the cross sections of $e^+e^-\to e^-\overline{\nu_e}u\overline{d}$ in the projection scheme only deviates from that in the schemes that restore the gauge invariance by considering a loop correction to the $WW\gamma$ vertex by less than 1%. A more careful calculation is in progress and the results will be published some time later [16].

Since the process $e^+e^-\to u\overline{d}e^-\overline{\nu_e}$ attracts much attention due to its significance for better understanding of top physics and precise tests of the standard model, further studies are on going [17]. Indeed, a convenient method that greatly simplifies analysis of data and at the same time obtains results deviating from the ''accurate'' values obtained by other more solid, but much more complicated methods only by a small fraction within the experimentally allowed tolerance, should be helpful and probably preferable. This projection scheme may be one of the appropriate and desired ones for both experimentalists and theoreticians.

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