Electric charge quantization from gauge invariance of a Lagrangian: A catalogue of baryon-number-violating scalar interactions

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In gauge theories such as the standard model, the electric charges of the fermions can be heavily constrained from the classical structure of the theory and from the cancellation of anomalies. There is, however, mounting evidence suggesting that these anomaly constraints are not as well motivated as the classical constraints. In light of this we discuss possible modifications of the minimal standard model that will give us complete electric charge quantization from classical constraints alone. Because these modifications to the standard model involve the consideration of baryon-number-violating scalar interactions, we present a complete catalogue of the simplest ways to modify the standard model so as to introduce explicit baryon number violation. This has implications for proton decay searches and baryogenesis. [S0556-2821(96)04123-9]

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I. INTRODUCTION AND MOTIVATION

Investigation of explicit baryon number violation in simple extensions of the standard model (SM) is interesting for a number of different reasons, including (a) the requirement of baryon number violation to explain baryogenesis, and (b) the continuing interest in terrestrial searches for baryon-number-violating processes.

The aim of this work is to provide a complete catalogue of the simplest ways to explicitly violate baryon number through extensions of the SM. A theoretical motivation for doing this arises also from the work done by one of us [4] on the possibility of obtaining complete electric charge quantization from classical constraints.

The quantization of the electric charges of the known fermions is a well-established experimental phenomenon. An approach to a theoretical understanding of this phenomenon has emerged in recent years based on the SM [1]. The SM is a gauge theory with gauge group

$$\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y, \tag{1}$$

which is assumed to be spontaneously broken by the vacuum expectation value (VEV) of a scalar doublet $\phi \sim (1,2,1)$. The $\mathrm{U}(1)_Y$ charge of ϕ can be normalized to 1 without loss of generality due to a scaling symmetry, $g \to \eta g, Y \to Y/\eta$, where g is the $\mathrm{U}(1)_Y$ coupling constant, and Y is the generator of the $\mathrm{U}(1)_Y$ gauge group. The gauge symmetry of the Lagrangian can be used to choose the standard form for the vacuum:

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ u \end{pmatrix}. \tag{2}$$

The VEV of ϕ breaks $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ leaving an unbroken U(1) symmetry, $\mathrm{U}(1)_Q$, which is identified with electromagnetism. Its generator Q is the linear combination that annihilates the VEV of Eq. (2):

$$Q = I_3 + Y/2. \tag{3}$$

The normalization of Q is not physically measurable, and we have adopted the convention of normalizing it so that the charged W bosons will have charge 1. The above reasoning shows that the electric charge quantization problem would be solved if a way could be found to deduce the Y charges of the fermions.

There are two quite distinct ways in which the standard model constrains the electric charges of the fermions. First, there is a set of constraints that follow from the definition of the theory at the classical level: the requirement that the Lagrangian be gauge invariant. Second, there are other constraints that are assumed to follow from the consistency of the theory at the quantum level: the anomaly cancellation conditions. The outcome of this is that charge quantization follows, provided that there is only one anomaly-free U(1) symmetry of the Lagrangian outside of those contained in $\mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L$. If it turns out that the generator of this U(1) symmetry is precisely standard weak-hypercharge Y, then not only is charge quantized but it is quantized correctly.

For instance, consider the minimal SM. In addition to standard Y, any one of $L_e - L_\mu$, $L_e - L_\tau$, and $L_\mu - L_\tau$ generates an anomaly-free U(1) symmetry of the Lagrangian. Therefore the minimal SM poses a charge quantization problem because the actual weak-hypercharge of the theory can be chosen to be $\cos\Theta Y_{\text{standard}} + \sin\Theta (L_i - L_j)$ where Θ is an arbitrary parameter and $i,j = e,\mu,\tau$ $(i \neq j)$. See Ref. [1] for more detailed reviews.

The above analysis assumes that the cancellation of gauge anomalies is a rigorous requirement for a consistent gauge theory. There are, however, several arguments that throw doubt on the validity of this requirement. For example there may be a set of as yet undetected mirror fermions that remove the anomaly cancellation requirement. There are also interesting arguments given by Kieu [2] in a series of papers to the effect that a properly analyzed ''anomalous'' gauge theory is not anomalous at all. (For other interesting work on the question of the consistency or otherwise of anomalous gauge theory see Ref. [3].) If gauge anomaly cancellation as routinely enforced is unnecessary, then there is no motivation to use these constraints in deriving electric charge quantization. Clearly one is then left with the following result: *Electric charge quantization will be a necessary outcome of* *the construction of a theory (i.e., a Lagrangian) provided that it displays only one unembedded U(1) invariance. If the generator of this single U(1) symmetry is standard weak-hypercharge, then not only is charge quantized but it is quantized correctly.*

The three-generation minimal SM has five U(1) invariances [aside from U(1) subgroups of SU(3)⊗SU(2)]. In addition to standard weak hypercharge, there is baryon number B and the three family lepton numbers L_e, L_μ, and L_τ. If gauge anomaly cancellation is not enforced, then the generator of the gauged U(1) in the minimal SM can be any linear combination of Y, B, and the L_i. This leads to a four-parameter charge quantization problem.

These simple observations provide strong motivation to construct extensions of the minimal SM that explicitly break B and the L_i (but of course leave Y exact). All such models would explain charge quantization in the sense that they simply could not be constructed unless charge was quantized (i.e., some terms in the Lagrangian would have to be absent in order to reinstate B or any of the L_i as a conserved charge). The purpose of this paper is to construct the simplest extensions of the minimal SM that explicitly break B and each of the L_i. Further, we will examine the most stringent phenomenological constraints on these models and thus determine those that are least constrained and hence of most experimental interest. This type of analysis was first performed in detail in Ref. [4]. We will extend the analysis of Ref. [4] and correct an important technical error. This is also a motivation for the present work.

The four parameter charge quantization problem of the minimal SM corresponds to there being four classically undetermined electric charges, which can be taken to be the three neutrino charges and the down quark charge [4]. From experimental data we know that three of these four charges are strongly constrained [5], with only $Q(\nu_\tau)$ being weakly constrained [4,6]. In the following work we seek to remove this four parameter uncertainty by means of simple extensions of the minimal standard model which explicitly break $U(1)_B$ and each of the $U(1)_{L_i}$.

The simplest and most phenomenologically interesting way to explicitly break the $U(1)_{L_i}$ is to introduce nonzero neutrino masses. This is most easily done by introducing right-handed neutrinos into the model. If we choose that our right- and left-handed neutrinos are related through Dirac mass terms, $\mathcal{L}=\lambda\overline{\nu_L}\nu_R+\text{H.c.}$, and if we assume that nontrivial mixing effects occur as in the quark sector, then we obtain the constraint $Q(\nu_e)=Q(\nu_\mu)=Q(\nu_\tau)$. This leaves just two undetermined electric charges, which can be taken to be $Q(\nu_e)$ and $Q(d)$, corresponding to the as yet unbroken global symmetries $U(1)_L$ and $U(1)_B$ where $L=L_e+L_\mu+L_\tau$ is total lepton number. If we then add a Majorana mass term $\mathcal{L}=\lambda\overline{\nu_R}(\nu_R)^c$, for one or more of the right-handed neutrinos we obtain the additional constraint $Q(\nu_e)=0$ [7]. Put another way, the Majorana mass terms explicitly break $U(1)_L$. This leaves just one undetermined electric charge, which can be taken to be the electric charge of the down quark, $Q(d)$. Our four-parameter uncertainty has therefore been reduced to a one-parameter uncertainty by this simple extension of the lepton sector.

Our remaining global symmetries are the hypercharge $U(1)_Y$ and the baryon number $U(1)_B$. Hence, assuming that anomaly cancellation is unnecessary, any combination of Y and B can be the U(1) symmetry that is gauged. To obtain complete electric charge quantization we require that this unwanted baryon number symmetry somehow be broken without affecting the $U(1)_Y$ hypercharge symmetry. This double requirement rules out the introduction of baryon number violating Majorana quark mass terms, because unlike their lepton counterparts such terms will result in the violation of standard hypercharge and color.

To achieve charge quantization in the simplest way, using only the standard model gauge symmetry, we therefore require the addition of a new scalar that incorporates the dual requirements of baryon number violation and hypercharge conservation [4]. The violation of baryon number requires that this new scalar interact with quarks, and assuming the usual dimension four (Yukawa-type) couplings there is a finite list of possible quantum numbers for this scalar. Since the scalar couples to a fermion bilinear, it follows from gauge invariance that the quantum numbers of the scalar are those of the fermion bilinears. For example a scalar σ_1 coupling via the interaction term $\mathcal{L}=\lambda\sigma_1^\dagger\overline{Q_L}(f_L)^c$ implies that σ_1 transforms as $\overline{Q_L}(f_L)^c$. Following such a procedure all possible scalars in terms of fermion bilinears can be found (see Ref. [4]). These scalars together with their $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$ representations are listed below:

$$\sigma_{1.1}\sim\overline{Q_L}(f_L)^c\sim\overline{u_R}(e_R)^c\sim\overline{d_R}(\nu_R)^c\sim(\overline{3},1,-y_d)(-1/3), \tag{4}$$

$$\sigma_{1.2}\sim\overline{Q_L}(f_L)^c\sim(\overline{3},3,-y_d)(-1/3),$$

$$\sigma_2\sim\overline{Q_L}e_R\sim\overline{u_R}f_L\sim(\overline{3},2,-3-y_d)(-1/3),$$

$$\sigma_{3.1}\sim\overline{Q_L}(Q_L)^c\sim\overline{u_R}(d_R)^c\sim(3,1,-2-2y_d)(-2/3),$$

$$\sigma_{3.2}\sim\overline{Q_L}(Q_L)^c\sim(3,3,-2-2y_d)(-2/3),$$

$$\sigma_{3.3}\sim\overline{Q_L}(Q_L)^c\sim\overline{u_R}(d_R)^c\sim(\overline{6},1,-2-2y_d)(-2/3),$$

$$\sigma_{3.4}\sim\overline{Q_L}(Q_L)^c\sim(\overline{6},3,-2-2y_d)(-2/3),$$

$$\sigma_4\sim\overline{u_R}(\nu_R)^c\sim(\overline{3},1,-2-y_d)(-1/3),$$

$$\sigma_5\sim\overline{d_R}f_L\sim\overline{Q_L}\nu_R\sim(\overline{3},2,-1-y_d)(-1/3),$$

$$\sigma_{6.1}\sim\overline{u_R}(u_R)^c\sim(3,1,-4-2y_d)(-2/3),$$

$$\sigma_{6.2}\sim\overline{u_R}(u_R)^c\sim(\overline{6},1,-4-2y_d)(-2/3),$$

$$\sigma_{7.1}\sim\overline{d_R}(d_R)^c\sim(3,1,-2y_d)(-2/3),$$

$$\sigma_{7.2}\sim\overline{d_R}(d_R)^c\sim(\overline{6},1,-2y_d)(-2/3),$$

$$\sigma_8\sim\overline{d_R}(e_R)^c\sim(\overline{3},1,2-y_d)(-1/3).$$

Note that we have included the baryon number of the fermion bilinear with which each scalar interacts as the last entry in each line above and we have used the following notation for the standard model fermions and right-handed neutrinos:

$$f_L \sim (1,2,-1), \quad e_R \sim (1,1,-2), \quad \nu_R \sim (1,1,0),$$

$$Q_L \sim (3,2,1+y_d), \quad u_R \sim (3,1,2+y_d), \quad d_R \sim (3,1,y_d), \quad (5)$$

It should also be pointed out that the fermion interactions $\overline{Q_L}(Q_L)^c$, $\overline{Q_c}(Q_L)^c$, $\overline{u_R}(u_R)^c$, $\overline{d_R}(d_R)^c$, associated with the $\sigma_{3.2}$, $\sigma_{3.3}$, $\sigma_{6.1}$, and $\sigma_{7.1}$ scalars are flavor antisymmetric.

Because of the fact that these proposed scalar particles may carry baryon number, the above interactions by themselves will not violate baryon number. Instead we can break baryon number by either proposing the existence of more than one quark-lepton interaction, or alternatively by proposing the existence of two or more scalar multiplets together with their associated interactions.

II. ONE SCALAR EXTENSIONS

In the interests of simplicity Ref. [4] considered the case where just one of these new exotic scalar particles existed, with $U(1)_B$ being broken explicitly in the Higgs potential. Because all of the scalars are either in the **3** or **6** representation of $SU(3)_c$, the only renormalizable terms that break baryon number and conserve $SU(3)_c$ and hypercharge are $\sigma^3\phi$ or $\sigma^3\phi^\dagger$. Since the Higgs doublet ϕ has hypercharge 1 (in our normalization) these scalar potentials require that our scalar particle σ has either a hypercharge of $-1/3$ or 1/3, respectively. Out of all the possibilities listed in Eq. (4) only σ_5 satisfies either of these constraints for the observed value of $y_d=-2/3$. It was thus concluded in Ref. [4] that under the assumption of one exotic scalar and one set of quark lepton interactions, that electric charge can be quantized classically.

Upon closer examination it is, however, found that the scalar potential term $\sigma_5^3\phi$ is in fact zero after antisymmetrization over the $SU(3)_c$ group (this is the error in Ref. [4] alluded to earlier). We must therefore broaden our search for baryon number violating extensions to the standard model that give the desired charge quantization.

We are primarily interested in simple extensions to the model. Thus we will initially continue to search for extensions that require the introduction of just one scalar particle. However, we know from the unsuccessful attempts made in Ref. [4] that the consideration of just one of the interactions shown in Eq. (4) will not provide the required charge quantization. In our quest for charge quantization we must therefore take the next step and consider pairs of interactions in Eq. (4) that can couple to the same scalar in a baryon number violating manner. Two different quark-lepton interactions can only couple to the one scalar if the group properties of this scalar are compatible with both interactions. If this compatibility is subject to the strict condition that $y_d=-2/3$, then not only do we have two sets of interactions which can be associated with the one scalar, but we also obtain charge quantization; i.e., the two sets of interactions provide us with the baryon number violation required for charge quantization. An example of such a pair is $\sigma_{1.2}$ and $\sigma_{3.2}^\dagger$, which are equal subject to the constraint $y_d=-2/3$ as desired. Thus $\sigma_{1.2}$ and $\sigma_{3.2}$ can be conjugate representations of the same particle, call it σ, with the lepton and quark interactions

$$\mathcal{L}=\lambda_1\overline{(f_L)^c}\sigma Q_L+\lambda_2\overline{Q_L}\sigma(Q_L)^c+\text{H.c.} \quad (6)$$

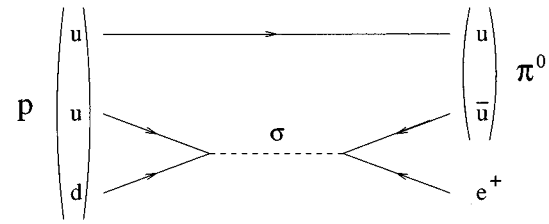

FIG. 1. The proton decay diagram resulting from conjugate pair $\sigma_{1.2}-\sigma_{3.2}^c$.

Because $\overline{(f_L)^c}\sigma Q_L$ and $\overline{Q_L}\sigma(Q_L)^c$ have different baryon numbers, $U(1)_B$ is explicitly broken.

All together there are three possible conjugate pairs, which we list below:

$$\sigma_{1.1}=\sigma_{3.1}^c \sim (\overline{3},1,2/3), \quad \sigma_{1.2}=\sigma_{3.2}^c \sim (\overline{3},3,2/3),$$

$$\sigma_4=\sigma_{7.1}^c \sim (\overline{3},1,-4/3). \quad (7)$$

For each of the above conjugate pairs there will be a set of quark lepton interactions obtainable from Eq. (4) [for example, the second pair has the interactions shown in Eq. (6)]. From these quark lepton interactions it is easy to see that baryon number violation of magnitude $\Delta B=1$ will occur, and as such these quark lepton interactions will give rise to nucleon decay [see Fig. 1]. The simplest Feynman diagrams leading to nucleon decay processes involve the production of one meson and one antilepton (Fig. 1) with decay width (evaluated from dimensional considerations only) of the form

$$\Gamma=O\left(\frac{\lambda^4 M_N^5}{M_\sigma^4}\right). \quad (8)$$

M_N represents the nucleon mass, M_σ the scalar particle mass, and the dimensionless $\lambda^4\equiv\lambda_1^2\lambda_2^2$ factor represents the contribution made by the Yukawa coupling constants [see Eq. (6)]. We can obtain a limit on the scalar mass by comparing the above decay width with the experimentally known lower limit on the proton lifetime $\approx 10^{32}$ yr [8]. This gives a lower limit on M_σ of

$$M_\sigma > \lambda \times 10^{16} \text{ GeV}. \quad (9)$$

This lower limit, however, does not hold true for the $\sigma_4-\sigma_{7.1}^c$ conjugate pair containing the left-handed antineutrino producing σ_4 scalar. The production of left-handed antineutrinos will result in there being an extra suppression factor in Eq. (8), which will consequently give rise to a less strict lower limit on the scalar mass; this special case will be considered later.

III. TWO SCALAR EXTENSIONS

The above lower limit on M_σ (which was obtained by proposing a one scalar particle extension to the standard model) is very large, and as such is phenomenologically uninteresting. In light of this it is desirable to consider the existence of two new scalar particles, in the hope of obtaining a phenomenologically more interesting model. Although this extension will complicate our model by requiring the introduction of scalar potential terms, there will still only be two sets of fermion-scalar interactions as in Eq. (6).

For example, a two scalar model can be used to resolve the antisymmetrization problem alluded to earlier; i.e., the zero value of the scalar potential term $\sigma_5^3\phi$. This is achieved by proposing the existence of two species of scalar with the same group transformations as σ_5, which we will call σ_a and σ_b. These particles will couple to leptons and quarks through the Lagrangian

$$\mathcal{L}=\lambda_1^a\overline{f_L}\sigma_a d_R+\lambda_1^b\overline{f_L}\sigma_b d_R+\lambda_2^a\overline{Q_L}\sigma_a^c\nu_R+\lambda_2^b\overline{Q_L}\sigma_b^c\nu_R+\text{H.c.}, \tag{10}$$

and will combine in the nonzero hypercharge constraining scalar potential term

$$\Delta V(\phi,\sigma_a,\sigma_b)=\lambda\sigma_a\sigma_b^2\phi+\text{H.c.} \tag{11}$$

The σ_a and σ_b scalar pair is just one of many combinations of scalar particles in Eq. (4) that violate baryon number while giving the correct hypercharge assignment to the down quark $y_d=-2/3$. However, unlike the σ_a and σ_b pair, these other combinations will in general involve scalars with different group transformation properties. By considering every possible scalar combination in Eq. (4), two lists of possible charge quantizing scalar potentials can be compiled, corresponding to $\Delta B=1$ baryon number violating processes and $\Delta B=2$ baryon number violating processes, respectively. The $\Delta B=1$ list is shown below:

$$\begin{aligned}
&\sigma_1,\sigma_2\rightarrow\sigma_{1.2}\sigma_{1.2}\sigma_2\phi,\\
&\sigma_1,\sigma_3\rightarrow\sigma_{1.1}\sigma_{3.1}+\sigma_{1.1}\sigma_{1.1}^c\sigma_{1.1}\sigma_{3.1}+\sigma_{1.1}\sigma_{3.1}\sigma_{3.1}^c\sigma_{3.1}\\
&\qquad+6_{1.1}6_{3.1}\phi^+\phi\\
&\qquad\rightarrow\sigma_{1.2}\sigma_{3.2}+\sigma_{1.2}\sigma_{1.2}^c\sigma_{1.2}\sigma_{3.2}+\sigma_{1.2}\sigma_{3.2}\sigma_{3.2}^c\sigma_{3.2}\\
&\qquad+6_{1.2}6_{3.2}\phi^+\phi,\\
&\sigma_1,\sigma_5\rightarrow\sigma_{1.1}\sigma_5\sigma_5+\sigma_{1.2}\sigma_{1.2}\sigma_5\phi^c,\\
&\sigma_1,\sigma_6\rightarrow\sigma_{1.2}\sigma_{6.1}\phi\phi,\quad \sigma_1,\sigma_7\rightarrow\sigma_{1.2}\sigma_{7.1}\phi^c\phi^c,\\
&\sigma_2,\sigma_3\rightarrow\sigma_2^c\sigma_{3.2}\sigma_{3.2}\phi^c,\quad \sigma_2,\sigma_7\rightarrow\sigma_2\sigma_{7.1}\phi,\\
&\sigma_3,\sigma_4\rightarrow\sigma_{3.2}\sigma_4\phi\phi,\quad \sigma_3,\sigma_5\rightarrow\sigma_{3.1}\sigma_5\phi+\sigma_{3.2}\sigma_{3.2}\sigma_5^c\phi,\\
&\sigma_3,\sigma_8\rightarrow\sigma_{3.2}\sigma_8\phi^c\phi^c,\\
&\sigma_4,\sigma_7\rightarrow\sigma_4\sigma_{7.1}+\sigma_4\sigma_4^c\sigma_4\sigma_{7.1}+\sigma_{7.1}\sigma_{7.1}^c\sigma_{7.1}\sigma_4\\
&\qquad+\sigma_4\sigma_{7.1}\phi^+\phi,\\
&\sigma_5,\rho\rightarrow\sigma_5\sigma_5\sigma_5\rho,\quad \sigma_5^a,\sigma_5^b\rightarrow\sigma_5^a\sigma_5^b\sigma_5^b\phi,\\
&\sigma_5,\sigma_7\rightarrow\sigma_5\sigma_{7.1}\phi^c,\quad \sigma_6,\sigma_8\rightarrow\sigma_{6.1}\sigma_8,\quad +6_{6.1}6_{6.1}^c6_{6.1}6_8\\
&\qquad+6_8 6_8^c 6_8 6_{6.1}+6_{6.1}6_8\phi^+\phi,
\end{aligned} \tag{12}$$

where ϕ represents the SM Higgs scalar $\phi\sim(1,2,1)$, and ρ represents a new Higgs-like scalar $\rho\sim(8,2,1)$. The scalar ρ is a new nonstandard model particle (like σ) which differs from the Higgs scalar in that it carries color charge; in fact it transforms as an **8** under $SU(3)_c$. This means that unlike the Higgs particle ϕ, ρ will not take part in symmetry breaking and will not form a VEV. Apart from this color structure, it has similar Yukawa couplings to ϕ.

The above scalar potential terms can be placed into groups consisting of quadratic, cubic, and quartic terms, with each subgroup giving rise to its own characteristic expression for the proton decay width. By comparing these decay widths with the known lower limits on the nucleon lifetimes, each subgroup will give its own particular constraint on the masses of the scalar particles involved. We will again be using dimensional arguments.

Note that in the following analysis we will initially be ignoring terms involving the σ_4 scalar, due to the complications associated with the production of the right-handed neutrino and left-handed antineutrino.

Our analysis begins with the quadratic potential terms $\mu^2\sigma_i\sigma_j$ [see Eq. (12)], where μ^2 is the coupling constant with dimensions of mass squared. These two particle interactions can be considered as constituting the off-diagonal elements of the exotic scalar mass matrix. The simplest nucleon decay processes that can be obtained from these bilinears involve the decay of the nucleon into a meson and an antilepton. For example, the proton decay diagram resulting from the $\sigma_{1.1}\sigma_{3.1}$ bilinear is shown in Fig. 2. From dimensional arguments these bilinears give rise to decay widths of the form

$$\Gamma\simeq O\left(\frac{\lambda^4\mu^4M_N^5}{M_{\sigma_i}^4M_{\sigma_j}^4}\right). \tag{13}$$

In this case the dimensionless constant $\lambda^4\equiv\lambda_i^2\lambda_j^2$, where λ_i and λ_j represent the Yukawa couplings associated with σ_i and σ_j. If we compare this decay width with experimental lower limits on the proton lifetime, i.e., $\approx10^{32}$ yr, we obtain a lower limit on M_σ of

$$M_\sigma>(\lambda\mu/M_\sigma)\times10^{16}\ \text{GeV}, \tag{14}$$

where we have expressed the coupling constant μ in terms of the scalar mass M_σ.

The simplest nucleon decay processes resulting from the Higgs doublet containing quadratic terms, $b\langle\phi\rangle\sigma_i\sigma_j$, involve the decay of a nucleon into a lepton and a meson [see Fig. 3 for example]. The decay widths for these processes take the form

$$\Gamma\simeq O\left(\frac{\lambda^4b^2\langle\phi\rangle^2M_N^5}{M_{\sigma_i}^4M_{\sigma_j}^4}\right). \tag{15}$$

In this case the coupling constant b has units of mass and the Yukawa coupling constants have again been taken into consideration via the $\lambda^4\equiv\lambda_i^2\lambda_j^2$ factor. The above decay processes are experimentally constrained by a lifetime lower limit of $\approx10^{31}$ yr [8], which gives rise to a constraint on M_σ of

$$M_\sigma>(\lambda^2b/M_\sigma)^{1/3}\times10^{11}\ \text{GeV}. \tag{16}$$

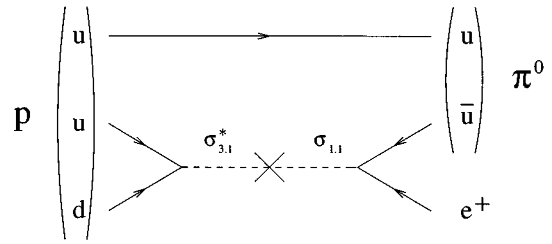

FIG. 2. Proton decay resulting from the $\sigma_{1.1}\sigma_{3.1}$ quadratic.

The simplest nucleon decay process resulting from the quadratic terms with two Higgs scalars, i.e., $\lambda\langle\phi\rangle\langle\phi\rangle\sigma_i\sigma_j$, involve the creation of a meson and an antilepton product. The decay widths for these processes take the form

$$\Gamma\simeq O\left(\frac{\lambda^6\langle\phi\rangle^4 M_N^5}{M_{\sigma_i}^4 M_{\sigma_j}^4}\right), \tag{17}$$

where the λ^6 factor represents the combined contribution made by the $\lambda^4\equiv\lambda_i^2\lambda_j^2$ Yukawa coupling and the λ^2 exotic scalar coupling. These nucleon decay processes are best constrained by the $\tau_n>10^{32}$ yr bound neutron decay limit [8], giving rise to the constraint

$$M_\sigma>\lambda^{3/4}\times 10^9 \text{ GeV}. \tag{18}$$

The above analysis is not, however, valid for the $\langle\phi\rangle\langle\phi\rangle\sigma_{1.2}\sigma_{6.1}$ bilinear, where we have an additional complication resulting from the fact that this bilinear necessarily gives rise to the production of charm quarks. The production of charm containing mesons from nucleon decay is of course kinematically forbidden. Thus our simplest nucleon decay process must involve an additional $c\rightarrow u$ conversion that will inhibit the decay width shown in Eq. (17), by an additional $G_F^2 M_N^4$ factor, where G_F is the Fermi coupling constant. By taking this additional complication into consideration it is found that the lower limit on M_σ in the case of the $\langle\phi\rangle\langle\phi\rangle\sigma_{1.2}\sigma_{6.1}$ bilinear is

$$M_\sigma>\lambda^{3/4}\times 10^8 \text{ GeV}. \tag{19}$$

We next consider the cubic terms, $b\sigma_i\sigma_j\sigma_j$ and $\lambda\langle\phi\rangle\sigma_i\sigma_j\sigma_j$. The simplest nucleon decay diagrams arising from the former of these terms involves the decay of a nucleon into one meson, two leptons, and one antilepton. On the other hand the latter, Higgs boson containing, cubic term gives rise to decays involving the production of two mesons, and one antilepton (see Fig. 4), or in the special case of $\langle\phi\rangle\sigma_2\sigma_{1.2}\sigma_{1.2}$, one meson, two antileptons, and one lepton. The order of magnitude decay widths for the $b\sigma_i\sigma_j\sigma_j$ and the $\lambda\langle\phi\rangle\sigma_i\sigma_j\sigma_j$ cubics are

$$\Gamma\simeq O\left(\frac{\lambda^6 b^2 M_N^{11}}{M_{\sigma_j}^8 M_{\sigma_i}^4}\right) \tag{20}$$

and

$$\Gamma\simeq O\left(\frac{\lambda^8\langle\phi\rangle^2 M_N^{11}}{M_{\sigma_j}^8 M_{\sigma_i}^4}\right), \tag{21}$$

respectively. Note that we have included the dimensionless Yukawa coupling constants as a $\lambda^6\equiv\lambda_i^2\lambda_j^4$ factor. The nucleon decay processes resulting from these cubic terms are constrained by an experimental limit of around $\approx 10^{31}$ yr [8]. Therefore the lower limits on M_σ for the $b\sigma_i\sigma_j\sigma_j$ and the $\lambda\langle\phi\rangle\sigma_i\sigma_j\sigma_j$ cubics are

$$M_\sigma>(\lambda^3 b/M_\sigma)^{1/5}\times 10^6 \text{ GeV} \tag{22}$$

and

$$M_\sigma>\lambda^{2/3}\times 10^5 \text{ GeV}, \tag{23}$$

respectively.

Finally we have the quartic terms $\lambda\sigma_i\sigma_i^c\sigma_i\sigma_j$. The simplest nucleon decay processes arising from these terms involves the decay of the nucleon into either one meson, two antileptons, and one lepton; or, depending on the scalars involved, two mesons and one antilepton. The decay width for these processes takes the form

$$\Gamma\simeq O\left(\frac{\lambda^{10} M_N^{17}}{M_{\sigma_i}^{12} M_{\sigma_j}^4}\right), \tag{24}$$

where $\lambda^{10}\equiv\lambda^2\lambda_i^6\lambda_j^2$. These quartic nucleon decay processes are experimentally constrained by an approximately 10^{31} yr lower limit, giving the constraint

$$M_\sigma>\lambda^{5/8}\times 10^4 \text{ GeV}. \tag{25}$$

The simplest nucleon decay processes resulting from the quartic $\lambda\sigma_5\sigma_5\sigma_5\rho$, and the closely related cubic $\lambda\sigma_5^a\sigma_5^b\sigma_5^b\langle\phi\rangle$, entail the creation of a one meson and three lepton product. The respective decay widths of these two nucleon decay processes are shown below:

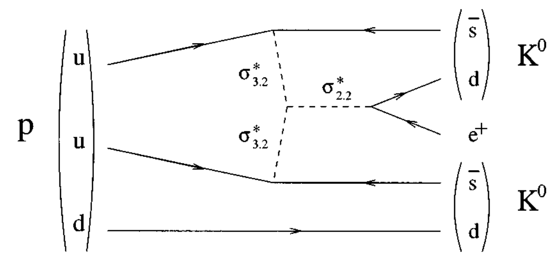

FIG. 4. Proton decay resulting from the $\langle\phi\rangle\sigma_{2.2}^c\sigma_{3.2}\sigma_{3.2}$ cubic.

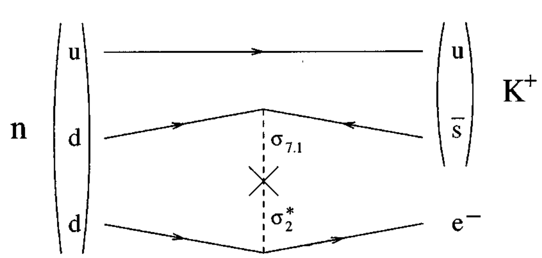

FIG. 3. Neutron decay resulting from the $\langle\phi\rangle\sigma_2\sigma_{7.1}$ quadratic.

$$\Gamma \simeq O\left(\frac{\lambda^{10} M_N^{17}}{M_{\sigma_5}^{12} M_\rho^4}\right), \tag{26}$$

$$\Gamma \simeq O\left(\frac{\lambda^{8}\langle\phi\rangle^2 M_N^{11}}{M_{\sigma_5^a}^{4} M_{\sigma_5^b}^8}\right). \tag{27}$$

Both of these decay widths have a dimensionless $\lambda^8 \equiv \lambda^2 \lambda_5^6$ contribution, with the former expression also containing a λ^2 contribution from the Yukawa constant associated with the ρ scalar. For both potentials the strictest experimental constraint on $M_{\sigma_5,\rho}$ comes from bound neutron decay processes (see Fig. 5), which have a lower lifetime limit of about 10^{31} yr [8]. From this constraint we obtain lower limits on $M_{\sigma,\rho}$ of

$$M_{\sigma,\rho} > \lambda^{5/8} \times 10^4 \text{ GeV} \tag{28}$$

and

$$M_\sigma > \lambda^{2/3} \times 10^5 \text{ GeV}, \tag{29}$$

respectively. This lower limit of $M_{\sigma,\rho} > \lambda^{5/8} \times 10^4$ GeV is the lowest constraint on M_σ for any of the $\Delta B = 1$ scalar pairs listed in Eq. (12).

Before leaving these $\Delta B = 1$ processes, we still have to consider the interesting case where one of the decay products is necessarily a right-handed neutrino or a left-handed antineutrino. This occurs for all of the σ_4 containing potentials in Eq. (12). We assume the usual seesaw model where ν_R gains a large Majorana mass M that leads to the usual hierarchy of masses [9],

$$m_{\nu_R} \simeq M \gg m \gg m_{\nu_L} \simeq \frac{m^2}{M}, \tag{30}$$

where m is the Dirac mass of the neutrinos, as given in the Dirac mass term $m\overline{\nu_L}\nu_R$, and M is the Majorana mass of the neutrinos, as given in the Majorana mass term $M\overline{\nu_R}(\nu_R)^c$. The massiveness of this right-handed neutrino means that any proton decay producing such a particle will be highly suppressed, as the amount of left-hand–right-hand neutrino mixing will be very small. By assuming that the Dirac mass of the neutrino is around the same as the Dirac masses of the other fermions, it is found that the suppression in the decay width will be of order $\approx 10^{-14}$.

By considering this additional attenuation, our lower limits on M_σ for the σ_4 containing interactions are found to reduce to $\lambda \times 10^{12}$ GeV for the conjugate pair σ_4-$\sigma_{7.1}^c$; to $(\lambda\mu/M_\sigma)\times 10^{12}$ GeV for the quadratic $\sigma_4\sigma_{7.1}$; to $\lambda^{3/4}\times 10^7$ GeV for the quadratic $\sigma_{3.2}\sigma_4\phi\phi$; to $\lambda^{5/8}\times 10^3$ GeV for the quartic $\sigma_{7.1}\sigma_{7.1}^c\sigma_{7.1}\sigma_4$; and to $\lambda^{5/8}\times 10^2$ GeV for the quartic $\sigma_4\sigma_4^c\sigma_4\sigma_{7.1}$. For the scalar combination $\sigma_4,\sigma_{7.1}$ our strongest constraint $M_\sigma > (\lambda\mu/M_\sigma)\times 10^{12}$ GeV, has thus been reduced in comparison to the $(\lambda\mu/M_\sigma)\times 10^{16}$ GeV constraint obtained for the quadratics in Eq. (12) without the σ_4 scalar.

The $\Delta B = 2$ scalar potentials, which consist of cubic and quartic terms are shown below:

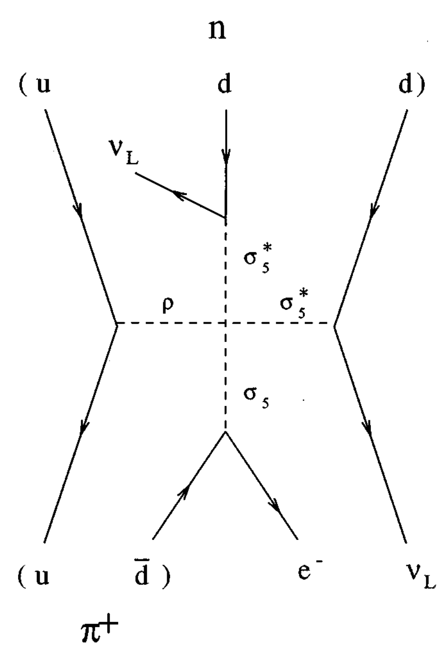

FIG. 5. Neutron decay resulting from the $\sigma_5\sigma_5\sigma_5\rho$ interaction.

$$\begin{aligned}
\sigma_1,\sigma_3 \to\ & \sigma_{1.1}\sigma_{3.1}\sigma_{1.1}\sigma_{3.1} + \sigma_{1.1}\sigma_{3.2}\sigma_{1.1}\sigma_{3.2} \\
& + \sigma_{1.1}\sigma_{3.3}\sigma_{1.1}\sigma_{3.3} + \sigma_{1.1}\sigma_{3.4}\sigma_{1.1}\sigma_{3.4} \\
& + \sigma_{1.2}\sigma_{3.1}\sigma_{1.2}\sigma_{3.1} + \sigma_{1.2}\sigma_{3.2}\sigma_{1.2}\sigma_{3.2} \\
& + \sigma_{1.2}\sigma_{3.3}\sigma_{1.2}\sigma_{3.3} + \sigma_{1.2}\sigma_{3.4}\sigma_{1.2}\sigma_{3.4}, \\
\sigma_3,\sigma_7 \to\ & \sigma_{3.1}\sigma_{3.1}\sigma_{7.2} + \sigma_{3.2}\sigma_{3.2}\sigma_{7.2} + \sigma_{3.3}\sigma_{3.3}\sigma_{7.2} \\
& + \sigma_{3.4}\sigma_{3.4}\sigma_{7.2}, \\
\sigma_4,\sigma_7 \to\ & \sigma_4\sigma_{7.1}\sigma_4\sigma_{7.1} + \sigma_4\sigma_{7.2}\sigma_4\sigma_{7.2}, \\
\sigma_6,\sigma_7 \to\ & \sigma_{6.2}\sigma_{7.1}\sigma_{7.1} + \sigma_{6.2}\sigma_{7.2}\sigma_{7.2}.
\end{aligned} \tag{31}$$

Unlike the $\Delta B = 1$ processes, these $\Delta B = 2$ processes will not give rise to nucleon decay. In order to obtain constraints on the scalar masses for these cases we must instead compare the above interactions with $\Delta B = 2$ experimental limits, such as binucleon decay measurements.

The cubic terms in Eq. (31), i.e., $b\sigma_i\sigma_j\sigma_j$, give rise to binucleon decay that in the simplest cases result in the production of two mesons. For example, the neutron-neutron decay diagram resulting from the $\sigma_{3.1}\sigma_{3.1}\sigma_{7.2}$ process is shown in Fig. 6. The decay widths for these processes, using a dimensional approach, take the form

$$\Gamma \simeq O\left(\frac{\lambda^6 b^2 M_N^{11}}{M_{\sigma_i}^4 M_{\sigma_j}^8}\right), \tag{32}$$

where M_N represents the mass of the nucleons $\simeq 1$ GeV, and the dimensionless constant $\lambda^6 \equiv \lambda_i^2 \lambda_j^4$, where λ_i and λ_j again represent the Yukawa couplings associated with σ_i and σ_j. There is an approximate 10^{31} yr [8] lower limit on these binucleon decay processes, thus for these trilinear terms we have a lower limit on M_σ of

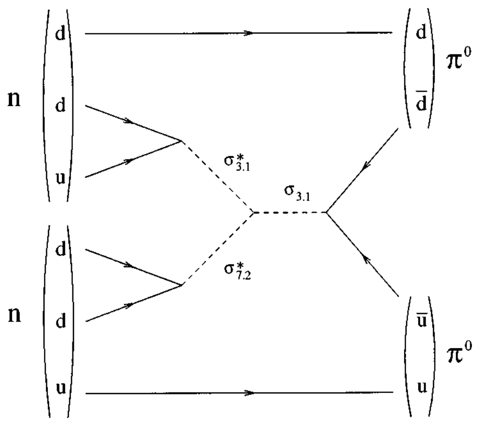

FIG. 6. Double neutron decay resulting from the $\sigma_{3.1}\sigma_{3.1}\sigma_{7.2}$ interaction.

$$M_\sigma > (\lambda^3 b/M_\sigma)^{1/5} \times 10^6 \;\; \text{GeV}. \tag{33}$$

The quartic terms in Eq. (31), i.e., $\lambda \sigma_i \sigma_j \sigma_i \sigma_j$, will give rise to binucleon decays that in the simplest cases will result in the creation of a meson and two antilepton product [see Fig. 7]. The decay widths of these processes take the form

$$\Gamma \simeq O\left(\frac{\lambda^{10} M_N^{17}}{M_{\sigma_i}^8 M_{\sigma_j}^8} \right), \tag{34}$$

where $\lambda^{10} \equiv \lambda^2 \lambda_i^4 \lambda_j^4$. These decay processes are again constrained by an approximate 10^{31} yr limit, giving a lower limit on M_σ of

$$M_\sigma > \lambda^{5/8} \times 10^4 \;\; \text{GeV}. \tag{35}$$

For the σ_4 containing quartics we have an extra supression resulting from the production of two left-handed antineutrinos. The lower limit on M_σ for these quartics is thus reduced to

$$M_\sigma > \lambda^{5/8} \times 10^2 \;\; \text{GeV}. \tag{36}$$

This is the least stringent constraint on M_σ that we have obtained. Therefore the $\sigma_4 - \sigma_{7.2}$ scalar combination is the combination with the weakest constraint on M_σ; the $\sigma_4 - \sigma_{7.1}$ combination is of course strictly constrained by its $\Delta B = 1$ processes.

IV. CONCLUSION

In this paper we have demonstrated how the observed charge quantization can be accounted for solely through classical constraints. In order to obtain complete charge quantization from classical constraints alone, we extended the minimal standard model to include right-handed neutrinos and baryon number violation. We have effectively suggested that the necessity of charge quantization from classical constraints provides a strong argument in favor of the existence of these baryon number violating processes. We considered only simple extensions of the SM Yukawa interaction where our new quark lepton interactions couple through new scalar particles σ.

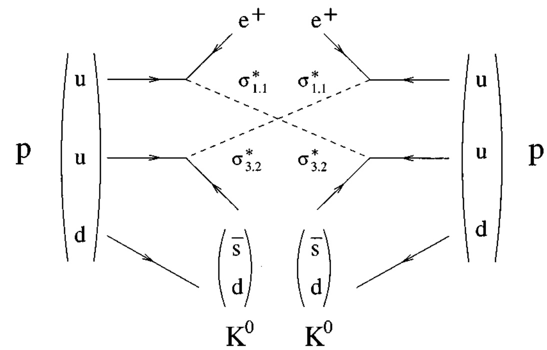

FIG. 7. Double proton decay resulting from the $\sigma_{1.1}\sigma_{3.2}\sigma_{1.1}\sigma_{3.2}$ interaction.

We know from experimental data on the decay of the proton and decay of nuclei that baryon number violation is very much inhibited. Thus, if these new baryon number violating Yukawa interactions exist, the masses of the associated scalars must be above a certain lower limit so as to evade detection by present day experiments. Using a dimensional approach, these lower limits were calculated for all of our proposed scalars and scalar combinations. From this exercise it was found that the scalar pair with the lowest constraints on M_σ is the $\sigma_4 - \sigma_{7.2}$ combination that gives rise to the quartic term listed in Eq. (31). This weakly constrained scalar combination is of interest as the possibility of these scalars appearing in low-energy interactions is not ruled out.

As a result of the fact that present day theories on baryogenesis suggest that baryon number violation must have occurred in the early universe, it would be of some interest to investigate the implications of these baryon number violating processes on baryogenesis.

ACKNOWLEDGMENTS

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an additional (nonlocal) term in the action that exactly cancels the gauge anomaly one calculates with the standard expansion of $\hat{U}$. We would like to emphasize that the additional term in the action is a quantity one *calculates* by rigorously solving the Tomonaga-Schwinger equation. It is not put in by hand, but rather it *has to be there*. One is not modifying the chiral Schwinger model in $1+1$ dimensions in any way; one is simply solving the equations of motion properly. Kieu also presents a plausible argument that the effect he has rigorously calculated in a $(1+1)$-dimensional theory should also occur in a properly solved ''anomalous'' chiral gauge theory in $3+1$ dimensions. In this case the calculation cannot be completed exactly because of technical difficulties. Nevertheless, his reasoning shows that the case for anomaly cancellation as a rigorous requirement for consistency in $3+1$ dimensions has yet to be made.

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