Scale dependence of quark mass matrices in models with flavor symmetries

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Numerical correlations between fermion masses and mixings could indicate the presence of a flavor symmetry at high energies. In general, the search for these correlations using low energy data requires an estimate of leading-log radiative corrections. We present a complete analysis of the evolution between the electroweak and the grand unification scales of quark mass parameters in minimal supersymmetric models. We take $M_t = 180$ GeV and consider all possible values of $\tan\beta$. We also analyze the possibility that the *top* and/or the *bottom* Yukawa couplings result from an intermediate quasifixed point (QFP) of the equations. We show that the quark mixings of the third family do *not* have a QFP behavior (in contrast with the masses, the renormalization of all the mixings is linear), and we evaluate the low energy value of V_{ub} which corresponds to $V_{ub}(M_X)=0$. Then we focus on the renormalization-group corrections to (i) typical relations obtained in models with flavor symmetries at the unification scale and (ii) a superstring-motivated pattern of quark mass matrices. We show that in most of the models the numerical prediction for V_{ub} can be *corrected* in both directions (by varying $\tan\beta$) due to *top* or *bottom* radiative corrections. [S0556-2821(96)00923-X]

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I. INTRODUCTION

The recent observation of the *top* quark, with a mass around 180 GeV [1], allows a more complete analysis of the Yukawa sector of the standard model. In particular, it allows an evaluation of the running to higher energy scales of the parameters in that sector. The perturbative unification of the gauge couplings obtained in minimal supersymmetric (SUSY) scenarios suggests that the (nonsinglet) matter and gauge contents do not change up to energies around 10^{16} GeV. If that is the case, the first step to understand the flavor structure of the standard model is to evolve it up to those energies. We present in the first part of this article an updated and complete analysis of the evolution from low energies to the unification scale M_X of the ten physical parameters in the quark mass matrices [six masses, three mixing angles, and the complex Cabibbo-Kobayashi-Maskawa (CKM) phase] in the minimal SUSY extension of the standard model (MSSM) [2]. We will study in detail the behavior of the mixing angles when the *top* and/or *bottom* Yukawa couplings $h_{t,b}$ are large at M_X. In this regime [3] the renormalization-group corrections *focus* any initial value of the coupling at M_X to a narrow interval δh_t [around $h_t(M_Z)\approx 1.2$] at low energy: $(\delta h_t/h_t)(M_X) \gg (\delta h_t/h_t)(M_Z)$. We will analyze how this nonlinear evolution of the couplings [quasifixed point (QFP) behavior] affects the mixings with the third generation; in particular, we will find the low energy value of the mixing s_{13} (with $V_{ub}=s_{13}e^{-i\delta_{13}}$) that corresponds to $s_{13}=0$ at M_X.

On the other hand, the observed pattern of fermion masses and mixings does not look accidental, and suggests a symmetry in the Yukawa matrices as the origin of the hierarchies. Obviously, such a flavor symmetry should be formulated at the unification scale. Although the Yukawa matrices contain more free parameters than physical observables, it is not easy to find simple structures that are able to accommodate without need of fine tuning the measured values of masses and mixings. As a matter of fact, the authors of Ref. [4] classify the symmetric quark mass matrices with a maximal number of texture zeros, and find that only five textures are acceptable experimentally. These matrices predict correlations between mass parameters that can be expressed in a simple (approximate) form; we will analyze how the correlations run from M_X to low energies. We will also analyze a particular scenario [5] derived from the heterotic string which has been recently proposed as the only realistic possibility among the models within its class. In this scenario (and also in the model proposed in [6], with one more texture zero than those in the cases in [4]), the renormalization-group corrections could be essential to obtain a predicted mixing V_{ub} within the experimental limits.

II. EVOLUTION OF QUARK MASSES AND MIXINGS

The quark Yukawa sector of the MSSM can be expressed in terms of the superpotential

$$P = h^u_{ij} H Q_i u^c_j + h^d_{ij} H' Q_i d^c_j \tag{1}$$

with $H\equiv(H^+H^0)$, $H'\equiv(H'^0H'^-)$, and $Q_i\equiv(u_id_i)$. In that sector there are ten independent physical parameters: once the Yukawa matrices are diagonalized, only the eigenvalues (three in the up and three in the down quark matrices) and the CKM matrix (with three mixing angles and a complex phase) appear in the Lagrangian. The procedure to obtain these parameters at M_X from the pole masses and the low energy mixings is the following. For the heavier quarks, the perturbative pole mass M_i is related to the running mass $m_i(M_i)$ in the modified minimal subtraction ($\overline{\text{MS}}$) scheme by a simple expression [7] (this change of scheme is numerically important due to the large size of α_s). Taking $M_t = 180$ GeV and $M_b = 4.7$ GeV we obtain

$$m_b(M_b) = 0.880 M_b, \quad m_t(M_t) = 0.946 M_t \tag{2}$$

The running masses of the lighter quarks are given at 1 GeV [8]: $m_u = 0.0056$ GeV; $m_d = 0.0099$ GeV; $m_s = 0.199$

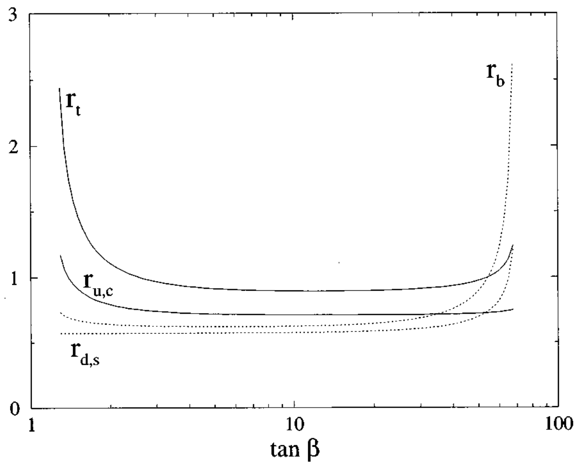

FIG. 1. Ratios $r_i = m_i(M_X)/m_i(M_Z)$ $(i = u,c,t,d,s,b)$ for different values of $\tan\beta$.

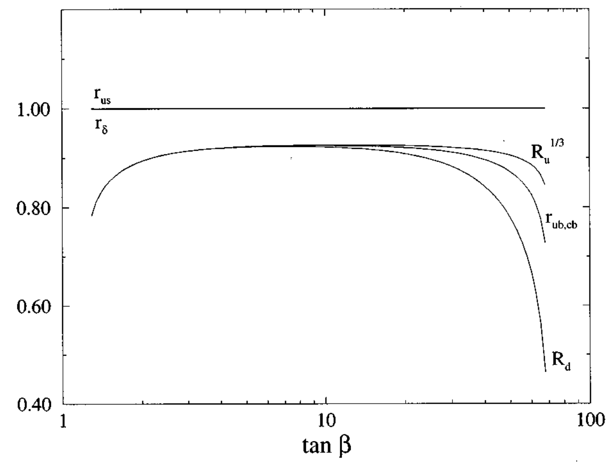

FIG. 2. Ratios $r_{ij} = V_{ij}(M_X)/V_{ij}(M_Z)$ $(ij = us,cb,ub)$, $r_\delta = \arg V_{ub}(M_X)/\arg V_{ub}(M_Z)$, $R_u = [m_{u,c}(M_X)/m_t(M_X)]/[m_{u,c}(M_Z)/m_t(M_Z)]$ and $R_d \equiv [m_{d,s}(M_X)/m_b(M_X)]/[m_{d,s}(M_Z)/m_b(M_Z)]$ for different values of $\tan\beta$.

GeV; $m_c = 1.35$ GeV. The masses evolve up to the lightest Higgs scale $m_h \approx M_Z$ due to gauge interactions only. For $\alpha_{\rm em}(0) = 1/137$ and $\alpha_s(M_Z) = 0.1165$ [8] this gives a factor 0.58 for the values at 1 GeV and 0.75 for $m_b(M_b)$. It will be also convenient to run the *top* quark mass down to that scale; we obtain $m_t(M_Z) = 1.05 m_t(M_t)$.

At M_Z we find the Yukawa couplings to the lightest neutral Higgs boson h, which correspond to the Yukawas in the standard model (we assume $m_h = M_Z \ll m_{\rm SUSY} = 250$ GeV)

$$\tilde{h}^u_{ij} = \frac{m_{u_i}}{174 \ \text{GeV}} \delta_{ij}, \quad \tilde{h}^d_{ij} = \frac{m_{d_i}}{174 \ \text{GeV}} V^*_{ij}, \tag{3}$$

V_{ij} being the CKM matrix at M_Z. From M_Z to M_t these Yukawas evolve due to the gauge and the Yukawa interactions of the light quark and leptons (all of them included in our equations); between M_t and $m_{\rm SUSY}$ the *top* Yukawa corrections are also important. From $m_{\rm SUSY}$ up to $M_X \approx 10^{16.2}$ GeV (the unification scale that corresponds to $\sin^2\theta_W = 0.23195$) we include the interactions of the SUSY particles and the extra Higgs scalars (the standard model and MSSM renormalization group equations can be found in Refs. [9,10], respectively). The Yukawa couplings to the two Higgs doublets are (at $m_{\rm SUSY}$)

$$h^u_{ij} = \frac{\tilde{h}^u_{ij}}{\sin\beta}, \quad h^d_{ij} = \frac{\tilde{h}^d_{ij}}{\cos\beta}, \tag{4}$$

where $\tan\beta$ is the ratio of vacuum expectation values (VEV's) giving mass to *down* and *up* quarks. At M_X we diagonalize the Yukawa matrices (we express the eigenvalues as h_i, with $i = u,d,\dots$) and find the CKM matrix. We will neglect corrections to mixing angles and light quark masses [11] proportional to the *soft* SUSY-breaking parameters, although these corrections can be significant in the large $\tan\beta$ regime [12].

We present in Figs. 1 and 2 the evolution of masses and mixings for values of $\tan\beta$ between 1.29 and 67.7, which correspond, respectively, to $h_t/4\pi$ and $h_b/4\pi$ equal to 0.25 at M_X (we analyze below in detail the quasifixed point regions). In Fig. 1 we plot $r_i \equiv m_i(M_X)/m_i(M_Z)$ for the six quarks (note that the ratios of masses and Yukawas coincide), whereas Fig. 2 expresses the ratios $r_{ij} \equiv V_{ij}(M_X)/V_{ij}(M_Z)$ $(ij = us,cb,ub)$ and $r_\delta \equiv \arg[V_{ub}(M_X)]/\arg[V_{ub}(M_Z)]$ (in the Maiani parametrization). In order to compare the relative renormalization of the mixings and different ratios of masses, we also plot $R_u^{1/3}$, R_d in Fig. 2, where $R_u \equiv [m_{u,c}(M_X)/m_t(M_X)]/[m_{u,c}(M_Z)/m_t(M_Z)]$ and $R_d \equiv [m_{d,s}(M_X)/m_b(M_X)]/[m_{d,s}(M_Z)/m_b(M_Z)]$. We take all the masses and mixings at M_Z in their central value ($V_{us} = 0.221 \pm 0.003$, $V_{cb} = 0.040 \pm 0.008$, and $|V_{ub}| = 0.0035 \pm 0.0015$), with $M_t = 180$ GeV and $\arg(V_{ub}) = -\pi/2$.

We observe the following behavior.

(i) The evolutions of m_u and m_c coincide at the 0.04%; the same happens for m_d and m_s (0.06%) and for $|V_{ub}|$ and V_{cb} (0.05%).

(ii) The running of the Cabibbo mixing V_{us} and of the CKM phase (in V_{ub}) are smaller than the 0.03% and 0.07%, respectively. The evolution of masses and mixings is insensitive to the value of the complex phase.

(iii) The approximation [4]

$$\frac{V_{ij}(M_X)}{V_{ij}(M_Z)} = \chi \quad (ij = us,cb,ub), \tag{5a}$$

$$\frac{m_{u,c}(M_X)/m_t(M_X)}{m_{u,c}(M_Z)/m_t(M_Z)} = \chi^3, \tag{5b}$$

$$\frac{m_{d,s}(M_X)/m_b(M_X)}{m_{d,s}(M_Z)/m_b(M_Z)} = \chi, \tag{5c}$$

is excellent (error smaller than 1%) for $\tan\beta \le 5$. These expressions, with $\chi = (M_X/M_Z)^{(h_t/4\pi)^2}$, are obtained assuming that the *top* Yukawa coupling is dominant and constant between M_Z and M_X.

As showed in [3], the low energy value of a large Yukawa coupling could be related to an intermediate QFP (previous to the Pendleton-Ross infrared fixed point [13]) of the renormalization-group equations. The effect of the QFP

TABLE I. Experimental value at M_Z of quark mass ratios (the central values) and CKM mixings (lower and upper limits).

$V_{us}(M_Z)$	0.218–0.224
$\sqrt{m_u/m_c}(M_Z)$	0.0609
$\sqrt{m_d/m_s}(M_Z)$	0.224
$V_{ub}(M_Z)$	0.002–0.005
$\sqrt{m_u/m_t}(M_Z)$	0.0040
$\sqrt{m_d/m_b}(M_Z)$	0.043
$V_{cb}(M_Z)$	0.032–0.048
$\sqrt{m_c/m_t}(M_Z)$	0.066
$\sqrt{m_s/m_b}(M_Z)$	0.193

would be to *focus* any large initial value (at M_X) of the coupling to a narrow region at M_Z. In the MSSM, the *top* coupling approaches a QFP value for $\tan\beta\approx 1$, and the *bottom* coupling may approach a QFP value for large $\tan\beta$. For a *top* mass on its upper experimental limit, *both* couplings would be attracted to QFP's [14] of the equations. The first case [large $h_t(M_X)$ and low or moderate $\tan\beta$] has been analyzed in [15], where an approximate expression is found for the QFP value of the *top* quark mass:

$$m_t(M_t)=196\ \text{GeV}\ \{1+2[\alpha_s(M_Z)-0.12]\}\ \sin\beta. \qquad (6)$$

For $\alpha_s(M_Z)=0.1165$ this expression gives $h_t(M_Z)=1.18$.

In Tables I–III we show in some detail the behavior of quark masses and mixings near the QFP. Table I expresses the initial values (at M_Z) of masses and mixings, whereas in Table II we write the values at M_X for different values of $\tan\beta$–which correspond to $h_t(M_X)=(2,8)$ [i.e., $h_t(M_X)/4\pi=(0.16,0.64)$] and $h_b(M_X)=(2,8)$–for $M_t=180$ GeV. In Table III both couplings are equal to 2 and/or 8 at M_X (this implies $M_t=190$–220 GeV and $\tan\beta=60$–69). In all cases, the large *top* (and/or *bottom*) Yukawa coupling at M_X is attracted to a QFP value around 1.2: $(\delta h_t/h_t)(M_Z)\approx 0.06$ for any value of $h_t/4\pi(M_X)$ larger than 0.32. Although for such large values of the couplings this attraction is quite effective, it is not *complete*; $h_t(M_Z)$ has some dependence on the initial (large) value of $h_t(M_X)$, giving variations that can increase its actual value by 10% with respect to the (approximate) QFP value given above. A more effective attraction would be obtained for smaller $h_t(M_X)$ or larger M_X. The evolution of the mixing angles for Yukawa couplings near the QFP value presents the following features.

(i) The running of the mixings with the third family (and also the Cabibbo mixing) is highly linear, in the sense that $(\delta V_{ij}/V_{ij})(M_X)=(\delta V_{ij}/V_{ij})(M_Z)$, with $ij=us,ub,cb$. This fact means that the angles do not have a QFP behavior. For example, for $h_t(M_X)=8$ the interval $V_{ub}(M_Z)=0.002$–0.005 evolves to $V_{ub}(M_X)=0.00135$–0.00337.

The evolution of V_{us} is still smaller than 0.04%. The difference between the running of V_{cb} and V_{ub} and of the *up* and *charm* (*down* and *strange*) Yukawas, smaller than 0.02% and 0.04% (0.03%), respectively, do not grow when increasing the Yukawa couplings at M_X.

(ii) The mixings with the third family V_{ib} and the ratios $\sqrt{m_{u,c}/m_t}$ and $\sqrt{m_{d,s}/m_b}$ tend to zero at M_X when $h_t(M_X)$ and/or $h_b(M_X)$ increase. However, when increasing for example h_t, the three quantities decrease at different rates. As a consequence, it will be always possible to *correct* a relation between masses and mixings by varying $\tan\beta$ and going to the adequate (*top* or *bottom*) QFP region (see next section).

(iii) To illustrate the size of the nonlinear effects (first point above), we will consider the case when V_{ub} vanishes at M_X. The running from M_X to M_Z generates then a nonzero mixing. In Fig. 3 we plot $V_{ub}(M_Z)$ for $\tan\beta$ between 1.29 and 67.7, with the fermion masses and the rest of mixings in their central values. We find a value much smaller than the experimentally preferred, although it grows when $\tan\beta$ decreases [i.e., for a fixed m_t, $h_t(M_X)$ increases]. We also show in Table II the low energy value of V_{ub} which corresponds in each case to a vanishing mixing at M_X. Note that

TABLE II. Value at M_X of quark mass ratios and CKM mixings for values of $\tan\beta$ which correspond to large h_t or h_b at M_X ($M_t=180$ GeV). In the last line we write the value of $V_{ub}(M_Z)$ which would correspond to $V_{ub}(M_X)=0$. We include the value of h_τ.

$\tan\beta$	1.22	1.44	65.0	69.5
$h_t(M_Z)$	1.33	1.25	1.03	1.03
$h_b(M_Z)$	0.028	0.031	1.16	1.24
$h_\tau(M_Z)$	0.016	0.018	0.653	0.697
$h_t(M_X)$	8	2	1.17	1.51
$h_b(M_X)$	0.024	0.021	2	8
$h_\tau(M_X)$	0.013	0.015	1.13	2.65
V_{us}	0.218–0.224	0.218–0.224	0.218–0.224	0.218–0.224
$\sqrt{m_u/m_c}$	0.0609	0.0609	0.0609	0.0608
$\sqrt{m_d/m_s}$	0.224	0.224	0.224	0.224
V_{ub}	0.00135–0.00337	0.00178–0.0042	0.00156–0.00390	0.00125–0.00313
$\sqrt{m_u/m_t}$	0.00223	0.00310	0.00325	0.00289
$\sqrt{m_d/m_b}$	0.0355	0.0396	0.0325	0.0236
V_{cb}	0.0216–0.0324	0.0269–0.0403	0.0249–0.0374	0.0200–0.0301
$\sqrt{m_c/m_t}$	0.0366	0.0509	0.0533	0.0474
$\sqrt{m_s/m_b}$	0.159	0.177	0.146	0.106
$V_{ub}(M_Z)$	6.56×10^{-6}	3.54×10^{-6}	1.37×10^{-6}	1.28×10^{-6}

TABLE III. Value at M_X of quark mass ratios and CKM mixings for large h_t and h_b couplings at M_X (M_t not fixed). We include the value of h_τ.

M_t	203	220	193	214
$\tan\beta$	63.5	59.8	68.8	66.8
$h_t(M_Z)$	1.17	1.27	1.10	1.24
$h_b(M_Z)$	1.13	1.07	1.23	1.19
$h_\tau(M_Z)$	0.638	0.600	0.691	0.671
$h_t(M_X)$	2	8	2	8
$h_b(M_X)$	2	2	8	8
$h_\tau(M_X)$	1.06	0.89	2.56	2.15
V_{us}	0.218–0.224	0.218–0.224	0.218—0.224	0.218–0.224
$\sqrt{m_u/m_c}$	0.0609	0.0609	0.0609	0.0609
$\sqrt{m_d/m_s}$	0.224	0.224	0.224	0.224
V_{ub}	0.00146–0.0036	0.0012–0.00301	0.00121–0.00303	0.00101–0.00251
$\sqrt{m_u/m_t}$	0.00276	0.00191	0.00264	0.00183
$\sqrt{m_d/m_b}$	0.0317	0.0295	0.0233	0.0218
V_{cb}	0.0234–0.0352	0.0192–0.0289	0.0194–0.0292	0.0161–0.0242
$\sqrt{m_c/m_t}$	0.0453	0.0315	0.0435	0.0301
$\sqrt{m_s/m_b}$	0.142	0.132	0.104	0.098

V_{ub}, the smallest mixing in the CKM matrix (s_{13} in the Maiani parametrization), is a physical parameter whose zero value at one loop is not protected by any symmetry. A flavor structure giving $V_{ub}=0$ at M_X would be characterized by $V_{ub}(M_Z)\approx 10^{-5}$.

Although we do not intend to discuss here the lepton sector (the possible *bottom–tau* unification has been analyzed in great detail in [16]), we include the *tau* Yukawa coupling at different scales in Tables II and III (as mentioned in the Introduction, a complete analysis of the large $\tan\beta$ regime requires an evaluation of soft SUSY-breaking corrections [12]).

III. EVOLUTION OF FLAVOR RELATIONS

We proceed now to study how the evolution from M_X to M_Z affects the relations between quark mass parameters obtained in models with flavor symmetries at the unification scale. First we will consider the five symmetric patterns with a maximal number of texture zeros found in Ref. [4]. These patterns depend on seven complex parameters which, after phase redefinitions, are reduced to seven moduli and two phases (three phases in solution 2 in [4]). The approximate analysis shows that it is possible to adjust the experimental masses and mixings without need of fine tuning. In particular, the absence of significant cancellations implies that the only role of the 2 complex phases in each pattern is to generate the CKM phase. As a consequence, the seven moduli fit the six quark masses and three mixings giving two relations. One relation [17] is shared by all the cases (solutions 1–5 in [4]):

$$V_{us}=\sqrt{\frac{m_d}{m_s}} \tag{7}$$

(with complex corrections of modulus $\sqrt{m_u/m_c}$ in solutions 1, 2, 4, 5). The second relation is

$$\frac{V_{ub}}{V_{cb}}=\sqrt{\frac{m_u}{m_c}} \tag{8}$$

for solutions 1, 2, 4 and

$$V_{ub}=\sqrt{\frac{m_u}{m_t}} \tag{9}$$

for solutions 3 and 5 [in the last case there are complex corrections of order $(m_t/2m_c)V_{cb}^2\approx 20\%$]. The masses and the rest of the mixings can be adjusted to their central values, with an arbitrary CKM phase.

Relations (7)–(9) are established at M_X, and one has to evolve the experimental quantities up to that scale in order to decide if they are acceptable. The running of the first two relations, however, is just a 0.2% (smaller than corrections to the approximate diagonalization of the matrices). Taking the

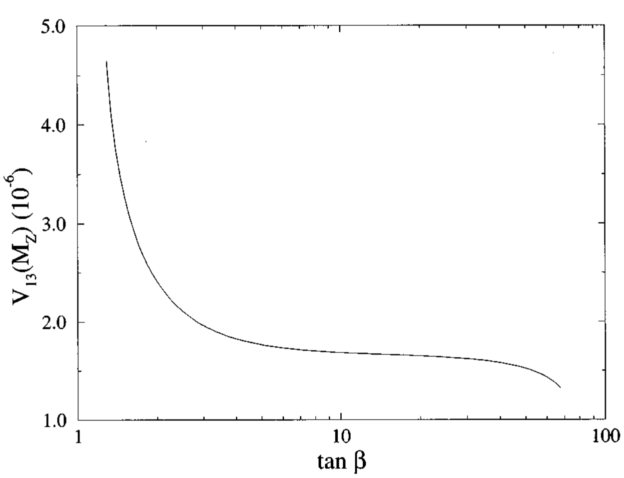

FIG. 3. Value of $V_{ub}(M_Z)$ that would correspond to a zero value of $V_{ub}(M_X)$, plotted for different values of $\tan\beta$. The masses and the mixings V_{us}, V_{cb} are taken at their central values.

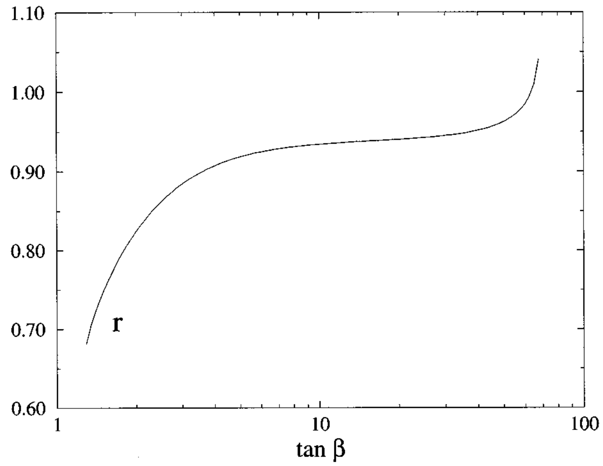

FIG. 4. Ratio r defined in Eq. (10) for different values of $\tan\beta$.

masses in their central values we have $\sqrt{m_d/m_s}=0.223$ and $\sqrt{m_u/m_c}=0.064$, which compare well with the data ($V_{12}=0.221\pm0.003$ and $V_{ub}/V_{cb}=0.08\pm0.02$ [8]). The relation $V_{ub}=\sqrt{m_u/m_t}$ suffers sizable renormalization-group corrections. At M_Z we have $V_{ub}=0.0035\pm0.0015$ and $\sqrt{m_u/m_t}\approx0.76\sqrt{m_u(1\text{ GeV})/M_t}=0.0040$. The running from M_Z to M_X can be expressed in terms of the ratio

$$r=\frac{\sqrt{m_u/m_t(M_X)}}{\sqrt{m_u/m_t(M_Z)}}\Bigg/\frac{V_{ub}(M_X)}{V_{ub}(M_Z)} \tag{10}$$

that we plot in Fig. 4 for different values of $\tan\beta$. Note that for $\tan\beta<62$, V_{13} diminishes less than $\sqrt{m_u/m_t}$ (i.e., $r<1$), while for larger values of $\tan\beta$ we observe the opposite behavior. If the masses and mixings were known with more accuracy, this fact could be used to *correct* the prediction

$$V_{ub}=0.0040r \tag{11}$$

in the preferred direction.

The different running of the mixings and the ratios of quark masses involving the third family would also affect the symmetric texture proposed in [6] (with one more zero than the textures in [4]). Those matrices predict $V_{cb}=\sqrt{m_c/m_t}$, a value that seems too large: the values of the masses at M_Z suggest $V_{cb}\approx0.76\sqrt{m_c(1\text{ GeV})/M_t}=0.066$, while the experimental upper bound is 0.048. Since the evolutions of V_{ub} (m_u) and V_{cb} (m_c) coincide, the running from M_X will be simply expressed by the same factor r in Fig. 4:

$$V_{ub}=0.066r \tag{12}$$

and the relation would be experimentally acceptable for $\tan\beta\leqslant1.4$ (with $M_t=180$ GeV).

We will finally analyze a pattern of quark matrices derived from the heterotic string. The matrices have been proposed [5] as the only realistic possibility in a class of models compactified in the Tian-Yau manifold. Their structure is[1]

$$M_u=\begin{pmatrix}0 & D_0 & C_0\\ D_0 & 0 & B_0\\ C_0 & B_0 & A_0\end{pmatrix},\quad M_d=\begin{pmatrix}D_0' & 0 & 0\\ 0 & C_0' & B_0''\\ 0 & B_0' & A_0'\end{pmatrix}. \tag{13}$$

By a redefinition of the quark fields we can put these matrices in a more convenient form:

$$M_u=\begin{pmatrix}0 & \widetilde{D} & C\\ \widetilde{D} & 0 & B\\ C & B & A\end{pmatrix},\quad M_d=\begin{pmatrix}D' & 0 & 0\\ 0 & C' & 0\\ 0 & \widetilde{B}' & A'\end{pmatrix}, \tag{14}$$

where $\widetilde{D}$ and $\widetilde{B}'$ are complex and the rest of the parameters are real and positive. As we will see, these eight moduli and two phases can fit all the masses and the larger mixings to their central values and predict acceptable (but *large*) values for V_{ub} and a nonzero (but *small*) complex CKM phase. The approximate diagonalization gives

$$m_t=A,\quad m_c=\frac{B^2}{A},\quad m_u=\left|\frac{2ABC\widetilde{D}-A^2\widetilde{D}^2}{AB^2}\right|, \tag{15}$$

$$m_b=A',\quad m_s=C',\quad m_d=D', \tag{16}$$

and a CKM matrix (in the Maiani parametrization) with

$$V_{us}=\left|\frac{BC-A\widetilde{D}}{B^2}\right|, \tag{17a}$$

$$V_{cb}=\left|\frac{\widetilde{B}'}{A'}-\frac{B}{A}\right|, \tag{17b}$$

$$|V_{ub}|=\left|\frac{(BC-A\widetilde{D})\widetilde{B}'}{AB^2}-\frac{\widetilde{D}}{B}\right|. \tag{17c}$$

In terms of physical quantities we have

$$V_{ub}=\left(V_{us}V_{cb}+V_{12}\sqrt{\frac{m_c}{m_t}}e^{i\alpha}-\frac{m_uV_{cb}}{2m_cV_{12}}e^{i\beta}\right), \tag{18}$$

where α and β are independent complex phases (the dominant phase α is related to the phase of $\widetilde{B}'$). At M_Z, for masses and mixings in their central values, the relation reads $V_{13}=0.0088+0.0146e^{i\alpha}-0.0004e^{i\beta}$. Then it seems that the small value of V_{ub} requires a cancellation between the first two terms, with a best value $|V_{ub}|>0.0054$ (for $\alpha=\pi$). Renormalization-group corrections affect this relation due to the different running of V_{cb} and $\sqrt{m_c/m_t}$, with the total effect captured again by the factor r plotted in Fig. 4:

$$V_{ub}=0.0088+r\ 0.0146e^{i\alpha}-0.00034e^{i\beta}. \tag{19}$$

For low values of r, the lower bound for the predicted value of $|V_{ub}|$ decreases. For example, for $\tan\beta=1.5$ we have

[1]We suppress an antisymmetric entry proportional to the VEV's of an extra Higgs doublet present in the model [5] since its presence would require a detailed analysis of flavor-changing neutral currents.

$|V_{ub}|>0.002$ and a CKM phase $\pi/2\leqslant\delta_{13}\leqslant 3\pi/2$, whereas $|V_{ub}|<0.005$ would imply $\tan\beta\leqslant 30$ (for all the quark masses and the rest of mixings in their central values). Note that for smaller values of V_{cb}, these bounds are relaxed.

IV. CONCLUSIONS

The observed value of M_t implies that h_t is the dominant term in the renormalization-group equations at large scales. As a consequence, the corrections to the quark masses lose universality and there appear nontrivial corrections to the CKM mixings of the light quarks with the third family. In addition, the low energy value of h_t could be related to a QFP of the equations: any *large* value of $h_t(M_X)$ seems to converge to a narrow interval around 1.2 at M_Z. In the MSSM with $M_t=180$ GeV this forces a low value of $\tan\beta$, whereas an analogous situation occurs for h_b in the large $\tan\beta$ regime. For M_t around 200 GeV and large $\tan\beta$, both low energy Yukawa couplings would result from any large value of the couplings at M_X (a large value of $\tan\beta$ could be also motivated by the possibility to relax the R_b anomaly [18]). In this framework, we perform an updated (with the new data for M_t) and complete (all values of $\tan\beta$) analysis of the evolution from M_Z and M_X of all the physical observables in the quark Yukawa sector of the MSSM. We study in detail the behavior of the smallest CKM mixing V_{ub} in the *top* and/or *bottom* QFP regions, and we show that the evolution is linear: $\delta V_{ub}/V_{ub}(M_X)\approx\delta V_{ub}/V_{ub}(M_Z)$. To illustrate the size of the nonlinear corrections we analyze the value of $V_{ub}(M_Z)$ which corresponds to $V_{ub}(M_X)=0$; we obtain $V_{ub}(M_Z)\leqslant 10^{-5}$ (this value increases going to nonperturbative values of h_t at M_X, i.e., lowering $\tan\beta$).

Then we analyze the renormalization-group corrections to fermion mass relations which appear in models with flavor symmetries at M_X. In particular, we discuss the relations obtained for symmetric mass matrices with a maximal number of zeros and in a superstring-motivated model. We show that in some relations the corrections can be numerically important (they are essential in some of the cases) and that they depend quite strongly on $\tan\beta$. In particular, for the relations analyzed the corrections can be expressed in terms of the ratio r in Eq. (10). We find it remarkable that, for a fixed M_t, r goes to zero decreasing $\tan\beta$ and grows $(r>1)$ for $\tan\beta$ large $(\tan\beta\geqslant 62)$. If the masses and mixings were measured with more accuracy, this fact could be used to conveniently *correct* the relations by varying $\tan\beta$, whereas if the Higgs sector of the MSSM were observed and $\tan\beta$ fixed, it could be used to exclude some of the quark mass matrix models.

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