PHYSICAL REVIEW D VOLUME 54, NUMBER 11 1 DECEMBER 1996

Extended isoscalar-flavor-spin symmetries for baryons with a single spectator isoscalar quark

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(Received 28 May 1996)

The flavor-spin symmetries of heavy quark effective theory imply that in the infinite heavy quark mass limit the magnetic moments, radiative decay rates, single pion emission decay rates, and the Σ $\Delta I=2$ isospin mass splittings for the heavy Σ baryon systems approach finite but undetermined values, independent of the spin of the Σ baryon and the flavor of the heavy quark. We propose, with supporting evidence, that the flavor-spin symmetries for those processes, with the isoscalar quark as a spectator, can be extended to all isoscalar quarks and thus overlap with the low-energy SU(6) symmetry through the strange quark. By interpolating between the flavor-spin symmetries at the heavy quark masses and the strange quark mass we determine these interesting properties of the heavy Σ baryons from the available low-energy data. We also discuss the interesting pattern of theoretical calculations and the experimental conflict on the Σ_c $\Delta I=2$ isospin mass splitting. [S0556-2821(96)04223-3]

PACS number(s): 11.30.Hv, 13.30.−a, 13.40.Dk

I. INTRODUCTION

In recent years much progress has been made in understanding the physics of hadrons with a single isoscalar heavy quark. The success is due to the effective heavy quark theory and the application of flavor-spin symmetries in the limit of infinitely heavy isoscalar quark masses [1]. The applicability to a wider class of phenomenology is limited largely by the difficulty of estimating the $\Lambda_{\mathrm{QCD}}/m_h$ corrections and QCD corrections for finite heavy quark mass m_h.

In this paper we attempt to control these corrections by limiting our attention to a special class of physical processes: the system of Σ-Λ types of baryons with a truly spectator isoscalar quark. For the purpose of this paper, we refer to *only* the isospin doublet quarks as light quarks q_l, and designate the isoscalar quarks excluding the strange quark (c, b, and t) as heavy quarks q_h. We use the Greek index ζ to denote isoscalar quarks in general. Being a spectator, the isoscalar quark does not participate directly in the interactions, and there will not be relative isoscalar-flavor-dependent QCD correction factors for different isoscalar flavors. As a spectator the contribution of the isoscalar quark on the matrix element is indirect through its influence to the baryon wave function. When a physical quantity is evaluated using that matrix element, the effect due to the mass of the isoscalar quark is suppressed. Effectively, the result would be approximately the same regardless of the single isoscalar quark mass. At the heavy quark mass limit $\Lambda_{\mathrm{QCD}}/m_\zeta \to 0$, the total spin of the two light quarks $\vec{s}_l$ and the spin of the heavy quark commute with the Hamiltonian. This leads to flavor-spin symmetries. The s_l remains a good quantum number for finite $m_\zeta \to m_s \gg m_d - m_u$ due to isospin symmetry and Fermi statistics, in spite of the fact that $\Lambda_{\mathrm{QCD}}/m_s$ is no longer small. The flavor-spin symmetries of the Σ-Λ baryon at $m_\zeta = m_s$ is the low-energy SU(6). The hyperfine splitting of the Σ baryons is comparable to that of the D mesons even though $\Lambda_{\mathrm{QCD}}/m_s \gg \Lambda_{\mathrm{QCD}}/m_c$. It is then not unreasonable to conjecture that the $\Lambda_{\mathrm{QCD}}/m_\zeta$ corrections are suppressed in the Σ-Λ baryon systems. The heavy quark flavor-spin symmetry can then be extended to all isoscalar quarks, including the strange quark. Via the strange quark, we can interpolate the flavor-spin symmetries between the heavy quark effective theory and the low-energy SU(6) symmetry. Such overlap due to the dual roles of the strange quark extends the two sets of the symmetry relations to a new covering set, which we shall refer to as the isoscalar-flavor-spin symmetries. When these conditions are satisfied, the matrix elements are independent of the single isoscalar quark flavor and they are related by the corresponding subset of SU(6) symmetry relations.

In order to prevent the isoscalar quark from participating in the interaction, we use the isospin conservation law to discriminate the isoscalar quark from the light quarks. We define ΔI for a given interaction Hamiltonian term if the quark part or the baryon part of this interaction Hamiltonian term (i.e., treating all other fields as classical external fields) transforms as the I irreducible representation of the isospin group. For each interaction which conserves the quark number and is bilinear in quark and antiquark fields, the isospin of a light quark can be changed by as much as one unit, $\Delta I \leqslant 1$, whereas the isospin of the isoscalar quark can be changed at most by a half unit, $\Delta I \leqslant \frac{1}{2}$. If we select processes such that $\Delta I=1$ for every interaction of quark, we can be sure that the isoscalar quark cannot participate. The necessary and sufficient condition for the isoscalar quark to be a spectator is that $\Delta I=1$ for first order interaction, $\Delta I=2$ for the second order interaction and, in general, $\Delta I=n$ for the nth order interaction. For the purpose of isospin consideration, baryon interaction can be viewed as a superposition of the corresponding quark interaction. First order baryon interaction is equivalent to first order quark interaction. This selection rule can be carried over directly to baryon processes with the same value of ΔI.

The physical baryon $\Delta I=1$ processes are the decays of the Σ_ζ^* and the Σ_ζ through single pion emission, magnetic dipole transition, and semileptonic weak decays. The $\Delta I=2$ second order process can be extracted from the isospin mass splittings. As the consequence of the heavy quark effective theory flavor-spin symmetries, each of these decay rates or mass splittings approaches a finite limit independent of the spin of the Σ baryon for infinitely heavy isoscalar quark mass. But their absolute values cannot be determined. Based on the validity of the isoscalar-flavor-spin symmetries hypothesis we can then extrapolate the isoscalar quark matrix elements continuously down to the strange quark domain to use the existing data to calibrate their asymptotic values. Thereby, we can derive more reliable predictions for the hyperfine splittings, magnetic moments, radiative decay rates, and single pion emission decay rates of the Σ_h and Σ_h^* heavy baryons than the estimates presently available.

However, the decay rates contain kinematic factors sensitively dependent on masses, some of which have not been measured experimentally. In particular, the phase space of the $\Sigma_h \to \Lambda_h + \pi$ decay may be small. Even isospin mass splittings may contribute non-negligible corrections. For this reason we shall begin exploring the applications of the isoscalar-flavor-spin symmetries hypothesis to the baryon masses and then use the estimated baryon masses for the decay rate calculations.

II. INTEGRAL ISOSPIN BARYON HYPERFINE SPLITTINGS

We assume that the low-lying baryons are three-quark s-wave ground states. Given that quark masses satisfy the inequalities $m_d - m_u \ll m_s - m_d \ll m_c - m_s \ll m_b - m_c$, we can always number the three quarks in any baryon so that

$$|m_1 - m_2| \ll 2m_3 - m_1 - m_2. \tag{1}$$

For all practical purposes, baryons are eigenstates of $s_{12}^2 = (\vec{s}_1 + \vec{s}_2)^2$ and $\vec{s}^2 = (\vec{s}_1 + \vec{s}_2 + \vec{s}_3)^2$. Consider the baryon with two light quarks and an isoscalar ζ quark. In the absence of the isospin symmetry breaking the two light quarks with mass m can have isospin $I=0$ or $I=1$. The $I=0$ state can only have $s_{12}=0$ and that state is the Λ_ζ. The $I=1$, $s_{12}=1$, and $s=\frac{1}{2}$ state is the Σ_ζ and the $I=1$, $s_{12}=1$, and $s=\frac{3}{2}$ state is the Σ_ζ^*.

The strong hyperfine splittings of baryons can be calculated perturbatively from the spin-spin interaction Hamiltonian

$$\sum_{i<j} -\frac{2}{3}\alpha_s\left(\frac{2}{3}\right)\nabla^2 V(\vec{r}_{ij})\frac{\vec{s}_i}{m_i}\cdot\frac{\vec{s}_j}{m_j} = \sum_{i<j}\frac{4}{9}\alpha_s f(\vec{r}_{ij})\frac{\vec{s}_i}{m_i}\cdot\frac{\vec{s}_j}{m_j}, \tag{2}$$

where $V(\vec{r}_{ij})$ is the interquark potential assumed to be flavor independent and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ is the relative position of the two quarks. The baryon masses are given by [2]

$$B(m_1,m_2,m_3;s_{12},s) = M_I(m_1,m_2,m_3) + \frac{1}{9}\alpha_s\left[\frac{\Delta(m_1,m_2;m_3)}{m_1 m_2}[2s_{12}(s_{12}+1)-3] + \left(\frac{\Delta(m_1,m_3;m_2)}{m_1 m_3} + \frac{\Delta(m_2,m_3;m_1)}{m_2,m_3}\right)\left(s(s+1) - s_{12}(s_{12}+1) - \frac{3}{4}\right)\right], \tag{3}$$

where $M_I(m_i,m_j,m_k)$, the spin-independent part of the baryon mass, is a symmetric function of the three-quark masses only, and

$$\Delta(m_i,m_j;m_k) = \Delta(m_j,m_i;m_k) = \langle m_i,m_j,m_k|f(\vec{r}_{ij})|m_i,m_j,m_k\rangle = \langle f(\vec{r}_{ij})\rangle_k, \tag{4}$$

where $\langle f(\vec{r}_{ij})\rangle_k$ denotes the expectation value of $f(\vec{r}_{ij})$ with respect to the s-wave ground state (ijk) baryon wave function which depends on the quark masses and is independent of the spin and isospin. It follows that [3,4]

$$\frac{2}{3}(\Sigma_\zeta^* - \Sigma_\zeta) + (\Sigma_\zeta - \Lambda_\zeta) = \frac{4\pi\alpha_s}{9m^2}\langle f(\vec{r}_{ll'})\rangle_\zeta, \tag{5}$$

$$\Sigma_\zeta^* - \Sigma_\zeta = \frac{2\pi\alpha_s}{3mm_\zeta}\langle f(\vec{r}_{l\zeta})\rangle_{l'}, \tag{6}$$

which can be combined to give

$$\frac{m}{m_h}\frac{\langle f(\vec{r}_{lh})\rangle_{l'}}{\langle f(\vec{r}_{ll'})\rangle_s} + \left[1 - \frac{\langle f(\vec{r}_{ll'})\rangle_h}{\langle f(\vec{r}_{ll'})\rangle_s}\right] = \frac{\frac{2}{3}(\Sigma^* - \Sigma) + (\Sigma - \Lambda) - (\Sigma_h - \Lambda_h)}{\frac{2}{3}(\Sigma^* - \Sigma) + (\Sigma - \Lambda)}. \tag{7}$$

The numerical value of the right-hand side for the charm flavor is 0.182 which is small and very close to m/m_c. The test of our hypothesis comes from the ratio

$$\frac{\langle f(\vec{r}_{ll'})\rangle_c}{\langle f(\vec{r}_{ll'})\rangle_s} = 1 + \frac{m}{m_c}\frac{\langle f(\vec{r}_{lc})\rangle_{l'}}{\langle f(\vec{r}_{ll'})\rangle_s} - 0.182, \tag{8}$$

which is indeed close to unity for a wide range of $\langle f(\vec{r}_{lc})\rangle_{l'}/\langle f(\vec{r}_{ll'})\rangle_s$. It is clear that while the ratio $\langle f(\vec{r}_{lc})\rangle_{l'}/\langle f(\vec{r}_{ll'})\rangle_s$ may differ from 1 by as much as 20%, the ratio $\langle f(\vec{r}_{ll'})\rangle_c/\langle f(\vec{r}_{ll'})\rangle_s$ can only be within at most a few percent deviation from 1. Therefore, it is consistent with our hypothesis that, as long as the two light quarks are light, the expectation value of $\langle f(\vec{r}_{ll'})\rangle_\zeta$ is independent of the ζ quark. The effect of the spectator isoscalar ζ quark is averaged out in the expectation value, even if the ζ quark is

TABLE I. Calculated and experimental values of the isospin mass differences of the Σ_c in MeV.

	Ref.	$\Sigma_c^{++}-\Sigma_c^0$	$\Sigma_c^{+}-\Sigma_c^0$	$\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^0$
Theory	Lichtenberg [7]	3.5	0.8	*1.8*[a]
	Itoh *et al.* [8]	6.5	2.4	*1.8*
	Ono [9]	6.1	2.2	*1.6*
	Lane and Weinberg [10]	−6	−4	*2.0*
	MIT bag [11]	−3.3	−2.5	*1.7*
	Wright [12]	−1.4	−2.0	*2.6*
	Hwang and Lichtenberg [14]	3.0	−0.5	*2.0*
	Sinha [13]	1.5	−0.3	*2.1*
	Capstick [15]	1.4	−0.2	*1.8*
	Cutkosky and Greger [16]	0.8	−0.4	*1.6*
	Chan [17,18]	0.3	−0.7	*1.8*
Experiment	CLEOI.5 [19]	$-0.1\pm0.6\pm0.1$		
	ARGUS [21]	$1.2\pm0.7\pm0.3$		
	CLEOII [20]	$1.1\pm0.4\pm0.1$	$1.4\pm0.4\pm0.1$	*−1.7±1*
	PDG average [6]	0.7 ± 0.6	$1.4\pm0.4\pm0.1$	*−2.1±1*
	Fermilab E791 [22]	$0.38\pm0.40\pm0.15$		-2.4 ± 1

[a]Calculated values are in italics.

not exactly heavy. An interesting consequence is the isoscalar-flavor symmetries for Eq. (5):

$$\frac{2}{3}(\Sigma_\zeta^*-\Sigma_\zeta)+(\Sigma_\zeta-\Lambda_\zeta)=205\ \text{MeV}\,\frac{\langle f(\vec r_{ll'})\rangle_\zeta}{\langle f(\vec r_{ll'})\rangle_s}$$
$$\approx 205\pm10\ \text{MeV}. \tag{9}$$

The value of 205.2 MeV is obtained from evaluating Eq. (5) with the strange flavor. The ratio $\langle f(\vec r_{l\zeta})\rangle_{l'}/\langle f(\vec r_{ll'})\rangle_s$ should be close 1 for $m_\zeta\approx m_s$ because SU(3) is a good symmetry. That ratio should approach a finite value as expected by heavy quark effective theory.

We now can apply the isoscalar-flavor symmetries to estimate some unknown $I=1$ heavy baryon masses. Using $\Sigma_c=2452.5$ MeV and $\Lambda_c=2285.1$ MeV [6] for the last equation, we obtain

$$\Sigma_c^*=2509\pm15\ \text{MeV}. \tag{10}$$

Similarly, we can apply Eqs. (6) and (9) for the bottom flavor to obtain

$$\Sigma_b-\Lambda_b=205\ \text{MeV}\left[\frac{\langle f(\vec r_{ll'})\rangle_b}{\langle f(\vec r_{ll'})\rangle_s}-\frac{m}{m_b}\,\frac{\langle f(\vec r_{lb})\rangle_{l'}}{\langle f(\vec r_{ll'})\rangle_s}\right], \tag{11}$$

$$\Sigma_b^*-\Lambda_b=205\ \text{MeV}\left[\frac{\langle f(\vec r_{ll'})\rangle_b}{\langle f(\vec r_{ll'})\rangle_s}+\frac{1}{2}\,\frac{m}{m_b}\,\frac{\langle f(\vec r_{lb})\rangle_{l'}}{\langle f(\vec r_{ll'})\rangle_s}\right] \tag{12}$$

isoscalar-flavor symmetries imply $\langle f(\vec r_{ll'})\rangle_b/\langle f(\vec r_{ll'})\rangle_s\approx1$. The large uncertainty of the ratio $\langle f(\vec r_{lb})\rangle_{l'}/\langle f(\vec r_{ll'})\rangle_s$ is suppressed by the small factor $m/m_c\approx1/15$. Given $\Lambda_b=5641$ MeV, we can estimate

$$\Sigma_b=5833\pm15\ \text{MeV}\quad\text{and}\quad\Sigma_b^*=5853\pm15\ \text{MeV}, \tag{13}$$

which is consistent with the recent predictions by Roncaglia *et al.*., [5]

$$\Sigma_b=5830\pm40\ \text{MeV}\quad\text{and}\quad\Sigma_b^*=5860\pm40\ \text{MeV}. \tag{14}$$

Experimental verification of these predicted masses would give support to our hypothesis.

III. $\Delta I=2$ BARYON MASS SPLITTINGS

In the standard model there are two contributions to the isospin violation: the electromagnetic interaction in the SU(2)×U(1) sector and the flavor-symmetry breaking $m_u\neq m_d$ in the QCD sector. These two sources have no apparent connection, yet their contributions to the hadron isospin mass splittings are comparable in magnitude. Nonetheless, the intrinsic light quark mass difference m_u-m_d is the dominant source of the isospin splittings. Unless the isospin splittings can be measured very accurately it would be difficult to distinguish among various phenomenological models. Lichtenberg pointed out that in the case of the isospin splitting $\Sigma_c^{++}-\Sigma_c^0$, there exists a large cancellation between the m_u-m_d and the Coulomb contribution to the isospin splitting and a measurement of this splitting would be the best hope for distinguishing among different models [7]. The predictions of various theoretical models and the experimental values are shown in Table I. The theoretical values spread over a very large range. The observed mass splitting can definitely rule out a large number of models. However, if one further takes into account the only measurement of $\Sigma_c^+-\Sigma_c^0$ by CLEOII [20] seriously, one finds that none of the theoretical models is consistent with the experimental mass splittings.

In the presence of isospin symmetry breaking, the Hamiltonian can be decomposed into irreducible isospin representations:

$$H=H_{I=0}+H_{I=1}+H_{I=2}. \tag{15}$$

We truncate terms with $I\geqslant 3$ because they do not apply in this case. Charge conservation implies $I_3=0$ for each term. The expectation values of the Hamiltonian between baryon states at rest define the corresponding mass matrix:

$$M=M_0+M_1Q+M_2Q^2, \tag{16}$$

where Q is the charge matrix. The masses of the Σ are given in term of their charge eigenvalues Q':

$$\begin{aligned}\Sigma^{Q'+1}&=M_0+(Q'+1)M_1+(Q'+1)^2M_2,\\ \Sigma^{Q'}&=M_0+Q'M_1+Q'^2M_2,\\ \Sigma^{Q'-1}&=M_0+(Q'-1)M_1+(Q'-1)^2M_2.\end{aligned} \tag{17}$$

The equal spacing of mass splittings is clearly seen for the $\Delta I=1$ contribution. The $\Delta I=2$ contribution can be isolated by the linear combination of the mass splittings,

$$\begin{aligned}2M_2&=\Sigma^{Q'+1}-2\Sigma^{Q'}+\Sigma^{Q'-1}\\ &=(\Sigma^{Q'+1}-\Sigma^{Q'})-(\Sigma^{Q'}-\Sigma^{Q'-1}),\end{aligned} \tag{18}$$

which measures the deviation from the equal-spacing mass splittings.

The inconsistency between the measurement and the predictions of theoretical models can be observed more directly by comparing the $\Delta I=2$ linear combination of the Σ_c isospin mass splittings, $\delta_{\Delta I=2}\Sigma_c=\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0}$, in Table I. While there are very large differences between predictions of various theoretical models for $\Sigma_c^{++}-\Sigma_c^{0}$, the $\Delta I=2$ part seems to be almost model independent and has the value ≈ 2 MeV, which is clearly inconsistent with the experimental value $\approx -2\pm 1$ MeV.

The fact that all different theoretical models give the same result is unlikely to be an accident. In this paper we attempt to derive this result basing on a set of general assumptions from our basic understanding of the isospin-breaking mechanism common to all these models. In that case we are facing a very serious situation that either the experimental determination of isospin splittings is highly questionable, or our basic understanding of the isospin-splitting source would have to undergo some fundamental modification.

We may treat the $\Delta I=1$ isospin symmetry-breaking term of the quark mass difference m_u-m_d as a perturbation term in QCD. The $\Delta I=2$ contribution to mass splitting must be at least second order perturbation and of the order of $(m_u-m_d)^2/\Lambda_{\mathrm{QCD}}\sim 0.05$ MeV. Therefore, we can virtually ignore the strong contribution to any $\Delta I=2$ isospin splitting. All $\Delta I=2$ isospin splittings must come from electromagnetic processes.

The electromagnetic interaction consists of $\Delta I=0$ and $\Delta I=1$ contributions. For isoscalar particles, only $\Delta I=0$ is possible. Therefore, the $\Delta I=2$ isospin splitting must come from electromagnetic interaction between two of the light quarks. The Coulomb interaction energy between two quarks q_i and q_j is given by $\alpha(Q_iQ_j/r_{ij})$. The magnetic interaction energy is given by

$$\sum_{i<j}\alpha\left(\frac{2\pi}{3}\right)\delta(\vec{r}_{ij})Q_iQ_j\frac{\vec{s}_i}{m_i}\cdot\frac{\vec{s}_j}{m_j}. \tag{19}$$

We assume that quarks have magnetic moments and the quark masses are the constituent quark masses. In the presence of isospin symmetry breaking, there will be mixing between the two states, Λ_ζ and the $I_3=0$ member of the Σ_ζ with $I=1$ and $s_{12}=1$. The mixing must come from the spin-spin interaction. The mixing, however, has a negligibly small effect on the mass eigenvalues and will be neglected in this paper. Then the $\Delta I=2$ part of the Σ_ζ isospin splittings is given by

$$\delta_{\Delta I=2}\Sigma_\zeta^{*}=\delta_{\Delta I=2}\Sigma_\zeta=\alpha\langle 1/r_{ll'}\rangle_\zeta-\frac{2\pi}{3m^2}\alpha\langle\delta(\vec{r}_{ll'})\rangle_\zeta, \tag{20}$$

where $m=\frac{1}{2}(m_u+m_d)$. The first term is the electric contribution and the second term is the magnetic contribution.

These matrix elements are the prime candidates for our isoscalar-flavor-spin symmetries hypothesis, which implies the $\Delta I=2$ isospin mass splittings are isoscalar-flavor independent:

$$\begin{aligned}\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0}&=\Sigma_c^{*++}-2\Sigma_c^{*+}+\Sigma_c^{*0}\\ &=\Sigma_b^{+}-2\Sigma_b^{0}+\Sigma_b^{-}=\Sigma_b^{*+}-2\Sigma_b^{*0}+\Sigma_b^{*-}\\ &\approx\Sigma^{+}-2\Sigma^{0}+\Sigma^{-}=\Sigma^{*+}-2\Sigma^{*0}+\Sigma^{*-}\\ &1.8\pm 0.15\ \ \mathrm{MeV}=2.6\pm 2\ \ \mathrm{MeV}.\end{aligned} \tag{21}$$

This result is consistent with all known theoretical calculations but disagrees with the experimental value -2.1 ± 1.0 MeV deduced from the CLEOII measurement of $\Sigma_c^{+}-\Sigma_c^{0}=1.4\pm 0.4$ MeV [20]. Even if one allows the matrix elements in Eq. (20) to vary in a reasonably smooth way from the strange quark to the bottom quark, it would take some very abnormal behavior of the quark mass dependence to switch the sign of the $I=2$ isospin splittings. In order to be consistent with the theoretical value [23], $\Sigma_c^{+}-\Sigma_c^{0}$ would have to be -0.55 ± 0.4 MeV, instead of the CLEOII value $1.4\pm 0.4\pm 0.1$ MeV. It is, therefore, crucial to independently verify this experimental value.

For physical baryons, the observed and the calculated (italic) masses are listed in Table II for the calculation of the decay rates. We list the predicted value of Σ_c^{+} instead of the experimental value 2453.5 ± 0.9 MeV because we prefer to use the theoretical value for a complete consistent calculation based on the extended symmetries.

IV. Σ_ζ AND Σ_ζ^{*} BARYON DECAYS

The isoscalar-flavor-spin symmetries hypothesis can be applied to other matrix elements of the baryons provided that the isoscalar quark is truly a spectator. This condition can be realized for the matrix elements of the $I=1$ vector current and the $I=1$ axial-vector current which can be used to relate various magnetic dipole electromagnetic transition rates and single pion emission transition rate of Σ_ζ and Σ_ζ^{*}.

TABLE II. Baryon masses in MeV.

	I_3	s	c	b
m_ζ		512.5	1681.9	5037.8
	1	1189.4	2453.1	*5829.7*
Σ_ζ	0	1192.6	*2451.9*	*5832.0*
	-1	1197.4	2452.4	*5835.9*
	1	1382.6	*2509.4*	*5850.4*
Σ_ζ^*	0	1384.1	*2508.6*	*5852.4*
	-1	1387.4	*2509.5*	*5856.2*
Λ_ζ	0	1115.7	2285.1	5641.0

[a]Calculated values are in italics.

A. Magnetic dipole electromagnetic transition

The magnetic dipole electromagnetic transition amplitudes are given by

$$A(\Sigma_\zeta \to \Lambda_\zeta \gamma) = \mu_{\Sigma_\zeta \Lambda_\zeta} k^\mu \epsilon^\nu \overline{u_{\Lambda_\zeta}} \sigma_{\mu\nu} u_{\Sigma_\zeta}, \tag{22}$$

$$\begin{aligned} A(\Sigma_\zeta^* \to \Lambda_\zeta \gamma) &= i\mu_{\Sigma_\zeta^* \Lambda_\zeta} \epsilon_{\mu\nu\lambda\kappa} k^\nu \epsilon^\mu \overline{u_{\Lambda_\zeta}} \gamma^\kappa u^\lambda_{\Sigma_\zeta^*} \\ &= \mu_{\Sigma_\zeta^* \Lambda_\zeta} \overline{u_{\Lambda_\zeta}} (k^\mu \not{\epsilon} - \epsilon^\mu \not{k}) \gamma_5 u^\mu_{\Sigma_\zeta^*}, \end{aligned} \tag{23}$$

$$A(\Sigma_\zeta^* \to \Sigma_\zeta \gamma) = i\mu_{\Sigma_\zeta^* \Sigma_\zeta} \epsilon_{\mu\nu\lambda\kappa} k^\nu \epsilon^\mu \overline{u_{\Sigma_\zeta}} \gamma^\kappa u^\lambda_{\Sigma_\zeta^*}, \tag{24}$$

where u and u^μ are the Dirac spinor and the Rarita-Schwinger vector spinor. Here, $k = p - q$ is the four-momentum of the photon, p is the momentum of the Σ_ζ^* and q is the momentum of the Λ_ζ. The $\Delta I = 1$ part of the magnetic dipole moments and the transition magnetic dipole moments are related by the isoscalar-flavor-spin symmetries:

$$\begin{aligned} \frac{1}{\sqrt{3}} \mu_{\Sigma_\zeta^* \Lambda_\zeta} &= \frac{1}{2\sqrt{3}} (\mu_{\Sigma_\zeta^{*+}} - \mu_{\Sigma_\zeta^{*-}}) \\ &= \sqrt{\frac{3}{4}} (\mu_{\Sigma_\zeta^+} - \mu_{\Sigma_\zeta^-}) = -3\mu_{\Sigma_\zeta \Lambda_\zeta} \\ & 1.56 \pm 0.03 \mu_N = 1.61 \pm 0.8 \mu_N, \end{aligned} \tag{25}$$

which are also isoscalar-flavor independent. The numerical values in Eq. (25) are the corresponding experimental values for $\zeta = s$. The decay rates are given by

$$\Gamma(\Sigma_\zeta \to \Lambda_\zeta \gamma) = \frac{1}{\pi} \mu^2_{\Sigma_\zeta \Lambda_\zeta} k^3 = \frac{1}{\pi} \mu^2_{\Sigma\Lambda} k^3, \tag{26}$$

$$\begin{aligned} \Gamma(\Sigma_\zeta^* \to \Lambda_\zeta \gamma) &= \frac{1}{\pi} \frac{\mu^2_{\Sigma_\zeta^* \Lambda_\zeta}}{3} \left(1 - \frac{1}{2} \frac{k}{m_{\Sigma_\zeta^*}}\right) k^3 \\ &= \frac{1}{\pi} \mu^2_{\Sigma\Lambda} \left(1 - \frac{1}{2} \frac{k}{m_{\Sigma_\zeta^*}}\right) k^3, \end{aligned} \tag{27}$$

where k is the magnitude of the photon three-momentum in the rest frame of the decay particle.

For very heavy quark mass, $\Sigma_\zeta, \Sigma_\zeta^*, \Lambda_\zeta \to \infty$, $\Sigma_\zeta - \Sigma_\zeta^* \to 0$ but according to Eq. (9), $k \to \Sigma_\zeta - \Lambda_\zeta \to 205 \pm 10$ MeV$= k_\infty$. Then, the $\Delta I = 1$ radiative decay widths

$$\Gamma_{\Sigma_\zeta^* \to \Lambda_\zeta \gamma} \approx \Gamma_{\Sigma_\zeta \to \Lambda_\zeta \gamma} \to \frac{\alpha}{m_N^2} \mu^2_{\Sigma\Lambda} k^3_\infty = 186 \pm 30 \ \text{keV} \tag{28}$$

are obviously spin symmetric.

B. Single pion emission transition

The single pion emission amplitudes are related to the amplitudes of the axial-vector current matrix elements:

$$A[B'(p) \to B(q)\pi(k)] = \frac{1}{\sqrt{2} f_\pi} k_\mu \langle B(q) | J_A^\mu | B'(p) \rangle |_{k\to 0} \tag{29}$$

and

$$A(\Sigma_\zeta \to \Lambda_\zeta \pi) = \frac{1}{\sqrt{2} f_\pi} k_\mu g_{\Sigma_\zeta \Lambda_\zeta} \overline{u_{\Lambda_\zeta}} \gamma^\mu \gamma_5 u_{\Sigma_\zeta}, \tag{30}$$

$$A(\Sigma_\zeta^* \to \Lambda_\zeta \pi) = \frac{1}{\sqrt{2} f_\pi} k_\mu g_{\Sigma_\zeta^* \Lambda_\zeta} \overline{u_{\Lambda_\zeta}} u^\mu_{\Sigma_\zeta^*}, \tag{31}$$

$$A(\Sigma_\zeta^{*a} \to \Sigma_\zeta^b \pi^c) = \frac{1}{\sqrt{2} f_\pi} k_\mu \epsilon_{abc} g_{\Sigma_\zeta^* \Sigma_\zeta} \overline{u_{\Sigma_\zeta}}(q) u^\mu_{\Sigma_\zeta^*}(p), \tag{32}$$

where $f_\pi = 92.4$ MeV is the pion decay constant and k is the magnitude of the pion three-momentum. The isoscalar-flavor-spin symmetries imply

$$-g_{\Sigma_\zeta \Lambda_\zeta} = \frac{1}{\sqrt{3}} g_{\Sigma_\zeta^* \Lambda_\zeta} = -g_{\Sigma_\zeta^* \Sigma_\zeta} = \text{const(independent of } \zeta). \tag{33}$$

The pion emission decay widths are given by

$$\Gamma(\Sigma_\zeta \to \Lambda_\zeta \pi) = \frac{g^2_{\Sigma\Lambda}}{4\pi f_\pi^2} k^3 \left(1 - \frac{\pi^2}{(\Sigma_\zeta + \Lambda_\zeta)^2}\right)^{-1}, \tag{34}$$

$$\Gamma(\Sigma_\zeta^* \to \Lambda_\zeta \pi) = \frac{g^2_{\Sigma^*\Lambda}}{12\pi f_\pi^2} k^3 \left(\frac{(\Sigma_\zeta^* + \Lambda_\zeta)^2 - \pi^2}{4\Sigma_\zeta^{*2}}\right), \tag{35}$$

$$\Gamma(\Sigma_\zeta^{*(i)} \to \Sigma_\zeta^{(j)} \pi^{(k)}) = \frac{g^2_{\Sigma^*\Lambda}}{12\pi f_\pi^2} k^3 \left(\frac{(\Sigma_\zeta^{*(i)} + \Sigma_\zeta^{(j)})^2 - \pi^{(k)2}}{4\Sigma_\zeta^{*2}}\right). \tag{36}$$

Using the total decay width $\Gamma_{\Sigma^*} = 37 \pm 2$ MeV and the branching ratio to the $\Lambda\pi$ mode $B^*_{\Sigma\to\Lambda\pi} = 88 \pm 2\%$, one determines $|g_{\Sigma^*\Lambda}| = 1.19 \pm 0.05$. Since Σ cannot decay into $\Lambda\pi$, $g_{\Sigma\Lambda}$ cannot be determined in a similar manner. However, it can be determined via the contribution of the axial current matrix element to the β decay $\Sigma^- \to \Lambda e^- \overline{\nu_e}$, assuming conserved vector current. Using the Σ^- lifetime

TABLE III. Σ_ζ and Σ_ζ^* baryon decay widths.

	I_3	s	c	b		Unit
m_ζ		512.5	1681.9	5037.8	∞	MeV
$\Sigma_\zeta \to \Lambda_\zeta \gamma$	0	8.9	90.2	143	186 ± 30	keV
$\Sigma_\zeta^* \to \Lambda_\zeta \gamma$	0	281	201	190	186 ± 30	keV
	1		3.18	8.45		
$\Sigma_\zeta \to \Lambda_\zeta \pi$	0		3.65	10.1	15 ± 2	MeV
	-1		3.06	10.3		
	1	30.8	18.8	15.2		
$\Sigma_\zeta^* \to \Lambda_\zeta \pi$	0	32.6	19.6	17.2	15 ± 2	MeV
	-1	32.7	18.8	17.5		

1.47×10^{-10} sec, the branching ratio to $\Lambda e^- \overline{\nu_e}$, $5.73\pm 0.27\times 10^{-5}$, and $V_{ud}=0.9747$ and neglecting the induced pseudoscalar and axial tensor contributions, one finds $|g_{\Sigma\Lambda}|=0.607\pm 0.016$. One can use these results to check the validity of the isoscalar-flavor-spin symmetries, Eq. (33), for $\zeta = s$. The ratio $\sqrt{3}|g_{\Sigma\Lambda}|/|g_{\Sigma*\Lambda}|=0.89\pm 0.07$ shows that the spin symmetry is a good approximation in a 5–10% range, which is consistent with our original expectation. We shall adopt the constant in Eq. (33) to be 1.19 in the following calculation since we expect that $g_{\Sigma*\Lambda}$ is closer to the symmetry limit.

In the heavy mass limit $k\to\sqrt{(\Sigma_\zeta-\Lambda_\zeta)^2-\pi^2}=152\pm 10$ MeV$=k_\infty$, Eqs. (34) and (35) approach to the spin symmetric limit:

$$\Gamma_{\Sigma_\zeta^*\to\Lambda_\zeta\pi}\approx\Gamma_{\Sigma_\zeta\to\Lambda_\zeta\pi}\to=\frac{g^2_{\Sigma*\Lambda}}{12\pi f_\pi^2}k_\infty^3=15\pm 2 \quad \text{MeV}. \tag{37}$$

The radiative and the pion emission decay widths calculated according to Eqs. (26), (27), (34), and (35) are given in Table III. The isoscalar-flavor-spin symmetry breakings are entirely due to the phase space kinematics. It should be pointed out that the physical masses also include isospin symmetry breaking which spoils the exact isoscalar-flavor-spin symmetries to the order of isospin violation even at the heavy mass limit. Our Σ_c decay widths are not very much different from those of Cheng *et al.* [24] and Yan *et al.* [25]. In the event that the measured value of Σ_c^*, Σ_b, or Σ_b^* becomes available and is different from our calculated value, the width should be recalculated according to the corresponding Eqs. (26), (27), (34), and (35).

It can be seen that Σ_ζ^* converges to the heavy mass spin symmetric limit much faster than Σ_ζ. The Σ_ζ^* decay widths approach the flavor-spin symmetric limit rapidly from above and reach rather close to the limit even at the charm quark mass. The Σ_ζ decay widths approach the flavor-spin symmetric limit much slowly from below.

The decay $\Sigma_\zeta^*\to\Sigma_\zeta\pi$ is kinematically allowed only for the strangeness flavor. The decay widths for $\Sigma^*\to\Sigma\pi$ can be calculated using $g_{\Sigma*\Sigma}$ determined by the isoscalar-flavor-spin symmetries relation, Eq. (33). These relations are part of the SU(6) relations. The results are summarized in Table IV. The agreement with the experimental observation is excellent.

V. SUMMARY

We have proposed isoscalar-flavor-spin symmetries which unify the flavor-spin symmetries for heavy quarks and the SU(6) symmetry for the light quarks applicable only for processes involving baryons with one spectator isoscalar quark. The (Σ-Λ) baryon system provides a unique and ideal environment for applications of these symmetries. In this paper we have carried out a complete and consistent analysis. We summarize the main contributions.

(1) The extended isoscalar-flavor-spin symmetries make possible consistent and reliable estimates on the masses and widths of the yet to be observed heavy baryon states Σ_c^*, Σ_b, and Σ_b^*. These relatively broad baryon resonances are hard to observe because of the weak signals in the presence of very large background. The reliable knowledge of their masses and widths may facilitate their detections.

(2) Tables II and III not only summarize useful new information about the masses and widths but also display some very interesting theoretical aspects of the (extended) isoscalar-flavor-spin symmetries. The extended isoscalar-flavor symmetry actually also implies that the masses and widths of the Σ-Λ baryon system are functions only of a

TABLE IV. Σ^* decay widths in MeV.

	Σ^{*+}	Σ^{*0}	Σ^{*-}	Average
$\Gamma(\Sigma^*\to\Lambda\pi)$	30.75	32.56	32.74	32.0
$\Gamma(\Sigma^*\to\Sigma\pi)$	4.84	4.41	4.92	4.72
$\Gamma_{\Sigma*}$	35.6	37.0	37.7	36.7
$\Gamma_{\Sigma*}$ (Expt.)	35.8 ± 0.8	36 ± 5	39.4 ± 2.1	36.7 ± 3
$B(\Sigma^*\to\Sigma\pi)$	0.14	0.12	0.13	0.13
$B(\Sigma^*\to\Sigma\pi)$(Expt.)				0.12 ± 0.02

continuous mass parameter of the isoscalar quark irrespective of the discrete isoscalar-flavor quantum number. In other words, we can very well remove the flavor headings s, c, and b, and replace the subscript ζ by the continuous parameter m. The heavy quark flavor-spin symmetries are reached as m becomes very large and the widths approach finite limits independent of the spin. Extending the isoscalar symmetry to the strange quark mass scale permits the extrapolation of these functions to the energy region where data are available to determine their normalizations.

(3) We have also shown that the Σ_ζ $\Delta I=2$ mass isospin splittings are flavor-spin symmetric, which explains why all theoretical models give approximately the same prediction $\Sigma_c^{++}-2\Sigma_c^{+}+\Sigma_c^{0}\approx 2$ MeV even though they differ substantially on other isospin mass splittings. The disagreement between this value and the CLEOII experimental measurement on $\Sigma_c^{+}-\Sigma_c^{0}$ is presently an outstanding puzzle in particle physics.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy.

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