Form factor relations for heavy-to-heavy and heavy-to-light meson transitions

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Relations between the form factors that parametrize the hadronic matrix elements, in spectator decays of heavy mesons, have been derived by Stech within the constituent quark model. In this paper, we examine these relations using a slightly modified description of the meson states. We find new and very general relations for some of the form factors. For the other form factors, we obtain small modifications to the relations previously derived by Stech, in the case of heavy-to-light transitions. [S0556-2821(96)00323-2]

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I. INTRODUCTION

In Ref. [1], Stech derives a set of relations between the form factors that parametrize the hadronic matrix elements, in the weak decays of heavy mesons. These relations follow from the constituent quark picture for the hadronic transitions, whenever spectator effects can be ignored, and they are independent of the particular model that is chosen for the wave functions of the mesons. For heavy-to-heavy meson transitions, the relations are consistent with those obtained from the heavy quark symmetry limit [2]. For heavy-to-light meson transitions, the relations obtained by Stech are important new results. For example, they can be useful in extracting the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ub}|$ (from the exclusive semileptonic decays $B\to\pi l\overline{\nu_l}$, $\rho l\overline{\nu_l}$, etc.), or $|V_{td}|$ (from the exclusive radiative decay $B\to\rho\gamma$); or they can help us understand better the color-suppressed hadronic decays, such as the B decays into the charmonium states.

In this paper, the derivation of the relations between the form factors is revisited, along steps similar to those in Ref. [1] (however, we adopt a slightly different description of the kinematics of the constituent quarks inside the mesons). We will show that new and very general relations can be derived for some of the form factors; they follow from the constituent quark description and the constraint of a spectator decay, but with no further assumptions. Additional relations between the other form factors are obtained in two limiting cases. For heavy-to-heavy transitions, we obtain the same results as Stech. For the heavy-to-light case, we analyze in more detail the conditions under which the approximations already used by Stech are valid. Using our description of the constituent kinematics, we confirm the surprising result that the static heavy quark approximation remains valid, even for decays with large recoil. On the other hand, we find that the approximation of a massless recoiling constituent is valid in a more restricted region of recoil momentum. When both approximations are used, the results of Stech are recovered, up to differences of the order of the ratio of the light to heavy meson masses.

The derivation is presented in some detail, although part of the results can already be found in Ref. [1]. For definiteness, we shall refer to the heavy decaying meson as a B meson, although part of the results may also apply to D decays.

II. FORM FACTOR RELATIONS

A. Hadronic matrix elements in the constituent quark model

The hadronic matrix elements of interest, in spectator decays of B mesons, are of the form

$$\langle X(\vec{p}\,')|\overline{q}\,\Gamma b|B(\vec{p})\rangle,$$
$$\Gamma=\gamma^\mu,\gamma^\mu\gamma_5,i\sigma^{\mu\nu}(p-p')_\nu,i\sigma^{\mu\nu}(p-p')_\nu\gamma_5. \tag{1}$$

In here, the meson X will be a pseudoscalar P or a vector meson V. It can be a heavy meson such as a D or D^*, or a light meson such as a π or ρ. In the constituent quark model, the B and X mesons are the quark-antiquark bound states $(b\overline{q_{\rm sp}})$ and $(q\overline{q_{\rm sp}})$, respectively. For a spectator decay, the operator $\overline{q}\,\Gamma b$ annihilates the b quark in the initial state, and creates the constituent quark q in the final state. It does not act on $\overline{q_{\rm sp}}$, which behaves as a spectator: its momentum and spin remain unchanged in the decay process.

The B meson state in Eq. (1) is

$$|B(\vec{p})\rangle=\sqrt{\frac{2E}{(2\pi)^3}}\int d^3\vec{k}\,\phi_B(|\vec{k}|)\sqrt{\frac{m_{\rm sp}}{E_{\rm sp}}}\sqrt{\frac{m_b}{E_b}}$$
$$\times\chi^{00}_{\sigma\overline{\sigma}}|\overline{q(\vec{p}_{\rm sp},\overline{\sigma})}\rangle|b(\vec{p}_b,\sigma)\rangle, \tag{2}$$

and similarly for the X meson state. The momentum wave function $\phi_B(|\vec{k}|)$ (with $L=0$) describes the distribution in internal momentum $\vec{k}$. It favors small values of $|\vec{k}|$ (of the order of a typical hadronic mass scale, $\Lambda_{\rm QCD}$), and falls off rapidly for large values of the internal momentum [3]. The spin wave function is $\chi^{00}_{\sigma\overline{\sigma}}=(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle)/\sqrt{2}$.

The hadronic matrix element of Eq. (1) is then [3]

$$\langle X(\vec{p}\,')|\overline{q}\,\Gamma b|B(\vec{p})\rangle=\sqrt{2E}\sqrt{2E'}\int d^3\vec{k}\,\phi'_B(|\vec{k}|)\phi^*_X(|\vec{k}'|)$$
$$\times\sqrt{\frac{m_b}{E_b}}\sqrt{\frac{m_q}{E_q}}\chi^{Jm_J}_{\sigma'\overline{\sigma}}\chi^{00}_{\sigma\overline{\sigma}}\overline{u_q}(\vec{p}_q,\sigma')$$
$$\times\Gamma u_b(\vec{p}_b,\sigma), \tag{3}$$

where the condition that the spin of the spectator $\overline{q_{\rm sp}}$ is not affected by the decay has been included. The condition that its momentum remains unchanged leads to a relation between the internal momenta $\vec{k}$ and $\vec{k}'$, respectively of the B

and X mesons. For definiteness, we work on the B rest frame, $\vec{p}=0$, and choose the z axis along the momentum of the X meson, $\vec{p}\,'=|\vec{p}\,'|\hat{z}$. In the absence of internal momentum, the constituents are at rest in the meson c.m. frame; but, in general,

$$p_{b,\perp}=k_{\perp}, \quad p_{b,z}=k_z, \tag{4}$$

$$p_{\mathrm{sp},\perp}=-k_{\perp}, \quad p_{\mathrm{sp},z}=-k_z \tag{5}$$

for the B meson, and

$$p_{q,\perp}=k'_{\perp}, \quad p_{q,z}=\gamma(k'_z+\beta\sqrt{m_q^2+\vec{k}'^2}), \tag{6}$$

$$p_{sp,\perp}=-k'_{\perp}, \quad p_{\mathrm{sp},z}=\gamma(-k'_z+\beta\sqrt{m_{\mathrm{sp}}^2+\vec{k}'^2}) \tag{7}$$

for the X meson, with $\beta=|\vec{p}\,'|/E'$ and $\gamma=1/\sqrt{1-\beta^2}$. This gives

$$k_{\perp}=k'_{\perp}, k_z=\frac{E'}{m_X}k'_z-\frac{|\vec{p}\,'|}{m_X}\sqrt{m_{\mathrm{sp}}^2+\vec{k}'^2}, \tag{8}$$

which is the relation between the internal momenta $\vec{k}$ and $\vec{k}'$, that is implicit in Eq. (3).

As in Ref. [1], we assume that the wave functions of the B and X mesons are strongly peaked (at $|\vec{k}|,|\vec{k}'|\simeq 0$). Then, the overlap of the wave functions in Eq. (3) will also be strongly peaked; and so, to a good approximation,

$$\langle X(\vec{p}\,')|\overline{q}\,\Gamma b|B(\vec{p})\rangle=\sqrt{4m_B E'\frac{m_b}{E_b}\frac{m_q}{E_q}}R_{B\to X}f_{\sigma'\sigma}\overline{u_q}\times(\vec{p}_q,\sigma')\Gamma u_b(\vec{p}_b,\sigma), \tag{9}$$

with $R_{B\to X}\equiv\int d^3\vec{k}'\,\phi_B(|\vec{k}|)\,\phi_X^*(|\vec{k}'|)$ and $f_{\sigma'\sigma}\equiv\chi_{\sigma'\bar{\sigma}}^{Jm_J}\chi_{\bar{\sigma}\sigma}^{00}$. This is the expression for the hadronic matrix elements that we will use to derive the form factor relations, in the next section.

In order to determine the range of recoil momentum $|\vec{p}\,'|$ where the form factor relations are valid, we will need to compare the quark momenta $|\vec{p}_b|$ and $|\vec{p}_q|$ with the corresponding quark masses m_b and m_q. The quark momenta in Eq. (9),

$$p_{b,\perp}=\underline{k}_{\perp}, \quad p_{b,z}=\underline{k}_z, \tag{10}$$

$$p_{q,\perp}=\underline{k}_{\perp}, \quad p_{q,z}=|\vec{p}|'+\underline{k}_z \tag{11}$$

are determined by the location $\underline{\vec{k}}$ of the peak in the overlap of the meson wave functions. The quark masses m_b and m_q are effective masses determined by the relations

$$m_B=\sqrt{m_b^2+\underline{\vec{k}}^2}+\sqrt{m_{\mathrm{sp}}^2+\underline{\vec{k}}^2}, \tag{12}$$

$$m_X=\sqrt{m_q^2+\underline{\vec{k}}'^2}+\sqrt{m_{\mathrm{sp}}^2+\underline{\vec{k}}'^2}, \tag{13}$$

and they too depend on $\underline{\vec{k}}$ [the corresponding value of $\underline{\vec{k}}'$ is given by the relations in Eq. (8)]. On the other hand, m_{sp} is a constant, and a parameter of the model; its maximum-allowed value is determined by Eqs. (12) and (13).

The transverse component of the internal momentum $\underline{\vec{k}}$ is $\underline{k}_{\perp}\simeq 0$, whereas the longitudinal component $\underline{k}_z$ is more sensitive to the relative shape of the B and X wave functions. For B and X wave functions that are similar (scenario **A**), both mesons share the internal momentum that is required for the spectator transition. Then,

$$\mathbf{A}{:}\underline{k}_z\simeq-\frac{1}{2}m_{\mathrm{sp}}\frac{|\vec{p}\,'|}{m_X}\sqrt{\frac{2m_X}{E'+m_X}}. \tag{14}$$

If the B wave function has a much narrower spread in internal momentum (scenario **B**), then

$$\mathbf{B}{:}\underline{k}_z\simeq 0. \tag{15}$$

If, on the contrary, the X wave function is much narrower (scenario **C**), then

$$\mathbf{C}{:}\underline{k}_z\simeq-m_{\mathrm{sp}}\frac{|\vec{p}\,'|}{m_X}. \tag{16}$$

B. Form factor relations

The hadronic matrix elements in Eq. (1) are parametrized in terms of Lorentz invariant form factors as

$$\langle P(\vec{p}\,')|\overline{q}\gamma^{\mu}b|B(\vec{p})\rangle=(p+p')^{\mu}f_1(q^2)+\frac{m_B^2-m_P^2}{q^2}q^{\mu}[f_0(q^2)-f_1(q^2)], \tag{17}$$

where $f_1(0)=f_0(0)$,

$$\langle P(\vec{p}\,')|\overline{q}i\sigma^{\mu\nu}q_{\nu}b|B(\vec{p})\rangle=s(q^2)[(p+p')^{\mu}q^2-(m_B^2-m_P^2)q^{\mu}], \tag{18}$$

$$\langle V(\vec{p}\,',\vec{\varepsilon})|\overline{q}\gamma^{\mu}b|B(\vec{p})\rangle=\frac{-1}{m_B+m_V}2i\epsilon^{\mu\alpha\beta\gamma}\varepsilon^*_{\alpha}p'_{\beta}p_{\gamma}V(q^2), \tag{19}$$

$$\begin{aligned}\langle V(\vec{p}\,',\vec{\varepsilon})|\overline{q}\gamma^{\mu}\gamma_5 b|B(\vec{p})\rangle &=(m_B+m_V)\varepsilon^{\mu *}A_1(q^2)\\ &-\frac{\varepsilon^*\cdot q}{m_B+m_V}(p+p')^{\mu}A_2(q^2)\\ &-2m_V\frac{\varepsilon^*\cdot q}{q^2}q^{\mu}[A_3(q^2)-A_0(q^2)],\end{aligned} \tag{20}$$

where $2m_VA_3(q^2)\equiv(m_B+m_V)A_1(q^2)-(m_B-m_V)A_2(q^2)$ and $A_0(0)=A_3(0)$,

$$\langle V(\vec{p}\,',\vec{\varepsilon})|\overline{q}i\sigma^{\mu\nu}q_{\nu}b|B(\vec{p})\rangle=i\epsilon^{\mu\alpha\beta\gamma}\varepsilon^*_{\alpha}p'_{\beta}p_{\gamma}F_1(q^2), \tag{21}$$

$$\langle V(\vec{p}\,',\vec{\varepsilon})|\overline{q}\,\overline{i}\sigma^{\mu\nu}q_\nu\gamma_5 b|B(\vec{p})\rangle$$
$$=[(m_B^2-m_V^2)\varepsilon^{\mu *}-\varepsilon^*\cdot q(p+p')^\mu]F_2(q^2)$$
$$+\varepsilon^*\cdot q\left[q^\mu-\frac{q^2}{m_B^2-m_V^2}(p+p')^\mu\right]F_3(q^2), \quad (22)$$

where $F_1(0)=2F_2(0)$. In all of the above, $q\equiv p-p'$.

From the result in Eq. (9), we obtain, for each one of the form factors,

$$f_1(q^2)=\left(\frac{m_B-E'}{|\vec{p}\,'|}P_++Q_+\right)NR_{B\to P}, \quad (23)$$

$$f_0(q^2)=\left[\left(\frac{m_B-E'}{|\vec{p}\,'|}P_++Q_+\right)-\frac{q^2}{m_B^2-m_P^2}\left(\frac{m_B+E'}{|\vec{p}\,'|}P_+-Q_+\right)\right]NR_{B\to P}, \quad (24)$$

$$s(q^2)=\frac{1}{|\vec{p}\,'|}P_-NR_{B\to P}, \quad (25)$$

$$V(q^2)=-\frac{m_B+m_V}{|\vec{p}\,'|}P_-NR_{B\to V}, \quad (26)$$

$$A_0(q^2)=\left(\frac{m_B-E'}{|\vec{p}\,'|}P_++Q_+\right)NR_{B\to V}, \quad (27)$$

$$A_1(q^2)=\frac{2m_B}{m_B+m_V}Q_-NR_{B\to V}, \quad (28)$$

$$A_2(q^2)=\frac{m_B+m_V}{m_B|\vec{p}\,'|}\left[\frac{m_BE'-m_V^2}{|\vec{p}\,'|}Q_- -m_V\left(P_++\frac{m_B-E'}{|\vec{p}\,'|}Q_+\right)\right]NR_{B\to V}, \quad (29)$$

$$F_1(q^2)=\left(\frac{m_B-E'}{|\vec{p}\,'|}P_++Q_+\right)2NR_{B\to V}, \quad (30)$$

$$F_2(q^2)=\frac{2m_B|\vec{p}\,'|}{m_B^2-m_V^2}\left(P_++\frac{m_B-E'}{|\vec{p}\,'|}Q_+\right)NR_{B\to V}, \quad (31)$$

$$F_3(q^2)=\left[-\frac{m_V(m_B^2-m_V^2)}{m_B|\vec{p}\,'|^2}Q_- +\frac{m_BE'+m_V^2}{m_B|\vec{p}\,'|}\left(P_++\frac{m_B-E'}{|\vec{p}\,'|}Q_+\right)\right]NR_{B\to V}, \quad (32)$$

with

$$Q_\pm\equiv 1\pm\frac{p_{q,z}p_{b,z}}{(E_q+m_q)(E_b+m_b)}, \quad (33)$$

$$P_\pm\equiv\frac{p_{b,z}}{E_b+m_b}\pm\frac{p_{q,z}}{E_q+m_q}. \quad (34)$$

The factor N is defined by

$$N\equiv\sqrt{\frac{E'}{m_B}\,\frac{E_q+m_q}{2E_q}\,\frac{E_b+m_b}{2E_b}}. \quad (35)$$

The overlap integrals $R_{B\to X}$, as well as the values of the quark momenta $p_{b,z}$ and $p_{q,z}$ that appear in $Q_\pm$, $P_\pm$, and N, depend on the detailed shape of the B and X wave functions. We are interested in deriving relations between form factors that are independent of these quantities, and thus free of the model dependence associated with them. From Eqs. (23)–(32), we have

$$F_1(q^2)=2A_0(q^2), \quad (36)$$

$$F_2(q^2)=\frac{m_B|\vec{p}\,'|}{m_V(m_B-m_V)}\left[\frac{m_BE'-m_V^2}{m_B|\vec{p}\,'|}A_1(q^2)-\frac{2m_B|\vec{p}\,'|}{(m_B+m_V)^2}A_2(q^2)\right], \quad (37)$$

$$F_3(q^2)=\frac{m_B+m_V}{2m_V}A_1(q^2)-\frac{m_BE'+m_V^2}{m_V(m_B+m_V)}A_2(q^2). \quad (38)$$

These relations follow from our initial assumption that spectator effects can be neglected, but no other approximations are necessary.

We now analyze the two special cases of heavy-to-heavy and heavy-to-light transitions; approximations for the b and q quark momenta will lead to additional model-independent relations between the form factors.

C. Heavy-to-heavy transitions

When X is a heavy meson such as a D or D^*, the internal momenta $|\vec{k}|$ and $|\vec{k}'|$, in the B and X mesons, are at most of order $m_{\rm sp}\simeq\Lambda_{\rm QCD}\ll m_B, m_X$. This is true for any of the scenarios **A**, **B**, or **C**, and for the entire range of the recoil momentum: $|\vec{p}\,'|=0$ – $(m_B^2-m_X^2)/2m_B$ or equivalently, $q^2=0$ – $(m_B-m_X)^2$. Then,

$$|\vec{p}_b|<<m_b, m_b\simeq m_B, \quad (39)$$

$$|\vec{p}_q|\simeq|\vec{p}\,'|, m_q\simeq m_X, \quad (40)$$

and so, $Q_\pm=1$, $P_\pm=\pm|\vec{p}\,'|/(E'+m_X)$. Using these results in the expressions for the form factors gives the additional relations

$$f_1(q^2)=f_0(q^2)\left[1-\frac{q^2}{(m_B+m_P)^2}\right]^{-1} \quad (41)$$

$$=-s(q^2)(m_B+m_P), \quad (42)$$

$$V(q^2)=A_0(q^2) \tag{43}$$

$$=A_2(q^2) \tag{44}$$

$$=A_1(q^2)\left[1-\frac{q^2}{(m_B+m_V)^2}\right]^{-1}. \tag{45}$$

These, together with the results in Eqs. (36)–(38), are the same results that follow from the heavy quark symmetry limit [2–5]. In that limit, the spin symmetry gives $m_P=m_V$ and so $R_{B\to P}=R_{B\to V}$ (since the P and V wave functions are then identical); then

$$f_1(q^2)=V(q^2). \tag{46}$$

Moreover, the flavor symmetry gives $R_{B\to X}=1$ for zero recoil (since the B and X wave functions are identical), and so

$$f_1(q^2_{\max})=V(q^2_{\max})=\frac{m_B+m_X}{2\sqrt{m_B m_X}}. \tag{47}$$

Both results are well-known consequences of the heavy quark symmetry, that can be derived independently of the constituent quark picture [2].

D. Heavy-to-light transition

When X is a light meson, such as a π or a ρ, there are two approximations that lead to model-independent relations between form factors. The first is that of a static b quark, i.e., $|\vec{p}_b|\ll m_b$. In that case, $Q_\pm=1$ and $P_+=-P_-$, which gives

$$2m_B s(q^2)=-f_1(q^2)+\frac{m_B^2-m_P^2}{q^2}[f_0(q^2)-f_1(q^2)], \tag{48}$$

$$V(q^2)=-\frac{(m_B+m_V)^2}{2m_B(m_B-E')}A_1(q^2)+\frac{m_B+m_V}{m_B-E'}A_0(q^2) \tag{49}$$

$$=\frac{(m_B+m_V)^3}{2m_B m_V(E'+m_V)}A_1(q^2)-\frac{m_B}{m_V}A_2(q^2). \tag{50}$$

We have found, as in Ref. [1], that a static b quark is a very good approximation over the entire range of the recoil momentum. The only exception is in the extreme case of scenario **C**, and for large recoil. There, the validity of the approximation depends significantly on the choice of $m_{\rm sp}$. In Fig. 1, we show the value of $|\vec{p}_b|/m_b$ for the $B\to\pi$ [Fig. 1(a)] and the $B\to\rho$ [Fig. 1(b)] transitions, in scenario **A**. The two curves correspond to the maximum and minimum values for the parameter $m_{\rm sp}$.

The second approximation is that of a nearly massless recoiling quark i.e., $|\vec{p}_q|\gg m_q$. Then, $Q_\pm=\pm P_\pm$, and so

$$f_1(q^2)=f_0(q^2)\left(1-\frac{q^2}{m_B^2-m_P^2}\,\frac{m_B+E'-|\vec{p}\,'|}{m_B-E'+|\vec{p}\,'|}\right)^{-1}, \tag{51}$$

$$V(q^2)=\frac{(m_B+m_V)^2}{2m_B|\vec{p}\,'|}A_1(q^2) \tag{52}$$

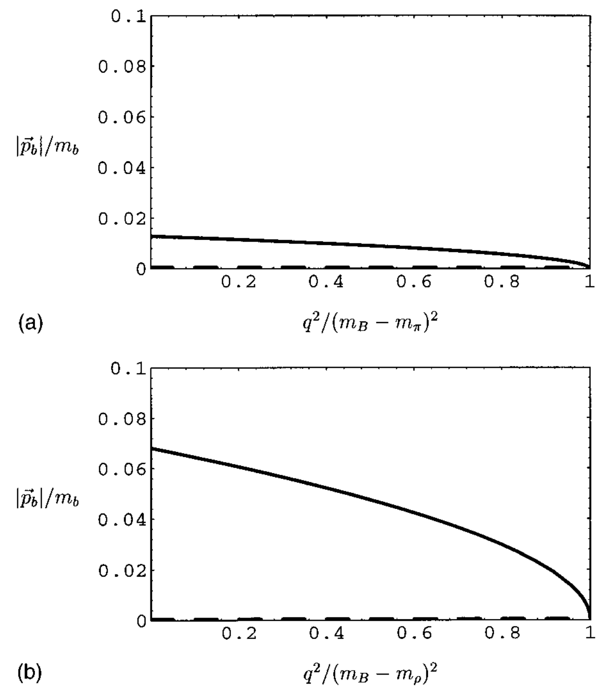

FIG. 1. Momentum vs mass of the b quarks (a) in $B\to\pi$ and (b) in $B\to\rho$, for $m_{\rm sp}$ maximum (full line) and minimum (dashed line).

$$=\frac{m_B|\vec{p}\,'|}{m_B E'-m_V^2}A_2(q^2)+\frac{m_V(m_B+m_V)}{m_B E'-m_V^2}A_0(q^2). \tag{53}$$

The region of validity of this approximation is more restricted than that of a static b quark: the approximation is valid at high recoil, but it breaks down close to lowest recoil; it is worse (in particular for scenario **C**) for a lower choice of $m_{\rm sp}$. For a very light meson, such as $X=\pi$, the validity region expands to most of the kinematic range. In Fig. 2, we show the value of $m_q/|\vec{p}_q|$ for the $B\to\pi$ [Fig. 2(a)] and the $B\to\rho$ [Fig. 2(b)] transitions, in scenario **A**. Again, the two curves correspond to the maximum and minimum values for the parameter $m_{\rm sp}$.

We have shown separately the consequences of the two approximations, given that they have different regions of validity. The overlap between the two regions is, however, significant, and in there we have

$$f_1(q^2)=f_0(q^2)\left(1-\frac{q^2}{m_B^2-m_P^2}\,\frac{m_B+E'-|\vec{p}\,'|}{m_B-E'+|\vec{p}\,'|}\right)^{-1} \tag{54}$$

$$=-(m_B-E'+|\vec{p}\,'|)s(q^2), \tag{55}$$

$$V(q^2)=\frac{(m_B+m_V)^2}{2m_B|\vec{p}\,'|}A_1(q^2) \tag{56}$$

$$=\frac{m_B+m_V}{m_B-E'+|\vec{p}\,'|}A_0(q^2) \tag{57}$$

$$=m_B|\vec{p}\,'|[E'(m_B+m_V)-m_V(m_B+m_V+|\vec{p}\,'|)]^{-1}A_2(q^2). \tag{58}$$

These reproduce the heavy-to-light relations of Ref. [1], up to differences in the terms of order m_X/m_B [6].

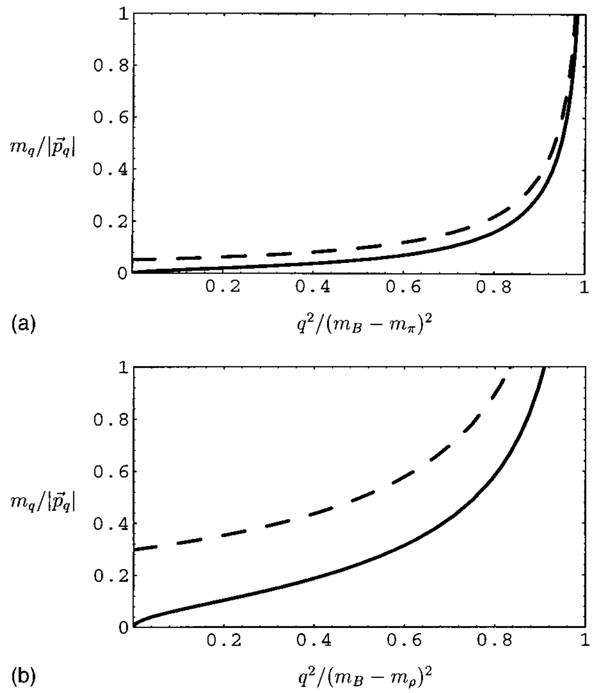

FIG. 2. Mass vs momentum of the q quarks (a) in $B\to\pi$ and (b) in $B\to\rho$ for $m_{\rm sp}$ maximum (full line) and minimum (dashed line).

III. CONCLUSION

The form factor relations in the constituent quark model, that were proposed by Stech in Ref. [1], were looked into in more detail, and using a slightly different description of the meson states. We found that the form factors for the $\langle V|\overline{q}\,\overline{i}\sigma^{\mu\nu}q_\nu b|B\rangle$ and $\langle V|\overline{q}\,\overline{i}\sigma^{\mu\nu}q_\nu\gamma_5 b|B\rangle$ matrix elements are related to the form factors for the $\langle V|\overline{q}\gamma^\mu\gamma_5 b|B\rangle$ matrix element [see Eqs. (36)–(38)]. These relations are independent of any assumptions, other than that no significant spectator effects are at play in the hadronic transition. Such relations can be used to study exclusive radiative decays, such as $B\to K^*\gamma$, but also semileptonic decays such as $B\to K^*e^+e^-$, or color-suppressed hadronic decays such as $B\to J/\psi K^*$.

The form factor relations for heavy-to-heavy transitions, in Eqs. (41)–(45), are those expected from the heavy quark symmetry limit, and they were also found in Ref. [1]. In the heavy-to-light case, we derived form factor relations that do not depend on the specific details of the meson wave functions. This is done with either one of two distinct approximations: that of a static b quark [see Eqs. (48)–(50)], and that of a nearly massless recoiling quark q [see Eqs. (51)–(53)]. In each case we have discussed the regions where the approximations are expected to hold. In particular, we have confirmed the result of Ref. [1], that the static b quark approximation remains valid throughout the entire kinematic range. For the case where both approximations are valid [see Eqs. (54)–(58)], our results differ from those of Ref. [1], by terms of order m_X/m_B.

ACKNOWLEDGMENTS

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[1] B. Stech, Phys. Lett. B **354**, 447 (1995).

[2] For a review of the heavy quark symmetry results for the form factors, see, for example, M. Neubert *et al.*, in *Heavy Flavors*, edited by A. J. Buras and L. Lindner (World Scientific, Singapore, 1992).

[3] The momentum wave function is normalized as

$$\int d^3\vec{k}\ |\phi_B(|\vec{k}|)|^2=1\ . \qquad (59)$$

The normalization of the meson and quark or antiquark states, as well as the normalization of the Dirac spinors, is as in Ref. [4].

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[5] N. Isgur and M. B. Wise, Phys. Rev. D **42**, 2388 (1990).

[6] As a simple check of the heavy-to-light form factor relations, in the limit $|\vec{p}_q|\gg m_q$, we can consider the hadronic matrix elements

$$A_\pm^{(\gamma)}\equiv\langle V(\lambda=\pm1)|\overline{q}\gamma^\mu(1-\gamma_5)b|B\rangle\varepsilon_\mu^* =(m_B+m_V)\left[-A_1(q^2)\pm V(q^2)\frac{2m_B|\vec{p}'|}{(m_B+m_V)^2}\right] \qquad (60)$$

and

$$A_\pm^{(\sigma)}\equiv\langle V(\lambda=\pm1)|\overline{qi}\sigma^{\mu\nu}q_\nu(1+\gamma_5)b|B\rangle\varepsilon_\mu^* =(m_B^2-m_V^2)\left[F_2(q^2)\mp F_1(q^2)\frac{m_B|\vec{p}'|}{m_B^2-m_V^2}\right]\ , \qquad (61)$$

for a B decay into a light vector meson V and a vector particle with polarization ε_μ, with helicities $\lambda=\pm1$. The operators $\overline{q}\gamma^\mu(1-\gamma_5)b$ and $\overline{qi}\sigma^{\mu\nu}q_\nu(1+\gamma_5)b$ create a left-handed quark q. In the limit $m_q\to0$, this corresponds to a quark with helicity $\lambda_q=-1/2$, and so the transition to a vector meson V with $\lambda=+1$ is not possible, if no spectator effects are allowed. Using the form factor relations for the heavy-to-light case, we can see immediately that $A_+^{(\gamma,\sigma)}=0$, as expected.