Decay constants of P- and D-wave heavy-light mesons

Siniša Veseli
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

Isard Dunietz
Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510
(Received 25 June 1996)

We investigate decay constants of P- and D-wave heavy-light mesons within the mock-meson approach. Numerical estimates are obtained using the relativistic quark model. We also comment on recent calculations of heavy-light pseudoscalar and vector decay constants. [S0556-2821(96)01723-7]

PACS number(s): 12.39.Hg, 12.39.Ki, 13.25.Hw

I. INTRODUCTION

Reliable estimates of heavy-light meson decay constants are important, since they appear in many processes from which fundamental quantities can be extracted [1]. Theoretical investigations have focused on estimating decay constants for the weakly decaying pseudo-scalar meson and its heavy quark effective theory (HQET) -related vector meson. Whereas the decay constant of the weakly decaying pseudoscalar meson is of paramount importance for determining fundamental quantities, the decay constant of the S-wave vector meson plays a role in exclusive $b\to ul\overline{\nu}$ transitions [2] and in radiative leptonic decays of heavy-light mesons [3].

While those decay constants have been and continue to be studied intensively, the decay constants of the more highly excited heavy-light states have been normally ignored. This paper attempts to rectify this situation, by predicting decay constants for many higher-excited resonances. That could be important phenomenologically on several accounts.

First, CLEO recently observed a significant wrong charm contribution in B decays [4],

$$\mathcal{B}(\overline{B}\to\overline{D}X)\approx 10\%, \tag{1.1}$$

governed essentially by the $b\to c\overline{c}\overline{s}'$ quark transitions.[1] The $B\to DX$ transitions were overlooked in all previous experimental analyses. Under the factorization assumption [5], wherein the virtual $W\to\overline{c}s$ hadronizes independently of the rest of the system, a quantitative modeling of the $\overline{B}\to\overline{D}X$ transitions can be undertaken once the theory provides the decay constants for D_s^{**}.

Second, reliable estimates for decay constants of D^{**} allow one to test whether color-allowed and color-suppressed decay amplitudes interfere constructively for the $B^-\to D^{**0}\{\pi^-,\rho^-,a_1^-,\dots\}$ modes, as has been seen for the $B^-\to D^{(*)0}\{\pi^-,\rho^-,a_1^-,\dots\}$ transitions [6]. Third, such estimates enable us to better predict subtle CP-violating phenomena.

[1]The prime indicates that the corresponding Cabibbo-suppressed mode is included.

Decay constants are defined through matrix elements of vector and pseudovector currents between meson states and the vacuum. Therefore, in order to calculate them, one has to find a way to evaluate hadronic matrix elements. The mock-meson method [7–10] has been frequently used in the literature for that purpose [7–17]. In this paper we follow the same approach, and use the mock-meson method in order to obtain expressions for the decay constants of heavy-light mesons, in terms of integrals over momentum-space bound-state wave functions. For numerical estimates we decided to use the simplest relativistic generalization of the Schrödinger equation [11,18–20], sometimes called the spinless Salpeter equation.

The rest of the paper is organized as follows. We begin with a brief description of the mock-meson method in Sec. II. Our approach is based on the j-j coupling scheme, since it is more appropriate for heavy-light mesons than the usual L-S scheme. Expressions for the decay constants of heavy-light meson states are given in Sec. III. The relativistic quark model and our numerical estimates are described in Sec. IV. There we also comment on recent calculations of pseudoscalar and vector decay constants [17]. Our conclusions are summarized in Sec. V.

II. THE MOCK-MESON METHOD

As already mentioned, the mock-meson approach [7–10] has been widely used for calculations of hadronic matrix elements [7–17]. The basic idea of the method is simple. The mock meson is defined as a collection of free quarks weighted with a bound-state wave function. The mock-meson matrix elements $\widetilde{\mathcal{M}}$ can then be calculated using full Dirac spinors. On the other hand, the physical matrix elements $\mathcal{M}$ can always be expressed in terms of Lorentz covariants with coefficients A_i, which are Lorentz scalars. In many simple cases, $\mathcal{M}$ and $\widetilde{\mathcal{M}}$ will be of the same form. The mock-meson prescription then says that in those cases one should simply take $A_i=\widetilde{A}_i$. Indeed, this correspondence is exact in the zero-binding limit and in the meson rest frame. Away from this limit the mock amplitudes are in general not invariant by terms of order p_i^2/m_i^2.

In this paper we are primarily concerned with the decay constants of heavy-light $q\overline{Q}$ mesons. In the $m_{\overline{Q}}\to\infty$ limit, heavy quark symmetry tells us that the angular momentum of the light degrees of freedom (LDF) in the heavy-light meson decouples from the spin of the heavy quark, and both are separately conserved by the strong interaction [21]. Therefore, total angular momentum j of the LDF is a good quantum number. For each j there are two degenerate heavy meson states ($J=j\pm\frac{1}{2}$), which can be labeled as J_j^P, where $P=(-1)^{L+1}$. This implies that in the case of heavy-light mesons the j-j coupling is more appropriate than the L-S coupling scheme. For this reason, we first define the LDF states $|j\lambda_j;L\frac{1}{2}\rangle$ as Clebsch-Gordan (CG) combinations of the eigenstates of orbital angular momentum $|LM_L\rangle$, and those of the spin of the light quark $|\frac{1}{2}s\rangle$, with CG coefficients denoted as $C^{j\lambda_j}_{LM_L;(1/2)s}$. Combining the LDF states with those of the heavy antiquark $|\frac{1}{2}\overline{s}\rangle$ (with CG coefficients $C^{JM_J}_{j\lambda_j;(1/2)\overline{s}}$), we get the $q\overline{Q}$ mock-meson state in its rest frame:

$$
\begin{aligned}
|J_j^P M_J;n\rangle = \sqrt{2\widetilde{M}}\frac{1}{\sqrt{3}}\sum_c \sum_{\lambda_j,M_L,s,\overline{s}} C^{JM_J}_{j\lambda_j;\frac{1}{2}\overline{s}} C^{j\lambda_j}_{LM_L;\frac{1}{2}s} \\
\times\int\frac{d^3\mathbf{p}}{(2\pi)^3}\sqrt{\frac{m_q m_{\overline{Q}}}{E_q E_{\overline{Q}}}}\phi_{nLM_L}(\mathbf{p})|q_c(\mathbf{p},s)\rangle \\
\times|\overline{Q}_{\overline{c}}(-\mathbf{p},\overline{s})\rangle. \qquad (2.1)
\end{aligned}
$$

In the above expression $E_i=\sqrt{m_i^2+\mathbf{p}^2}$, $\widetilde{M}$ is the mock-meson mass, and the color wave function (subscript c denotes color) is written explicitly. Also, $\phi_{nLM_L}(\mathbf{p})$ is a normalized momentum wave function, where n denotes all other quantum numbers of a state not connected to angular momentum (e.g., radial quantum number). The factor $1/(2\pi)^3\sqrt{m_q m_{\overline{Q}}/E_q E_{\overline{Q}}}$ appears due to our normalization convention for creation and annihilation operators [22], $\{b_\alpha(k),b^\dagger_{\alpha'}(k')\}=(2\pi)^3 k_0/m\,\delta^3(\mathbf{k}-\mathbf{k}')\,\delta_{\alpha\alpha'}$, etc. The mock-meson states as given in Eq. (2.1) are normalized to $2\widetilde{M}$.

As already observed in [15], the mock-meson approach suffers from a number of ambiguities, such as the choice for quark masses, or the definition of the mock-meson mass $\widetilde{M}$. In the spirit of the method, the mock-meson mass should be defined as $\widetilde{M}=\langle E_q\rangle+\langle E_{\overline{Q}}\rangle$. However, as pointed out in [15], the mock-meson mass has been introduced to give the correct relativistic normalization of the meson's wave function, and hence the use of the physical meson mass M instead of $\langle E_q\rangle+\langle E_{\overline{Q}}\rangle$ may be more appropriate. We adopt the same approach, and write $\widetilde{M}=M$. We also note that the heavier the mesons are, the less important it is how the mock-meson mass is defined, since the relativistic effects and binding energies become less significant. As far as quark masses are concerned, the self-consistency of the model requires the use of constituent quark masses. In our error estimates we have included variations of constituent light quark masses over a range of about 200 MeV, and also of heavy quark masses over a range of about 400 MeV, so that we believe that uncertainties introduced by a particular choice of quark masses are being properly taken into account.

III. DECAY CONSTANTS

Decay constants of heavy-light mesons are defined through matrix elements of vector V^μ and pseudovector A^μ currents between a meson state and the vacuum. Following standard definitions in the literature [11], for pseudoscalar (P), vector (V), scalar (S), and pseudovector (A) mesons, we write

$$\langle 0|A^\mu(0)|0^-_{1/2}(k)\rangle=\frac{1}{(2\pi)^{3/2}}f_P k^\mu, \qquad (3.1)$$

$$\langle 0|V^\mu(0)|1^-_{1/2}(\epsilon,k)\rangle=\frac{1}{(2\pi)^{3/2}}f_{V_{1/2}}M_{V_{1/2}}\epsilon^\mu, \qquad (3.2)$$

$$\langle 0|V^\mu(0)|0^+_{1/2}(k)\rangle=\frac{1}{(2\pi)^{3/2}}f_S k^\mu, \qquad (3.3)$$

$$\langle 0|A^\mu(0)|1^+_{1/2}(\epsilon,k)\rangle=\frac{1}{(2\pi)^{3/2}}f_{A_{1/2}}M_{A_{1/2}}\epsilon^\mu, \qquad (3.4)$$

$$\langle 0|A^\mu(0)|1^+_{3/2}(\epsilon,k)\rangle=\frac{1}{(2\pi)^{3/2}}f_{A_{3/2}}M_{A_{3/2}}\epsilon^\mu, \qquad (3.5)$$

$$\langle 0|V^\mu(0)|1^-_{3/2}(\epsilon,k)\rangle=\frac{1}{(2\pi)^{3/2}}f_{V_{3/2}}M_{V_{3/2}}\epsilon^\mu. \qquad (3.6)$$

Note that in the heavy quark limit $0^-_{1/2}$ and $1^-_{1/2}$ states are degenerate (S waves), and so are $0^+_{1/2}$ and $1^+_{1/2}$ (P-wave states). The spin-2 members of P-wave ($1^+_{3/2},2^+_{3/2}$) and D-wave ($1^-_{3/2},2^-_{3/2}$) doublets do not couple leptonically due to conservation of angular momentum.

In order to obtain expressions for decay constants in terms of integrals over momentum-space meson wave functions, we evaluate the matrix elements (3.1)–(3.6) in the meson rest frame using Eq. (2.1). Of course, any choice of polarization for spin-1 mesons should yield the same result. As mentioned earlier, current matrix elements between states defined in Eq. (2.1) and the vacuum can be evaluated exactly with full Dirac spinors. Because of spherical symmetry, the momentum-space wave function can be written in the form

$$\phi_{nLM_L}(\mathbf{p})=R_{nL}(p)Y_{LM_L}(\hat{\mathbf{p}}). \qquad (3.7)$$

In the above Y_{LM_L} are the usual spherical harmonics, and $R_{nL}(p)$ is the radial part of the wave function, where p denotes $|\mathbf{p}|$ henceforth. Using Eq. (3.7), and keeping track of the relevant CG coefficients, we find that all heavy-light meson decay constants in the mock-meson approach can be written in the form

$$f_i=\frac{2\sqrt{3}}{\sqrt{M}}\sqrt{4\pi}\int_0^\infty\frac{p^2dp}{(2\pi)^{3/2}}\sqrt{\frac{(m_q+E_q)(m_{\overline{Q}}+E_{\overline{Q}})}{4E_qE_{\overline{Q}}}}F_i(p), \qquad (3.8)$$

where

$$F_P(p)=\left[1-\frac{p^2}{(m_q+E_q)(m_{\overline{Q}}+E_{\overline{Q}})}\right]R_{n0}(p), \qquad (3.9)$$

$$F_{V_{1/2}}(p)=\left[1+\frac{1}{3}\frac{p^2}{(m_q+E_q)(m_{\bar{Q}}+E_{\bar{Q}})}\right]R_{n0}(p), \tag{3.10}$$

$$F_S(p)=\left[\frac{1}{(m_q+E_q)}-\frac{1}{(m_{\bar{Q}}+E_{\bar{Q}})}\right]pR_{n1}(p), \tag{3.11}$$

$$F_{A_{1/2}}(p)=\left[\frac{1}{(m_q+E_q)}+\frac{1}{3}\frac{1}{(m_{\bar{Q}}+E_{\bar{Q}})}\right]pR_{n1}(p), \tag{3.12}$$

$$F_{A_{3/2}}(p)=\left[\frac{2\sqrt{2}}{3}\frac{1}{(m_{\bar{Q}}+E_{\bar{Q}})}\right]pR_{n1}(p), \tag{3.13}$$

$$F_{V_{3/2}}(p)=\left[\frac{2\sqrt{2}}{3}\frac{1}{(m_q+E_q)}\frac{1}{(m_{\bar{Q}}+E_{\bar{Q}})}\right]p^2R_{n2}(p). \tag{3.14}$$

Expressions (3.9) and (3.10) were found in [12] and [17], respectively.

It is interesting to observe that in the limit $m_{\bar{Q}}\to\infty$, Eqs. (3.8)–(3.14) become

$$f_i^{HL}=\frac{2\sqrt{3}}{\sqrt{M}}\sqrt{4\pi}\int_0^\infty\frac{p^2dp}{(2\pi)^{3/2}}\sqrt{\frac{(m_q+E_q)}{2E_q}}F_i^{HL}(p), \tag{3.15}$$

with

$$F_P^{HL}(p)=F_{V_{1/2}}^{HL}(p)=R_{n0}(p), \tag{3.16}$$

$$F_S^{HL}(p)=F_{A_{1/2}}(p)=\frac{1}{(m_q+E_q)}pR_{n1}(p), \tag{3.17}$$

$$F_{A_{3/2}}^{HL}(p)=0, \tag{3.18}$$

$$F_{V_{3/2}}^{HL}(p)=0. \tag{3.19}$$

Equality of f_P and $f_{V_{1/2}}$, and also that of f_S and $f_{A_{1/2}}$, as well as vanishing of $f_{A_{3/2}}$ and $f_{V_{3/2}}$, are in the heavy quark limit expected from the heavy quark symmetry.

IV. RELATIVISTIC QUARK MODEL

In order to obtain numerical estimates for the decay constants of heavy-light mesons, we consider the simplest and widely used generalization of the nonrelativistic Schrödinger equation [11,18–20] with Hamiltonian given by

$$H=\sqrt{m_q^2+p^2}+\sqrt{m_{\bar{Q}}^2+p^2}+V(r), \tag{4.1}$$

where for $V(r)$ we take the QCD-motivated Coulomb-plus-linear potential [11]

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+br+c. \tag{4.2}$$

TABLE I. Relativistic quark model predictions compared to experimental spin-averaged heavy-light meson masses. Parameters of the model are $m_{u,d}=0.300$ GeV, $m_s=0.483$ GeV, $m_c=1.671$ GeV, $m_b=5.121$ GeV, $\alpha_s=0.498$, $b=0.142$ GeV^2, and $c=-0.350$ GeV. The unknown D_0 and D_1' mesons ($0^+_{1/2}$ and $1^+_{1/2}$ states) were assumed to have a spin-averaged mass of 2400 MeV. Heavy quark symmetry arguments then lead to the spin-averaged mass of 2502 MeV for the corresponding D_{s0} and D_{s1}' mesons.

Meson	State	Experiment [MeV]	Theory [MeV]	Error [MeV]
$D(1867)$	$0^-_{1/2}$	$1S(1974)$	1971	-3
$D^*(2009)$	$1^-_{1/2}$			
$D_0(\sim 2400)$	$0^+_{1/2}$	$1P(2431)$	2434	$+3$
$D_1'(\sim 2400)$	$1^+_{1/2}$			
$D_1(2425)$	$1^+_{3/2}$			
$D_2^*(2459)$	$2^+_{3/2}$			
$D_s(1969)$	$0^-_{1/2}$	$1S(2076)$	2079	$+3$
$D_s^*(2112)$	$1^-_{1/2}$			
$D_{s0}(\sim 2502)$	$0^+_{1/2}$	$1P(2540)$	2537	-3
$D_{s1}'(\sim 2502)$	$1^+_{1/2}$			
$D_{s1}(2535)$	$1^+_{3/2}$			
$D_{s2}^*(2573)$	$2^+_{3/2}$			
$B(5279)$	$0^-_{1/2}$	$1S(5314)$	5314	$+0$
$B^*(5325)$	$1^-_{1/2}$			
$B_s(5374)$	$0^-_{1/2}$	$1S(5409)$	5409	-0
$B_s^*(5421)$	$1^-_{1/2}$			

For the sake of simplicity,[2] we take α_s to be a fixed, effective, short range coupling constant. The effective string tension of the model can be determined from the requirement that the linear Regge structure of the model in the light-light limit agrees with the observed slope of the ρ trajectory [23]. Fixing $m_{u,d}$, other parameters can be chosen so that the model reproduces the observed spin-averaged spectrum of the known heavy-light states. One such set of parameters includes constituent quark masses $m_{u,d}=0.300$ GeV, $m_s=0.483$ GeV, $m_c=1.671$ GeV, and $m_b=5.121$ GeV, and also $\alpha_s=0.498$, $b=0.142$ GeV^2, and $c=-0.350$ GeV.[3] As can be seen in Table I, these parameters yield an excellent description of the observed spin-averaged heavy-light spectrum.

We now turn to the discussion of pseudoscalar and vector decay constants. Recently, Ref. [17] used Eq. (4.1) with six different potentials, and with current quark masses from [24], minimized the Hamiltonian with respect to the variational parameter β of a single harmonic oscillator (HO) wave function,

[2]The running coupling constant was used in [11].

[3]This particular parameter set corresponds to the spin-averaged mass of the unknown D_0 and D_1' mesons ($0^+_{1/2}$ and $1^+_{1/2}$ states) of about 2400 MeV.

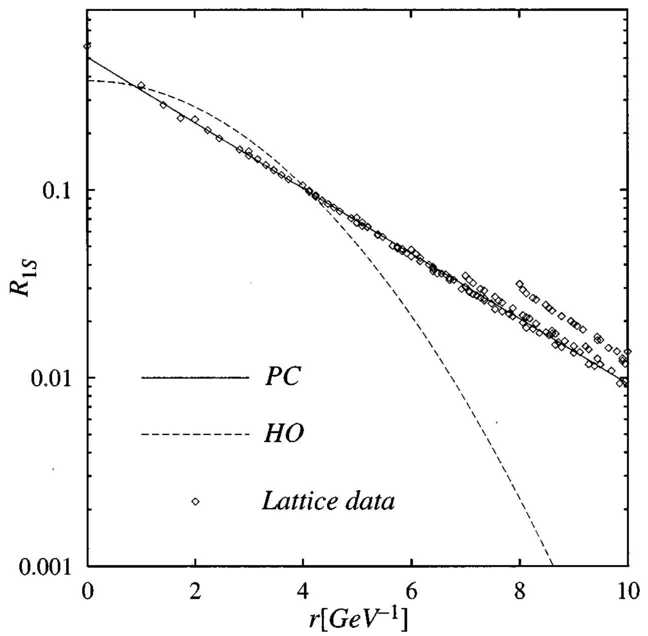

FIG. 1. Comparison of the pseudo-Coulombic [PC, $R_{1S}(r) = 2\beta^{3/2}e^{-\beta r}$], and the harmonic oscillator [HO, $R_{1S}(r) = 2\beta^{3/2}/\pi^{1/4}e^{-\beta^2 r^2/2}$], $1S$ configuration space wave functions with the lattice data [25]. For both PC and HO wave functions we used $\beta = 0.40$ GeV.

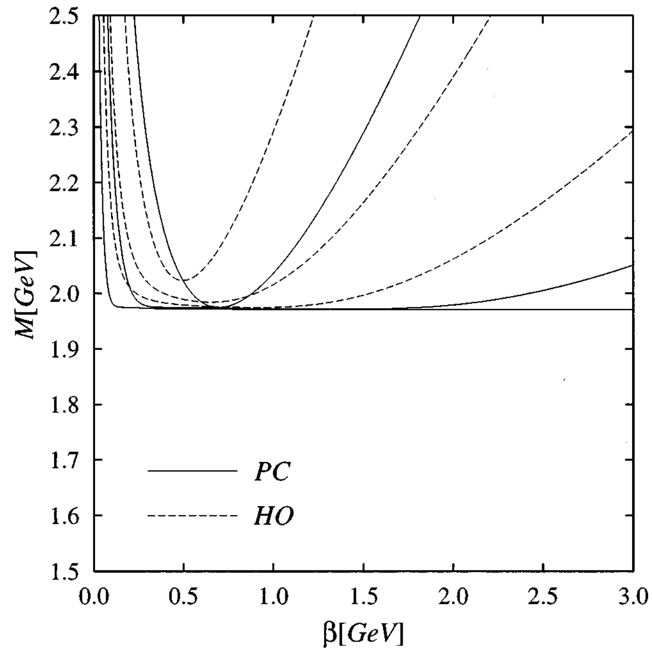

FIG. 2. Convergence of the $1S$ state mass of Eqs. (4.1) and (4.2), with $m_1 = m_{u,d} = 0.300$ GeV, $m_2 = m_c = 1.671$ GeV, $b = 0.142$ GeV^2, $c = -0.350$ GeV, and $\alpha_s = 0.498$. Pseudo-Coulombic (PC, full lines) and harmonic oscillator (HO, dashed lines) wave functions with $N = 1$, 5, and 15 basis states.

$$R_{1S}(p) = \frac{2}{\pi^{1/4}\beta^{3/2}}\exp[-p^2/(2\beta^2)], \tag{4.3}$$

and then used the wave function obtained in this way to get pseudoscalar and vector decay constants from Eqs. (3.8)–(3.10). However, a single harmonic oscillator (HO) basis state is not a suitable approximation for the meson wave function. Namely, lattice simulations [25] show that heavy-light wave functions fall exponentially with large r ($\sim e^{-\beta r}$) and therefore, HO wave functions ($\sim e^{-\beta^2 r^2/2}$) cannot be expected to reflect the correct dynamics of heavy-light mesons. If single basis states are used, a much better choice would be pseudo Coulombic (PC) basis states [26] which fall exponentially with large r and appear to be in a good agreement with the lattice data, as can be seen in Fig. 1.

Models such as the one we are using here are usually solved by diagonalizing the Hamiltonian matrix in a particular (truncated) basis, with basis states depending on some variational parameter [27]. As one increases the number of basis states, the dependence of eigenvalues and eigenfunctions on the variational parameter should vanish for the lowest states. In the case of QCD-motivated potentials the solutions obtained with the PC wave functions converge much more rapidly with an increase in the number of basis states, than those obtained with the HO wave functions. We illustrate that in Fig. 2, by plotting the dependence of energy of the lowest $1S$ state on the variational parameter for $N = 1$, 5, and 15 basis states, for both PC and HO wave functions. One can clearly see that the lowest $1S$ HO wave function is not a very good trial wave function in a variational calculation of Eq. (4.1) (with QCD-motivated potentials). Furthermore, even if one believes that the $N = 1$ HO result for a state energy is acceptable (it is roughly 50 MeV higher than the exact solution, as can be seen in Fig. 2), that still does not justify the use of a single HO basis state as a meson wave function. This issue is clearly important in calculations where a correct description of meson dynamics is needed, such as calculations of meson decay constants. Results obtained by varying a single HO basis state are thus to be interpreted as nonrelativistic estimates of some effective harmonic oscillator potential, and not as the results of a QCD-motivated relativistic quark model.

One can now observe that if one uses enough basis states, the choice of basis wave functions should not matter, and pseudoscalar and vector decay constants should be obtainable from the relativistic quark model considered here. The problem is, however, that the $1S$ wave function is divergent at the spatial origin [28]: i.e.,

$$\psi_{1S} \sim r^{-4\alpha_s/(3\pi)}. \tag{4.4}$$

The singularity for $r \to 0$ is related to the singularity of the short-range Coulomb potential. By increasing the number of (usually finite at $r = 0$) basis states, one is gradually beginning to see that singularity [20]. Furthermore, from Eq. (4.4) one can see that the degree of divergence highly depends on the choice of α_s. Because of that, one can expect that pseudoscalar and vector decay constants cannot be reliably estimated within the model we are considering. In Figs. 3 and 4 we demonstrate the dependence of the pseudoscalar (D-meson) and vector (D^*-meson) decay constants on the number of basis states (N), for both PC and HO wave functions. As one can see, for small N both f_P and $f_{V_{1/2}}$ are significantly increasing with an increase in N. By including enough basis states, the dependence on N would eventually

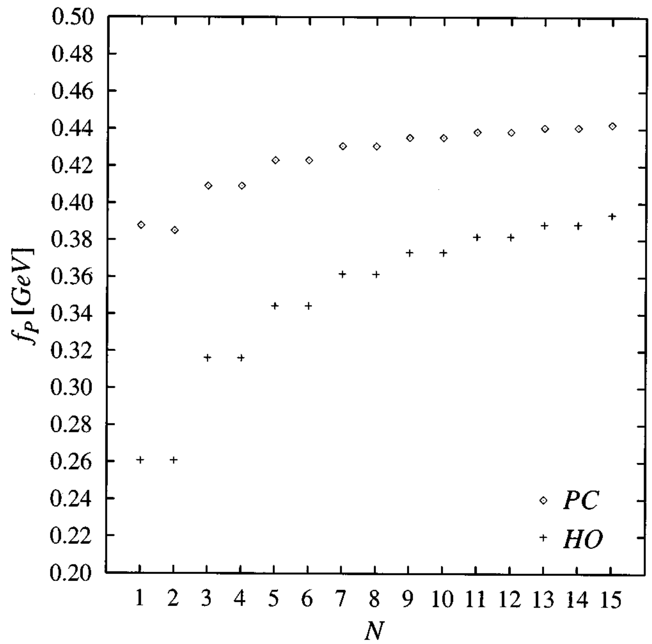

FIG. 3. Dependence of the pseudoscalar (D meson, $0^-_{1/2}$ state) decay constant f_P on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) basis states. We have used parameters given in the text, i.e., $m_1=m_{u,d}=0.300$ GeV, $m_2=m_c=1.671$ GeV, $b=0.142$ GeV^2, $c=-0.350$ GeV, and $\alpha_s=0.498$.

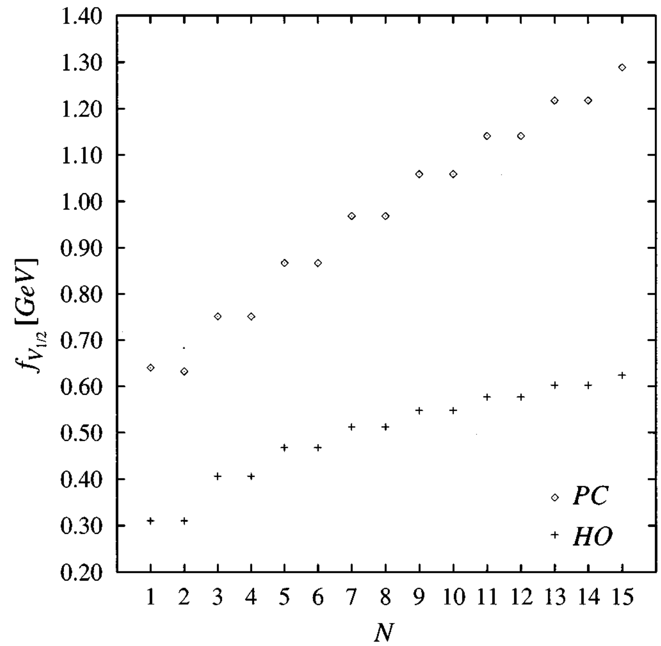

FIG. 4. Dependence of the vector (D^* meson, $1^-_{1/2}$ state) decay constant $f_{V_{1/2}}$ on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) basis states. We have used $m_1=m_{u,d}=0.300$ GeV, $m_2=m_c=1.671$ GeV, $b=0.142$ GeV^2, $c=-0.350$ GeV, and $\alpha_s=0.498$.

vanish.[4] However, as implied by Eq. (4.4), both f_P and $f_{V_{1/2}}$ are quite sensitive to the particular choice of parameters of the model. In our calculations we have observed that results obtained with fixed N can vary up to a few hundred MeV. Because of that, we were not able to obtain reliable estimates of f_P and $f_{V_{1/2}}$ from the model considered in this paper.[5]

One possible solution of the problem discussed above would be to replace the $1/r$ potential with the one-loop single gluon exchange potential, i.e., $\alpha_s \rightarrow \alpha_s(r)$. The $1S$ solution of Eq. (4.1) in that case is still divergent, but the divergence is only logarithmic [28]. This should lead to much more stable results than the ones shown in Figs. 3 and 4. These results should also be much less dependent on the specific choice of the model parameters. In fact, such a calculation for f_P (for D, D_s, B, and B_s mesons) was already performed by Capstick and Godfrey in [15] using the model of [11]. The dependence of their results on the number of basis states was not shown, but the authors of [15] stated that they believed that the model overestimates pseudoscalar decay constants (e.g., for D meson they found $f_P=301$ MeV with uncertainty of 20%). Even though it is important to investigate what really happens with both f_P and $f_{V_{1/2}}$ in such a model, we shall not consider it in the present paper.

We next discuss the heavy-light P- and D-wave decay constants. While we were not able to obtain reliable results from Eqs. (4.1) and (4.2) for the S waves, the situation for P and D waves is completely different. In Figs. 5–8 we show the dependence on the number of basis states (N), for scalar (S), two pseudovector ($A_{1/2}$ and $A_{3/2}$), and vector ($V_{3/2}$) decay constants, respectively. All the results shown are for the D^{**} mesons. As one can see in those figures, in general only a few basis states are needed for results to become independent of N, even though the derivatives of the actual $1P$ and $1D$ wave functions are singular at spatial origin [20].[6] Furthermore, as N increases the HO results approach the PC results (always from below) which shows that the difference between the two basis sets is slowly vanishing. However, even with 15 basis states (when the state energy obtained from the model is essentially equal for both PC and HO wave functions), we can still see the difference for $f_{A_{1/2}}$ (Fig. 6) and for $f_{A_{3/2}}$ (Fig. 7). This reflects the difference in the wave functions obtained from the two basis sets. The reason why both PC and HO basis states yield almost the same results for f_S (Fig. 5), even though $0^+_{1/2}$ state is also a

[4]Because of the minus sign in Eq. (3.9) the results for f_P are better behaved than those for $f_{V_{1/2}}$. For example, f_P obtained with $N=50$ PC states are usually larger than those obtained with $N=25$ by only a few MeV. On the other hand, the same increase in N in general leads to increase in $f_{V_{1/2}}$ by several hundred MeV.

[5]From Figs. 3 and 4 it should be clear that in the model considered here the ratio $f_P/f_{V_{1/2}}$ also cannot be determined with reasonable errors.

[6]By fixing all input parameters, the sensitivity of the decay constants on the number of basis states was investigated. To achieve an accuracy of 0.1 MeV for f_S and $f_{V_{3/2}}$ as little as 10 PC basis states usually were needed, while to achieve the same accuracy for $f_{A_{1/2}}$ and $f_{A_{3/2}}$ requires in general about 50–75 PC basis states.

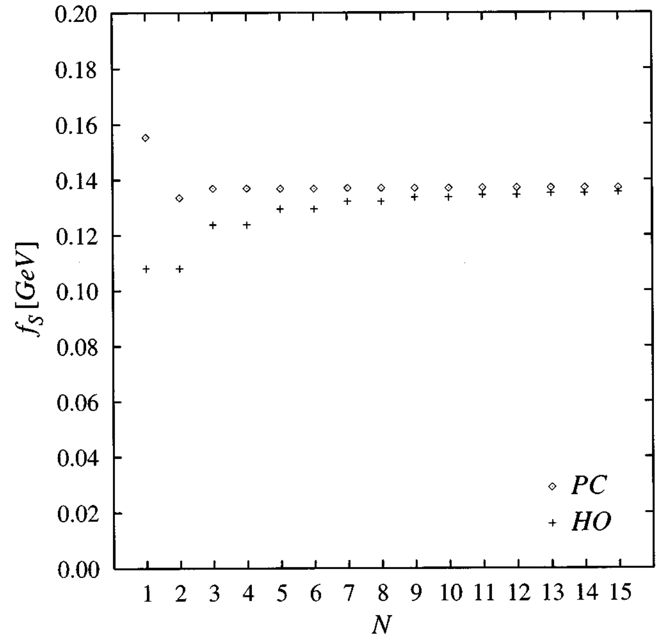

FIG. 5. Dependence of the scalar (D_0 meson, $0^+_{1/2}$ state) decay constant f_S on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) basis states. We have used $m_1 = m_{u,d} = 0.300$ GeV, $m_2 = m_c = 1.671$ GeV, $b = 0.142$ GeV^2, $c = -0.350$ GeV, and $\alpha_s = 0.498$.

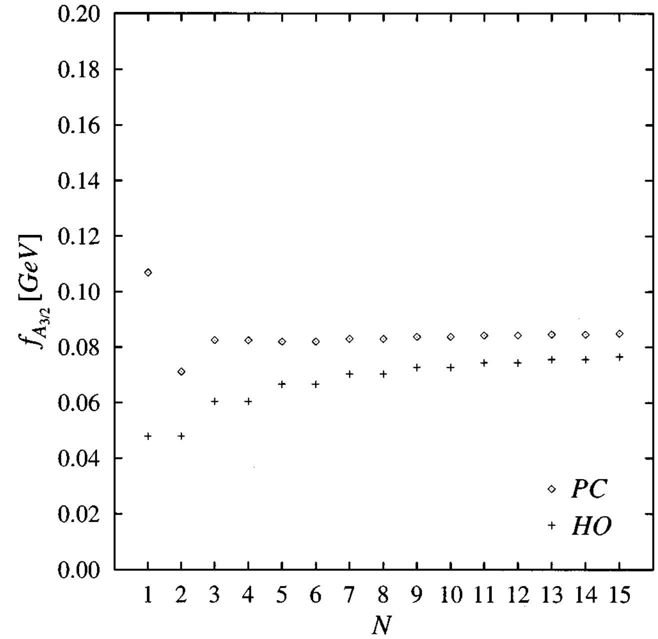

FIG. 7. Dependence of the pseudovector (D_1 meson, $1^+_{3/2}$ state) decay constant $f_{A_{3/2}}$ on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) wave functions. We have used $m_1 = m_{u,d} = 0.300$ GeV, $m_2 = m_c = 1.671$ GeV, $b = 0.142$ GeV^2, $c = -0.350$ GeV, and $\alpha_s = 0.498$.

P wave, is the minus sign in Eq. (3.11). Of course, because of the much more rapid convergence, the PC results are to be preferred over the HO results.

Our calculations of P- and D-wave decay constants showed that their dependence on the particular choice of the model parameters is significantly smaller than the corresponding dependence of f_S and $f_{V_{1/2}}$. We present the results for D^{**}, D_s^{**}, B^{**}, and B_s^{**} mesons in Tables II, III, IV,

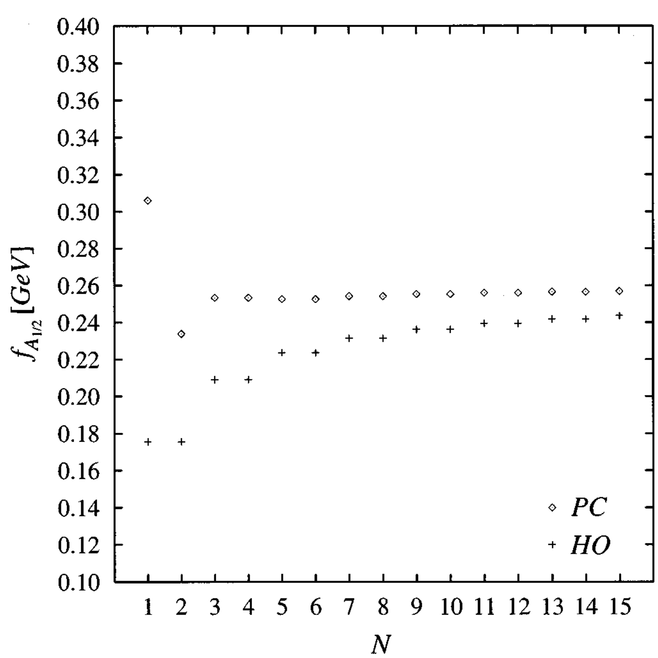

FIG. 6. Dependence of the pseudovector (D_1' meson, $1^+_{1/2}$ state) decay constant $f_{A_{1/2}}$ on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) wave functions. We have used $m_1 = m_{u,d} = 0.300$ GeV, $m_2 = m_c = 1.671$ GeV, $b = 0.142$ GeV^2, $c = -0.350$ GeV, and $\alpha_s = 0.498$.

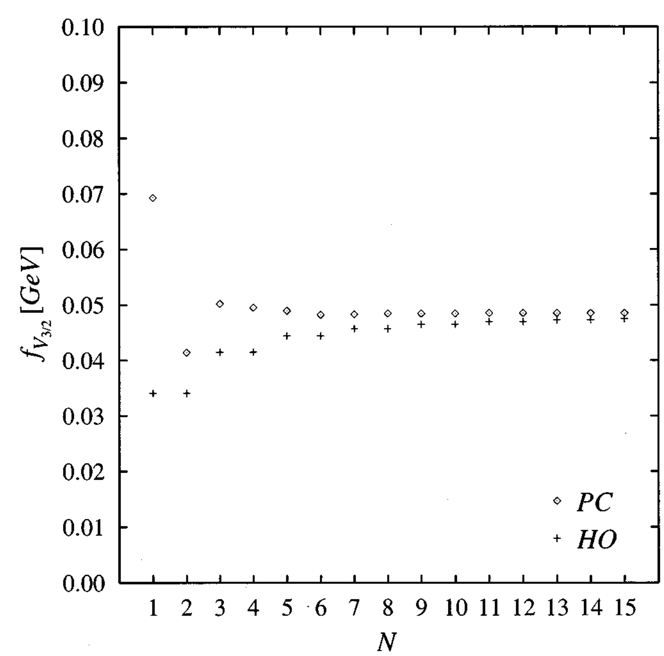

FIG. 8. Dependence of the vector (D_1'' meson, $1^-_{3/2}$ state) decay constant $f_{V_{3/2}}$ on the number (N) of pseudo-Coulombic (PC), and harmonic oscillator (HO) wave functions. We have used $m_1 = m_{u,d} = 0.300$ GeV, $m_2 = m_c = 1.671$ GeV, $b = 0.142$ GeV^2, $c = -0.350$ GeV, and $\alpha_s = 0.498$.

TABLE II. Decay constants of heavy-light D^{**} states, as obtained from the relativistic quark model. Whenever possible we used experimental meson masses. If these were unknown, we used model predictions for the spin-averaged masses.

Meson	State	f_i [MeV]
$D(1867)$	$1S,0^-_{1/2}$	Not reliable
$D^*(2009)$	$1S,1^-_{1/2}$	Not reliable
$D_0(2410\pm40)$	$1P,0^+_{1/2}$	139 ± 30
$D'_1(2410\pm40)$	$1P,1^+_{1/2}$	251 ± 37
$D_1(2425)$	$1P,1^+_{3/2}$	77 ± 18
$D''_1(2700\pm55)$	$1D,1^-_{3/2}$	48 ± 7

and V, respectively.[7] To obtain these results the effective string tension b of the model was determined from the observed slope of the ρ trajectory. For a fixed $m_{u,d}$ other parameters were obtained from the spectrum of the known heavy-light states. Experimental meson masses were used in Eq. (3.8) only when their quantum numbers were unambiguously determined. Else, we used model predictions for the appropriate spin-averaged masses, which are also shown in Tables II through V.

In order to estimate uncertainties introduced by a particular choice of the constituent mass of u and d quarks, we have varied $m_{u,d}$ in the range from 150 MeV to 350 MeV. For a given $m_{u,d}$, by adjusting c we have also varied constituent heavy quark masses in the range of about 400 MeV (e.g., m_c was varied in the range from about 1.3 GeV to about 1.7 GeV). We emphasize that a good description of the spin-averaged heavy-light meson spectrum was always maintained.

Results for the decay constants obtained in this way depend on the assumption for the unknown spin-averaged mass of D_0 and D'_1 mesons ($0^+_{1/2}$ and $1^+_{1/2}$ states). To take into account ambiguities introduced in our results in that way, we have repeated all calculations for this unknown mass in the range from 2200 MeV to 2450 MeV. Errors quoted in Tables II through V reflect the uncertainty due to the unknown P-wave mass, as well as the uncertainties related to the choice of constituent quark masses discussed above.

TABLE III. Decay constants of heavy-light D_s^{**} states, as obtained from the relativistic quark model. Whenever possible we used experimental meson masses. If these were unknown, we used model predictions for the spin-averaged masses.

Meson	State	f_i [MeV]
$D_s(1969)$	$1S,0^-_{1/2}$	Not reliable
$D_s^*(2112)$	$1S,1^-_{1/2}$	Not reliable
$D_{s0}(2510\pm45)$	$1P,0^+_{1/2}$	110 ± 18
$D'_{s1}(2510\pm45)$	$1P,1^+_{1/2}$	233 ± 31
$D_{s1}(2535)$	$1P,1^+_{3/2}$	87 ± 19
$D''_{s1}(2795\pm55)$	$1D,1^-_{3/2}$	45 ± 6

[7] All results given in Tables II through V were obtained with 25 PC basis states, which was more than enough for the accuracy of less than 1 MeV in all cases considered.

TABLE IV. Decay constants of heavy-light B^{**} states, as obtained from the relativistic quark model. Whenever possible we used experimental meson masses. If these were unknown, we used model predictions for the spin-averaged masses.

Meson	State	f_i [MeV]
$B(5279)$	$1S,0^-_{1/2}$	Not reliable
$B^*(5325)$	$1S,1^-_{1/2}$	Not reliable
$B_0(5765\pm60)$	$1P,0^+_{1/2}$	162 ± 24
$B'_1(5765\pm60)$	$1P,1^+_{1/2}$	206 ± 29
$B_1(5765\pm60)$	$1P,1^+_{3/2}$	32 ± 10
$B''_1(6040\pm70)$	$1D,1^-_{3/2}$	18 ± 3

As one can see from those tables, in spite of the fact that our calculations are performed for a broad range of constituent quark masses, and also for a wide range of the unknown P-wave mass, as long as a good description of the observed heavy-light meson spectrum is maintained, the P- and D-wave heavy-light decay constants are all predicted rather precisely. It is also interesting to observe that the decay constants of strange $0^+_{1/2}$, $1^+_{1/2}$, and $1^-_{3/2}$ states are slightly smaller than those of the corresponding nonstrange states. The main reason is [besides the $1/\sqrt{M}$ dependence of Eq. (3.8)] the light quark dependence of Eqs. (3.11), (3.12), and (3.14). On the other hand, Eq. (3.13) does not depend on the light quark mass, so that $\sqrt{(m_q+E_q)/E_q}$ factor in Eq. (3.8) plays a much more significant role, and as a result $f_{A_{3/2}}$ for the strange states are larger than the ones for nonstrange states. Also, note that f_S for B_0 and B_{s0} are larger than those of the corresponding D_0 and D_{s0} states, while it is the other way around in the case of $f_{A_{1/2}}$. The reason for this are the minus and plus signs in Eqs. (3.11) and (3.12), respectively. Finally, the fact that $1^+_{3/2}$ and $1^-_{3/2}$ B^{**} states have decay constants smaller than those of the corresponding D^{**} states, can be easily explained with the $1/(m_{\bar{Q}}+E_{\bar{Q}})$ dependence of Eqs. (3.13) and (3.14).

V. CONCLUSION

In this paper we have examined decay constants of heavy-light mesons within the mock-meson approach [7–10]. We

TABLE V. Decay constants of heavy-light B_s^{**} states, as obtained from the relativistic quark model. Whenever possible we used experimental meson masses. If these were unknown, we used model predictions for the spin-averaged masses.

Meson	State	f_i [MeV]
$B_s(5374)$	$1S,0^-_{1/2}$	Not reliable
$B_s^*(5421)$	$1S,1^-_{1/2}$	Not reliable
$B_{s0}(5860\pm65)$	$1P,0^+_{1/2}$	146 ± 19
$B'_{s1}(5860\pm65)$	$1P,1^+_{1/2}$	196 ± 26
$B_{s1}(5860\pm65)$	$1P,1^+_{3/2}$	36 ± 10
$B''_{s1}(6130\pm75)$	$1D,1^-_{3/2}$	17 ± 3

obtained all the relevant expressions in the j-j coupling scheme. For numerical estimates we employed a simple and widely used relativistic quark model [11,18–20]. It is based on a spinless Salpeter equation with QCD-motivated Coulomb-plus-linear potential. The effective string tension is chosen so that the Regge structure of the model in the light-light limit is consistent with experiment, and other parameters are based on the good description of the known spin-averaged heavy-light meson masses.

Due to the singular nature of the $L=0$ wave functions at spatial origin [28], we were not able to obtain reliable estimates of pseudoscalar and vector decay constants. On the other hand, even though we have allowed for large variations of input parameters, our results show that the model predicts a rather narrow range for all lowest P- and D-wave heavy-light decay constants.

Such precisely predicted decay constants allow us to estimate the $D_{(s)}^{**}$ production fractions in b decays governed by the $b \to c\overline{cs}'$ transitions under the factorization assumption. Quantitative predictions regarding the interference of color-allowed and color-suppressed amplitudes in $B^- \to D^{**0}\{\pi^-,\rho^-,a_1^-,\ldots\}$ modes can now be formulated. These and some other consequences of our findings will be discussed elsewhere [29].

ACKNOWLEDGMENTS

We thank J. F. Amundson, E. Eichten, and D. Zeppenfeld for discussions. S.V. would also like to thank the theory group for hospitality during his visit to the Fermilab. This work was supported in part by the U.S. Department of Energy under Contract Nos. DE-AC02-76CH03000 and DE-FG02-95ER40896, and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

[1] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, ''Weak decays beyond leading logarithms,'' Report No. SLAC-PUB-95-7009 (hep-ph/9512380) (unpublished).

[2] See, for instance, N. Isgur and M. B. Wise, Phys. Rev. D **41**, 151 (1990); M. B. Wise, *ibid.* **45**, 2188 (1992); G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992); G. Burdman and J. Kambor, Fermilab Report No. FERMILAB-Pub-96/033-T, hep-ph/9602353 (unpublished).

[3] G. Burdman, T. Goldman, and D. Wyler, Phys. Rev. D **51**, 111 (1995).

[4] CLEO Collaboration, Y. Kwon, seminar presented at Moriond, 1996 (unpublished).

[5] D. Fakirov and B. Stech, Nucl. Phys. **B133**, 315 (1978); L. L. Chau, Phys. Rep. **95**, 1 (1983); M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).

[6] T. E. Browder and K. Honscheid, Prog. Part. Nucl. Phys. **35**, 81 (1995).

[7] A. Le Yaouanc, L. Oliver, O. Péne, and J.-C. Raynal, Phys. Rev. D **9**, 2636 (1974).

[8] M. J. Ruiz, Phys. Rev. D **12**, 2922 (1975).

[9] N. Isgur, Phys. Rev. D **12**, 3666 (1975); Acta Phys. Pol. B **6**, 1081 (1977).

[10] C. Hayne and N. Isgur, Phys. Rev. D **25**, 1944 (1982).

[11] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[12] S. Godfrey, Phys. Rev. D **33**, 1391 (1986).

[13] B. Grinstein, M. B. Wise, and N. Isgur, Phys. Rev. Lett. **56**, 298 (1986).

[14] N. Isgur, D. Scora, B. Grinstein, and M. B. Wise, Phys. Rev. D **39**, 799 (1989).

[15] S. Capstick and S. Godfrey, Phys. Rev. D **41**, 2856 (1990).

[16] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995).

[17] D. S. Hwang and G.-H. Kim, ''Decay Constants of B, B^* and D, D^* Mesons in Relativistic Mock Meson Model,'' Report No. hep-ph/9601209 (unpublished); Phys. Rev. D **53**, 3659 (1996).

[18] B. Durand and L. Durand, Phys. Rev. D **25**, 2312 (1982); **30**, 1904 (1984).

[19] D. B. Lichtenberg, W. Namgung, E. Predazzi, and J. G. Wills, Phys. Rev. Lett. **48**, 1653 (1982).

[20] S. Jacobs, M. G. Olsson, and C. J. Suchyta, III, Phys. Rev. D **33**, 3338 (1986).

[21] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991).

[22] C. Itzykson and J. B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).

[23] S. Veseli and M. G. Olsson, Phys. Lett. B **383**, 109 (1996).

[24] C. A. Dominguez and E. de Rafael, Ann. Phys. (N.Y.) **174**, 372 (1987).

[25] A. Duncan, E. Eichten, and H. Thacker, Phys. Lett. B **303**, 109 (1993).

[26] E. J. Weniger, J. Math. Phys. (N.Y.) **26**, 276 (1985).

[27] M. G. Olsson, S. Veseli, and K. Williams, Phys. Rev. D **51**, 5079 (1995); **52**, 5141 (1995).

[28] J. F. Amundson, Phys. Rev. D **52**, 2926 (1995).

[29] I. Dunietz and S. Veseli (in progress).