$K\to\pi\nu\bar\nu$ and high precision determinations of the CKM matrix

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We investigate the future determination of the CKM matrix using theoretically clean quantities, such as $B(K^+\to\pi^+\nu\bar\nu)$, $B(K_L\to\pi^0\nu\bar\nu)$ or $\sin 2\beta$, $\sin 2\alpha$ as extracted from CP violation studies in B physics. The theoretical status of $K\to\pi\nu\bar\nu$ is briefly reviewed and their phenomenological potential is compared with that of CP asymmetries in B decays. We stress the unique opportunities provided by measuring the CP-violating rare decay $K_L\to\pi^0\nu\bar\nu$. It is pointed out that this mode is likely to offer the most precise determination of $\mathrm{Im}\, V_{ts}^*V_{td}$ and the Jarlskog parameter J_{CP}, the invariant measure of CP violation in the standard model. [S0556-2821(96)03523-0]

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I. INTRODUCTION

The standard model (SM) provides an economical and elegant description of CP violation. Within the Cabibbo-Kobayashi-Maskawa (CKM) framework [1], the violation of CP symmetry is accounted for by a single phase, naturally emerging in the three-generation model, and CP violation is intimately connected with the physics of quark mixing. Until today, this theoretical ansatz has been consistent with all known weak decay phenomena, but some of the CKM parameters are only rather loosely constrained and the information on CP violation is limited to the K^0-$\bar K^0$ system.

One of the most important goals of particle physics in the coming years will be to precisely determine all parameters of the CKM matrix and to check the SM picture for consistency by using as many independent observables as possible.

In the standard parametrization of the CKM matrix [2], the four basic parameters are s_{12}, s_{23}, s_{13} and the phase δ. A convenient alternative representation uses the Wolfenstein parameters λ, A, ϱ, and η [3], which can be defined by [4,5]

$$s_{12}=\lambda,\quad s_{23}=A\lambda^2,\quad s_{13}e^{-i\delta}=A\lambda^3(\varrho-i\eta),\qquad (1)$$

Here $|s_{13}|=|V_{ub}|$ and, to an accuracy of $\sim 10^{-5}$, $s_{12}=V_{us}$, $s_{23}=V_{cb}$. In the Wolfenstein parametrization $\lambda=0.22$ can be used as an expansion parameter to simplify expressions for CKM elements. The representation is particularly convenient for the unitarity triangle, which graphically displays the unitarity relation

$$1+\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}=-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\equiv\bar\varrho+i\bar\eta\qquad (2)$$

in the $(\bar\varrho,\ \bar\eta)$ plane (Fig. 1). To an accuracy of better than 0.1%, one has

$$\bar\varrho=\varrho\left(1-\frac{\lambda^2}{2}\right),\quad \bar\eta=\eta\left(1-\frac{\lambda^2}{2}\right).\qquad (3)$$

The difference between the Wolfenstein parameters (ϱ,η), defined in Eqs. (1), and the vertex of the normalized unitarity triangle $(\bar\varrho,\ \bar\eta)$ in Fig. 1 is about 2.4%, which will have to be taken into account in future high precision studies. It is customary to denote the angles of the unitarity triangle by α, β, and γ as shown in Fig. 1.

In general, λ and A can be determined from decays allowed at the tree level. The parameter λ is measured in $K\to\pi e\nu$ or hyperon decays, and $A=V_{cb}/\lambda^2$ can be extracted from either exclusive or inclusive $b\to c$ transitions. On the other hand, determinations of ϱ and η have to rely largely on rare processes, which are typically loop induced and may involve CP violation. Observables that have been used so far to constrain these parameters, such as ε_K, $b\to ul\nu$, and Δm_{B_d}, suffer from considerable theoretical uncertainties. These will ultimately limit the accuracy of CKM determinations, even with continuing progress on the experimental side. In order to achieve decisive tests, it is mandatory to consider observables where theoretical uncertainties are very well under control.

Among the quantities best suited for this purpose are the CP-violating asymmetry in $B_d(\bar B_d)\to J/\psi K_S$, measuring $\sin 2\beta$, and the branching ratio of $K_L\to\pi^0\nu\bar\nu$, determining η. These observables have essentially no theoretical uncertainties and can be pursued at future B factories and at dedi-

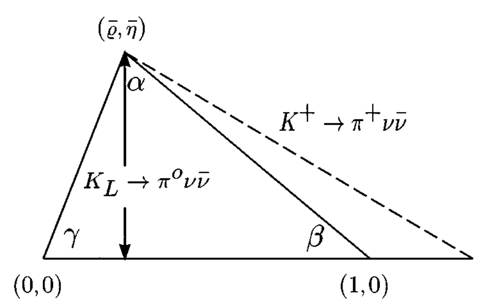

FIG. 1. Unitarity triangle.

cated kaon experiments, respectively.

Some other processes, such as $K^+\to\pi^+\nu\bar{\nu}$, the ratio of mixing parameters, $x=\Delta m/\Gamma$, in the B_s and the B_d system x_s/x_d, and, potentially, also CP asymmetries in $B_d\to\pi\pi$, have only slightly larger theoretical ambiguities. Still they are extraordinarily clean and therefore prime candidates for precisely testing the CKM paradigm.

The purpose of this article is to discuss the prospects for high precision determinations of the CKM matrix using clean observables with very small theoretical uncertainty. We consider various strategies that will measure the CKM parameters and allow unambiguous standard model tests. We compare the potential of CP violation measurements in B physics with that of $K_L\to\pi^0\nu\bar{\nu}$ and $K^+\to\pi^+\nu\bar{\nu}$. Both ways allow one to determine the unitarity triangle with comparable accuracy. A combination of these complementary results promises detailed insight into the physics of quark mixing and CP violation.

This paper is organized as follows. Theoretical uncertainties are discussed and summarized in Sec. II. Section III reviews briefly the theoretical status of CP asymmetries in $B_d\to J/\psi K_S$ and $B_d\to\pi\pi$ in the context of measuring $\sin 2\beta$ and $\sin 2\alpha$. Various strategies to determine the unitarity triangle $(\bar{\varrho},\ \bar{\eta})$ are considered and compared in Sec. IV. Finally, Sec. V contains our conclusions.

II. THEORETICAL UNCERTAINTIES IN $K\to\pi\nu\bar{\nu}$

The rare decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ are loop-induced flavor-changing neutral current (FCNC) processes in the standard model. Being semileptonic and short-distance dominated, these channels are theoretically exceptionally well under control. They are therefore sensitive probes of the physics at high energy scales and allow in particular to access the CKM couplings of the top quark in a very clean way. In the present section we shall briefly review the theoretical status of the $K\to\pi\nu\bar{\nu}$ decay modes.

A. $K^+\to\pi^+\nu\bar{\nu}$

The branching fraction of $K^+\to\pi^+\nu\bar{\nu}$ can be written as

$$B(K^+\to\pi^+\nu\bar{\nu})=\kappa\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X(x_t)\right)^2+\left(\frac{\mathrm{Re}\lambda_c}{\lambda}P_0(K^+)+\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)\right)^2\right], \tag{4}$$

$$\kappa=r_{K^+}\frac{3\alpha^2 B(K^+\to\pi^0e^+\nu)}{2\pi^2\sin^4\Theta_W}\lambda^8=4.11\times 10^{-11}. \tag{5}$$

Here $x_t=m_t^2/M_W^2$, $\lambda_i=V_{is}^*V_{id}$, and $r_{K^+}=0.901$ summarizes isospin-breaking corrections in relating $K^+\to\pi^+\nu\bar{\nu}$ to the well-measured leading decay $K^+\to\pi^0e^+\nu$. In the standard parametrization λ_c is real to an accuracy of better than 10^{-3}. The function X is given by

$$X(x)=\eta_X\cdot\frac{x}{8}\left[\frac{x+2}{x-1}+\frac{3x-6}{(x-1)^2}\ln x\right],\quad \eta_X=0.985, \tag{6}$$

where η_X is the next to leading order (NLO) correction calculated in [6]. With $m_t\equiv\bar{m}_t(m_t)$ the QCD factor η_X is practically independent of m_t. Next,

$$P_0(K^+)=\frac{1}{\lambda^4}\left[\tfrac{2}{3}X_{\mathrm{NL}}^e+\tfrac{1}{3}X_{\mathrm{NL}}^\tau\right] \tag{7}$$

represents the charm quark contribution with X_{NL}^l calculated in [7]. The central value of $P_0(K^+)$ for $\Lambda_{\overline{\mathrm{MS}}}^{(4)}=325$ MeV, $m_c=\bar{m}_c(m_c)=1.3$ GeV, and the renormalization scale $\mu_c=m_c$ is $P_0(K^+)=0.400$. We remark that in writing $B(K^+\to\pi^+\nu\bar{\nu})$ in the form of Eq. (4) a negligibly small term $\sim(X_{\mathrm{NL}}^e-X_{\mathrm{NL}}^\tau)^2$ has been omitted (0.2% effect on the branching ratio).

In general, a measurement of $B(K^+\to\pi^+\nu\bar{\nu})$ alone yields a constraint on $\mathrm{Re}\lambda_t$ and $\mathrm{Im}\lambda_t$ according to Eq. (4). This relationship is very clean and uncertainties arise only from the branching fraction, the charm quark contribution, and the top quark mass, where the latter error is almost negligible.

Using in addition information from A (or V_{cb}), the relation between $\mathrm{Re}\lambda_t$ and $\mathrm{Im}\lambda_t$ can be translated into a constraint in the $(\bar{\varrho},\bar{\eta})$ plane. For fixed input parameters this constraint is approximately an ellipse centered at $\bar{\varrho}=1+P_0(K^+)/[A^2X(x_t)]$, $\bar{\eta}=0$, which is shifted from $(\bar{\varrho},\bar{\eta})=(1,0)$ by the presence of the charm contribution as indicated in Fig. 1.

To learn more about the CKM parameters from $B(K^+\to\pi^+\nu\bar{\nu})$ requires additional input. One suitable further piece of information, such as $|V_{ub}/V_{cb}|$ or $B(K_L\to\pi^0\nu\bar{\nu})$, is, however, sufficient to determine the CKM matrix completely. All CKM elements are then given in both magnitude and phase, in particular V_{td}.

In the following we briefly address the most important uncertainties in the theoretical treatment of $K^+\to\pi^+\nu\bar{\nu}$.

The top quark contribution is characterized by high energy scales of $O(m_t)$ where QCD perturbation theory is a very reliable tool. The inclusion of $O(\alpha_s)$ corrections essentially eliminates the sizable renormalization scale dependence of the leading order result. This analysis indicates that the residual uncertainty in $X(x_t)$, for fixed m_t, is merely at the level of $\sim$1% and thus practically irrelevant.

For the charm quark contribution the situation is less favorable, since QCD perturbation theory cannot be expected to be as accurate at the rather low scale of m_c. This case further requires the resummation of large logarithms $\ln M_W/m_c$ using renormalization group methods. Still the reliability of the calculation can be much improved by performing a next-to-leading logarithmic analysis where, in addition to the leading logarithms of $O(x_c\alpha_s^n\ln^{n+1}x_c)$, the terms of $O(x_c\alpha_s^n\ln^n x_c)$ are included in the charm quark function $X_c=X_{\mathrm{NL}}$. The calculation of the NLO corrections allows a better assessment of the applicability of perturbation theory. In fact, the NLO correction turns out to be sufficiently small for this approach to make sense in the present context. Furthermore, the sensitivity to the unphysical renormalization scale $\mu_c=O(m_c)$ is reduced at NLO. The remaining ambiguity is to be interpreted as a theoretical uncertainty, due to the use of a truncated perturbation series, and is about $\pm$10% in $P_0(K^+)$.

This ambiguity corresponds to part of the neglected higher order corrections and thus provides a quantitative estimate for their order of magnitude. Of course, knowledge of the complete $O(x_c \alpha_s^{n+1} \ln^n x_c)$ terms, appearing at the order beyond next-to-leading logarithms, could, strictly speaking, give a more rigorous estimation of the residual error. This order has, however, not yet been fully calculated. Given that the perturbative expansion for $X_{\rm NL}$ appears rather well behaved after renormalization group improvement and the NLO value is within the range obtained by varying the scale (1 GeV$\leq \mu_c \leq$3 GeV) in the LO result, we expect the error estimate based on the scale dependence to give a fair account of the actual uncertainty. In fact, additional support for this procedure comes from considering the term $O(x_c \alpha_s)$ in the charm quark function $P_0(K^+)$ [7]. This term is a contribution beyond NLO and therefore not included in $X_{\rm NL}$. It is, however, known from the calculation of $X(x_t)$. It provides another, independent estimate of the typical size of neglected higher order terms. Quantitatively, its size is $\sim$10% in $P_0(K^+)$, compatible with the error estimate based on the μ_c dependence.

Besides through top and charm quark loops, which are short distance in character due to m_t, $m_c \gg \Lambda_{\rm QCD}$, $K^+ \to \pi^+ \nu \bar{\nu}$ may also proceed through second order weak interactions involving up quarks. This mechanism is the source of long-distance contributions to $K \to \pi \nu \bar{\nu}$, which are determined by nonperturbative low energy QCD dynamics and difficult to calculate reliably. Of crucial importance for the high accuracy that can be achieved in the theoretical treatment of $K \to \pi \nu \bar{\nu}$ is the fact that such contributions are very small. The reason for this is a hard Glashow-Iliopoulos-Maiani (GIM) suppression of the electroweak $\bar{s} d \to \nu \bar{\nu}$ amplitude. This means that the charm contribution behaves as $m_c^2 \ln M_W/m_c$ for $m_c \to 0$, rather than, say, just logarithmically $\sim \ln M_W/m_c$. Hence the size of the short-distance-dominated charm quark sector is essentially determined by m_c^2, while the long-distance up quark contribution is characterized by the QCD scale $\Lambda^2_{\rm QCD}$ (the up quark mass being negligible). The long-distance part is therefore suppressed by $\Lambda^2_{\rm QCD}/m_c^2$ relative to the charm quark amplitude. Detailed estimates [8–11] quantify this general suppression pattern. Using chiral perturbation theory the authors of [10] find $X_{\rm LD} \lesssim (g_8 \pi^2/3)(f_\pi/M_W)^2$, with g_8=5.1. This estimate is based on the amplitude involving one W-boson and one Z-boson exchange, which is enhanced by the ΔI=1/2 rule and can be expected to be dominant. The result is less than about 5% of the charm quark contribution and negligible in view of the perturbative uncertainty in the charm quark sector. A further long-distance mechanism, involving two W-boson exchanges, is $K^+ \to \nu_l l^{+*} \to \nu_l \pi^+ \bar{\nu}_l$, $l = e$, μ, calculated in [9]. It amounts to $\approx$2% of the charm quark amplitude and is likewise negligible. Similar conclusions on the long-distance contribution have been reached in [8,11].

To eliminate the hadronic matrix element $\langle \pi | (\bar{s} d)_V | K \rangle$ in the calculation of $K^+ \to \pi^+ \nu \bar{\nu}$, the branching ratio $B(K^+ \to \pi^+ \nu \bar{\nu})$ can be related to $B(K^+ \to \pi^0 e^+ \nu)$ using isospin symmetry. Corrections to the strict isospin limit have been considered in [12]. They arise from phase space effects due to differences in the mass of π^+ and π^0 (or K^0 and K^+ for the neutral mode $K_L \to \pi^0 \nu \bar{\nu}$), isospin violation in the $K \to \pi$ form factors, and electromagnetic radiative corrections that affect the $\bar{s} \to \bar{u} e^+ \nu$ transition, but not $\bar{s} \to \bar{d} \nu \bar{\nu}$. Ultimately, these effects stem from the usual sources of isospin breaking, the electromagnetic interaction, or the u-d mass difference. For the correction factor r_{K^+} in Eq. (5) [12], obtain $r_{K^+} = 0.9614 \times 0.9574 \times 0.979 = 0.901$, where the first factor is from phase space, the second from the $K \to \pi$ form factors, and the last from QED radiative corrections. Since the meson masses are known precisely, the phase space effect has essentially no uncertainty. The QED correction factor is calculated in the leading logarithmic approximation and given by $[1 + 2\alpha/\pi \ln(M_Z/\mu_h)]^{-1} = 0.979$ for $\alpha = 1/137$ and $\mu_h = m_p = 0.938$ GeV. Taking into account the various ambiguities in this calculation, from nonlogarithmic $O(\alpha)$ corrections, the fact that α could be $\alpha(M_Z)$ rather than $\alpha(m_e)$, replacing M_Z by M_W, or varying μ_h between 0.5 and 2 GeV, one finds typically an uncertainty of $\pm$0.5%. Finally, the expression for $B(K^+ \to \pi^+ \nu \bar{\nu})$ receives a small error from the use of $B(K^+ \to \pi^0 e^+ \nu) = 0.0482$, which is currently measured to 1% accuracy.

To summarize, the theoretical uncertainty in $K^+ \to \pi^+ \nu \bar{\nu}$ is dominated by the charm quark contribution. The latter is estimated to be $P_0(K^+) = 0.40 \pm 0.047$, where the error bar represents the symmetrized range obtained by varying the renormalization scale μ_c between 1 and 3 GeV. This uncertainty translates into a $\pm$5% variation of the branching ratio. The other errors, such as those from the scale dependence in the top quark sector or from the long-distance contribution, are small in comparison and can be neglected.

The intrinsic theoretical uncertainties we have discussed so far should be distinguished from uncertainties in basic standard model parameters. Among these are the errors in V_{cb} and m_t that will be specified later on. In the charm quark sector one has the charm quark mass and the QCD scale for which we shall take $m_c = \bar{m}_c(m_c) = (1.30 \pm 0.05)$ GeV (the running $\overline{\rm MS}$ mass) and $\Lambda^{(4)}_{\overline{\rm MS}} = (325 \pm 75)$ MeV. Here we have anticipated that by the time $K^+ \to \pi^+ \nu \bar{\nu}$ will be measured, the precision in m_c should have improved over the current status. Combining the uncertainties from theory, m_c and Λ, we finally obtain

$$\begin{aligned} P_0(K^+) &= 0.400 \pm 0.047(th) \pm 0.035(m_c) \pm 0.026(\Lambda) \\ &= 0.40 \pm 0.06, \end{aligned} \tag{8}$$

which we will use in the analysis below.

B. $K_L \to \pi^0 \nu \bar{\nu}$

Because of the CP properties of K_L, π^0, and the relevant hadronic, short-distance transition current, the mode $K_L \to \pi^0 \nu \bar{\nu}$ proceeds in the SM almost entirely through direct CP violation. In explicit terms the branching fraction per neutrino flavor is given by

$$\begin{aligned} B(K_L \to \pi^0 \nu_l \bar{\nu}_l) = r_{K_L} \frac{\tau_{K_L}}{\tau_{K^+}} \frac{\alpha^2 B(K^+ \to \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \Theta_W |V_{us}|^2} \\ \times \frac{1}{2} \frac{|\xi - \xi^* (1-\bar{\varepsilon})/(1+\bar{\varepsilon})|^2}{1 + |(1-\bar{\varepsilon})/(1+\bar{\varepsilon})|^2}. \end{aligned} \tag{9}$$

Here $\xi=\Sigma_{i=u,c,t}\lambda_i X_i$ and $r_{K_L}=0.944$ is the isospin-breaking correction [12] from relating $K_L\to\pi^0\nu\bar{\nu}$ to $K^+\to\pi^0 e^+\nu$. The factor

$$\frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}}=\frac{M_{12}^*-i\Gamma_{12}^*/2}{(\Delta m-i\Delta\Gamma/2)/2} \tag{10}$$

derives from $|K_L\rangle\sim(1+\bar{\varepsilon})|K^0\rangle+(1-\bar{\varepsilon})|\bar{K}^0\rangle$, with M_{12} and Γ_{12} denoting the off-diagonal elements in the neutral kaon mass and decay constant matrix, respectively. $\Delta m=m_L-m_S$ ($\Delta\Gamma=\Gamma_L-\Gamma_S$) is the difference in mass (decay rate) between the eigenstates K_L and K_S. We use the CP phase conventions $CP|K^0\rangle=-|\bar{K}^0\rangle$, $CP(\bar{d}s)_V CP^{-1}=-(\bar{s}d)_V$. (The neutral pion has negative CP parity $CP|\pi^0\rangle=-|\pi^0\rangle$.)

In principle, arbitrary phases could be introduced in the CP transformation of K^0 and the current $(\bar{d}s)_V$. These phases would multiply the factor ξ^* in Eq. (9). However, compensating phases would then be present in the hadronic matrix elements of M_{12} and Γ_{12}, assuring that the physics remains unchanged. Note further that the expression in Eq. (9) is manifestly invariant under rephasing of the quark fields, since $(1-\bar{\varepsilon})/(1+\bar{\varepsilon})\sim\lambda_i^2$. In particular, one has $(\lambda_u^*/\lambda_u)(1-\bar{\varepsilon})/(1+\bar{\varepsilon})=1$ up to a few times 10^{-3}, which is independent of the CKM matrix phase convention. It then follows that

$$\left|\xi-\xi^*\,\frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}}\right|^2=\left|\xi-\xi^*\,\frac{\lambda_u}{\lambda_u^*}\right|^2=4\,\frac{(\mathrm{Im}\lambda_t\lambda_u^*)^2}{|\lambda_u|^2}\,(X_t-X_c)^2, \tag{11}$$

where we have neglected long-distance contributions. This expression is manifestly rephasing invariant.

Neglecting the charm quark contribution, which affects the branching ratio by only 0.1%, specializing to the standard CKM parametrization where $\lambda_u^*=\lambda_u$, and summing over the three neutrino species, one obtains the familiar result

$$B(K_L\to\pi^0\nu\bar{\nu})=\kappa_L\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}\,X(x_t)\right)^2,\quad \mathrm{Im}\lambda_t=\eta A^2\lambda^5, \tag{12}$$

$$\kappa_L=r_{K_L}\,\frac{\tau_{K_L}}{\tau_{K^+}}\,\frac{3\alpha^2 B(K^+\to\pi^0 e^+\nu)}{2\pi^2\sin^4\Theta_W}\,\lambda^8=1.80\times10^{-10}. \tag{13}$$

Equation (12) provides a very accurate relationship between the observable $B(K_L\to\pi^0\nu\bar{\nu})$ and fundamental SM parameters. The high precision that can be achieved in the theoretical calculation of this decay mode is rather unique among rare decay phenomena.

$K_L\to\pi^0\nu\bar{\nu}$ shares many features with the charged mode $K^+\to\pi^+\nu\bar{\nu}$, which make it already a very clean process. This situation is still improved considerably by the CP-violating nature of $K_L\to\pi^0\nu\bar{\nu}$, since here only the top quark contribution is significant and all the uncertainties associated with the charm quark sector are eliminated. After including NLO corrections, the intrinsic theoretical uncertainty in $X^2(x_t)$ from truncating the perturbation series is estimated to be $\pm1\%$.

Long-distance contributions to $K_L\to\pi^0\nu\bar{\nu}$ are still further suppressed compared to the case of $K^+\to\pi^+\nu\bar{\nu}$ due to the CP-violating property of the neutral mode. They are likewise completely negligible [8].

The factor $B(K^+\to\pi^0 e^+\nu)\tau_{K_L}/\tau_{K^+}$ serves to eliminate the hadronic matrix element required for the calculation of $K_L\to\pi^0\nu\bar{\nu}$. The combined experimental error in this quantity is $\pm1.5\%$, dominated by the uncertainty in $B(K^+\to\pi^0 e^+\nu)$ [2]. This error can be further reduced by improved measurements in the future.

The isospin-breaking correction r_{K_L} is here $r_{K_L}=1.0522\times0.9166\times0.979=0.944$ [12]. The first factor comes from the difference in phase space between $K^0\to\pi^0$ and $K^+\to\pi^0$ decay and does not introduce any significant error. The short-distance QED correction 0.979 is the same as in the case of $K^+\to\pi^+\nu\bar{\nu}$ and has an uncertainty of probably below $\pm0.5\%$.

From the full expression given in Eq. (9), one can derive the contribution of indirect CP violation to $B(K_L\to\pi^0\nu\bar{\nu})$. For this purpose it is convenient to use the CKM phase conventions of the standard parametrization. We further approximate

$$\bar{\varepsilon}\approx\varepsilon=\frac{1+i}{\sqrt{2}}\,|\varepsilon|,\quad |\varepsilon|=(2.282\pm0.019)\times10^{-3}, \tag{14}$$

where ε is the parameter describing indirect CP violation in $K^0\to\pi\pi$ decays [2]. Expanding to first order in $|\varepsilon|$, one finds that the effect of indirect CP violation in $K_L\to\pi^0\nu\bar{\nu}$ is to multiply the branching ratio in Eq. (12) by a factor of

$$1+\sqrt{2}|\varepsilon|\,\frac{\mathrm{Re}\xi}{\mathrm{Im}\xi}\quad\text{where}\quad\frac{\mathrm{Re}\xi}{\mathrm{Im}\xi}=-\,\frac{1+P_0(K^+)/A^2X(x_t)-\varrho}{\eta}. \tag{15}$$

Since $\mathrm{Re}\xi/\mathrm{Im}\xi$ is typically -4, we find that indirect CP violation reduces the branching fraction $B(K_L\to\pi^0\nu\bar{\nu})$ by $\approx1\%$. We shall neglect this small correction for simplicity. The effect can of course be taken into account in the future, should such a high precision be required.

III. *CP* ASYMMETRIES IN B_d DECAYS

The observation of CP-violating asymmetries in neutral B decays to CP eigenstates will test the standard model and allow one to determine angles of the unitarity triangle in Fig. 1. Among the most promising candidates for these experiments are the decay mode $B_d(\bar{B}_d)\to J/\psi K_S$ and, to a lesser extent, also $B_d(\bar{B}_d)\to\pi^+\pi^-$, which will be pursued in particular at the upcoming B factories. The corresponding time-dependent or time-integrated (at hadron colliders) CP asymmetries in the decay of tagged B_d, compared to $\bar{B}_d$, measure $\sin2\beta$ and $\sin2\alpha$, respectively. This subject has been extensively discussed in the literature. Here we content ourselves with recalling a particular aspect, the effect of penguin contributions, which is important for the theoretical accuracy in inferring $\sin2\phi$, $\phi=\alpha,\beta$, from measured asymmetries. In the absence of a penguin amplitude, the time-dependent asymmetry oscillates as $\sin\Delta m_{B_d}t$ with an amplitude given by $\sin2\phi$. When a small penguin contribution is present in addition to the dominant tree-level amplitude, the amplitude of $\sin\Delta mt$ does not in general measure $\sin2\phi$ alone, but the combination [13]

TABLE I. Illustrative example of the determination of CKM parameters from $K\to\pi\nu\bar{\nu}$ and from CP-violating asymmetries in B decays. The relevant input is as described in the text. Shown in parentheses are the errors one obtains using $V_{cb}=0.040\pm0.001$ instead of $V_{cb}=0.040\pm0.002$.

	$K\to\pi\nu\bar{\nu}$	$B\to\pi\pi,\ J/\psi K_S$ (I)	$B\to\pi\pi,\ J/\psi K_S$ (II)
$\lvert V_{td}\rvert/10^{-3}$	$10.3\pm1.1(\pm0.9)$	$8.8\pm0.5(\pm0.3)$	$8.8\pm0.5(\pm0.2)$
$\lvert V_{ub}/V_{cb}\rvert$	$0.089\pm0.017(\pm0.011)$	$0.087\pm0.009(\pm0.009)$	$0.087\pm0.003(\pm0.003)$
$\bar{\varrho}$	$-0.10\pm0.16(\pm0.12)$	$0.07\pm0.03(\pm0.03)$	$0.07\pm0.01(\pm0.01)$
$\bar{\eta}$	$0.38\pm0.04(\pm0.03)$	$0.38\pm0.04(\pm0.04)$	$0.38\pm0.01(\pm0.01)$
$\sin2\beta$	$0.62\pm0.05(\pm0.05)$	$0.70\pm0.06(\pm0.06)$	$0.70\pm0.02(\pm0.02)$
$\mathrm{Im}\lambda_t/10^{-4}$	$1.37\pm0.07(\pm0.07)$	$1.37\pm0.19(\pm0.15)$	$1.37\pm0.14(\pm0.08)$

$$\sin2\phi-2\left|\frac{A_2}{A_1}\right|\cos2\phi\cos(\delta_1-\delta_2)\sin(\phi_1-\phi_2), \qquad (16)$$

where A_i, δ_i, and ϕ_i are the amplitude, the strong phase, and the weak phase, respectively, of the tree ($i=1$) and the penguin contribution ($i=2$) for $B_d\to f$. The strong phases are unknown, and $\cos(\delta_1-\delta_2)$ could be one in the worst case.

The ratio of $|A_2/A_1|$ is expected to be typically $\sim$3–5% for $B_d\to J/\psi K_S$ and $\sim$10–20% for $B_d\to\pi^+\pi^-$. The penguin amplitude is slightly enhanced in the latter case through the ratio of CKM angles, $|V_{tb}^*V_{td}/(V_{ub}^*V_{ud})|\sim3$, whereas in $B_d\to J/\psi K_S$ this factor is $|V_{tb}^*V_{ts}/(V_{cb}^*V_{cs})|\sim1$. It should be remarked that these estimates of $|A_2/A_1|$ are highly uncertain due to the poor knowledge of hadronic matrix elements.

For $B_d\to J/\psi K_S$ this potential problem is, however, practically eliminated since the tree and the penguin amplitudes have almost identical weak phases. More quantitatively, $\sin(\phi_1-\phi_2)\simeq\lambda^2\eta\approx0.02$ and the penguin contamination in Eq. (16) is estimated to be below ±0.002.

The situation is not as fortunate for $B_d\to\pi^+\pi^-$, where $\sin(\phi_1-\phi_2)=\sin\alpha$. As pointed out in [14], if $\alpha\approx\pi/2$, which cannot be excluded at present, $\sin2\alpha\approx0$. However, the asymmetry coefficient (16), which is supposed to measure $\sin2\alpha$, could at the same time be as large as $\sim$0.4 due to penguin effects. For larger $\sin2\alpha$ the impact of the penguin contribution is smaller. A detailed discussion can be found in [14]. More recently, this problem has also been addressed in [15].

As shown in [16,17], the penguin contamination could be eliminated in principle by an isospin analysis. This, however, requires the measurement of the rates for $B^+\to\pi^+\pi^0$ and $B_d\to\pi^0\pi^0$, and their CP conjugates, which will be difficult to achieve, in particular in view of the fact that the branching ratio for $B_d\to\pi^0\pi^0$ is expected to be below 10^{-6} [18].

In summary, while CP asymmetries in $B_d\to J/\psi K_S$ are a very clean measure of $\sin2\beta$, the extraction of $\sin2\alpha$ from $B_d\to\pi^+\pi^-$ is somewhat more problematic. If the difficulties related to penguin contributions can be overcome, also this channel will be a very useful observable for CKM matrix determinations. A recent discussion of alternative methods for the extraction of α can be found in [19].

IV. DETERMINATIONS OF THE UNITARITY TRIANGLE

We shall now describe several applications of the observables we have discussed above for precise determinations of the CKM matrix. Four independent pieces of information are needed to fix the four parameters of quark mixing, λ, A, ϱ, and η. This determination is the necessary first step towards a comprehensive test of this important standard model sector. Those physical quantities should be chosen for this purpose that allow one to define the most accurate set of CKM parameters and therefore constitute a firm basis for any further tests and comparisons. Which observables will eventually turn out to provide the optimal set of CKM matrix input is not yet completely clear at present, but theoretically clean processes such as $K\to\pi\nu\bar{\nu}$ and the CP asymmetries, both time dependent and time integrated, in the ''gold-plated'' mode $B_d\to J/\psi K_S$ are certainly prime candidates.

In the following we illustrate several scenarios for determining the CKM matrix and show what degree of accuracy can be expected in the future.

A. Unitarity triangle from $K\to\pi\nu\bar{\nu}$ and from $\sin2\alpha$ and $\sin2\beta$

The most obvious source for two of the parameters is weak decays allowed at the tree level: $K\to\pi e\nu$ and hyperon decays give λ, and $A=V_{cb}/\lambda^2$ can be extracted from exclusive and inclusive semileptonic $b\to c$ transitions. Measuring $\sin2\alpha$ and $\sin2\beta$ from CP asymmetries in B decays allows one, in principle, to fix the remaining two parameters $\bar{\eta}$ and $\bar{\varrho}$, which can be expressed as [20]

$$\bar{\eta}=\frac{r_-(\sin2\alpha)+r_+(\sin2\beta)}{1+r_+^2(\sin2\beta)}, \quad \bar{\varrho}=1-\bar{\eta}r_+(\sin2\beta), \qquad (17)$$

where $r_\pm(z)=(1\pm\sqrt{1-z^2})/z$. In general, the calculation of $\bar{\varrho}$ and $\bar{\eta}$ from $\sin2\alpha$ and $\sin2\beta$ involves discrete ambiguities. As described in [20], they can be resolved by using further information, e.g., bounds on $|V_{ub}/V_{cb}|$, so that eventually the solution (17) is singled out.

Alternatively, $\bar{\varrho}$ and $\bar{\eta}$ may also be determined from $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ alone [21,22]. An interesting feature of this possibility is in particular that the extraction of $\sin2\beta$ from these two modes is essentially independent of m_t and V_{cb} [22]. This fact enables a rather accurate determination of $\sin2\beta$ from $K\to\pi\nu\bar{\nu}$.

A comparison of both strategies is displayed in Table I, where the following input has been used:

$$V_{cb}=0.040\pm0.002, \quad m_t=(170\pm3)\ \text{GeV}, \qquad (18)$$

TABLE II. $\mathrm{Im}\lambda_t/10^{-4}$ as determined from CP asymmetries in B decays. Scenario I assumes $\sin 2\alpha = 0.40 \pm 0.10$, $\sin 2\beta = 0.70 \pm 0.06$. For scenario II we take $\sin 2\alpha = 0.40 \pm 0.04$, $\sin 2\beta = 0.70 \pm 0.02$. We use $V_{cb} = 0.040 \pm 0.002$ and, for the results in square brackets, $V_{cb} = 0.040 \pm 0.001$.

		$\Delta(\sin 2\alpha)$	$\Delta(\sin 2\beta)$	$\Delta(V_{cb})$	Δ_{total}
I	1.370	± 0.030	± 0.131	± 0.137 [± 0.069]	± 0.192 [± 0.151]
II	1.370	± 0.012	± 0.044	± 0.137 [± 0.069]	± 0.144 [± 0.083]

$$B(K_L \to \pi^0 \nu\bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11},$$

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = (1.0 \pm 0.1) \times 10^{-10}. \tag{19}$$

The charm quark contribution in $K^+ \to \pi^+ \nu\bar{\nu}$ is assumed to be known to $\pm 15\%$, $P_0(K^+) = 0.40 \pm 0.06$.

The measurements of CP asymmetries in $B_d \to \pi\pi$ and $B_d \to J/\psi K_S$, expressed in terms of $\sin 2\alpha$ and $\sin 2\beta$, are taken to be

$$\sin 2\alpha = 0.40 \pm 0.10, \quad \sin 2\beta = 0.70 \pm 0.06 \quad \text{(scenario I)} \tag{20}$$

$$\sin 2\alpha = 0.40 \pm 0.04, \quad \sin 2\beta = 0.70 \pm 0.02 \quad \text{(scenario II).} \tag{21}$$

Scenario I corresponds to the accuracy being aimed for at B factories prior to the LHC era. An improved precision can be anticipated from LHC experiments, which we illustrate with our choice of scenario II.

As can be seen in Table I, the CKM determination using $K \to \pi\nu\bar{\nu}$ is competitive with the one based on CP violation in B decays, except for $\bar{\varrho}$, which is less constrained by the rare kaon processes. On the other hand, $\mathrm{Im}\lambda_t$ is better determined in the kaon scenario. It can be obtained from $K_L \to \pi^0 \nu\bar{\nu}$ alone and does not require knowledge of V_{cb}, which enters $\mathrm{Im}\lambda_t$ when derived from $\sin 2\alpha$ and $\sin 2\beta$. We have displayed the extraction of $\mathrm{Im}\lambda_t$ from CP asymmetries in B decays in more detail in Table II.

This should be compared with the results for $\mathrm{Im}\lambda_t$ that could be obtained using $B(K_L \to \pi^0 \nu\bar{\nu})$. Taking $B(K_L \to \pi^0 \nu\bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11}$, $m_t = (170 \pm 3)$ GeV [case (a)] and $B(K_L \to \pi^0 \nu\bar{\nu}) = (3.0 \pm 0.15) \times 10^{-11}$, $m_t = (170 \pm 1)$ GeV [case (b)], we find

$$\mathrm{Im}\lambda_t/10^{-4} = 1.368 \pm 0.069 \pm 0.028 = 1.368 \pm 0.074 \quad \text{(a)}, \tag{22}$$

$$\mathrm{Im}\lambda_t/10^{-4} = 1.368 \pm 0.035 \pm 0.009 = 1.368 \pm 0.036 \quad \text{(b)}. \tag{23}$$

The comparison suggests that $K_L \to \pi^0 \nu\bar{\nu}$ should eventually yield the most accurate value of $\mathrm{Im}\lambda_t$. This would be an important result since $\mathrm{Im}\lambda_t$ plays a central role in the phenomenology of CP violation in K decays and is furthermore equivalent to the Jarlskog parameter J_{CP} [23], the invariant measure of CP violation in the standard model, $J_{CP} = \lambda(1 - \lambda^2/2)\mathrm{Im}\lambda_t$.

B. Unitarity triangle from $K_L \to \pi^0 \nu\bar{\nu}$ and $\sin 2\alpha$

Next, results from CP asymmetries in B decays could also be combined with measurements of $K \to \pi\nu\bar{\nu}$. As an illustration, we would like to discuss a scenario where the unitarity triangle is determined by λ, V_{cb}, $\sin 2\alpha$, and $B(K_L \to \pi^0 \nu\bar{\nu})$ (see Fig. 2). In this case $\bar{\eta}$ follows directly from $B(K_L \to \pi^0 \nu\bar{\nu})$, Eq. (12), and $\bar{\varrho}$ is obtained using [20]

$$\bar{\varrho} = \frac{1}{2} - \sqrt{\frac{1}{4} - \bar{\eta}^2 + \bar{\eta} r_-(\sin 2\alpha)}, \tag{24}$$

where $r_-(z)$ is defined after Eq. (17). The advantage of this strategy is that most CKM quantities are not very sensitive to the precise value of $\sin 2\alpha$. Moreover, a high accuracy in the Jarlskog parameter and in $\mathrm{Im}\lambda_t$ is automatically guaranteed. As shown in Table III, very respectable results can be expected for other quantities as well, with only modest requirements on the accuracy of $\sin 2\alpha$. It is conceivable that theoretical uncertainties due to penguin contributions could eventually be brought under control at least to the level assumed in Table III. As an alternative, $\sin 2\beta$ from $B_d \to J/\psi K_S$ could be used as independent input instead of $\sin 2\alpha$. Unfortunately, the combination of $K_L \to \pi^0 \nu\bar{\nu}$ and $\sin 2\beta$ tends to yield somewhat less restrictive constraints on the unitarity triangle. On the other hand, it has of course the advantage of being practically free of any theoretical uncertainties.

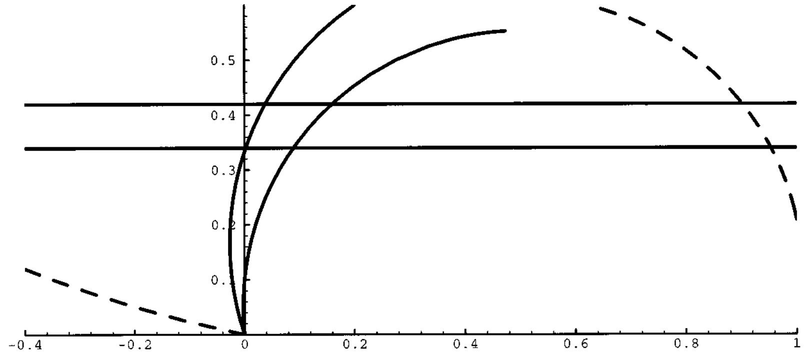

FIG. 2. Constraints in the $(\bar{\varrho}, \bar{\eta})$ plane from $\sin 2\alpha = 0.4 \pm 0.2$ (''half moon'') and from $B(K_L \to \pi^0 \nu\bar{\nu})$ [horizontal band, input as specified in Eqs. (18) and (19)]. The dashed curves illustrate the discrete ambiguities involved in determining $\bar{\varrho}$ and $\bar{\eta}$ from $\sin 2\alpha$ for the central value $\sin 2\alpha = 0.4$. They can be elminated by information on ε_K and $|V_{ub}/V_{cb}|$. Note that even for a quite loosely determined $\sin 2\alpha$, as in the present example, the resulting constraint in the $(\bar{\varrho}, \bar{\eta})$ plane is rather tight.

TABLE III. Determination of the CKM matrix from λ, V_{cb}, $K_L\to\pi^0\nu\bar{\nu}$, and $\sin 2\alpha$ from the CP asymmetry in $B_d\to\pi^+\pi^-$. Scenario A (B) assumes $V_{cb}=0.040\pm0.002$ (±0.001) and $\sin 2\alpha=0.4\pm0.2$ (±0.1). In both cases we take $B(K_L\to\pi^0\nu\bar{\nu})\times10^{11}=3.0\pm0.3$ and $m_t/\mathrm{GeV}=170\pm3$.

		A	B
$\bar{\eta}$	0.380	±0.043	±0.028
$\bar{\varrho}$	0.070	±0.058	±0.031
$\sin 2\beta$	0.700	±0.077	±0.049
$\|V_{td}\|/10^{-3}$	8.84	±0.67	±0.34
$\|V_{ub}/V_{cb}\|$	0.087	±0.012	±0.007

C. Unitarity triangle and V_{cb} from $\sin 2\alpha$, $\sin 2\beta$, and $K_L\to\pi^0\nu\bar{\nu}$

In [20] an additional strategy has been proposed that could offer unprecedented precision for all basic CKM parameters. While λ is obtained as usual from $K\to\pi e\nu$, $\bar{\varrho}$ and $\bar{\eta}$ could be determined from $\sin 2\alpha$ and $\sin 2\beta$ as measured in CP-violating asymmetries in B decays. Given η, one could take advantage of the very clean nature of $K_L\to\pi^0\nu\bar{\nu}$ to extract A or, equivalently, V_{cb}. This determination benefits further from the very weak dependence of A on the $K_L\to\pi^0\nu\bar{\nu}$ branching ratio, which is only with a power of 0.25. Moderate accuracy in $B(K_L\to\pi^0\nu\bar{\nu})$ would thus still give a high precision in V_{cb}. As an example, we take $\sin 2\alpha=0.40\pm0.04$, $\sin 2\beta=0.70\pm0.02$, and $B(K_L\to\pi^0\nu\bar{\nu})=(3.0\pm0.3)\times10^{-11}$, $m_t=(170\pm3)$ GeV. This yields

$$\bar{\varrho}=0.07\pm0.01,\quad \bar{\eta}=0.38\pm0.01,\quad V_{cb}=0.0400\pm0.0013, \tag{25}$$

which would be a truly remarkable result.

D. $B\to X_{d,s}\nu\bar{\nu}$, $B_{d,s}\to\mu^+\mu^-$, and x_d/x_s

Finally, we would like to mention a few additional observables that are theoretically very well under control and which are therefore also potential candidates for precise CKM determinations. These are the ratios

$$\frac{B(B\to X_d\nu\bar{\nu})}{B(B\to X_s\nu\bar{\nu})}=\left|\frac{V_{td}}{V_{ts}}\right|^2, \tag{26}$$

$$\frac{B(B_d\to\mu^+\mu^-)}{B(B_s\to\mu^+\mu^-)}=\frac{\tau_{B_d}}{\tau_{B_s}}\frac{m_{B_d}}{m_{B_s}}\frac{f_{B_d}^2}{f_{B_s}^2}\left|\frac{V_{td}}{V_{ts}}\right|^2, \tag{27}$$

$$\frac{x_d}{x_s}=\frac{\tau_{B_d}}{\tau_{B_s}}\frac{m_{B_d}}{m_{B_s}}\frac{B_{B_d}}{B_{B_s}}\frac{f_{B_d}^2}{f_{B_s}^2}\left|\frac{V_{td}}{V_{ts}}\right|^2, \tag{28}$$

which all measure

$$\left|\frac{V_{td}}{V_{ts}}\right|^2=\lambda^2\,\frac{(1-\bar{\varrho})^2+\bar{\eta}^2}{1+\lambda^2(2\bar{\varrho}-1)}. \tag{29}$$

The cleanest quantity is Eq. (26), which is essentially free of hadronic uncertainties. Next comes Eq. (27), involving SU(3)-breaking effects in the ratio of B meson decay constants. Finally, SU(3) breaking in the ratio of bag parameters, B_{B_d}/B_{B_s}, enters in addition in Eq. (28). These SU(3)-breaking effects should eventually be calculable with reasonable precision from lattice QCD.

In order to extract $|V_{td}|$ from either of the quantities in Eqs. (26)–(28) with an accuracy competitive to the one in the first column of Table I, the combined theoretical and experimental uncertainty for $|V_{td}/V_{ts}|^2$, as determined from Eqs. (26)–(28), should be brought below $\pm20\%$.

V. CONCLUSIONS

We have discussed the phenomenological potential of theoretically clean observables that promise to provide precise determinations of the CKM matrix and detailed tests of standard model flavor dynamics. CP violation experiments at e^+e^- B factories and hadron colliders will pursue the measurement of $\sin 2\beta$ and $\sin 2\alpha$. The former is essentially free of theoretical uncertainties, and the latter will also give important and rather clean information, provided the penguin contributions can be sufficiently well controlled.

Besides this class of phenomena, the following, theoretically very clean processes can give additional pieces of information that will be crucial for concise tests of the CKM description of quark mixing: (a) $B(K_L\to\pi^0\nu\bar{\nu})$; (b) $B(B\to X_d\nu\bar{\nu})/B(B\to X_s\nu\bar{\nu})$; (c) $B(K^+\to\pi^+\nu\bar{\nu})$; (d) $B(B_d\to\mu^+\mu^-)/B(B_s\to\mu^+\mu^-)$; (e) x_d/x_s. This list is essentially ordered according to increasing theoretical uncertainties. In principle, quantity (b) has basically, like $B(K_L\to\pi^0\nu\bar{\nu})$, no such uncertainties, but is presumably even more difficult to measure.

We have considered several strategies to determine the CKM matrix. In particular, we have pointed out that a measurement of $\sin 2\alpha$ with only rather moderate precision combined with $B(K_L\to\pi^0\nu\bar{\nu})$ could give a very respectable determination of CKM parameters. This emphasizes the great importance to also succeed in measuring $\sin 2\alpha$.

Since the number of theoretically clean processes is quite limited, it is mandatory that all of them be pursued experimentally as far as possible, irrespective of which quantities will ultimately turn out to give the best determination of CKM parameters. After all, the goal is not just to measure, but eventually to overconstrain the CKM matrix.

We stress that the rare decays $K^+\to\pi^+\nu\bar{\nu}$ and $K_L\to\pi^0\nu\bar{\nu}$ are excellent probes of flavor physics. They are not only clean measures of CKM parameters in their own right, but in addition complement CP violation studies in B decays due to, in general, different sensitivity to new physics and entirely different experimental systematics. In particular, we emphasize the unique role that can be played by $K_L\to\pi^0\nu\bar{\nu}$. This decay probes directly and unambiguously the nature of CP violation. Its branching fraction is one of the best measures of CKM parameters. Especially, $\mathrm{Im}\lambda_t$ and the Jarlskog parameter J_{CP} can be determined from a $\pm10\%$ measurement of $B(K_L\to\pi^0\nu\bar{\nu})$ with a precision that cannot even be achieved from CP violation studies in B decays in the CERN Large Hadron Collider (LHC) era. Of course, the detection of $K_L\to\pi^0\nu\bar{\nu}$ is experimentally very challenging, but it is not unrealistic. The current upper limit on the branching fraction is 5.8×10^{-5} [24]. Possibilities for future experiments have been discussed in [25,26]. Recently, a very interesting proposal has been made, aiming at a ~10% measurement of $B(K_L\to\pi^0\nu\bar{\nu})$ at the Brookhaven Alternating Gradient Synchrotron (AGS) by the year 2000 [27]. These developments are rather encouraging. The theoretical motivation clearly warrants all efforts necessary to reach this goal. It can be expected that further progress will also be achieved for other quantities, such as CP asymmetries in B decays at the LHC [28] and $B(K^+\to\pi^+\nu\bar{\nu})$ [29,30], where the current upper limit of 2.4×10^{-9} [31] is already rather close to the standard model expectation of $(1.0\pm0.4)\times10^{-10}$ [5].

The combined use of all available processes will then improve considerably our understanding of quark mixing and lead to new, and possibly unexpected, insights into this important area of high energy physics.

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