Ponderomotive force due to neutrinos

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We derive the ponderomotive force for an arbitrary distribution of neutrinos in a plasma using quantum statistical field theory. The ponderomotive force has two components. One component, due to gradients in the neutrino and plasma densities, reduces to a known expression. The second component is due to a coupling between anisotropies in the neutrino and plasma densities. Simple estimates suggest that the ponderomotive force is too small to play an important role in the acceleration mechanism for plasma in type II supernovae. [S0556-2821(96)01322-7]

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I. INTRODUCTION

Recently, Melrose and Hardy [1] derived a covariant expression for the ponderomotive force due to a distribution of photons in a medium (plasmons) using a synthesis of quantum electrodynamics and the (covariant) kinetic theory of plasmas, called quantum plasmadynamics (QPD). The expression derived reproduces the well-known classical (nonquantum) limit and has a clear physical interpretation involving the self-energy of the electron in a medium. Here we apply the same procedure to determine a covariant expression for the ponderomotive force due to a distribution of neutrinos in a plasma. We show that the resulting expression reproduces the known expression for isotropic neutrinos and electron distributions.

QPD [2] is a prescription for including the effects of a medium in QED in a manner similar to the more well-known finite temperature field theory (FTFT) [3]. The main advantage of QPD in these calculations is that it allows the use of arbitrary distribution functions to represent the medium, rather than the thermal distribution functions used in FTFT. Thus QPD allows the analysis of the effects of anisotropic media, such as a beamed distribution of neutrinos or an electron distribution with a heat-flux-induced anisotropy.

The main motivation for the investigation reported here is the application to the regions near the core of a type II supernova (SN) during the brief (≈ 3 sec) neutrino burst, where an intense neutrino flux occurs and passes through very dense plasma. These conditions are the most favorable known for the ponderomotive force due to neutrinos to be physically significant.

The ponderomotive force due to a distribution of photons in a plasma is the force that acts on the background plasma as a result of the average effect of the high frequency oscillations of the electromagnetic field [4].

The ponderomotive force due to a distribution of photons in a medium is identified via the following steps [1]: The self-energy of the electron is calculated and the contribution due to the photon distribution is identified; the effective mass correction due to this self-energy term is calculated; this mass correction is integrated over the electron distribution and is interpreted as a potential in which the electrons lie; finally, the ponderomotive force is identified as the gradient of this potential. This prescription reproduces known covariant expressions for the ponderomotive force due to an arbitrary distribution of photons in a medium.

Here we apply the same procedure to calculate the ponderomotive force due to a beamed distribution of neutrinos via the electron self-energy diagrams shown in Fig. 1, where the neutrino propagators are replaced by their propagators averaged over the neutrinos in the medium. All calculations are performed to lowest order in $1/M_W$ where M_W is the mass of the W boson. Thus calculated, the ponderomotive force separates into two terms. The first of these is the force due to gradients in the electron and neutrino densities and reduces to a known expression [5]. This term is proportional to the difference between the neutrino and antineutrino densities. The second term couples asymmetries in the neutrino distribution to asymmetries in the electron distribution and is proportional to the sum of the neutrino and antineutrino densities. This second term has not been identified previously.

It should be noted that the ponderomotive force calculated here is due solely to resonant neutrino-electron interactions and should not be confused with the force due to elastic neutrino-electron scattering.

In Sec. II we introduce our notation and one fundamental equation of QPD, the particle propagator statistically averaged over the medium. We then calculate the contributions of the self-energy operators shown in Fig. 1. From these we obtain an expression for the ponderomotive force due to neutrinos which is accurate to lowest order in $1/M_W$. This theory is applied to the neutrino burst of a type II SN in Sec. III, and the dynamical significance of the ponderomotive force is discussed. Natural units ($\hbar = c = 1$) are used throughout unless otherwise stated.

II. PONDEROMOTIVE FORCE

The form of the statistically averaged fermion propagator used here is that derived by Hayes and Melrose [6]. This

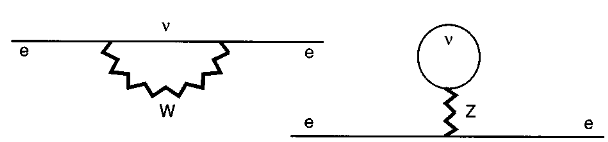

FIG. 1. Feynman diagrams representing the self-energy of the electron due to neutrinos.

derivation is outlined in the Appendix. The resulting expression for the propagator for a massive fermion is

$$G(P)=(\gamma_\mu P^\mu+m)\left[\frac{1}{P^2-m^2+i0}+\frac{i\pi}{\varepsilon}\sum_{\epsilon=\pm 1} n^\epsilon(\epsilon\mathbf{p})\,\delta(P^0-\epsilon\varepsilon)\right], \tag{1}$$

where $n^\epsilon(\epsilon\mathbf{p})$ are the fermion occupation numbers, with $\epsilon=+1$ corresponding to particles, $\epsilon=-1$ corresponding to antiparticles, and where the four-momenutm $P=(E,\mathbf{P})$ is related to the physical four-momentum $p=(\varepsilon,\mathbf{p})$ through $P=\epsilon p$. Note that the physical four-momentum is defined such that the particle's energy ε is always positive. Clearly, if $n^\epsilon(\epsilon\mathbf{p})$ are evaluated for thermal distributions, Eq. (1) reduces to the expressions derived through FTFT [3].

Following Hayes and Melrose [6], we relate this propagator to a four-dimensional distribution function $N(P)$ given by

$$N(P)=\sum_{\epsilon=\pm 1}\frac{2\pi m}{\varepsilon}\,\delta(E-\epsilon\varepsilon)\,n^\epsilon(\epsilon\mathbf{p}), \tag{2}$$

which may be written

$$N(P)=\sum_{\epsilon=\pm 1} 4\pi m\,\delta(P^2-m^2)H(\epsilon\varepsilon)n^\epsilon(\epsilon\mathbf{p}), \tag{3}$$

where $H(x)$ is the Heaviside step function. Equation (3) is useful in performing averages over the distribution of fermions through

$$\overline{A}=\mathrm{Tr}\int\frac{d^4P}{(2\pi)^4}N(P)A, \tag{4}$$

where ''Tr'' denotes the trace over the Dirac matrices and corresponds to a sum over the spin states of the fermions.

The resonant part of the statistically averaged propagator, Eq. (1), for a massless neutrino may be written

$$\mathrm{Res}G_{\mathrm{nu}}(q)=2\pi i\not{q}\sum_{\epsilon=\pm 1}\delta(q^2)H(\epsilon\varepsilon)n^\epsilon_{\mathrm{nu}}(\epsilon\mathbf{q}). \tag{5}$$

The nonresonant part of the propagator is identical to the vacuum propagator.

We turn now to the evaluation of the self-energy diagrams shown in Fig. 1. We present the derivation for the W-boson self-energy term Σ^W and state the analogous result for the Z-boson self-energy term Σ^Z. Analyzing the first Feynman diagram shown in Fig. 1, one obtains

$$\Sigma^W(p)=\frac{-ig^2}{8}\gamma_\mu(1-\gamma_5)\int\frac{d^4q}{(2\pi)^4}\times\mathrm{Res}G_{\mathrm{nu}}(q)\,\gamma_\nu(1-\gamma_5)G_W^{\mu\nu}(p-q), \tag{6}$$

where

$$G_W^{\mu\nu}(p)=\frac{g^{\mu\nu}-p^\mu p^\nu/M_W^2}{p^2-M_W^2} \tag{7}$$

is the W-boson propagator. Assuming that all momenta are small compared to the mass of the W boson, Eq. (6) becomes

$$\Sigma^W=\frac{-iG_F}{\sqrt{2}}\gamma_\mu(1-\gamma_5)\int\frac{d^4q}{(2\pi)^4}\mathrm{Res}G_{\mathrm{nu}}(q)\,\gamma^\mu(1-\gamma_5). \tag{8}$$

Inserting Eq. (5), expanding, and integrating over q^0 leads to

$$\Sigma^W=\frac{-iG_F}{\sqrt{2}}4(1+\gamma_5)\gamma_\mu\int\frac{d^3\mathbf{q}}{(2\pi)^3}[(1,\hat{\mathbf{q}})^\mu n^+_{\mathrm{nu}}(\mathbf{q})+(-1,\hat{\mathbf{q}})^\mu n^-_{\mathrm{nu}}(-\mathbf{q})], \tag{9}$$

where we use the energy-momentum relation for massless neutrinos, $\varepsilon=|\mathbf{q}|$, and where $\hat{\mathbf{q}}=\mathbf{q}/|\mathbf{q}|$. Performing the same procedure for the Z-boson diagram, the second diagram of Fig. 1, leads to

$$\Sigma^Z=(g_V+g_A)\Sigma^W, \tag{10}$$

with

$$g_V=-\tfrac{1}{2}+2\sin^2\theta_W,\quad g_A=-\tfrac{1}{2}, \tag{11}$$

and where θ_W is the Weinberg angle.

Hence, the resonant contribution of a neutrino medium to the self-energy of an electron is given by

$$\Sigma=\frac{-iG_F}{\sqrt{2}}8\sin^2\theta_W(1+\gamma_5)\gamma_\mu N^\mu, \tag{12}$$

where

$$N^\mu=\int\frac{d^3\mathbf{q}}{(2\pi)^3}[(1,\hat{\mathbf{q}})^\mu n^+_{\mathrm{nu}}(\mathbf{q})+(-1,\hat{\mathbf{q}})^\mu n^-_{\mathrm{nu}}(-\mathbf{q})]. \tag{13}$$

A. Mass excess

In the electron propagator, the self-energy term appears in the denominator as $1/(\not{p}+\Sigma-m)$. This leads to a correction $m\to m+\delta m$ in the electron mass given by $[\not{p}+\Sigma]^2=(m+\delta m)^2$. Assuming $|\delta m|\ll m$, this gives

$$\delta m=\frac{-1}{2m}[\not{p}\Sigma+\Sigma\not{p}]. \tag{14}$$

Inserting Eq. (12) one obtains the correction to the mass of a single electron due to a medium of neutrinos:

$$\delta m=\frac{\sqrt{2}G_F}{m_e}[2(pN)+\gamma_5(\not{N}\not{p}-\not{p}\not{N})], \tag{15}$$

where we use standard properties of the γ matrices.

The mass excess of Eq. (15) is averaged over the electron distribution as in Eq. (4). Interpreting this average as an energy density we write

$$U=\frac{\sqrt{2}G_F}{m_e}\mathrm{Tr}\int\frac{d^4P}{(2\pi)^4}2\pi m_e\delta(p^2-m_e^2)\sum_\epsilon H(\epsilon\varepsilon)n_e^\epsilon(\boldsymbol{\epsilon}\mathbf{p})\times[2(pN)+\gamma_5(\not{N}\not{p}-\not{p}\not{N})]. \tag{16}$$

Performing the trace over the γ matrices and carrying out the p^0 integral over the δ function leads to

$$U=\frac{G_F}{\sqrt{2}}2\sin^2\theta_W N_\mu M^\mu, \tag{17}$$

where

$$M^\mu=\int\frac{d^3\mathbf{p}}{(2\pi)^3}[(1,\hat{\mathbf{p}})^\mu n_e^+(\mathbf{p})+(-1,\hat{\mathbf{p}})^\mu n_e^-(-\mathbf{p})]. \tag{18}$$

In a covariant treatment of the ponderomotive force [7], the energy-momentum tensor $T_B^{\mu\nu}$ for the background plasma experiences a four-force such that one has $F^\nu=\partial_\mu T_B^{\mu\nu}=\partial^\nu U$. Thus the ponderomotive three-force per unit volume due to a distribution of neutrinos may be written

$$\tilde{\mathbf{F}}=-\frac{G_F}{\sqrt{2}}2\sin^2\theta_W\boldsymbol{\nabla}(N_\mu M^\mu). \tag{19}$$

In the following section we evaluate Eq. (19) for some simple distributions.

B. Axisymmetric distributions

We assume, for simplicity, that the neutrino and electron distribution functions are separable, axisymmetric, and that their axes of symmetry are aligned. We write

$$n_p^\epsilon(\boldsymbol{\epsilon}\mathbf{p})=\eta_p^\epsilon(|\mathbf{p}|)\Phi_p^\epsilon(\cos\alpha), \tag{20}$$

where the subscript p denotes the particle species (e or nu) and where we introduce spherical polar momentum coordinates $(|\mathbf{p}|,\alpha,\varphi)$. We denote the number densities of a given species as n_p^ϵ given by

$$n_p^\epsilon=\int\frac{d^3\mathbf{p}}{(2\pi)^3}n_p^\epsilon(\mathbf{p}). \tag{21}$$

We also define a measure of the anisotropy of the particle distributions through

$$\langle\alpha_p^\epsilon\rangle=\frac{1}{n_p^\epsilon}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\cos\alpha\, n_p^\epsilon(\boldsymbol{\epsilon}\mathbf{p}). \tag{22}$$

For separable distributions of the form (20), one has that

$$\langle\alpha_p^\epsilon\rangle=\frac{1}{4\pi}\int_{-1}^{1}d(\cos\alpha)\cos\alpha\,\Phi_p^\epsilon(\alpha), \tag{23}$$

where we assume that the angular distribution is normalized to 4π.

Evaluating Eqs. (18) and (13) leads to

$$N^\mu=(n_{\mathrm{nu}}^+-n_{\mathrm{nu}}^-,0,0,\langle\alpha_{\mathrm{nu}}^+\rangle n_{\mathrm{nu}}^++\langle\alpha_{\mathrm{nu}}^-\rangle n_{\mathrm{nu}}^-)^\mu \tag{24}$$

and

$$M^\mu=(n_e^+-n_e^-,0,0,\langle\alpha_e^+\rangle n_e^++\langle\alpha_e^-\rangle n_e^-)^\mu. \tag{25}$$

On evaluating Eq. (19), the ponderomotive force per unit volume has two components:

$$\tilde{\mathbf{F}}_{\mathrm{pond}}=\tilde{\mathbf{F}}+\tilde{\mathbf{F}}', \tag{26}$$

where

$$\tilde{\mathbf{F}}=-\frac{G_F}{\sqrt{2}}2\sin^2\theta_W\boldsymbol{\nabla}[(n_{\mathrm{nu}}^+-n_{\mathrm{nu}}^-)(n_e^+-n_e^-)] \tag{27}$$

and

$$\tilde{\mathbf{F}}'=+\frac{G_F}{\sqrt{2}}2\sin^2\theta_W\boldsymbol{\nabla}[(\langle\alpha_{\mathrm{nu}}^+\rangle n_{\mathrm{nu}}^++\langle\alpha_{\mathrm{nu}}^-\rangle n_{\mathrm{nu}}^-)\times(\langle\alpha_e^+\rangle n_e^++\langle\alpha_e^-\rangle n_e^-)]. \tag{28}$$

The component (27) is the force due to any distribution of interacting neutrinos and electrons. This expression reduces to that given by [5] in the limit $n_e^-=n_{\mathrm{nu}}^-=0$, n_e^+ constant, and $\theta_W=\pi/2$. Thus, Eq. (27) represents a generalization of the known result to include the effects of positrons and antineutrinos. The other component (28) corresponds to a previously unidentified force due to coupling of the anisotropies in the electron and neutrino distributions. Clearly, the form of Eq. (28) is dependent on the assumption that the electron and neutrino distributions are axisymmetic and that their axes are aligned. If the axes of symmetry of the distributions are orthogonal, the inner product of Eq. (19) has no contribution from the asymmetry of the electrons, and Eq. (28) is identically zero.

III. APPLICATIONS

The motivation for this investigation is the possible application to the acceleration of plasma in type II SNe. The explosion mechanism for type II SNe suffers a well-known problem in that the bounce shock, which should blow off the outer layers of the star, is insufficiently energetic to overcome disassociation energy losses near the center of the SN [8]. This problem has been largely overcome in recent times through consideration of neutrino driven convective overturn [9]. This convective instability leads to much larger density irregularities in the center of SNe than previously considered. In view of Eqs. (27) and (28), which involve the rate of change of the electron number density, these density irregularities may be crucial to the physical significance of this work. We restrict ourselves here to general considerations of sources of anisotropies and density irregularities which might contribute to the ponderomotive force and simple estimates of its magnitude.

The neutrinos emitted during a SN explosion are generated within the core and diffuse outwards to a region which is transparent to neutrino transport. Thus the neutrinos appear to be radiating from a ‘‘neutrinosphere’’ approximately 70 km in diameter. All species of neutrinos and antineutrinos are thought to be produced in the neutrino burst in roughly equal quantities. Thus we see that the first component of the ponderomotive force, Eq. (27), is suppressed as it is proportional to the difference of the neutrino and antineutrino number densities.

A. Anisotropies

We turn now to the second component of the ponderomotive force, Eq. (28) which is nonzero only if both the electron and neutrino distributions are anisotropic.

At any point outside the neutrinosphere the neutrinos propagate in a cone with opening angle subtended by the neutrinosphere. For a cone with an opening angle α_0 ($\ll 1$),

$$\langle \alpha_{\rm nu}^{\pm} \rangle = \frac{1}{2}(1+\cos\alpha_0) \approx 1 - \frac{\alpha_0^2}{4}, \tag{29}$$

implying a large neutrino anisotropy.

Anisotropies in the electron distribution may be produced by a variety of effects, though the level of anisotropy near the core of a SN is expected to be small. We now briefly discuss some possible sources of anistropic electron distributions.

(1) One possible source of anisotropy relates to the bulk flows of the electron medium. Any differential flow over regions of the plasma leads to an anisotropic component of the distribution. This has been characterized for plasma in the solar wind by Dung [10].

(2) Another cause of an anisotropic plasma distribution is a heat flux through the plasma surrounding the SN core. This leads to a level of anisotropy in the electron distribution which is related to the thermal conductivity of the plasma medium [11,12].

(3) Finally, anisotropies are often encountered in magnetized plasmas where the temperature perpendicular to the magnetic field may be different to that parallel to the magnetic field [12].

We provide no specific measures of the level of anisotropy caused by these effects—as will be seen below, the actual magnitude is irrelevant to the conclusions of this work.

B. Simple estimates

We estimate the magnitude of Eq. (28) for two limiting cases. In the first, the anisotropic component of the ponderomotive force is assumed to arise solely through the density gradient of the neutrinos caused by their radial expansion. The electron distribution is assumed constant. In the second case, the reverse is assumed, and the contribution to Eq. (28) from a change in the electron density is evaluated assuming that any variation in the neutrino distribution is negligible. The true physical scenario may lie between these extremes, and there may also be a contribution from the change in the measure of the anisotropy of the electron density. A relevant force per unit volume for comparison is that of direct neutrino-electron scattering which is approximately $10^{11}\ \mathrm{N\,m^{-3}}$ for the neutrino and plasma properties used below.

Now, assuming that the electron density and anisotropy are constant in space, and that the neutrino density has a $1/r^2$ dependence due to the radial expansion, Eq. (28) reduces to

$$\widetilde{F}' \approx \frac{G_F}{\sqrt{2}} 4\sin^2\theta_W \langle \alpha_e^+ \rangle n_e \frac{n_{\rm nu}}{r}. \tag{30}$$

For plasma and neutrino parameters relevant to the core of a type II SN, $n_e = 10^{36}\ \mathrm{m^{-3}}$, $n_{\rm nu} = 10^{38}\ \mathrm{m^{-3}}$, and $r = 10^5$ m, we find

$$\widetilde{F}' \approx \langle \alpha_e^+ \rangle 10^7\ \mathrm{N\,m^{-3}}, \tag{31}$$

where we have used $G_F = 10^{-62}\ \mathrm{N\,m^4}$. This force is many orders of magnitude smaller than that due to neutrino-electron scattering, regardless of the level of anisotropy of the electron distribution.

For the second case, it is assumed that the neutrino density is constant and that the electron density varies a certain amount Δn_e over some length scale Δx. This might represent large scale variations due to convective motions in the plasma or smaller scale variations due to plasma instabilities or plasma oscillations. For the same plasma parameters used above we have

$$\widetilde{F}' \approx \langle \alpha_e^+ \rangle \frac{\Delta n_e}{\Delta x} 10^{-25}\ \mathrm{N\,m}. \tag{32}$$

Thus, for any reasonable levels of density fluctuations, this contribution to the ponderomotive force is even smaller than that of Eq. (31), and is not significant.

Given that Eqs. (31) and (32) both show that the anisotropic contribution to the ponderomotive force is dynamically unimportant in their respective applications, we conclude that the ponderomotive force appears not to play a role in the explosion mechanism of type II SNe.

IV. CONCLUSION

An expression for the ponderomotive force on a plasma due to a distribution of neutrinos is derived, allowing for anisotropies in both the electron and neutrino distribution. The inclusion of anisotropies here is possible due to the use of propagators that are averaged over arbitrary distributions of electrons and neutrinos, whereas in earlier calculations of neutrino interactions only finite temperature propagators, averaged over thermal distributions, were used [3]. The ponderomotive force appears to be dynamically unimportant in its most promising application, the explosion mechanism of type II SNe.

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APPENDIX

We present here the derivation of the fermion propagator statistically averaged over the medium, as in [6]. The statistical averaging is performed by replacing the vacuum density matrix $\hat{w}_V$ by a particle density matrix

$$\hat{w}_P = \sum_q \prod_\epsilon w_{\epsilon q} |\epsilon q\rangle\langle\epsilon q| \tag{A1}$$

in the expression for the electron propagator:

$$G(x-x') = -i\mathrm{Tr}[\hat{w}_P \mathcal{T}\{\hat{\Psi}(x)\overline{\hat{\Psi}}(x')\}]. \tag{A2}$$

In Eq. (A1), the states are denoted by the set of quantum numbers $\{\epsilon q\}$ where $\epsilon=+1$ for particles, $\epsilon=-1$ for antiparticles, and where q denotes the remaining quantum numbers. Writing the density matrix in this form assumes that it is diagonal; that is, it is assumed that there are no phase correlations between the states. In Eq. (A2), $\mathcal{T}$ denotes the chronological operator, and $\hat{\Psi}$ is a wave function operator.

The statistical average K of any operator $\hat{K}$ is

$$\overline{K} = \mathrm{Tr}(\hat{w}_P \hat{K}) = \sum_{q\epsilon} w_{q\epsilon}\langle\epsilon q|\hat{K}|\epsilon q\rangle. \tag{A3}$$

The particle number operators

$$\hat{n}_q^\epsilon = \hat{a}_q^{\epsilon\dagger}\hat{a}_q^\epsilon \tag{A4}$$

give

$$\overline{n_q^\epsilon} = \mathrm{Tr}[\hat{w}\hat{n}_q^\epsilon], \tag{A5}$$

with

$$\mathrm{Tr}[\hat{w}\hat{a}_q^\epsilon \hat{a}_q^{\epsilon\dagger}] = 1 - \overline{n_q^\epsilon}. \tag{A6}$$

The statistically averaged form of the fermion propagator is obtained by first expanding the chronological operator in Eq. (A2), and inserting the second-quantized form of the the wave functions. This leads to

$$\begin{aligned} G(x-x') = &-iH(t-t')\mathrm{Tr}\left[\hat{w}_P \sum_q \sum_{q'} \hat{a}_q \hat{a}_{q'}^\dagger \psi_q^+(x)\overline{\psi_{q'}^+}(x')\exp\{-i\varepsilon_q t + i\varepsilon_{q'}t'\} + \hat{b}_q^\dagger \hat{b}_{q'}\psi_q^-(x)\overline{\psi_{q'}^-}(x')\exp\{i\varepsilon_q t - i\varepsilon_{q'}t'\}\right] \\ &+ iH(t'-t)\mathrm{Tr}\left[\hat{w}_P \sum_q \sum_{q'} \hat{a}_{q'}^\dagger \hat{a}_q \overline{\psi_{q'}^+}(x')\psi_q^+(x)\exp\{-i\varepsilon_q t + i\varepsilon_{q'}t'\} + \hat{b}_{q'}\hat{b}_q^\dagger \overline{\psi_{q'}^-}(x')\psi_q^-(x)\exp\{i\varepsilon_q t - i\varepsilon_{q'}t'\}\right], \end{aligned} \tag{A7}$$

where $\hat{a}$ and $\hat{b}$ are particle and antiparticle annihilation operators, respectively, ψ_q^+ and ψ_q^- are particle and antiparticle wave functions, respectively, and ε_q denotes the energy eigenvalue of a particle with given quantum numbers q.

The trace over the density matrices is performed through Eqs. (A5) and (A6), and the integral expression for the step operator,

$$H(t'-t) = \int \frac{d\omega}{2\pi} \frac{\exp\{-i\omega(t-t')\}}{\omega + i0}, \tag{A8}$$

is inserted. This leads to

$$\begin{aligned} G(x-x') = \sum_q \int \frac{d\omega}{2\pi}\frac{1}{\omega+i0}(&\psi_q^+(x)\overline{\psi_q^+}(x')\{(1-n_q^+)\exp[i(\omega+\varepsilon_q)(t'-t)] - n_q^+\exp[i(\varepsilon_q-\omega)(t'-t)]\} \\ &+ \psi_q^-(x)\overline{\psi_{q'}^-}(x')\{n_q^-\exp[i(\varepsilon_q-\omega)(t-t')] + (1-n_q^-)\exp[i(\omega+\varepsilon_q)(t-t')]\}). \end{aligned} \tag{A9}$$

In each of the terms above, the integral is shifted in origin to eliminate ε_q from the argument of the exponential, and we substitute plane wave solutions

$$\psi_q^\epsilon = \varphi_s^\epsilon \exp(i\epsilon\mathbf{p}\cdot\mathbf{x}), \tag{A10}$$

leading to

$$\begin{aligned} G(x-x') = V\sum_s \int \frac{dp^0}{2\pi}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\exp\{ip^0(t'-t)\}\Bigg[&\left\{\frac{1-n^+(\mathbf{p})}{p^0-\varepsilon+i0} - \frac{n^+(\mathbf{p})}{p^0-\varepsilon-i0}\right\}\varphi_s^+(\mathbf{p})\overline{\varphi_s^+}(\mathbf{p})\exp\{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')\} \\ &+ \left\{\frac{n^-(\mathbf{p})}{p^0+\varepsilon+i0} - \frac{1-n^-(\mathbf{p})}{p^0+\varepsilon-i0}\right\}\varphi_s^-(-\mathbf{p})\overline{\varphi_s^-}(-\mathbf{p})\exp\{i\mathbf{p}\cdot(\mathbf{x}'-\mathbf{x})\}\Bigg]. \end{aligned} \tag{A11}$$

Summing over spin states through

$$\sum_s \varphi_s^\epsilon(\mathbf{p})\overline{\varphi_s^\epsilon}(\mathbf{p}) = \frac{\not{p} + \epsilon m}{\varepsilon} \tag{A12}$$

and taking the Fourier transform leads to

$$G(P)=\frac{1}{2\varepsilon}\left[\left\{\frac{1-n^{+}(\mathbf{p})}{p^{0}-\varepsilon+i0}-\frac{n^{+}(\mathbf{p})}{p^{0}-\varepsilon-i0}\right\}(\gamma_{\mu}P^{\mu}+m)+\left\{\frac{n^{-}(\mathbf{p})}{p^{0}+\varepsilon+i0}-\frac{1-n^{-}(\mathbf{p})}{p^{0}+\varepsilon-i0}\right\}(\gamma_{\mu}P^{\mu}+m)\right]. \tag{A13}$$

This expression may be reduced to

$$G(P)=(\gamma_{\mu}P^{\mu}+m)\left[\frac{1}{P^{2}-m^{2}+i0}+\frac{i\pi}{\varepsilon}\sum_{\epsilon=\pm1}n^{\epsilon}(\epsilon\mathbf{p})\,\delta(P^{0}-\epsilon\varepsilon)\right], \tag{A14}$$

which is the propagator for a fermion statistically averaged over a medium. Clearly, if the occupation numbers in Eq. (A14) are those for a thermal distribution of fermions, this expression reduces to the expression commonly encountered in the literature derived via finite temperature field theory [3].

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