Tensor-multiscalar theories from multidimensional cosmology

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Inhomogeneous multidimensional cosmological models with a higher-dimensional space-time manifold $M=M_0\times\prod_{i=1}^{n}M_i$ $(n\geqslant 1)$ are investigated under dimensional reduction to tensor-multiscalar theories. In the Einstein conformal frame, these theories take the shape of a flat σ model. For the singular case where M_0 is two dimensional, the dimensional reduction to dilaton gravity is preformed with different distinguished representations of the action. [S0556-2821(96)03320-6]

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I. INTRODUCTION

In recent years, scalar-tensor theories have received renewed interest. There are two reasons for this. First, extended inflation [1,2] which originally was based on standard Brans-Dicke (BD) theory [3] revives the scenario of inflation via a first-order transition. It provides a natural (non-fine-tuned) way to restore the original ideas of inflation while avoiding the cosmological difficulties coming from vacuum-dominated exponential expansion obtained in general relativity. Second, scalar-tensor theories, generalizing standard BD theory, can satisfy the solar system criteria [4] and other present observations [5] to arbitrary accuracy, but still diverge from general relativity in the strong field limit. Thus, in the future, such theories may provide an important test of general relativity. It should be noted in that context that, via conformal transformation of the metric, we can write scalar-tensor models equally well in the Jordan-Brans-Dicke frame or in the Einstein-Pauli frame, and the question which frame is a physical one is still open.

Several ways to generalize standard BD theory were proposed. These theories can be split into three main groups. First, there are the theories with a BD parameter ω depending on the dilatonic scalar field [6–14]. The second class is represented by theories with more than one dilaton field [4,15,16]. All other possible variations of the standard BD model form the third group, containing, e.g., models in which the dilaton couples with different strengths to both visible matter and conjectured ''dark'' matter [5].

Scalar-tensor theories follow naturally as the low-energy limit of various Kaluza-Klein theories. Among them, multidimensional cosmological models with a space-time consisting of $n\geqslant 2$ Einstein spaces are most popular. Usually, theories with one internal Einstein space $(n=2)$ are considered [17–19]. The dimensional reduction of these theories yields only one dilaton field (such as in the original BD theory). Here, we show that this case is exceptional, with a midisuperspace metric of degenerate signature. The model with $n\geqslant 2$ was considered in the paper [20], where the emphasis was on the problem of the internal dimensions compactification. In our paper we shall give a more elegant way of dimensional reduction to tensor-multiscalar theories which reveals explicitly the nature of the dilaton fields. For example, the dilaton field with opposite sign in the kinetic term of the Lagrangian is connected with a dynamical volume of the whole internal space. In a conformal Einstein-Pauli frame, a σ-model representation of the theory under consideration can easily be obtained.

In Sec. II, it is shown, for space-time dimension $D_0>2$ after dimensional reduction, that, in the σ-model representation of our model, the metric on the space of scalar fields is the flat Euclidean one. If $D_0=2$, there is no conformal Einstein(-Pauli) frame. This is actually no fault of the theory here, because in two dimensions the Einstein action is a topological invariant, whence it does not yield a dynamics of the two-geometry.

Nevertheless, it is worthwhile to consider here two-dimensional (2D) dilation gravity, which has been a subject of intensive investigations recently [21–23]. In Sec. III we obtain the action of 2D dilaton gravity under dimensional reduction from cosmological models. Different representations are given, which correspond to different choices of conformal frames.

II. EFFECTIVE BD MODELS FROM MULTIDIMENSIONAL MODELS

Let us now consider a multidimensional space-time manifold

$$M=M_0\times\prod_{i=1}^{n}M_i\,. \tag{2.1}$$

The metric on M can be decomposed as

$$g=g^{(0)}+\sum_{i=1}^{n}e^{2\beta^i(x)}g^{(i)}, \tag{2.2}$$

where x are some coordinates of M_0, and

$$g^{(0)}=g^{(0)}_{\mu\nu}(x)dx^\mu\otimes dx^\nu. \tag{2.3}$$

With the Laplace-Beltrami operator on M_0 defined by

$$\Delta[g^{(0)}]=\frac{1}{\sqrt{|\det g^{(0)}|}}\frac{\partial}{\partial x^\mu}\left(\sqrt{|\det g^{(0)}|}g^{(0)\mu\nu}\frac{\partial}{\partial x^\nu}\right), \tag{2.4}$$

we get the Ricci curvature scalar [24]

$$R[g]=R[g^{(0)}]+\sum_{i=1}^{n}e^{-2\beta^i}R[g^{(i)}]-\sum_{i,j=1}^{n}(D_i\delta_{ij}+D_iD_j)\times g^{(0)\mu\nu}\frac{\partial\beta^i}{\partial x^\mu}\frac{\partial\beta^j}{\partial x^\nu}-2\sum_{i=1}^{n}D_i\Delta[g^{(0)}]\beta^i. \tag{2.5}$$

With total dimension $D:=\Sigma_{i=0}^{n}D_i$, κ^2 a D-dimensional gravitational coupling constant, and $S_{\rm GH}$ the standard Gibbons-Hawking boundary term, we consider an action of the form

$$S=\frac{1}{2\kappa^2}\int_M d^Dx\sqrt{|\det g|}R[g]+S_{\rm GH}. \tag{2.6}$$

In the following we assume that $R[g^{(i)}]$ is finite on $(M_i,g^{(i)})$. Let us now consider the volumes μ_i of $(M_i,g^{(i)})$ and the total internal space volume μ, satisfying

$$\mu_i=\int_{M_i}d^{D_i}y\sqrt{|\det g^{(i)}|},\quad \mu=\prod_{i=1}^{n}\mu_i. \tag{2.7}$$

If all of the spaces M_i $(i=1,\ldots,n)$ are compact, then the volumes μ_i and μ are finite, and so are also the numbers

$$\rho_i=\int_{M_i}d^{D_i}y\sqrt{|\det g^{(i)}|}R[g^{(i)}]. \tag{2.8}$$

However, a noncompact M_i might have infinite volume μ_i or infinite ρ_i. Nevertheless, for the following we have to assume only that all ratios ρ_i/μ_i, $i=1,\ldots,n$ are finite. This is, in particular, the case when M_i is *homogeneous*. Then,

$$\frac{\rho_i}{\mu_i}=R[g^{(i)}]$$

is always constant and finite. In the special case, where M_i is an *Einstein* manifold $R^k_j[g^{(i)}]=\lambda^i\delta^k_j$ with constant λ^i, it is

$$\frac{\rho_i}{\mu_i}=R[g^{(i)}]=\lambda^iD_i,$$

and, more specially, when M_i has *constant curvature* k,

$$\frac{\rho_i}{\mu_i}=R[g^{(i)}]=kD_i(D_i-1).$$

However, here we do not restrict *a priori* to Einstein or constant curvature spaces. For convenience and beauty, in the following we will exemplify the dimensional reduction just for the case of homogeneous spaces $M_1,\ldots,M_n$, although the proceedure could be easily generalized for the case of inhomogeneous $M_1,\ldots,M_n$. (Then, one has to work with ρ_i/μ_i in place of $R[g^{(i)}]$.) The bare gravitational coupling constant

$$\kappa^2=\kappa_0{}^2\cdot\mu \tag{2.9}$$

might become infinite, while the true D_0-dimensional coupling constant $\kappa_0{}^2$ is always finite. If $D_0=4$, then $\kappa_0{}^2=8\pi G_N$, where G_N is the Newton constant. Then, the action (2.6) remains well defined, even when some of the volumes μ_i are infinite. After rewriting

$$\begin{aligned}\frac{1}{\kappa^2}\int_M d^Dx\sqrt{|\det g|}\sum_{i=1}^{n}D_i\Delta[g^{(0)}]\beta^i &=\frac{\mu}{\kappa^2}\sum_{i=1}^{n}D_i\int_{M_0}d^{D_0}x\sqrt{|\det g^{(0)}|}\prod_{l=1}^{n}e^{D_l\beta^l}\\&\quad\times\frac{1}{\sqrt{|\det g^{(0)}|}}\frac{\partial}{\partial x^\lambda}\left(\sqrt{|\det g^{(0)}|}g^{(0)\lambda\nu}\frac{\partial}{\partial x^\nu}\beta^i\right)\\&=\frac{1}{\kappa_0^2}\sum_{i=1}^{n}D_i\int_{M_0}d^{D_0}x\\&\quad\times\left[\frac{\partial}{\partial x^\lambda}\left(\sqrt{|\det g^{(0)}|}g^{(0)\lambda\nu}\prod_{l=1}^{n}e^{D_l\beta^l}\frac{\partial}{\partial x^\nu}\beta^i\right)\right.\\&\quad\left.-\sqrt{|\det g^{(0)}|}g^{(0)\lambda\nu}\frac{\partial\beta^i}{\partial x^\nu}\prod_{l=1}^{n}e^{D_l\beta^l}\sum_{j=1}^{n}D_j\frac{\partial\beta^j}{\partial x^\lambda}\right]\\&=S_{\rm GH}-\frac{1}{\kappa_0^2}\int_{M_0}d^{D_0}x\sqrt{|\det g^{(0)}|}\prod_{l=1}^{n}e^{D_l\beta^l}\\&\quad\times\sum_{i,j=1}^{n}D_iD_jg^{(0)\lambda\nu}\frac{\partial\beta^i}{\partial x^\lambda}\frac{\partial\beta^j}{\partial x^\nu},\end{aligned} \tag{2.10}$$

the action is

$$\begin{aligned}S=\frac{1}{2\kappa_0^2}\int_{M_0}d^{D_0}x\sqrt{|\det g^{(0)}|}\prod_{l=1}^{n}e^{D_l\beta^l}&\times\left\{R[g^{(0)}]-\sum_{i,j=1}^{n}G_{ij}g^{(0)\lambda\nu}\frac{\partial\beta^i}{\partial x^\lambda}\frac{\partial\beta^j}{\partial x^\nu}\right.\\&\left.+\sum_{i=1}^{n}R[g^{(i)}]e^{-2\beta^i}\right\},\end{aligned} \tag{2.11}$$

where

$$G_{ij}:=D_i\delta_{ij}-D_iD_j. \tag{2.12}$$

Let us first consider the exceptional case $n=1$:

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det g^{(0)}|}e^{D_1\beta^1}\times\left\{R[g^{(0)}]+D_1(D_1-1)g^{(0)\lambda\nu}\frac{\partial\beta^1}{\partial x^\lambda}\frac{\partial\beta^1}{\partial x^\nu}+R[g^{(1)}]e^{-2\beta^1}\right\}. \quad (2.13)$$

Here, for $D_1>1$ the kinetic term has a different sign than usual, and for $D_1=1$ there is no kinetic term at all. Setting

$$\phi:=e^{D_1\beta^1}, \quad (2.14)$$

it is

$$\frac{\partial\beta^1}{\partial x^\lambda}=\frac{1}{D_1}\frac{1}{\phi}\frac{\partial\phi}{\partial x^\lambda},$$

and, hence,

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det g^{(0)}|}\left\{\phi R[g^{(0)}]-\omega g^{(0)\lambda\nu}\frac{1}{\phi}\frac{\partial\phi}{\partial x^\lambda}\frac{\partial\phi}{\partial x^\nu}+R[g^{(1)}]\phi^{1-2/D_1}\right\}, \quad (2.15)$$

with BD parameter $\omega=\omega(D_1)=1/D_1-1$, depending on the present extra dimension D_1. It is remarkable that the conformal coupling constant $\xi_{c,d+1}$ in dimension $d+1$ determines the BD parameter for general extra dimension d as

$$\omega(d):=\frac{1}{d}-1\equiv-4\xi_{c,d+1}. \quad (2.16)$$

Let us now examine the general case $n>1$. Here it is useful to diagonalize the metric tensor (2.12). For the midisuperspace metric

$$G:=G_{ij}d\beta^i\otimes d\beta^j=\eta_{kl}dz^k\otimes dz^l=-dz^1\otimes dz^1+\sum_{i=2}^{n}dz^i\otimes dz^i, \quad (2.17)$$

the diagonalizing transformation

$$z^i=T^i{}_j\beta^j, \quad i=1,\dots,n \quad (2.18)$$

is given by (see also [25])

$$z^1=q^{-1}\sum_{j=1}^{n}D_j\beta^j,$$

$$z^i=[D_{i-1}/\Sigma_{i-1}\Sigma_i]^{1/2}\sum_{j=i}^{n}D_j(\beta^j-\beta^{i-1}), \quad (2.19)$$

$i=2,\dots,n$, where

$$q:=[D'/(D'-1)]^{1/2}=\frac{1}{2\sqrt{\xi_{c,D'+1}}}, \quad D':=D-D_0,$$

$$\Sigma_k:=\sum_{i=k}^{n}D_i.$$

Especially, we have

$$T^1{}_i=\frac{D_i}{q}, \quad i=1,\dots,n. \quad (2.20)$$

Let us determine $U=T^{-1}$ inverting Eq. (2.18) to

$$\beta^i=U^i{}_jz^j, \quad i,j=1,\dots,n. \quad (2.21)$$

Equations (2.17) and (2.18) imply $G_{ij}=\eta_{kl}T^k{}_iT^l{}_j$, $i,j=1,\dots,n$, and hence,

$$U^i{}_j=G^{ik}T^l{}_k\eta_{lj}=G^{ik}(T^t)_k{}^l\eta_{lj}, \quad i,j=1,\dots,n, \quad (2.22)$$

where the tensor components of the inverse midisuperspace metric are given as

$$G^{ij}=\frac{\delta^{ij}}{D_i}+\frac{1}{1-D'}. \quad (2.23)$$

With Eq. (2.20), we obtain especially

$$U^i{}_1=G^{ij}T^k{}_j\eta_{k1}=-G^{ij}T^1{}_j=\frac{1}{q(D'-1)}=\frac{1}{\sqrt{D'(D'-1)}}, \quad i=1,\dots,n. \quad (2.24)$$

Using that, we can rewrite the action (2.11) as

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det g^{(0)}|}\prod_{l=1}^{n}e^{D_l\beta^l}\times\left\{R[g^{(0)}]+g^{(0)\lambda\nu}\frac{\partial z^1}{\partial x^\lambda}\frac{\partial z^1}{\partial x^\nu}-\sum_{i=2}^{n}g^{(0)\lambda\nu}\frac{\partial z^i}{\partial x^\lambda}\frac{\partial z^i}{\partial x^\nu}+(e^{qz^1})^{-(2/D')}\sum_{i=1}^{n}R[g^{(i)}]e^{-2\Sigma_{k=2}^{n}U^i_kz^k}\right\}. \quad (2.25)$$

Let us define the BD field as

$$\phi:=e^{qz^1}=\prod_{l=1}^{n}e^{D_l\beta^l}\equiv v_{\text{int}}, \quad (2.26)$$

where

$$v_{\text{int}}:=V_{\text{int}}/\mu \quad (2.27)$$

is a scale which renormalizes the internal space volume $V_{\text{int}}:=\int_{M_1\times\cdots\times M_n}d^{D'}y\sqrt{|\det(g/g^{(0)})|}$. Its corresponding logarithmic scale factor is the dilaton field z^1. The derivative of the latter is

$$\frac{\partial}{\partial x^\mu}z^1=\frac{1}{q\phi}\partial_\mu\phi. \quad (2.28)$$

So, we can write the action (2.25) as

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det g^{(0)}|}\Bigg\{\phi R[g^{(0)}] -\omega g^{(0)\lambda\nu}\frac{1}{\phi}\frac{\partial\phi}{\partial x^\lambda}\frac{\partial\phi}{\partial x^\nu}-\phi\sum_{i=2}^{n} g^{(0)\lambda\nu}\frac{\partial z^i}{\partial x^\lambda}\frac{\partial z^i}{\partial x^\nu} +\phi^{1-(2/D')}\sum_{i=1}^{n} R[g^{(i)}]e^{-2\Sigma_{k=2}^{n}U_k^i z^k}\Bigg\}, \tag{2.29}$$

where now $\omega=\omega(D')=1/D'-1\equiv-4\xi_{c,D'+1}$ is the BD parameter, depending now on the total extra dimension D'. In the action (2.11) all scalar fields β^i, $i=1,\ldots,n$ couple to the curvature $R[g^{(0)}]$. After the diagonalization (2.18), only one of the scalar fields, namely, the BD field ϕ, is coupled to the curvature. In the action (2.29) scalar fields z^i play the role of normal scalar matter fields coupled to the dilaton BD scalar ϕ. Note that the kinetic terms for the fields z^i, $i=2,\ldots,n$, have the usual normal sign. In contrast to the action (2.15) with respect to its field ϕ, Eq. (2.29) contains *no* self-interaction terms for any of its fields ϕ and z^i, $i=2,\ldots,n$. Rather, it contains $\phi-z^i$ cross terms $(i=2,\ldots,n)$. These cross terms are, such as the fields z^i and ϕ themselves, of purely geometric nature. The exceptional case (2.15) corresponds formally to the case $z^k\equiv0$, $D_k\equiv0$ $(k=2,\ldots,n)$ of Eq. (2.29).

For $D_0\neq2$, the action (2.29) can be written in a σ-model representation [15]. We define a new metric $\hat{g}^{(0)}_{\mu\nu}$ which yields the so-called *Einstein conformal frame*, and new scalar fields φ^i $(i=1,\ldots,n)$ by

$$\hat{g}^{(0)}_{\mu\nu}=\phi^{(2/D_0-2)}g^{(0)}_{\mu\nu},\quad \varphi^1=-A\ln\phi,\quad \varphi^i=z^i,\quad i=2,\ldots,n, \tag{2.30}$$

where $A:=\pm[\omega(D')+(D_0-1)/(D_0-2)]^{1/2}$. Note that this transformation is regular for $\omega(D')\neq\omega_{c,D_0}$, where $\omega_{c,D_0}:=-(D_0-1)/(D_0-2)\equiv-\frac{1}{4}\xi^{-1}_{c,D_0}$ is the conformal parameter for dimension D_0. Taking into account that $-1<\omega(D')\leqslant0$ for $D'\geqslant1$ and $\omega(0)=\infty$, one obtains: If $D_0>2$, Eq. (2.30) is regular for any $D'>0$, with $A^2>0$. For $D_0=2$ or $D'=0$, Eq. (2.30) is singular. It is singular with $A^2=0$ if $(D_0,D')=(0,2)$. If $D_0=1$, Eq. (2.30) is singular for $D'=1$, but regular for any $D'>1$. In the latter case $A^2<0$, and a real redefinition of the complex field, $\varphi^1\to|\varphi^1|$, yields again a Minkowskian metric in the space of scalar fields.

For $D_0>2$, with the flat σ-model metric

$$d\sigma=\sigma_{ij}d\varphi^i\otimes d\varphi^j,\quad (\sigma_{ij})=\mathrm{diag}(+1,\ldots,+1), \tag{2.31}$$

where $i,j=1,\ldots,n$, and the potential

$$V(\varphi^i)=-e^{-(B/A)\varphi^1}\sum_{i=1}^{n}R[g^{(i)}]e^{-2\Sigma_{k=2}^{n}U_k^i\varphi^k}, \tag{2.32}$$

where $B:=1-2/D'-D_0/(D_0-2)$, the action (2.29) then reads

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det\hat{g}^{(0)}|}\times\Bigg\{\hat{R}[\hat{g}^{(0)}]-\sum_{i,j=1}^{n}\sigma_{ij}\hat{g}^{(0)\lambda\nu}\frac{\partial\varphi^i}{\partial x^\lambda}\frac{\partial\varphi^j}{\partial x^\nu}-V(\varphi^i)\Bigg\}. \tag{2.33}$$

Note that the σ-model metric (2.31) is flat such as the midisuperspace metric (2.17); however, while Eq. (2.17) is Minkowskian, Eq. (2.31) is Euclidean. So we found equivalent representations Eqs. (2.11) and (2.33) of the same action S, but with different signature in their respective space of scalar fields.

In the case $n=1$, with just one dilaton φ, the action (2.33) is equal to

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det\hat{g}^{(0)}|}\Bigg\{\hat{R}[\hat{g}^{(0)}] -\hat{g}^{(0)\mu\nu}\frac{\partial\varphi}{\partial x^\mu}\frac{\partial\varphi}{\partial x^\nu}+R[g^{(1)}]e^{-(B/A)\varphi}\Bigg\}. \tag{2.34}$$

This action can be written in the ''stringlike'' form (see e.g., [26–28] and references therein)

$$S=\frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|\det\hat{g}^{(0)}|}\Bigg\{\hat{R}[\hat{g}^{(0)}] -\hat{g}^{(0)\mu\nu}\frac{\partial\varphi}{\partial x^\mu}\frac{\partial\varphi}{\partial x^\nu}-2\Lambda e^{-2\lambda\varphi}\Bigg\}, \tag{2.35}$$

where the constants are fixed by the conditions

$$2\Lambda:=-R[g^{(1)}],\quad \lambda^2:=\lambda_c^2=\frac{D-2}{D_1(D_0-2)}. \tag{2.36}$$

In Eq. (2.35) λ is the dilatonic coupling constant. For $D_0=10$ and $\Lambda=0$ (e.g., for a Ricci flat internal space), this action describes the scalar-tensor (i.e., Yang-Mills-free) part of the bosonic sector from the ten-dimensional Einstein-Yang-Mills supergravity that occurs as low energy limit from superstring theory.

For arbitrary $\Lambda\neq0$, the action (2.35) corresponds to the scalar-tensor sector of an effective string action in dimension D_0, only if the dilatonic coupling is fixed to

$$\lambda^2:=\lambda_s^2=\frac{1}{D_0-2}. \tag{2.37}$$

The coupling (2.37) is obtained for our models with Eq. (2.36) only in the limit of infinite internal dimension:

$$\lambda_c^2\to\lambda_s^2=\frac{1}{D_0-2}\quad\text{for } D_1\to\infty. \tag{2.38}$$

Especially for the ten-dimensional effective action, the required value of $\lambda^2=\frac{1}{8}$ is obtained just in this limit, while, for $\Lambda=0$ above, the value of λ was completely arbitrary. Indeed, $\Lambda=0$ is a critical value for the string theories, whence $\Lambda\neq0$ occurs just for noncritical string theories.

The action (2.35) can equivalently be obtained from a multidimensional cosmological model with a usual cosmological term Λ, if the internal space M_1 is a Ricci flat Einstein space, i.e., $R[g^1]=0$. Then, the equivalence to our previous model is given by exchanging $D-1 \leftrightarrow 1-D_1$, which obviously leaves D_0 invariant. In this case $\lambda_c^2 = D_1/[(D-2)(D_0-2)]$, and the correspondence to noncritical string theories is again given in the limit (2.38).

Finally, note that for $D_0 \to 2$, both couplings λ_s^2 and λ_c^2 become asymptotically equal to $(D_0-2)^{-1}$. Hence, in the limit $D_0 \to 2$, our models become, independently from the internal dimension D_1, equivalent to effective low energy models of string theory, for any scalar curvature $-\Lambda/2$ of the internal space.

III. 2D DILATON GRAVITY FROM INHOMOGENEOUS COSMOLOGY

Let us now consider in more detail the dimensional reduction to a space-time of dimension $D_0 = 2$. In this case the conformal transformation (2.30) is singular, whence the model of Eq. (2.29) can not be expressed in a conformal Einstein-Pauli frame. This is not a fault of the theory, but rather corresponds to the well-known fact that two-dimensional Einstein equations are empty, i.e., they do not imply a dynamics [21,22]. Thus, we shall consider two-dimensional dilaton gravity only.

We start with the case with one dilaton, $n=1$. The action (2.13) can be written in the ''stringlike'' form [29–31]

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det g^{(0)}|}\, e^{-2\sigma}\left\{ R[g^{(0)}] + 4m g^{(0)\lambda\nu}\frac{\partial\sigma}{\partial x^\lambda}\frac{\partial\sigma}{\partial x^\nu} - 2\Lambda e^{-2[(1/k)+m]\sigma}\right\}, \quad (3.1)$$

where

$$\sigma := -\frac{1}{2}D_1\beta^1,$$

$$m := \frac{D_1-1}{D_1},$$

$$k := -\frac{D_1}{D_1+1},$$

$$2\Lambda := -R[g^{(1)}]. \quad (3.2)$$

By a conformal transformation of $g^{(0)}_{\mu\nu}$ to

$$\tilde{g}^{(0)}_{\mu\nu} = e^{-2m\sigma} g^{(0)}_{\mu\nu}, \quad (3.3)$$

we can formulate the action without kinetic dilation term, as

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det\tilde{g}^{(0)}|}\, e^{-2\sigma}\{\tilde{R}[\tilde{g}^{(0)}] - 2\Lambda e^{-(2/k)\sigma}\}. \quad (3.4)$$

The 2D actions (3.1) and (3.4) are invariant under homogeneous conformal transformations

$$\check{g}^{(0)}_{\mu\nu} := \Omega^{-2}\tilde{g}^{(0)}_{\mu\nu}, \quad \check{g}^{(1)}_{\mu\nu} := \Omega^{-2} g^{(1)}_{\mu\nu}, \quad (3.5)$$

where Ω is constant. Applying Eq. (3.5) with

$$\Omega^2 := -\frac{D_1}{(D_1+1)^{1+1/D_1}}\frac{1}{2\Lambda}$$

yields

$$2\check{\Lambda} := -\check{R}[\check{g}^{(1)}] = -\frac{D_1}{(D_1+1)^{1+1/D_1}} = \frac{k}{(k+1)^{1+1/k}} \quad (3.6)$$

and the action (3.4) now reads

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det\check{g}^{(0)}|}\, e^{-2\sigma}\{\check{R}[\check{g}^{(0)}] - 2\check{\Lambda} e^{-(2/k)\sigma}\}. \quad (3.7)$$

If we assume that the dilaton field is specifically given through the geometry on M_0 and the dimension D_1 of M_1, according to

$$e^{-2\sigma} := (k+1)(\check{R}[\check{g}^{(0)}])^k, \quad (3.8)$$

then the action (3.7) takes the form [23,30–32]

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det\check{g}^{(0)}|}(\check{R}[\check{g}^{(0)}])^{k+1} = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det\check{g}^{(0)}|}(\check{R}[\check{g}^{(0)}])^{1/(D_1+1)}. \quad (3.9)$$

In the general case of multiscalar fields, the kinetic term of the dilaton can be removed by an obvious analogous procedure. The ''stringlike'' form of the action (2.25) is

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det g^{(0)}|}\, e^{-2\sigma} \times\left\{ R[g^{(0)}] + 4m g^{(0)\lambda\nu}\frac{\partial\sigma}{\partial x^\lambda}\frac{\partial\sigma}{\partial x^\nu} - \sum_{i=2}^{n} g^{(0)\lambda\nu}\frac{\partial z^i}{\partial x^\lambda}\frac{\partial z^i}{\partial x^\nu} - e^{-2(1/k+m)\sigma}\sum_{i=1}^{n} 2\Lambda_i e^{-2\Sigma_{j=2}^{n} U_j^i z^j}\right\}, \quad (3.10)$$

where now

$$\sigma := -\frac{1}{2}qz^1,$$

$$m := \frac{1}{q^2} = \frac{D'-1}{D'},$$

$$k := -\frac{D'}{D'+1},$$

$$2\Lambda_i := -R[g^{(i)}]. \quad (3.11)$$

With Eq. (3.11), the conformal transformation (3.3) yields

$$S = \frac{1}{2\kappa_0^2}\int_{M_0} d^2x \sqrt{|\det\tilde{g}^{(0)}|}\, e^{-2\sigma} \times\left\{ \tilde{R}[\tilde{g}^{(0)}] - \sum_{i=2}^{n}\tilde{g}^{(0)\lambda\nu}\frac{\partial z^i}{\partial x^\lambda}\frac{\partial z^i}{\partial x^\nu} - e^{-(2/k)\sigma}\sum_{i=1}^{n} 2\Lambda_i e^{-2\Sigma_{j=2}^{n} U_j^i z^j}\right\}. \quad (3.12)$$

In Eq. (3.12) there is no kinetic term of the dilaton field. The kinetic terms of all extra scalar fields z^i have the normal sign. The extra fields z^i play the role of usual matter, coupling to the dilaton field σ.

IV. CONCLUSIONS AND DISCUSSION

We started from multidimensional cosmology. The corresponding metric is, from one side, a generalization of the Friedmann metric, which corresponds here to the special case where all $M_0, M_1, \ldots, M_n$ are spaces of constant curvature. From another side, our metric generalizes the anisotropic Kasner metric. In contrast to the (spatially) homogeneous Friedmann and Kasner metrics, our multidimensional metric is, in general, a (spatially) inhomogeneous one with scale factors depending on spatial coordinates of M_0. We obtained effective BD formulations for multidimensional models via dimensional reduction on M_0. Self-interaction terms appear exclusively in the degenerate case (2.15) where there is only one scalar field. For $n \geqslant 2$ scalar fields, the BD-like effective action (2.29) contains $\phi - z^i$ cross terms, between the BD field ϕ and the other scalar fields z^i, $i=2, \ldots, n$, instead.

In the case of only one internal space M_1, the actions obtained after dimensional reduction of the multidimensional Einstein-Hilbert one may be written in stringlike form. Thus, the associated field equations have the same form as for the (scalar) bosonic sector of the superstring theory in the low energy limit. The corresponding effective models of string theory are obtained from our models in the limit of infinite internal dimension $D_1 \rightarrow \infty$.

The BD-like effective action (2.29), which has a Minkowskian metric in the space of scalar fields, is (with few exceptional cases) equivalent to a conformal Einstein σ-model action (2.33), which has an Euclidean metric in the space of scalar fields. The case of a one-dimensional Universe is exceptional: There, the metric in the space of (real) scalar fields of a conformal Einstein σ-model is also Minkowskian.

With the effective dimension D_0 of the Universe and the total extra dimension D', the singular cases of the conformal transformation are given by $D_0 = 2$ or $D' = 0$, where Eq. (2.30) is undefined, or by $D = D_0 + D' = 2$, where A in Eq. (2.30) is zero. In these exceptional cases our model is not conformal to an Einsteinian one. However, one should also keep in mind that Einstein equations in a two-dimensional space-time do not imply any dynamics (see [21,22]). For a space-time with $D_0 = 2$, the dimensional reduction of the multidimensional model can be written as a ''stringlike'' dilaton gravity, representable in the form (3.12), where the dilaton appears without kinetic term, and all extra fields couple to the dilaton with normal signs of their kinetic terms. If there are no fields besides the dilaton, then the action can be represented in the form (3.9) (see also [23,30–32]), which has a nontrivial variation only for nonvanishing extra dimension $D_1 > 0$.

A conformal equivalence transformation between two scalar-tensor Lagrangian models becoming singular at specific parameters (here given by the exceptional dimensions) is a familiar effect. Such singularities yielding inequivalent models were also discussed in [33].

Although, in the exceptional dimensions, the models (2.29) and (2.33) are mathematically inequivalent, the question remains, as for all other dimensions, which model is the physical one. The difference in the exceptional cases is that, in principle, this question could be decided by experiments on a *classical* level. For the dimension $D_0 > 2$, the two models are mathematically equivalent; so on the classical level it cannot be decided which is the physical one. However, if one demands that the gravitational interaction is generated by a pure massless spin-two graviton (without scalar spin-zero admixture), then, reasoning similar as in [34], Eq. (2.33) rather than Eq. (2.29) has to be taken as the physical model.

Taking into account the conformal relation of scalar-tensor theories to fourth-(or higher-) order gravity (see, e.g., [35]), the recent debate on the physical metric [36–39] concerns also the corresponding scalar-tensor theories. The result of this purely classical debate was rather poor: It mainly confirms Brans [40], who pointed out that, once the weak equivalence principle holds true in a given frame (in [40] it is the frame of the original higher-order gravity), it will be violated in any nontrivially conformally related frame. However, the choice of the frame with respect to which a test particle of ordinary matter moves along geodesics remains arbitrary for classical scalar-tensor theories. So, the frame of Brans and Dicke [3] might be the physical one, giving geodesic paths for minimally coupled test matter, or likewise the Einstein-Pauli frame might be the physical one. In [41], Hawking argued that black holes might follow geodesics in the Einstein-Pauli frame but violate the *strong* equivalence principle in the BD frame, while the latter provides geodesic paths for usual test matter. For massive objects such as black holes, this phenomenon is known as the Nordtvedt effect [42]. Furthermore, Cho [43] showed that in the BD frame, quantum corrections enforce also a violation of the *weak* equivalence principle. We believe, therefore, that the issue of the physical frame will be resolved finally only by a quantum theory of gravity. Since such a theory might not be subject to any equivalence principle, the latter might no longer serve as the guiding principle for the physical metric. However, generalized arguments of Cho [34,43] give some hint that the Einstein-Pauli frame (when quantum corrections are small enough not to destroy any frame at all) might then, nevertheless, be taken as the physical one.

It should, however, be noted that our multiscalar-tensor theories differ essentially from usual scalar-tensor theories: There, some ''ordinary'' matter field is minimally coupled to the geometry, either in the Jordan-Brans-Dicke frame or, equally well, in the Einstein-Pauli frame (see also [35]). We saw above that, arguing on the basis of a classical equivalence principle for the ordinary matter only, there is no way to select the physical frame. However, in our models all scalar fields are derived from a multidimensional geometry, which determines *all* couplings of *all* scalar fields to the geometry and among one another. These couplings can be tested, in principle, by experiments, thus selecting the physically admissible multiscalar-tensor theories and their corresponding multidimensional counterparts. Because of this predictive power, it is tempting to postulate that any multiscalar-tensor model should derive its (scalar) fields from a higher-dimensional geometry, i.e., all (scalar) matter should have some geometric origin.

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