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Cosmic vortons and particle physics constraints

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We investigate the cosmological consequences of particle physics theories that admit stable loops of superconducting cosmic string—*vortons*. General symmetry-breaking schemes are considered, in which strings are formed at one energy scale and subsequently become superconducting in a secondary phase transition at what may be a considerably lower energy scale. We estimate the abundances of the ensuing vortons, and thereby derive constraints on the relevant particle physics models from cosmological observations. For a range of values of the dimensionless parameters in the analysis, these constraints significantly restrict the category of admissible grand unified theories. However, we demonstrate that the constraints are quite compatible with recently proposed effects whereby superconducting strings may have been formed close to the electroweak phase transition. [S0556-2821(96)00922-8]

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I. INTRODUCTION

In the past few years it has become clear that topological defects produced in the early Universe may have a considerably richer microstructure than what had previously been imagined [1]. In particular, the core of a defect acquires additional features at each subsequent symmetry breaking which preserves the topology of the object. The new microphysics associated with additional core structure has been exploited by several authors to provide a new, defect-based scenario for electroweak baryogenesis [2,3].

The purpose of the present paper is to constrain general particle physics theories by demanding that the microphysics of defects in these models be consistent with the requirements of the standard cosmology. The basic idea, due originally to Davis and Shellard [4–6], is as follows. If a spontaneously broken field theory admits linear topological defects, *cosmic strings*, which subsequently become superconducting, then an initially weak current on a closed string loop will automatically tend to amplify as the loop undergoes dissipative contraction. This current may become sufficiently strong to modify the dynamics and halt the contraction so that the loop settles down in an equilibrium state known as a *vorton*.

The population of vorton states produced by such a mechanism is tightly constrained by empirical cosmological considerations. It was first pointed out by Davis and Shellard that to avoid obtaining a present-day cosmological closure factor Ω greatly exceeding unity, any theory giving rise to stable vorton creation by superconductivity that sets in during string formation is ruled out if the symmetry-breaking scale is above some critical value. One of the first attempts to estimate this critical scale [7] indicated that it probably could not exceed that of electroweak symmetry breaking at about 10^2 GeV by more than a few orders of magnitude. Such strong limits are, of course, dependent on the supposition that the vortons are absolutely stable on time scales as long as the present age of the Universe. However, even if the vortons only survive for a few minutes, this would be sufficient to significantly affect primordial nucleosynthesis and hence provide limits of a weaker but nevertheless still interesting kind [8].

In all previous works it was supposed that the relevant superconductivity sets in during or very soon after the primary phase transition in which the strings are formed. What is new in the present work is an examination of the extent to which the limits discussed above are weakened if it is supposed that superconductivity sets in during a distinct secondary phase transition occurring at what may be a very much lower temperature than the string formation scale [9].

The structure of the paper is as follows. In Sec. II A we shall, for completeness, give a brief introductory review of string superconductivity. In Sec. II B we describe the mechanism of formation of a vorton from an originally distended string loop and in Sec. II C we summarize the basic properties of vorton equilibrium states. In Sec. III A we first comment on how it can be that the formation scale and the superconductivity scale be separated by many orders of magnitude. We then demonstrate, using a suitably simplified statistical description of the string network, how to estimate the vorton abundance for a generic theory as a function of the temperature and the symmetry-breaking scales. In Sec. III B we apply this procedure to the relatively simple case when the superconductivity develops during the early period when dissipation is mainly due to the friction of the ambient medium. In Sec. III C we go on to treat the more compli-

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cated situation that arises if the superconductivity does not develop until the much later stage in which dissipation is mainly due to gravitational radiation, and in Sec. III D we briefly comment on stability issues. In Sec. IV we consider the comparatively weak bounds that are obtained if it is supposed that the vortons are stable only for a few minutes. Finally, in Sec. V we consider rather the stronger bounds that are obtained if the vortons are of a kind that is sufficiently stable to survive as a constituent of the dark matter in the Universe at the present epoch. We conclude in Sec. VI.

II. CONSEQUENCES OF COSMIC STRING SUPERCONDUCTIVITY

A. Currents in the Witten model

In so far as it has been developed at the present time, the quantitative theory of vorton structure has been entirely based on the supposition that the essential features are describable in terms of a simple bosonic superconductivity model of the kind introduced by Witten [10]. This category of models consists of spontaneously broken gauged $U(1)\times U(1)$ field theories, which generalize the even simpler category of spontaneously broken gauged U(1) field theories on which the standard Kibble description of nonsuperconducting cosmic strings is based.

The Kibble model is characterized by a potential V with a quartic dependence on a complex Higgs field ϕ of the familiar form $\tilde{\lambda}(|\phi|^2-v^2)^2$. Here, $\tilde{\lambda}$ is a dimensionless coupling and v is a mass scale of order the ''Kibble mass'' m_x, whose square is identifiable with the string tension $\mathcal{T}$, which is (in this model) constant and equal to the mass per unit length. The string is defined as the region in which $|\phi|$ is topologically excluded from its vacuum value as given by $v \simeq m_x$.

In addition to ϕ, the Witten model contains a second complex scalar field σ and is characterized by a quartic potential depending on several dimensionless parameters that, such as $\tilde{\lambda}$, are assumed to be of order unity. Witten's potential function also depends on a second mass parameter, say m_σ, which determines the temperature scale below which σ gives rise to a current carrying condensate on the vortex. In the Witten model the vortex defects are cosmic strings in which the tension $\mathcal{T}$ is no longer constant but variable, as a function of the current magnitude $|j|$, attaining its maximum (Kibble) value only when the current magnitude vanishes, so that in general one has $\mathcal{T} \leq m_x^2$.

In earlier discussions [4–8] of vorton physics it was implicitly or explicitly supposed that the magnitude of m_σ was not very different from that of the original symmetry-breaking mass parameter m_x. In that case, the formation of the σ condensate could be considered as a part of the same symmetry-breaking phase transition as that by which the strings themselves were formed. Our purpose here is to consider scenarios in which m_σ may be very much smaller than m_x, as will occur when successive phase transitions at two entirely distinct cosmological epochs are involved [9]. It is only after the second phase transition that a condensate with amplitude $|\sigma|$ and angular variable phase, say θ, will form on the string world sheet. There then exists an identically conserved world sheet phase current with components

$$\tilde{j}^a = \frac{1}{2\pi}\varepsilon^{ab}\partial_b\theta, \tag{1}$$

where ε^{ab} are the components of the antisymmetric unit surface element tensor induced on the two-dimensional world sheet. As well as this topologically conserved phase current, there will also be a dynamically conserved particle number current with components given [6] by

$$j_a = 2\tilde{\Sigma}(\partial_a\theta - eA_a), \tag{2}$$

where $\tilde{\Sigma}$ is the surface integral of $|\sigma|^2$ over the vortex core cross section. Here, A_a are the induced components of the electomagnetic background potential and e is the coupling constant associated with the carrier field if it is gauged. When e is nonzero, $\tilde{j}^a$ will be gauge dependent, but j^a is physically well defined and will determine a corresponding electric surface current given by

$$I^a = e j^a. \tag{3}$$

B. Formation of vorton states

In the case of a closed string loop the conserved surface currents characterized above will determine a corresponding pair of integral quantum numbers that are expressible in terms of circuit integration round the loop. These are given by

$$N = \oint \tilde{j}^a d\ell_a, \quad Z = \oint j^a d\ell_a, \tag{4}$$

where $d\ell_a$ are the components of the length element normal to the circuit in the world sheet. Note that even when $\tilde{j}^a$ is gauge dependent, N is well defined. A nonconducting Kibble-type string loop must ultimately decay by radiative and frictional drag processes until it disappears completely. However, since a Witten-type conducting string loop is characterized by the classically conserved quantum numbers N and Z, such a loop may be saved from disappearance by reaching a state in which the energy attains a minimum for given nonzero values of these numbers.

In view of a widespread misunderstanding about this point, it is to be emphasized that the existence of such energy minimizing ''vorton'' states does not require that the carrier field be gauge coupled. If there is indeed a nonvanishing charge coupling then the loop will, of course, be characterized by a corresponding total electric charge

$$Q = \oint I^a d\ell_a, \tag{5}$$

in terms of which the particle number will be expressible directly as $Z = Q/e$. However, the important point is that even in the uncoupled case, for which I^a, and hence also Q, vanish, the quantum number Z will nevertheless remain perfectly well defined.

Although the essential physical properties of a vorton state will be fully determined by the specification of the relevant pair of integers N and Z, it is not true that any arbitrary choice of these two numbers will characterize a viable vorton configuration. This is because the requirement that the strictly classical string description should remain valid turns out to be rather restrictive. To start with, it is evident that to avoid decaying completely like a nonconducting loop, a conducting loop must have a nonzero value for at least one of the numbers N and Z. In fact, one would expect that both these numbers should be reasonably large compared with unity to diminish the likelihood of quantum decay by barrier tunneling. However, even for moderately large values of N and Z there will be further restrictions on the admissible values of their ratio Z/N due to the necessity of avoiding spontaneous particle emission as a result of current saturation.

The existence of a maximum amplitude for the string current was originally predicted by Witten himself [10]. However, quantitative knowledge about the current saturation phenomenon remained undeveloped until the appearance of an important pioneering investigation by Babul, Piran, and Spergel [11] who undertook a detailed numerical analysis of the mechanism whereby the presence of a current tends to diminish the string tension $\mathcal{T}$. In a nonconducting string, the tension $\mathcal{T}$ is identifiable with the energy per unit length $\mathcal{E}$, but in the conducting case the diminution of $\mathcal{T}$ is accompanied by an augmentation of $\mathcal{E}$ such that

$$\mathcal{T} \leqslant m_{\mathrm{x}}^2 \leqslant \mathcal{E}. \tag{6}$$

The analysis of [11] provided an empirical ‘‘equation of state’’ specifying $\mathcal{T}$ as a nonlinear function of the current magnitude $|j|$ and hence of the energy density $\mathcal{E}$. The minimum of $\mathcal{T}$, and hence the maximum allowed value for $\mathcal{E}$, is obtained when the current amplitude $|j|$ reaches a critical saturation value with order of magnitude

$$|j|^2 \approx \mathcal{E} - \mathcal{T} \lesssim m_\sigma^2. \tag{7}$$

Such knowledge of the equation of state is precisely what is required for investigating string dynamics.

We now apply this to vorton equilibrium states. The phase angle θ is expressible in terms of the background time coordinate t and a coordinate ℓ representing arc length round the loop by

$$\theta = \omega t - k\ell, \tag{8}$$

where ω and k are constants. If the total circumference of the vorton configuration is denoted by ℓ_v then it can be seen that these constants will be given in terms of the corresponding quantum numbers by

$$\omega = \frac{Z}{2\tilde{\Sigma}\ell_{\mathrm{v}}}, \quad k = \frac{2\pi N}{\ell_{\mathrm{v}}}. \tag{9}$$

The general condition [13] for purely centrifugally supported equilibrium is that the rotation velocity should be given by a formula of the same simple form as the one [12] for the speed of extrinsic wiggle propagation: namely,

$$v^2 = \frac{\mathcal{T}}{\mathcal{E}}. \tag{10}$$

Note that this relationship applies when the electrical coupling is absent or, as will commonly be the case, negligible because of the smallness of the fine structure constant [14].

C. Properties of vortons

The random distribution of initial values of the quantum numbers N and Z leads to the formation of a range of qualitatively different kinds of vorton states. We shall now briefly review the reasons why it is to be expected that initially the most numerous will be of approximately *chiral*-type, meaning that their rotation velocity is comparable with the speed of light as given by $v=1$. Note, however, that it may be that vortons of the very different *subsonic*-type, with $v \ll 1$, will be favored by natural selection in the long run.

In the case of a *spacelike* current, with $\omega^2 < k^2$ (the only possibility envisaged in [11]), the velocity given by Eq. (10) is to be interpreted simply as the *phase speed*:

$$v = \frac{\omega}{k} = \frac{\pi Z}{\tilde{\Sigma} N}. \tag{11}$$

Since we are assuming that m_σ is small compared with m_{x}, the saturation limit (7) implies

$$\frac{\mathcal{E} - \mathcal{T}}{\mathcal{T}} \ll 1, \tag{12}$$

which obviously, by Eq. (10), implies the property of approximate chirality, $v \simeq 1$. It is expected on dimensional grounds that, although the sectional integral $\tilde{\Sigma}$ is a function of the current, it will not get extremely far from the order unity. This behavior has been observed numerically in particular cases [11,15] (see also [16–18]). It can, therefore, be deduced that approximate chirality requires the vorton to be characterized by a pair of quantum numbers having roughly comparable orders of magnitude,

$$|Z| \approx N, \tag{13}$$

where we use an orientation convention such that N is always positive.

On purely statistical grounds one would expect that the two quantum numbers would most commonly be formed with comparable order of magnitude, so as to satisfy Eq. (13), though not necessarily with an exact ratio small enough to give a spacelike vorton current. In fact, although the limit (7) definitely excludes the possibility of vorton states with $N \gg |Z|$, it turns out that there is nothing to prevent the existence of nonchiral vortons having a current that is not just marginally timelike ($\omega^2 > k^2$), but which have $|Z| \gg N$. In the *timelike* case the velocity v in Eq. (10) is not the phase speed but the *current velocity* given by

$$v = k/\omega. \tag{14}$$

A preliminary application [19] of the principles described in the following sections suggests that subsonic vortons, which will necessarily be characterized by $|Z| \gg N$, will initially be formed in much smaller numbers than those of the more familiar chiral variety described by Eq. (13). This means that if ordinary chiral vortons are sufficiently stable to survive over cosmologically significant time scales (a question that must be left open for future research) then they can provide us with much stronger constraints on admissible particle theories than the more exotic subsonic variety. In order to keep our discussion as clear and simple as possible, we shall, therefore, say no more about subsonic vortons and restrict our attention to the chiral variety.

Chiral vortons are only marginally affected by the electromagnetic coupling e [14]. Therefore, they can be described by the simplest kind of elastic string formalism in which electromagnetic effects are ignored altogether. This means [13] that the mass energy $E_{\rm v}$ of such a vorton will be given in terms of its circumference $\ell_{\rm v}$ by

$$E_{\rm v}=\ell_{\rm v}(\mathcal{E}+\mathcal{T})\simeq\ell_{\rm v}m_{\rm x}^2. \tag{15}$$

In order to evaluate this quantity all that remains is to work out $\ell_{\rm v}$. In the earliest work on vorton states it was always presumed that they would be circular, with radius, therefore, given by $R_{\rm v}=\ell_{\rm v}/2\pi$ and angular momentum quantum number J given [13] by $J=NZ$ or equivalently by $J^2=\mathcal{E}\mathcal{T}\ell_{\rm v}^4/4\pi^2$. Thus, eliminating J, one obtains the required result as

$$\ell_{\rm v}=(2\pi)^{1/2}|NZ|^{1/2}(\mathcal{E}\mathcal{T})^{-1/4}\simeq(2\pi)^{1/2}|NZ|^{1/2}m_{\rm x}^{-1}. \tag{16}$$

More recent work [20] has established that even in cases where the vorton configuration is strongly distorted, the expression (16) will remain perfectly valid. Combining this with Eq. (15), and recalling that for chiral vortons $|Z|\approx N$, we thus obtain a final estimate of the vorton mass energy as

$$E_{\rm v}\simeq(2\pi)^{1/2}|NZ|^{1/2}m_{\rm x}\approx Nm_{\rm x}. \tag{17}$$

The preceding formulas are based on a classical description of the string dynamics. This is valid only if the length $\ell_{\rm v}$ is large compared with the relevant quantum wavelengths, of which the longest is the Compton wavelength associated with the carrier mass m_σ. It can be seen from Eq. (16) that this condition, namely,

$$\ell_{\rm v}\gg m_\sigma^{-1}, \tag{18}$$

will only be satisfied if the product of the quantum numbers N and Z is sufficiently large. A loop that does not satisfy this requirement will never stabilize as a vorton. After its length has been reduced to the order of magnitude (18) by a classical contraction process, it will presumably undergo a rapid quantum decay whereby it will finally disappear completely just as if there were no current.

III. THE VORTON ABUNDANCE

A. Basic postulates: A scheme based on two mass scales

The present analysis will be carried out within the framework of the usual FRW model in which the Universe evolves in approximate thermal equilibrium with a cosmological background temperature T. The effective number of massless degrees of freedom at temperature T is denoted by g^*. Note that $g^*\approx 1$ at low temperatures but that in the range where vorton production is likely to occur, from the electroweak scale through to grand unification, $g^*\simeq 10^2$ is a reasonable estimate.

Any vorton formation process must occur during the radiation-dominated era which ended when the temperature of the Universe dropped below 10^{-2} GeV and became effectively transparent. The relevant cosmological quantities are the age of the Universe, given by $t\approx H^{-1}$ where H is the Hubble parameter, and the radiation-dominated, time-temperature relationship

$$t\approx\frac{m_{\rm P}}{\sqrt{g^*}T^2}, \tag{19}$$

where $m_{\rm P}$ is the Planck mass.

During this cosmological evolution, the particle physics gauge group is assumed to undergo a series of successive spontaneous symmetry-breaking phase transitions, expressible schematically as

$$\mathcal{G}_{\rm GUT}\mapsto\cdots\mathcal{H}\cdots\mapsto\mathcal{G}_{\rm EW}\mapsto{\rm SU(3)}\times{\rm U(1)}. \tag{20}$$

Here, $\mathcal{G}_{\rm GUT}$ is the "grand unified" group, $\mathcal{H}$ is some hypothetical intermediate symmetry group (such as that of the axion phase), and $\mathcal{G}_{\rm EW}$ is the standard model group ${\rm SU(3)}\times{\rm SU(2)}\times{\rm U(1)}$ or one of its nonstandard (e.g., supersymmetric) extensions. The role of the Witten model is to provide an approximate description of an evolution process dominated by two distinct steps in this chain.

When a semisimple symmetry group, say $\mathcal{G}$, is broken down to a subgroup, say $\mathcal{H}$, the topological criterion for cosmic string formation is that the first homotopy group of the quotient should be nontrivial:

$$\pi_1\{\mathcal{G}/\mathcal{H}\}\neq 1. \tag{21}$$

Our primary supposition is that such a process occurs at some particular cosmological temperature $T_{\rm x}$, which we assume to be of the same order of magnitude as the relevant Kibble mass scale $m_{\rm x}$. This mass scale is interpretable as being of the order of the mass of the Higgs particle responsible for the symmetry breaking according to the simple model discussed in the previous section.

Our next basic postulate is that a current-carrying field, characterized by the independent mass scale m_σ, condenses on the ensuing string defect at a subsequent stage, when the background temperature has dropped to a lower value T_σ, which we assume to have the same order of magnitude as the mass scale m_σ.

The formation of a condensate with finite amplitude characterized by the dimensionless sectional integral $\tilde{\Sigma}$ does not in itself imply a nonzero expectation value for the corresponding local current vector j. However, one expects that thermal fluctuations will give rise to a nonzero value for its squared magnitude $|j|^2$ and hence a random walk process will result in a spectrum of finite values for the corresponding string loop quantum numbers N and Z. Therefore, in the long run, those loops for which these numbers satisfy the minimum length condition (18), are predestined to become stationary vortons, provided of course that the quantum numbers are strictly conserved during the subsequent motion, a requirement whose validity depends on the condition that string crossing processes later on are statistically negligible. We describe these loops as *protovortons*.

Note that the protovortons will not become vortons in the strict sense until a lower temperature, the vorton relaxation temperature, say $T_{\rm r}$ (whose value will not be relevant for our present purpose), since the loops must first lose their excess energy. Whereas frictional drag and electromagnetic radiation losses will commonly ensure rapid relaxation, there may be cases in which the only losses are due to the much weaker mechanism of gravitational radiation.

As the string network evolves, the distribution rarifies due to damping out of its fine structure first by friction and later by radiation reaction. However, not all of its lost energy goes directly into the corresponding frictional heating of the background or emitted radiation. There will always be a certain fraction, say ε, that goes into loops which evolve without subsequent collisions with the main string distribution. It is this process that provides the raw material for vorton production. Such loops will ultimately be able to survive as vortons if the current induced by random fluctuations during the carrier condensation process is sufficient for the condition (16) to be satisfied, i.e., provided its winding number and particle number are large enough to satisfy

$$|NZ|^{1/2} \gg \frac{T_{\rm x}}{T_\sigma}. \tag{22}$$

Any loop that fails to satisfy this condition is doomed to lose all its energy and disappear.

In favorable circumstances (namely, those considered in Sec. III B), most of the loops that emerge in this way at T_σ will satisfy the condition (22) and thus be describable as protovortons. However, in other cases (namely, those considered in Sec. III C), the majority of the loops that emerge during the period immediately following the carrier condensation will be too small to have acquired sufficiently large quantum numbers by this stochastic mechanism. These loops will, therefore, not be viable in the long run and are classified as *doomed loops*. Nevertheless, even in such unfavorable circumstances, the monotonic increase of the damping length $L_{\rm min}$ will ensure that at a lower temperature $T_{\rm f} < T_\sigma$ a later, and less prolific, generation of emerging loops will after all be able to qualify as protovortons. We refer to $T_{\rm f}$ as the protovorton formation temperature.

The scenario summarized above is based on the accepted understanding of the Kibble mechanism [6], according to which, after the temperature has dropped below $T_{\rm x}$, the effect of various damping mechanisms will remove most of the structure below an effective smoothing length $L_{\rm min}$, which will increase monotonically as a function of time, so that nearly all the surviving loops will have a length $L = \oint d\ell$ that satisfies the inequality

$$L \gtrsim L_{\rm min}. \tag{23}$$

There will thus be a distribution of string loops, of which the most numerous will be relatively short ones, with $L \approx L_{\rm min}$, that are on the verge of emerging, or that have already emerged, as protovortons or doomed loops as the case may be.

Whereas on larger scales closed loops and wiggles on very long string segments will be tangled together, on the shortest scales, characterized by the lower cutoff $L_{\rm min}$, loops will be of a relatively smooth form. It is these smallest loops that are candidates for subsequent transformation into vortons.

The total number density of small loops with length and radial extension of the order of $L_{\rm min}$ will (due to the rapid falloff of the spectrum that is expected for larger scales) be not much less than the number density of all closed loops and so will be given by an expression of the form

$$n \approx \nu L_{\rm min}^{-3}, \tag{24}$$

where ν is a time-dependent parameter which we will discuss later.

The theory reviewed above was originally developed on the assumption that the string evolution is governed by Goto-Nambu-type dynamics. In the kind of scenario we are considering, this condition will obviously be satisfied as long as the cosmological temperature T is greater than or comparable to the carrier condensation temperature T_σ. Moreover, the usual Goto-Nambu–type description, and its consequences as described above, will remain valid for a while after the strings have become superconducting since the currents will initially be too weak to have any significant dynamical effect. The Goto-Nambu theory inevitably breaks down at some temperature above $T_{\rm r}$. However, there may be cases for which such a description will break down even before the protovorton formation temperature $T_{\rm f}$ is reached.

The typical length scale of string loops at the transition temperature $L_{\rm min}(T_\sigma)$ is considerably greater than relevant thermal correlation length T_σ^{-1}, that will presumably characterize the local current fluctuations at that time. It is because of this that string loop evolution is modified after current carrier condensation. The inequality

$$L_{\rm min}(T_\sigma) \gg T_\sigma^{-1}, \tag{25}$$

and the fact that, by Eq. (23), the length of any loop present at the time of the condensation will satisfy $L \gtrsim L_{\rm min}(T_\sigma)$, means that the random walk effect can build up reasonably large, and typically comparable initial values of the quantum numbers $|Z|$ and N. The reason is that for a loop of length L, the expected root mean square values produced in this way from carrier field fluctuations of wavelength λ can be estimated as

$$|Z| \approx N \approx \sqrt{\frac{L}{\lambda}}. \tag{26}$$

At the time of the condensation, a typical loop is characterized by

$$L \approx L_{\rm min}(T_\sigma) \tag{27}$$

and

$$\lambda \approx T_\sigma^{-1} \tag{28}$$

so that one obtains the estimate

$$|Z| \approx N \approx \sqrt{L_{\rm min}(T_\sigma) T_\sigma}, \tag{29}$$

which, by Eq. (25), is large compared with unity.

For current condensation during the friction-dominated regime discussed in Sec. III B, we shall see that this will always be sufficient to satisfy the requirement Eq. (22). However, this condition will not hold for condensation later on in the radiation damping regime discussed in Sec. III C. In the latter case, typical small loops that free themselves from the main string distribution at or soon after the time of current condensation will be doomed loops since they do not satisfy Eq. (22). However, there will always be a minority of longer loops for which Eq. (22) will be satisfied, namely, those exceeding a minimum length given, according to Eq. (29), by

$$L \approx \frac{T_{\mathrm{x}}^{2}}{T_\sigma^3}. \tag{30}$$

This condition is still not quite sufficient to qualify them as protovortons since such exceptionally long loops will be very wiggly and collision prone. It is not until a later time at a lower temperature T_{f} that free protovorton loops will emerge. In this case, the typical wavelength of the carrier field will be given by Eq. (28) and the final value of the length of a typical loop is

$$L \approx L_{\mathrm{min}}(T_{\mathrm{f}}). \tag{31}$$

The new estimate for the values of the quantum numbers is

$$|Z| \approx N \approx \sqrt{\frac{L_{\mathrm{min}}(T_{\mathrm{f}})T_\sigma}{\mathcal{Z}_{\mathrm{f}}}}, \tag{32}$$

where we have included a blueshift factor $\mathcal{Z}_{\mathrm{f}}$, whose value is not immediately obvious but that is needed to allow for the net effect on the string of weak stretching due to the cosmological expansion and stronger shrinking due to wiggle damping during the period as the temperature cools from T_σ to T_{f}. In the earlier friction-dominated regime T_{f} is identifiable with T_σ so the problem does not arise, and in the radiation damping era cosmological stretching will in fact be negligible. Therefore, the net effect is that $\mathcal{Z}_{\mathrm{f}}$ will be small compared with unity, the hard part of the problem being to estimate how much so.

The value given by Eq. (32) will increase monotonically as T_{f} diminishes. The required value of T_{f}, at which the formation of the protovorton loops will actually occur, is that for which the function in Eq. (32) reaches the minimum qualifying value given by Eq. (22). This value is thus obtainable in principle by solving the equation

$$\frac{L_{\mathrm{min}}(T_{\mathrm{f}})}{\mathcal{Z}_{\mathrm{f}}} \approx \frac{T_{\mathrm{x}}^2}{T_\sigma^3}, \tag{33}$$

but this can only be done in practice when we have found the T_{f} dependence of $L_{\mathrm{min}}(T_{\mathrm{f}})$ and $\mathcal{Z}_{\mathrm{f}}$. We discuss the T_{f} dependence of $\mathcal{Z}_{\mathrm{f}}$ shortly.

The number density of protovorton loops at the temperature T_{f} will be comparable to the total loop number density at the time, so that by Eq. (24) it will be expressible as

$$n_{\mathrm{f}} \approx \varepsilon \nu_{\mathrm{f}} L_{\mathrm{min}}(T_{\mathrm{f}})^{-3}, \tag{34}$$

where ε is an efficiency factor of order unity, and ν_{f} is the value of the dimensionless parameter ν at that time. If the current condenses in the friction-dominated regime ν_{f} will simply have an order of unity value. However, if the condensation does not occur until later on, in the radiation-dominated era, ν_{f} will have a lower value which is not so easy to evaluate.

The number of protovorton loops in a comoving volume will be approximately conserved during their subsequent evolution. Therefore, since volumes will scale proportionally to the inverse of the entropy density it follows that the number density n_v of the resulting vortons at a lower temperature T will be given in terms of the number density n_{f} of the protovorton loops at the time of condensation by

$$\frac{n_v}{n_{\mathrm{f}}} \approx f\left(\frac{T}{T_{\mathrm{f}}}\right)^3. \tag{35}$$

Here, f is a dimensionless adjustment factor that we expect to be small but not very small compared with unity, and that will be given by

$$f \simeq \frac{\varepsilon g^*}{g_{\mathrm{f}}^*}, \tag{36}$$

where g_{f}^* is the value of g^* at the protovorton formation temperature T_{f}.

Using Eq. (17) the corresponding mass density will be given by

$$\rho_v \approx N m_{\mathrm{x}} n_v. \tag{37}$$

Thus, the mass density of the distribution of the protovortons in the range $T_{\mathrm{f}} \gtrsim T \gtrsim T_{\mathrm{r}}$, and of the mature vortons after their formation in the range $T \lesssim T_{\mathrm{r}}$, is given by the general formula

$$\rho_v \approx f \nu_{\mathrm{f}} \frac{T_{\mathrm{x}} T_\sigma^{1/2}}{\mathcal{Z}_{\mathrm{f}}^{1/2} L_{\mathrm{min}}(T_{\mathrm{f}})^{5/2}} \left(\frac{T}{T_{\mathrm{f}}}\right)^3. \tag{38}$$

When the dependences of ν_{f}, $L_{\mathrm{min}}(T_{\mathrm{f}})$, and T_{f} on the fundamental parameters T_{x} and T_σ are known, the formula (38) will allow us to place limits on T_σ by determining how the presence of the corresponding population of remnant vortons would affect the course of cosmic evolution. In the following sections we shall derive several constraints on such a population by demanding that it not significantly interfere with the cornerstones of the standard cosmology. However, before we can do so it remains to obtain at least rough estimates of the required values of the dependent variables. This turns out to be fairly easy in the case of condensation during the friction-dominated era that will be discussed in the next section. However, the derivation of firm conclusions is less straightforward for the kind of scenario discussed in Sec. III C, in which the current condensation occurs in the radiation-dominated regime.

B. Condensation in the friction damping regime

According to the standard picture [21], the evolution of a cosmic string network is initially dominated by the frictional drag of the thermal background. The relevant dynamical damping time scale τ during this period is approximately given by

$$\tau \approx \frac{T_x^2}{\beta T^3}, \tag{39}$$

where β is a dimensionless drag coefficient that depends on the details of the underlying field theory but that is typically expected [21,22] to be of order unity. In this regime the large scale structure is frozen and retains the Brownian random walk form (24). However, the microstructure is smoothed out below a correlation length $L_{\rm min}$ given by

$$L_{\rm min} \approx \sqrt{\tau t}, \tag{40}$$

where t is the Hubble time. Neglecting the very weak g^* dependence, the required correlation length is thus found to be given by

$$L_{\rm min} \approx \left(\frac{m_{\rm P}}{\beta}\right)^{1/2} \frac{T_x}{T^{5/2}}. \tag{41}$$

The friction-dominated regime continues until the temperature T drops below a critical value $T_\star$ given by

$$T_\star \approx \frac{T_x^2}{\beta m_{\rm P}}, \tag{42}$$

at which τ is comparable to t.

Setting T equal to T_σ in Eq. (41) in order to obtain the relevant value of $L_{\rm min}(T_\sigma)$, and using Eqs. (29) and (27), the required expectation value for the quantum number N can be estimated as

$$N \approx \left(\frac{m_{\rm P}}{\beta T_\sigma}\right)^{1/4} \left(\frac{T_x}{T_\sigma}\right)^{1/2}. \tag{43}$$

It follows from Eqs. (17) and (16) that a typical vorton in this relic distribution will have a mass energy given by

$$E_v \approx \left(\frac{m_{\rm P}}{\beta T_\sigma}\right)^{1/4} \left(\frac{T_x^3}{T_\sigma}\right)^{1/2}, \tag{44}$$

which corresponds to a vorton circumference

$$\ell_v \approx \left(\frac{m_{\rm P}}{\beta T_\sigma}\right)^{1/4} (T_x T_\sigma)^{-1/2}. \tag{45}$$

It can thus be confirmed using Eq. (42) that the postulate $T_\sigma > T_\star$ automatically ensures that these vortons will indeed satisfy the minimum length requirement (18), though only marginally when T is at the lower end of this range.

From Eqs. (34) and (41) the number density of these protovorton loops, at formation, is

$$n_{\rm f} \approx \nu_\star \left(\frac{\beta T_\sigma}{m_{\rm P}}\right)^{3/2} \left(\frac{T_\sigma^2}{T_x}\right)^3. \tag{46}$$

It follows from Eq. (35) that at later times the number density of their mature vorton successors will be

$$n_v \approx \nu_\star f \left(\frac{\beta T_\sigma}{m_{\rm P}}\right)^{3/2} \left(\frac{T_\sigma T}{T_x}\right)^3. \tag{47}$$

Thus, after the temperature has fallen below the value $T_{\rm r}$, the resulting mass density of the relic vorton population will be

$$\rho_v \approx \nu_\star f N \left(\frac{\beta T_\sigma}{m_{\rm P}}\right)^{3/2} \left(\frac{T_\sigma}{T_x}\right)^2 T_\sigma T^3, \tag{48}$$

which by Eq. (43) gives our final estimate as

$$\rho_v \approx \nu_\star f \left(\frac{\beta T_\sigma}{m_{\rm P}}\right)^{5/4} \left(\frac{T_\sigma}{T_x}\right)^{3/2} T_\sigma T^3. \tag{49}$$

C. Condensation in the radiation damping regime

For strings formed at low energies, for example in some nonstandard electroweak symmetry-breaking transition, the scenario of the preceding subsection is the only one that needs to be considered. However, for strings formed at much higher energies, in particular for the commonly considered case of grand unified theory (GUT) strings, current condensation could occur during the extensive temperature range below $T_\star$. The minimum length requirement is only marginally satisfied by typical loops when condensation occurs near the end of the friction-dominated regime. Therefore, if $T_\sigma < T_\star$, typical loops present during the transition will not be long enough to qualify as protovortons. This means that the vorton formation temperature $T_{\rm f}$ will not coincide with T_σ as it did in the friction-dominated regime, but rather will have a distinctly lower value.

In these scenarios the final stage of protovorton formation will be preceded by a period of evolution in the temperature range $T_\star \gtrsim T \gtrsim T_{\rm f}$. During this interval friction will be negligible and the only significant dissipation mechanism will be that of radiation reaction. Moreover, during the first part of this period, in the range $T_\star > T > T_\sigma$, the only radiation mechanism will be gravitational, which is so weak that to begin with it will have no perceptible effect at all. Thus, there will be an interval during which the smoothing length remains roughly constant at its value at the end of the friction-dominated era, given, according to Eqs. (41) and (42), by

$$L_{\rm min}(T_\star) \approx \frac{\beta^2 m_{\rm P}^3}{T_x^4}. \tag{50}$$

During the last stage before the protovortons are formed, in the range $T_\sigma > T > T_{\rm f}$, there will already be currents on the strings. However, in practice, even in the coupled case the expected currents will be too weak for electromagnetic radiation damping to be important. Therefore, gravitational radiation is the only important effect throughout the range $T_\star > T > T_{\rm f}$.

The resulting gravitational smoothing scale will be the length of the shortest loop for which the survival time exceeds the cosmological time scale (19). From dimensional considerations this can be estimated by the expression

$$t \approx \frac{L_{\rm min}}{\Gamma G \mathcal{E}}, \tag{51}$$

where $\mathcal{E}\simeq\mathcal{T}\simeq T_x^2$ is the mass-energy density of the string. Here, Γ is a dimensionless coefficient of order unity and, for the GUT strings, the gravitational factor will be given by $G\mathcal{E}\simeq(m_x/m_P)^2\approx10^{-6}$. The validity of the formula (51) has been confirmed in many particular cases by numerical simulations [6], though the value of the coefficient turns out to be typically $\Gamma\approx10^2$.

Equating Eq. (51) to the cosmological time scale we have

$$L_{\rm min}\approx\frac{\Gamma}{\sqrt{g^*}m_P}\left(\frac{T_x}{T}\right)^2. \tag{52}$$

This formula is valid when its value becomes larger than that given by Eq. (50). This occurs at a critical value $T_\dagger<T_\star$ which, from Eq. (52), is given by

$$T_\dagger\approx\left(\frac{\Gamma}{\sqrt{g^*}}\right)^{1/2}\frac{T_x^3}{\beta m_P^2}. \tag{53}$$

The relation (52) may also be expressed as

$$L_{\rm min}\approx\kappa t, \tag{54}$$

where κ is a constant given by

$$\kappa\approx\Gamma\left(\frac{T_x}{m_P}\right)^2. \tag{55}$$

To avoid ambiguity we can, of course, simply use the formula (54) as a defining relation to specify the parameter κ during the Hubble damping ''doldrum'' regime $T_\star\gtrsim T_\dagger$. However, with this convention κ will have to be considered as a function of time, starting with unit value at $T_\star$ and decreasing to the very low value (55) at which it levels off at $T\approx T_\dagger$. This description of the network evolution is illustrated graphically in Fig. 1.

In order to apply the formula (38) for the final vorton mass density we must evaluate the dimensionless coefficient ν that determines the protovorton number density. It was easy to do this for the friction-dominated case for which we could assume a constant value $\nu_\star$ of the order of unity. However, ν may subsequently be reduced by an amount whose estimation is not so clearly evident. It is reasonable to expect that in the radiation-dominated regime the string distribution tends towards a scale-invariant form, albeit one that will not be quite so simple as the Brownian form that prevailed in the friction-dominated era. Although there may already be approximate scaling for large scales, it is clear that scaling on the smaller scales that matter for our present purpose can only be obtained after the parameter κ defined by Eq. (54) has settled down to a constant value. This occurs at temperatures below $T_\dagger$ for which a detailed analysis is beyond the scope of the simulations that have been achieved so far. For simplicity we shall use a crude, but we hope sufficiently robust, description based on simple and quite natural physical considerations.

In so far as ν is concerned, the plausible conjecture that it should be describable by a scaling solution is to be interpreted as meaning that it should be a function only of the dimensionless ratio R/t (where, it is to be recalled, R denotes the radial scale under consideration and t is the Hubble time). It is reasonable to expect that for values of R in the range $\kappa\ll R/t\ll1$, the value of ν should be given by a simple power law of the form

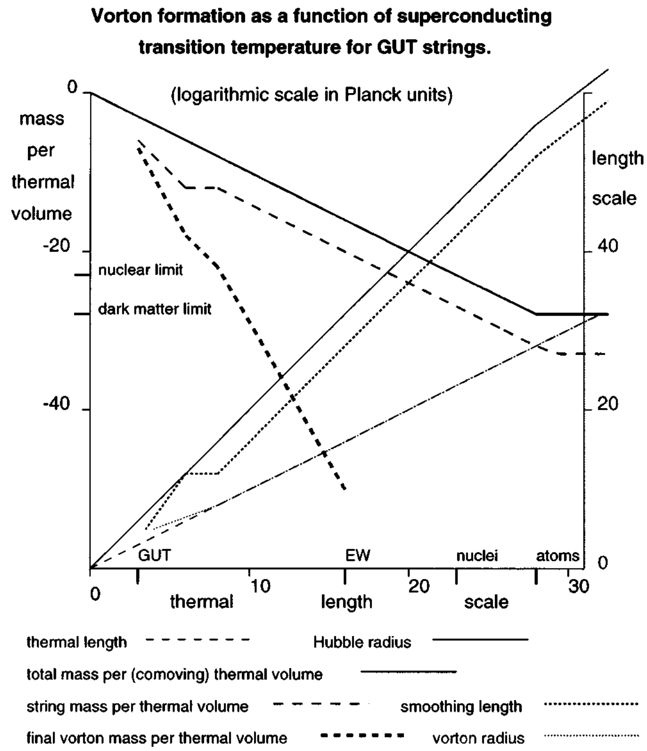

FIG. 1. The evolution of relevant physical quantities as a function of the thermal length scale in the early Universe.

$$\nu\approx\nu_\star\left(\frac{R}{t}\right)^\zeta, \tag{56}$$

with constant index ζ. We expect that, in the radiation-dominated regime, the appropriate value of the index should be close to, but perhaps slightly greater than, a lower limit given by [23]

$$\zeta=\frac{3}{2}. \tag{57}$$

(The analogue for the matter era in which we are situated today is a value slightly greater than a lower limit given by $\zeta=2$.)

Assuming that the formula (56) still gives the right order of magnitude at the lower end of its range, ν will be given in the radiation-dominated regime by the constant value

$$\nu\approx\nu_\star\kappa^\zeta \tag{58}$$

with κ given by Eq. (55]). This means that the corresponding value of the loop-number density itself will be given according to Eq. (54) by

$$n\approx\nu_\star\kappa^{\zeta-3}t^{-3}. \tag{59}$$

Before evaluating the required result (38), it remains to obtain the value $T_{\rm f}$. To do this we have to solve Eq. (33) that results from the minimum length requirement, which, from Eq. (52), reduces to

$$\mathcal{Z}_{\rm f} T_{\rm f}^2 \approx \frac{\Gamma T_\sigma^3}{\sqrt{g^*} m_{\rm P}}. \tag{60}$$

However, before we can solve this deceptively simple equation in practice, we need to know the T dependence of the factor $\mathcal{Z}$. This is the most delicate part of the calculation, since it involves competing effects of comparable magnitude.

As the string distribution evolves, the time-dependent blueshift factor $\mathcal{Z}$ is the factor by which the supporting string length has shrunk since the time of the condensation at the temperature T_σ. This shrinking can be accounted for as the net result of three main effects, two of which are comparatively easy to evaluate.

The weakest of these effects is the stretching due to the expansion of the Universe. This will always be more than compensated (except in the ‘‘doldrum’’ period in which compensation is only marginal) by shrinking due to the steady damping out of the short wavelength wiggles that give the most important contribution to the total string length per unit volume in the radiation-dominated era. If these two effects were the only ones it would be relatively easy to estimate $\mathcal{Z}$ since it would simply be proportional to the total string length, say Σ, in a comoving volume that can conveniently be taken to be a cubic thermal wavelength. This is given by

$$\Sigma \approx \Lambda T^{-3}, \tag{61}$$

where Λ is the total string length per unit volume. Now, note that the main contribution to the total length of the string distribution is provided by short wavelength modes with scale of order the smoothing length $L_{\rm min}$. Therefore, Λ can be estimated as being $nL_{\rm min}$, where n is given by Eq. (24). This implies

$$\Lambda \approx \nu L_{\rm min}^{-2}, \tag{62}$$

in which ν is given by (58).

If the only effect of the damping were to smooth out the short wavelength wiggles on the main part of the string distribution, it would be possible to identify $\mathcal{Z}$ with the ratio of Σ to its value Σ_σ at the time of the current condensation. However, this would not allow for a third important effect, namely, the losses of small loops which are continually liberated from the string distribution at the lower end of the spectrum. Whether these loops survive as vortons or disappear altogether, the effect on the main part of the string distribution is that the corresponding string lengths must be subtracted at each stage. Thus, the total shrinking factor $\mathcal{Z}$ will be the ratio of Σ, not to its original known value Σ_σ, but to a value that is considerably reduced in such a way as to take account of this new effect.

In terms of the variation $\delta\Sigma$ of Σ, the variation $\delta\mathcal{Z}$ of $\mathcal{Z}$ is given by

$$\frac{\delta\mathcal{Z}}{\mathcal{Z}} \simeq \frac{\delta\Sigma + \Delta\Sigma}{\Sigma}, \tag{63}$$

where $\Delta\Sigma$ is the length of string irreversibly chopped off in the form of small loops per comoving thermal volume within the short time interval δt under consideration. The delicate question is that of quantifying $\Delta\Sigma$. It is not hard to obtain an order of magnitude estimate, but since the final result is rather sensitively dependent on this quantity a more accurate estimate would be desirable. In the absence of a detailed numerical investigation, we express $\Delta\Sigma$ as a fraction of the total lost length

$$\Delta\Sigma \simeq -\varepsilon\,\delta\Sigma, \tag{64}$$

where ε is a dimensionless efficiency factor that must lie in the range $0<\varepsilon<1$. This factor is roughly identifiable with the coefficient introduced, using the same notation, in Eq. (34). However, in that context it was sufficient to know that it should be of order unity.

Whatever the exact value of ε, the substitution of Eq. (64) in Eq. (63) provides a differential equation that can be solved to give

$$\mathcal{Z} \approx \left(\frac{\Sigma}{\Sigma_\sigma}\right)^{1-\varepsilon} \tag{65}$$

and using Eqs. (52) and (62) we finally obtain

$$\mathcal{Z} \approx \left(\frac{T}{T_\sigma}\right)^{1-\varepsilon}. \tag{66}$$

It is to be remarked that if the efficiency ε of loop production were zero, this would mean that the carrier field would be blueshifted by a factor that would be precisely the inverse of that by which the background radiation is redshifted. However, if substantial loop production occurs there will be a blueshift by a moderate factor.

Assuming the particle number weighting factor can be taken to have the fixed value g_σ^*, Eqs. (60) and (66) then give

$$\frac{T_{\rm f}}{T_\sigma} = \left(\frac{\Gamma T_\sigma}{\sqrt{g_\sigma^*} m_{\rm P}}\right)^{1/(3-\varepsilon)}. \tag{67}$$

Using this result in conjunction with the estimates (52) and (58), the formula (38) gives the mass density of the resulting vorton distribution as

$$\rho_v \approx \varepsilon g^{*5/4} \nu_\star \Gamma^{\zeta-5/2} \left(\frac{m_{\rm P}}{T_{\rm x}}\right)^{5-2\zeta} \left(\frac{T_\sigma}{m_{\rm P}}\right)^{5/2} \times \left(\frac{\Gamma T_\sigma}{\sqrt{g_\sigma^*} m_{\rm P}}\right)^{(3+\varepsilon)/(6-2\varepsilon)} T_{\rm x} T^3. \tag{68}$$

If we adopt the value (57) for ζ, this simplifies to

$$\frac{\rho_v}{T_{\rm x} T^3} \approx \varepsilon g^{*5/4} \nu_\star \Gamma^{-1} \left(\frac{m_{\rm P}}{T_{\rm x}}\right)^2 \left(\frac{T_\sigma}{m_{\rm P}}\right)^{5/2} \left(\frac{\Gamma T_\sigma}{\sqrt{g_\sigma^*} m_{\rm P}}\right)^{(3+\varepsilon)/(6-2\varepsilon)}, \tag{69}$$

but there remains an unsatisfactory degree of sensitivity to the uncertain efficiency factor ε. This is because although this index will always be quite small, the factor $T_\sigma/m_{\rm P}$ is very tiny in the cases of interest.

D. Stability issues

Before we consider cosmological constraints we would like to say a few words about stability. One of our postulates is that superconducting current conservation is sufficiently effective to allow the protovortons that emerge at the temperature $T_{\rm f} \lesssim T_\sigma$ to settle down as dynamically stable bodies, at a possibly lower relaxation temperature $T_{\rm r} \lesssim T_{\rm f}$. This does not exclude the possibility that in the very long run they may finally decay by quantum tunneling or other ''secular'' instability mechanisms. In this case they would ultimately disappear when the thermal background reached a corresponding vorton death temperature with an even lower value, say $T_{\rm d}$. This means that a complete analysis of vorton formation and evolution could involve five successive temperature scales related by

$$T_{\rm x} \gtrsim T_\sigma \gtrsim T_{\rm f} \gtrsim T_{\rm r} \gtrsim T_{\rm d}. \tag{70}$$

Protovortons are small compared with the ever-expanding scales characterizing the rest of the string distribution and hence undergo few extrinsic collisions. Also, they are sufficiently smooth to avoid destructive fragmentation by self-collisions. It, therefore, follows that in most cases the relevant quantum numbers N and Z will be conserved. As a consequence, the statistical properties of the future vorton population will be predetermined by those of the corresponding protovorton loops at the time of their emergence at the temperature $T_{\rm f}$. It is, therefore, unnecessary for our present purpose to consider how long it takes for the protovorton loops to settle down and become proper stationary vortons. Thus, the value of $T_{\rm r}$ will not play any role in the discussion that follows. This is convenient because the details of protovorton loop decay have not yet been adequately studied, and will obviously be sensitively dependent on whether the current is electromagnetically coupled, in which case would expect the later stages of the protovorton loop contraction to be relatively rapid.

The final decay temperature $T_{\rm d}$ is also absent from the quantitative formulas that we will derive. Indeed, its only role is to characterize the two principal kinds of scenario that we consider. In Sec. V, we discuss vortons which survive until the present epoch, which requires that $T_{\rm d}$ should not much exceed 10^{-12} GeV (corresponding to the observed 3° radiation temperature). However, in Sec. IV we adopt the weaker supposition that the vortons survive at least till the time of nucleosynthesis, which requires only that $T_{\rm d}$ should not much exceed 10^{-4} GeV. This means that the only temperature scales that remain as variable parameters in the analysis that follows are the Higgs-Kibble temperature $T_{\rm x}$, the condensation temperature T_σ, and the protovorton formation temperature $T_{\rm f}$ which will not be truly independent but is a function of T_σ and $T_{\rm x}$.

IV. THE NUCLEOSYNTHESIS CONSTRAINT

One of the most robust predictions of the standard cosmological model is the abundances of the light elements that were fabricated during primordial nucleosynthesis at a temperature $T_{_N} \approx 10^{-4}$ GeV.

In order to preserve this well-established picture, it is necessary that the energy density in vortons at that time, $\rho_v(T_{_N})$, should have been small compared with the background energy density in radiation, $\rho_{_N} \approx g^* T_{_N}^4$. Assuming that carrier condensation occurs during the friction-damping regime and that g^* has dropped to a value of order unity by the time of nucleosynthesis, it can be seen from Eq. (49) that this restriction,

$$\rho_v(T_{_N}) \ll \rho_{_N}, \tag{71}$$

is expressible as

$$\varepsilon \nu_\star g_\sigma^{*\,-1} \beta^{5/4} m_{\rm P}^{-5/4} T_{\rm x}^{-3/2} T_\sigma^{15/4} \ll T_{_N}. \tag{72}$$

Below we apply this constraint to some specific examples.

A. Case $\mathbf{T_x \approx T_\sigma}$

The case for which carrier condensation occurs at or very soon after the time of string formation has been studied previously and yields rather strong restrictions for very long-lived vortons [7]. If it is only assumed that the vortons survive for a few minutes, which is all that is needed to reach the nucleosynthesis epoch, we obtain a much weaker restriction. Setting T_σ equal to $T_{\rm x}$ in Eq. (72) gives

$$\left(\frac{\varepsilon \nu_\star}{g_\sigma^*}\right)^{4/9} T_{\rm x} \ll \left(\frac{m_{\rm P}}{\beta}\right)^{5/9} T_{_N}^{4/9}. \tag{73}$$

Taking $g_\sigma^* \approx 10^2$ yields the inequality

$$T_{\rm x} \lesssim (\varepsilon \nu_\star)^{-4/9} \beta^{-5/9} \times 10^9 \ \text{GeV}. \tag{74}$$

This is the condition that must be satisfied by the formation temperature of *cosmic strings that become superconducting immediately*, subject to the rather conservative assumption that the resulting vortons last for at least a few minutes. It is to be observed that, if we assume that the net efficiency factor $(\varepsilon \nu_\star)^{-4/9}$ and drag factor $\beta^{-5/9}$ are of order unity (in view of the low value of the index), this condition rules out the formation of such strings during any conceivable GUT transition, but is consistent by a wide margin with their formation at temperatures close to that of the electroweak symmetry-breaking transition.

B. Case $\mathbf{T_x \simeq T_{GUT} \approx 10^{16}}$ GeV

Here, we wish to calculate the highest temperature at which GUT strings can become superconducting without violating the nucleosynthesis constraints. Setting $T_{\rm x}$ equal to $T_{\rm GUT}$ in Eq. (72), and again using $g_\sigma^* \approx 10^2$, we obtain

$$T_\sigma \lesssim (\varepsilon \nu_\star)^{-4/15} (g_\sigma^* T_{_N})^{4/15} T_{\rm GUT}^{2/5} \left(\frac{m_{\rm P}}{\beta}\right)^{1/3}$$
$$\approx (\varepsilon \nu_\star)^{-4/15} \beta^{-1/3} \times 10^{12} \ \text{GeV}. \tag{75}$$

It can be checked, using the Kibble formula Eq. (42), that the maximum value given by (75) is at least marginally consistent with the assumption that current condensation occurs in the friction-dominated regime. The validity of our derivation is thereby confirmed. It follows that the nucleosynthesis constraint will always be satisfied when T_σ lies in the radiation damping epoch.

Therefore, subject again to the rather conservative assumption that the resulting vortons last for at least a few minutes and assuming order of unity values for $\varepsilon\nu_\star$ and β, theories in which GUT cosmic strings become superconducting above 10^{12} GeV are inconsistent with the observational data.

V. THE DARK MATTER CONSTRAINT

In this section we consider the rather stronger constraints that can be obtained if at least a substantial fraction of the vortons are sufficiently stable to last until the present epoch. It is generally accepted that the virial equilibrium of galaxies and particularly of clusters of galaxies requires the existence of a cosmological distribution of ''dark'' matter. This matter must have a density considerably in excess of the baryonic matter density, $\rho_{\rm b}\approx 10^{-31}$ g/cm^3. On the other hand, on the same basis, it is also generally accepted that to be consistent with the formation of structures such as galaxies it is necessary that the total amount of this ''dark'' matter should not greatly exceed the critical closure density, namely,

$$\rho_{\rm c}\approx 10^{-29}\ \text{g cm}^{-3}. \tag{76}$$

As a function of temperature, the critical density scales like the entropy density so that it is given by

$$\rho_{\rm c}(T)\approx g^* m_{\rm c} T^3, \tag{77}$$

where $m_{\rm c}$ is a constant mass factor. Since $g^*\approx 1$ at the present epoch, the required value of $m_{\rm c}$ (which is roughly the critical mass per black body photon) can be estimated as

$$m_{\rm c}\approx 10^{-28} m_{\rm P}\approx 1\ \text{eV}. \tag{78}$$

However, for comparison with the density of vortons that were formed as a result of current condensation at an earlier epoch characterized by T_σ, what one needs is the corresponding factor $g^*_\sigma m_{\rm c}$, which can be estimated to be

$$g^*_\sigma m_{\rm c}\approx 10^{-26} m_{\rm P}\approx 10^2\ \text{eV}. \tag{79}$$

(This distinction was obscured in the previous derivations of the dark matter constraint [7,19], in which the value quoted for $m_{\rm c}$ should be interpreted as meaning the value of $g^*_\sigma m_{\rm c}$, which is what one actually needs.)

The general dark matter constraint is

$$\Omega_v \equiv \frac{\rho_v}{\rho_{\rm c}} \lesssim 1. \tag{80}$$

In the case of vortons formed as a result of condensation during the friction-damping regime the relevant estimate for the vortonic dark matter fraction is obtainable from Eq. (49) as

$$\Omega_v \approx \beta^{5/4}\left(\frac{\varepsilon\nu_\star m_{\rm P}}{g^*_\sigma m_{\rm c}}\right)\left(\frac{T_\sigma}{m_{\rm P}}\right)^{9/4}\left(\frac{T_\sigma}{T_{\rm x}}\right)^{3/2}. \tag{81}$$

In particular, this formula applies to the case in which the carrier condensation occurs very soon after the strings themselves are formed, as was supposed in earlier work.

However, if we want to strengthen the nucleosynthesis limit (75) for the general category of strings formed at the GUT scale, then we are obliged to consider the case of vortons formed as a result of condensation during the gravitational radiation damping regimes. In this case, Eq. (69) gives the relevant estimate for the vortonic dark matter fraction as

$$\Omega_v \approx \varepsilon g^{*1/4}\nu_\star \Gamma^{-1}\left(\frac{m_{\rm P}^2}{m_{\rm c} T_{\rm x}}\right)\left(\frac{T_\sigma}{m_{\rm P}}\right)^{5/2}\left(\frac{\Gamma T_\sigma}{\sqrt{g^*_\sigma} m_{\rm P}}\right)^{(3+\varepsilon)/(6-2\varepsilon)}. \tag{82}$$

Let us now again examine some specific examples.

A. Case $T_{\rm x}\approx T_\sigma$

The formula (81) is applicable to the case considered in earlier work [7], in which it was supposed that vortons sufficiently stable to last until the present epoch were formed as the result of the carrier condensation occurring at or very soon after the time of string formation. This example provides the strongest limits on $T_{\rm x}$. Setting T_σ equal to $T_{\rm x}$ in Eq. (81) one obtains

$$\beta^{5/9}\frac{T_{\rm x}}{m_{\rm P}}\left(\frac{\varepsilon\nu_\star m_{\rm P}}{g^*_\sigma m_{\rm c}}\right)^{4/9} \lesssim 1. \tag{83}$$

Substituting the estimates above we obtain

$$T_{\rm x} \lesssim (\varepsilon\nu_\star)^{-4/9}\beta^{-5/9}\times 10^7\ \text{GeV}. \tag{84}$$

This result is based on the assumptions that the vortons in question are stable enough to survive until the present day. Thus, this constraint is naturally more severe than its analogue in the previous section. It is to be remarked that vortons produced in a phase transition occurring at or near the limit that has just been derived would give a significant contribution to the elusive dark matter in the Universe. However, if they were produced at the electroweak scale, i.e., with $T_{\rm x}\approx T_\sigma\approx T_{\rm EW}$, where $T_{\rm EW}\approx 10^2$ GeV, then they would constitute such a small dark matter fraction, $\Omega_v\approx 10^{-9}$, that they would be very difficult to detect.

B. Case $T_{\rm x}\simeq T_{\rm GUT}\approx 10^{16}$ GeV

For the most commonly considered case, namely, that of strings formed during the GUT transition, the nucleosynthesis limit (75) is already sufficient for the exclusion of carrier condensation in the friction-damping regime. To obtain the stronger limit that is applicable if the vortons are sufficiently stable to survive as a dark matter constituent, we need to consider the case in which the condensation occurs during the regime of gravitational damping. In this case, the relevant dark matter fraction is given by Eq. (82). Setting $T_{\rm x}$ equal to $T_{\rm GUT}$ in this formula, and dropping the order of unity coefficients we obtain the corresponding limit

$$\frac{T_\sigma}{m_{\rm P}} \lesssim \left(\frac{m_{\rm c} T_{\rm GUT}}{\varepsilon\nu_\star m_{\rm P}^2}\right)^{(3-\varepsilon)/(9-2\varepsilon)}. \tag{85}$$

If the loop production were extremely efficient, $\varepsilon \simeq 1$, this would already give the numerical limit

$$T_\sigma \lesssim \nu_\star^{-2/7} \times 10^{10} \text{ GeV}, \tag{86}$$

which is significantly stronger than the more conservative limit (75) that pertains if the vortons only survive for a few minutes.

However, contrary to what one might have guessed, the highest conceivable loop production efficiency is not what maximizes ultimate vorton production. This is because it merely tends to enhance the charge and current loss rate by production not of protovortons but of doomed loops. Thus if, instead of supposing that the loop production efficiency ε is close to a hundred percent, one makes the plausible supposition that it does not much exceed fifty percent, then one obtains

$$T_\sigma \lesssim \nu_\star^{-5/16} \times 10^{9} \text{ GeV}. \tag{87}$$

This limit is still compatible by a very large margin with one of the most obvious, albeit rather extreme, possibilities [9] that come to mind, namely, that in which the strings are formed at the GUT level, $T_x \approx T_{\text{GUT}}$, but no current carrier condenses until the electroweak level, $T_\sigma \approx T_{\text{EW}}$.

VI. CONCLUSIONS

We have explored the constraints and implications both for particle physics and cosmology arising from the existence of populations of remnant vortons of more general types than have previously been considered. Specifically, we have envisaged the possibility of cosmic string superconductivity by condensation of the relevant carrier field at energy scales significantly below that of the string formation. We have seen that there are two qualitatively very different possibilities. In scenarios for which the carrier condensation occurs at comparatively high energy, during the friction-damping regime, a substantial majority of the superconducting string loops will ultimately survive as vortons. Such scenarios are more easily excluded on observational grounds than the alternative possibility, which is that superconductivity does not set in until a later stage, in which case only a minority of the loops initially present ultimately becomes vortons.

We have shown that large classes of particle physics models can provisionally be ruled out as incompatible by these cosmological considerations, and in particular, we have shown that models admitting GUT strings must not allow any string superconductivity giving stable vortons to set in much above 10^9 GeV (for order of unity dimensionless parameters). The excluded regions of parameter space are shown in Fig. 2.

Our conclusions are, however, dependent on a number of more or less ''conventional'' assumptions, whose validity will need to be systematically scrutinized in future work.

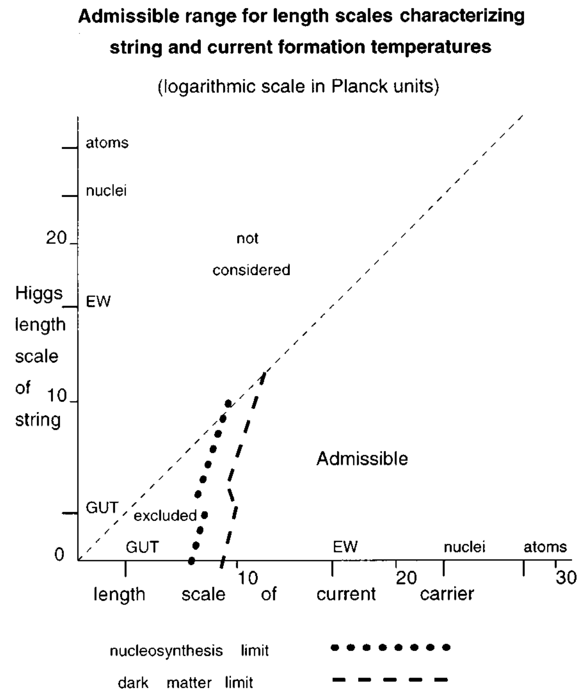

FIG. 2. A summary of our results. This diagram shows how the appropriate length scales in particle physics theories are constrained. Both the nucleosynthesis and dark matter bounds are shown for the case where the undetermined parameters ε, $\nu_\star$, and β are set to unity. If the values of these parameters are much smaller than unity then the bounds above are relaxed.

Invalidation of these conventional assumptions, particularly those concerning the long term stability of the vortons, in specific theoretical contexts would mean that in such circumstances the constraints given here might need to be considerably relaxed. On the other hand, our constraints may be considerably tightened by the use of more detailed observational data and the ensuing limits on the populations of various kinds of vortons that can exist today. On the constructive side, we have shown that it is possible for various conceivable symmetry-breaking schemes to give rise to a remnant vorton density sufficient to make up a significant portion of the dark matter in the Universe.

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