

Density perturbations and black hole formation in hybrid inflation

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We investigate the recently proposed hybrid inflation models with two stages of inflation. We show that quantum fluctuations at the time corresponding to the phase transition between the two inflationary stages can trigger the formation of a large number of inflating topological defects. In order to study density perturbations in these models we further develop a method to calculate density perturbations in a system of two scalar fields. We show that density perturbations in hybrid inflation models of the new type can be very large on the scale corresponding to the phase transition. The resulting density inhomogeneities lead to a copious production of black holes. This could be an argument against hybrid inflation models with two stages of inflation. However, we find a class of models where this problem can be easily avoided. The number of black holes produced in these models can be made extremely small, but in general it could be sufficiently large to have important cosmological and astrophysical implications. In particular, for certain values of parameters these black holes may constitute the dark matter in the Universe. It is also possible to have hybrid models with two stages of inflation where the black hole production is not suppressed, but where the typical masses of the black holes are very small. Such models lead to a completely different thermal history of the Universe, where postinflationary reheating occurs via black hole evaporation. [S0556-2821(96)00522-X]

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I. INTRODUCTION

A period of ‘‘inflation’’ or accelerated expansion in the early Universe is an attractive idea in modern cosmology. Acceleration of the scale factor could drive the Universe towards homogeneity, isotropy, and spatial flatness. However it is the ability of quantum fluctuations in the fields driving inflation to produce a nearly scale-invariant spectrum of quantum fluctuations that provides the most powerful test of the inflationary paradigm and may allow us to constrain the physics involved. Cosmological observations allow us to measure the amplitude and tilt of the primordial density and, possibly, gravitational wave spectra on scales that would have left the horizon during inflation.

The first inflationary models such as the old and the new inflationary universe scenario presumed that inflation began in the false vacuum state after the high temperature phase transitions in the early Universe [1,2]. Later it was proposed that all possible initial conditions should be considered without necessarily assuming initial thermal equilibrium, and see whether some of these conditions may lead to inflation. This scenario was called chaotic inflation [3]. For many years the idea of chaotic initial conditions seemed too radical, since it implied a considerable deviation from the idea of the hot big bang. It was argued that for a successful realization of inflationary theory one should satisfy so-called ‘‘thermal constraints’’ [4]. However, gradually it was understood that the assumption of thermal initial conditions is neither natural nor helpful for inflationary theory [5]. As a result, most of the

models investigated now belong to the class of chaotic inflation, which provides the most general framework for the development of inflationary cosmology.

The simplest models of chaotic inflation include theories with potentials $V(\phi)$ such as $m^2\phi^2/2$ or $\lambda\phi^4/4$. Inflation occurs in these theories at $\phi > M_P$. However, one may also consider chaotic inflation near $\phi = 0$ in models with potentials which could be used for implementation of the new inflation scenario, such as $-m^2\phi^2/2 + \lambda\phi^4/4$ [6]. For brevity, one may call inflation in such models ‘‘new inflation,’’ to distinguish it from inflation at large ϕ , but strictly speaking these models also belong to the general class of chaotic inflation models: the original new inflationary universe scenario based on the theory of high temperature phase transitions have never been successfully implemented in realistic theories.

The simplest models of chaotic inflation such as the model $m^2\phi^2/2$ have many advantages, including natural initial conditions near the Planck density and the existence of the regime of eternal self-reproduction of the Universe [5]. Normalizing the mass scale by the fluctuations in the microwave background observed by the Cosmic Background Explorer (COBE) [7] gives $m \sim 10^{13}$ GeV and the energy density at the end of inflation is $V(\phi) \approx (10^{16}\text{GeV})^4$. At this energy gravitational waves contribute about 10% of the microwave background fluctuations. The tilt of the density perturbation spectrum in this model is $n - 1 \approx -0.03$.

However, inflation occurs in such models only for $\phi \lesssim M_P$. It is quite possible to have inflation at $\phi > M_P$ in

models with polynomial potentials, but in string theory and supergravity one often encounters potentials which are extremely steep at $\phi > M_P$. It is not an unsolvable problem, see, e.g., [8], but it would be nice to have a simple model where inflation may occur at $\phi < M_P$ as well. It is possible to achieve this, for instance, in versions of “new inflation” with $V(\phi) \sim -m^2 \phi^2/2 + \lambda \phi^4/4$. However, in the simplest models of such type one has an unacceptably large negative tilt of the spectrum, unless the amplitude of spontaneous symmetry breaking is much greater than M_P [9]. Thus we return to the problem of having successful inflation at $\phi < M_P$.

There has recently been a lot of interest in the hybrid inflation scenario [10–15]. Initial conditions for inflation in this scenario are not determined by thermal effects, and thus hybrid inflation also belongs to the general class of chaotic inflation models. However, hybrid inflation may occur at values of the scalar fields much smaller than M_P . The tilt of the spectrum in hybrid inflation typically is very small and positive, giving rise to so-called “blue spectra” [16,17]. The contribution of gravitational waves to the microwave background anisotropies is usually negligible. The reheating temperature in this scenario is typically large enough to ensure the possibility of electroweak baryogenesis, but small enough to avoid the problem of primordial gravitinos.

It is still a challenge to obtain a natural implementation of this scenario in the context of supergravity and string theory, but in globally supersymmetric theories this scenario appears in a very natural way. A very interesting version of the hybrid inflation scenario recently proposed by Randall, Soljačić, and Guth was even called “supernatural” [18].

A distinctive feature of hybrid inflation is that it describes the evolution of two scalar fields, ϕ and ψ . In the beginning one of these fields (field ϕ) moves very slowly, and the second field may not move at all (though this second condition is not necessary [13,19]). The energy density supporting inflation is dominated by the false vacuum energy of the field ψ . At the moment when the slowly moving field ϕ reaches some critical value ϕ_c , it triggers a rapid motion of the field ψ , inducing a transition to a “waterfall” regime. Then the energy density of the field ψ rapidly decreases, and inflation ends.

Care is needed in evaluating the spectrum of density perturbations produced by inflation in the presence of more than one field. Many of the usual simplifying assumptions break down. Perturbations may no longer be purely adiabatic and hence curvature perturbations depend not only on the field fluctuations at horizon crossing but also upon their subsequent evolution up to the end of inflation, or even beyond. In the first versions of hybrid inflation the mass of the field ψ whose false vacuum energy density drives inflation was much larger than that of the slowly rolling field ϕ and so the single-field approximation was quite sufficient. Also, inflation ended abruptly when the field ϕ reached its critical value ϕ_c , and the field ψ began its motion.

This regime is certainly not the most general. Recently attention has been drawn to the possibility that both fields ϕ and ψ could have masses close to the supersymmetry-(SUSY-) breaking scale $m \approx 1$ TeV, while the symmetry-breaking scale in the ψ direction may be as large as M_P [18]. In this case the process of rolling of the field ψ towards its

minimum may take a lot of time even if its mass is a few times greater than the Hubble constant at the end of inflation. This may be attractive in the context of supersymmetric theories but raises new issues about the generation of density perturbations in the two-field model. We will consider these issues in this paper and spell out the dangers of ending hybrid inflation by a slow phase transition.

The main problem associated with this scenario can be explained in the following way. The effective potential of the field ψ used in [18] is symmetric with respect to the change $\psi \rightarrow -\psi$. As a result, at the moment of the phase transition the field ψ can roll with equal probability towards its positive and negative values. This leads to the usual domain wall problem. To avoid this problem in the original hybrid inflation model [10] it was suggested to change the topology of the vacuum manifold and couple the field ψ to gauge fields. In such a case instead of domain walls one may obtain either strings or monopoles, or (as in the electroweak theory) no stable topological defects at all. Monopoles should be avoided, but strings do not lead to any cosmological problems in theories with a relatively small scale of symmetry breaking, as studied in Ref. [10]. Alternatively, one may consider versions of the hybrid inflation scenario considered in Ref. [13], where no topological defects are produced.

In the model proposed in Ref. [18] topological defects do appear. In the simplest realization of this model one gets domain walls, which should be avoided at all costs. If one modifies the model to produce strings instead, one also has a problem, since strings corresponding to the scale of spontaneous symmetry breaking $\sim M_P$ by themselves produce density perturbations $\delta\rho/\rho \sim 1$ on all scales. One could expect that monopoles would not lead to any trouble since the distance between them grows exponentially during the second stage of inflation. However, because of inflation, which occurs in this model during the long stage of rolling of the field ψ to its minimum, all topological defects in this model appear to be inflating, as in [20]. Independently of the nature of these defects (domain walls, strings, monopoles, either topologically stable or not) their exponential expansion leads to density perturbations $\delta\rho/\rho \sim 1$ on the exponentially large scale corresponding to the moment of the phase transition. This may result in a copious black hole formation. However, inflating topological defects in our model are rather specific, because they appear in the theory with $|m_\psi| > H$. For this reason the possibility of black hole formation by such defects requires separate investigation. This problem is extremely interesting since here the issue of inflating topological defects appears in the context of observational cosmology.

Independently of this issue, the appearance of inflating topological defects clearly demonstrates that the existence of the second stage of inflation in the hybrid scenario may lead to large density perturbations. As pointed out in Ref. [18], the phase transition at $\phi = \phi_c$ leads to the appearance of a characteristic spike in the spectrum of density perturbations. The existence of such a spike was first found in a similar model by Kofman and Pogosyan [21]. In the “supernatural” hybrid inflation model it is difficult to calculate the amplitude of the peak of the spectrum; in Ref. [18] it was done slightly away from the point of the phase transition, where the amplitude of the density perturbations has already diminished. In order to perform the calculation, the evolution of

the fluctuating field in [18] was divided into several parts, and different approximations were used at every new step. However, the results of calculations of the amplitude of density perturbations near the narrow peak can be very sensitive to the choice of the approximation, especially in a situation where one may expect density perturbations to be large. Therefore we developed a more direct method of calculations of density perturbations in this model.

One of the most interesting aspects of the model of Ref. [18] is the existence of a regime in which quantum diffusion of the coarse-grained background fields dominates over its classical evolution and determines prominent features within our present cosmological horizon. Previously such phenomena were confined to scales much beyond our present horizon and were usually ignored. In Ref. [18], the machinery of stochastic inflation, see Ref. [22], was used to estimate the behavior of the fields close to the phase transition, where large quantum fluctuations make the stochastic formalism necessary. In this paper we will use this formalism to find whether or not most of inflationary trajectories come through the region where large density perturbations are generated. We believe that the method which we developed may be of interest in its own right and can be applied to a more general class of models with many scalar fields.

Our final results agree with the conclusion based on the topological defect analysis: density perturbations created at the moment of the phase transition are very large. In particular, in the model of Ref. [18] with the parameters given there corresponding to the second stage of inflation lasting for 20–30 Hubble times H^{-1} , one has density perturbations $\delta\rho/\rho \sim 1$. In such a situation one can expect copious production of huge black holes, which should lead to disastrous cosmological consequences.

However, this is not an unsolvable problem. For a suitable choice of parameters the second stage of inflation can be completely eliminated, and in this respect the model can be made very similar to the original hybrid inflation model of Ref. [10], where the problem of black holes does not appear at all. A very interesting possibility appears if the second stage of inflation does exist, but is very short, lasting only two or three Hubble times H^{-1} . Then the black holes formed from the large density perturbations may be small enough to evaporate quickly. With the parameters of the model of Ref. [18] the evaporation time is still very large even for the smallest black holes. However, if one studies hybrid inflation with a larger Hubble constant, the black holes produced during inflation can be made very small, so that they evaporate before nucleosynthesis. Even if the probability of formation of such black holes is suppressed, the fraction of matter in such micro black holes at the moment of their evaporation may be quite substantial, since the fraction of energy in radiation rapidly decreases in an expanding universe. This may lead to crucial modifications of the thermal history of the Universe and may rejuvenate the possibility that the baryon asymmetry of the Universe was produced in the process of black hole evaporation [23,24].

Finally, one may consider the models where the second stage of inflation lasts for about ten Hubble times. As we will see, in this case the probability of the black hole formation may be sufficiently small, so that the amount of black holes does not contradict the cosmological bounds on the black

hole abundance. It raises a very interesting possibility (see also [25]) that the black holes produced in the hybrid inflation scenario may serve as the dark matter candidates. In other words, dark matter may indeed be black.

As we already mentioned, there exist some versions of the hybrid inflation scenario where topological defects do not appear at all. In this paper we will suggest another version of such a scenario, which we will call “natural” hybrid inflation. This scenario is a hybrid of the simplest version of “natural inflation” [26,27], and the model of Ref. [18]. It shares some attractive features of “natural inflation” such as the natural origin of small parameters appearing in the theory. On the other hand, unlike the original “natural inflation,” our scenario does not require the radius of the “Mexican hat” potential to be greater than the Planck scale, which causes problems when one attempts to implement “natural inflation” in string theory [26]. We will show that in the models of natural hybrid inflation one can easily avoid the problem of large density perturbations.

The plan of the paper is as follows. In Sec. II we will briefly describe the simplest hybrid inflation model [10] and its relation to the model of Ref. [18]. We will find classical solutions describing the evolution of the fields ϕ and ψ in this model. Most of our investigation will be fairly general, but since the original hybrid inflation model [10] is already well investigated [12], we will concentrate on the model of Ref. [18] where an additional stage of inflation occurs after the phase transition. In Sec. III we will evaluate the amplitude of quantum fluctuations of each scalar field. In Sec. IV we analyze the issue of inflating topological defects and the associated density perturbations. In Sec. V we study density perturbations both before and after the phase transition and then discuss the important issue of quantum diffusion at the phase transition. In Sec. VI we analyze the probability of primordial black hole formation due to large density perturbations. We will also discuss the possibility of reheating of the universe by evaporation of small black holes. In Sec. VII we propose and briefly describe the “natural” hybrid inflation model. We will discuss our results and summarize our conclusions in Sec. VIII.

II. CLASSICAL FIELD DYNAMICS

The simplest realization of chaotic hybrid inflation is provided by the potential [10]

$$V(\phi, \psi) = \left(M^2 - \frac{\sqrt{\lambda}}{2} \psi^2 \right)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} \gamma \phi^2 \psi^2. \quad (1)$$

For comparison, we will write here the effective potential of one of the models considered in Ref. [18]:

$$V(\phi, \psi) = M^4 \cos^2 \left(\frac{\psi}{\sqrt{2}f} \right) + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda^2 \phi^2 \psi^2. \quad (2)$$

In the region $\psi < f$, where inflation occurs in the model (2), the potentials (1) and (2) practically coincide, with the redefinition of parameters, $f^2 \rightarrow M^2/2\sqrt{\lambda}$ and $\lambda^2 \rightarrow 2\gamma$. (Note that the fields denoted ϕ and ψ in Ref. [18] correspond to our fields ψ and ϕ . In this respect we have followed the notation of Ref. [12].) In what follows we will study the

model (1), but we will be most interested in values of the parameters close to those of Ref. [18].

The equations of motion for the homogeneous fields are then

$$\ddot{\phi} + 3H\dot{\phi} = -(m^2 + \gamma\psi^2)\phi, \quad (3)$$

$$\ddot{\psi} + 3H\dot{\psi} = (2\sqrt{\lambda}M^2 - \gamma\phi^2 - \lambda\psi^2)\psi, \quad (4)$$

subject to the Friedmann constraint

$$H^2 = \frac{8\pi}{3M_p^2} \left[V(\phi, \psi) + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 \right]. \quad (5)$$

Although we can always integrate the equations of motion numerically for arbitrary initial conditions, the classical motion of the homogeneous field is not necessarily a good representation of the coarse-grained field on superhorizon scales. If the classical motion is sufficiently slow it can become dominated by quantum diffusion caused by wave modes crossing outside the Hubble radius.

In particular we wish to consider the case when ϕ is much larger than ψ at early times so that ψ has a large positive mass $\approx \gamma\phi^2$ and rolls rapidly to $\psi=0$.¹ For values of ϕ above a critical value ϕ_c , $\psi=0$ is a stable minimum. Thus ψ remains zero while ϕ slowly rolls (for $m^2 \ll H^2$) down to the critical value, $\phi = \phi_c$, where the ψ field becomes massless. For smaller values, $\psi=0$ is an unstable local maximum and quantum diffusion initiates a second-order phase transition from the false vacuum to the true vacuum state.

In the simplest version of hybrid inflation, where the couplings λ and γ are of order unity, this is essentially all the dynamical evolution that matters. The bare mass of the ψ field, $-m_\psi^2 = 2\sqrt{\lambda}M^2$, must be much larger than H^2 [12] and the phase transition occurs rapidly and inflation ends. The perturbation constraints on large scales are then readily derived from the usual single-field results where the role of ψ at early times can be neglected.

But what if the bare mass of the ψ field is not very much larger than H ? In particular, what if this field has the type of potential we might expect for a moduli field with a minimum at $\psi \sim M_p$ and a negative mass squared $-m_\psi^2$ of order $(1 \text{ TeV})^2$ about $\psi=0$? The false vacuum energy density at $\psi=0$ is then $M^4 \sim m^2 M_p^2 \approx (10^{11} \text{ GeV})^4$ and the Hubble constant $H \approx m_\psi \sim 1 \text{ TeV}$, as discussed in Ref. [18]. It lies outside the range of parameters originally considered for hybrid inflation since it corresponds to an exceedingly flat potential for ψ with effective coupling constant $\lambda \sim m_\psi^4/M^4 \sim 10^{-30}$. The rolldown of the ψ field need no longer be fast and we must consider the complicated evolution of both the fields ϕ and ψ during this stage.

When $\psi=0$ the potential simply reduces to $V(\phi) = M^4 + m^2\phi^2/2$. For the range of parameters and field values that we are interested in the constant term always dominates and the Hubble expansion can be taken to be de

Sitter expansion with $H = H_0 \equiv \sqrt{8\pi/3}M^2/M_p$. It is useful to then write the bare masses of the two fields ϕ and ψ relative to the Hubble scale as

$$\alpha \equiv \frac{m^2}{H_0^2} \quad \text{and} \quad \beta \equiv 2\sqrt{\lambda} \frac{M^2}{H_0^2}. \quad (6)$$

In the case of a single scalar field σ evolving during inflation, one usually resorts to the slow-roll, $\dot{\sigma}^2 \ll V(\sigma)$, and quasimassless, $V'' \ll H^2$, approximations to make analytic progress. This allows one to reduce the equations of motion for the scalar field to a first-order equation. However in our case the mass of the ψ field is less than H only for a short interval, $\phi_c\sqrt{1-1/\beta} < \phi < \phi_c\sqrt{1+1/\beta}$. Even then, we wish to consider values of α not much below unity so the quasimassless approximation may not be very good for ϕ either.

Fortunately the fact that the potential energy, and hence the Hubble rate, are so nearly constant (and this really is a very good approximation for the parameters of Ref. [18]) allows us to integrate the second-order equations in two regimes.

A. Region I: Small ψ

The first approximation regime will be for $\psi^2/H_0^2 \ll \alpha/\gamma$.² This leaves the mass of the ϕ field constant and the equation of motion becomes

$$\phi'' + 3\phi' + \alpha\phi = 0, \quad (7)$$

where a prime denotes a derivative with respect to $N = H_0(t - t_e)$, the number of e -folds from the end of inflation.³ This can be readily integrated to give

$$\phi(N) = \phi_+ \exp(-r_+ N) + \phi_- \exp(-r_- N),$$

$$r_\pm = \frac{3}{2} \mp \sqrt{\frac{9}{4} - \alpha}. \quad (8)$$

The asymptotic solution is $\phi = \phi_+ \exp(-rN)$ where $r \equiv r_+ > 0$ which approaches the slow-roll solution $\phi = \phi_+ \exp(-\alpha N/3)$ for $\alpha \ll 1$.

For $\phi > \phi_c$, the ψ field remains trapped in the stable minimum at $\psi=0$ and we have effectively single-field inflation and the above solution gives the exact evolution of $\phi(N)$.

Below $\phi \approx \phi_c$ we can no longer take the ψ field to remain fixed at $\psi=0$. The ψ field equation of motion is

$$\psi'' + 3\psi' = \left(\beta - \gamma \frac{\phi^2}{H_0^2} \right) \psi. \quad (9)$$

We see that the field becomes massless at $\phi_c^2 \equiv (\beta/\gamma)H_0^2$. Rewriting this as an equation for ψ as a function of ϕ gives

¹If instead ϕ rolls rapidly to $\phi=0$ it remains there and the problem reduces to the usual case where the ψ field rolls directly to the global minimum at $V=0$.

²We also require $\psi^2/H_0^2 \ll \beta/\lambda$ but in practice this is a much weaker condition for the parameters of [18].

³Note that we have defined N so that it increases with time and so we have $N < 0$ for $t < t_e$.

$$\phi^2 \frac{d^2 \psi}{d\phi^2} + (1-2q)\phi \frac{d\psi}{d\phi} + (\kappa^2 \phi^2 + q^2 - \nu^2)\psi = 0, \quad (10)$$

where

$$q \equiv \frac{3}{2r}, \quad \kappa \equiv \frac{\sqrt{\beta}}{r} \frac{1}{\phi_c}, \quad \nu \equiv \frac{1}{r} \sqrt{\frac{9}{4} + \beta}. \quad (11)$$

The exact solution is a linear combination of Bessel functions,

$$\psi(\phi) = \phi^q [-c_1 Y_\nu(\kappa\phi) + c_2 J_\nu(\kappa\phi)]. \quad (12)$$

For $\phi \ll \phi_c$ the growing mode is given by the small angle approximation

$$\psi(\phi) = -c_1 \phi^q Y_\nu(\kappa\phi) \approx c_1 A \phi^{-(\nu-q)} \quad (13)$$

$$= c_1 A \phi_+^{-(\nu-q)} e^{(\nu-q)rN}, \quad (14)$$

where the numerical coefficient $A = (2/\kappa)^\nu \Gamma(\nu)/\pi$.

B. Region II: Small ϕ

As ϕ decreases, the effective potential for the ψ field becomes dominated by its bare (tachyonic) mass $\sqrt{\beta}H_0$. Thus for $\phi \ll \phi_c$, and still assuming $\psi^2/H_0^2 \ll \beta/\lambda$, we have

$$\psi'' + 3\psi' - \beta\psi = 0, \quad (15)$$

which has the general solution

$$\psi(N) = \psi_+ \exp(s_+ N) + \psi_- \exp(s_- N),$$

$$s_\pm = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \beta}. \quad (16)$$

Matching to the asymptotic solution in region I, Eq. (14), we see that only the growing mode ($s = s_+ > 0$) exists in region II. Recalling that N is measured from the end of inflation, it is simply given by

$$\psi(N) = \psi_e \exp(sN), \quad (17)$$

for all trajectories. The total amount of inflation after a given point is determined solely by the ratio ψ_e/ψ , as noted in Ref. [18]. Even though the effective mass of ψ has become large and negative, inflation will only end when $\dot{\psi}^2 \approx V$, which implies⁴

$$\psi^2 \approx \psi_e^2 \equiv \frac{M_P^2}{4\pi s(s+1)}. \quad (18)$$

From a given value ψ , the evolution will take

$$N(\psi) \approx \frac{1}{s} \ln\left(\frac{M_P}{s\psi}\right) \quad (19)$$

e -folds to the end of inflation. As we shall see, if the initial value of ψ is only of order $H_0 \sim 1$ TeV there must be a further $(32/s)$ e -folds before inflation ends.

The growing value of ψ increases the effective mass of the ϕ field through the interaction term and it also becomes larger than H . However, the ϕ field is much closer to the minimum of its potential than the ψ field ($\phi_c/M_P \sim 10^{-12}$) and ϕ soon starts to execute damped oscillations about $\phi=0$, as noted in Ref. [18].

This time we have a parametric equation for the inflaton field in terms of the triggering field, for which the general solution is

$$\phi(\psi) = \psi^p [\bar{c}_1 J_\mu(\rho\psi) + \bar{c}_2 Y_\mu(\rho\psi)], \quad (20)$$

where

$$p \equiv -\frac{3}{2s}, \quad \rho \equiv \frac{\sqrt{\beta}}{s} \frac{1}{\phi_c}, \quad \mu \equiv \frac{1}{s} \sqrt{\frac{9}{4} - \alpha}. \quad (21)$$

Matching to the asymptotic solution in Eq. (13), we find that only the J_μ solution is selected. We can see this from the small $\rho\psi$ expansion of Eq. (20) with $\bar{c}_2 = 0$,

$$\phi(\psi) = \bar{c}_1 \psi^p J_\mu(\rho\psi) \approx \bar{c}_1 B \psi^{p+\mu}, \quad (22)$$

where $B = (\rho/2)^\mu / \Gamma(\mu+1)$ and $p+\mu \equiv -(\nu-q)^{-1} \equiv -r/s$. Comparing this with the limiting behavior of Eq. (13) we find a relation between the coefficients

$$\bar{c}_1 B = (c_1 A)^{r/s}. \quad (23)$$

Therefore, as long as the solutions pass through the overlapping region, our solutions of region I evolve smoothly into region II.

III. QUANTUM FIELD FLUCTUATIONS

In this section we discuss the evolution of quantum fluctuations of the fields. In the slow-roll approximation, the amplitude of quantum fluctuations of a massless field at horizon crossing ($k = aH$) is approximately $H/2\pi$. However, in our case the masses of the fields are not necessarily much smaller than the Hubble scale, and corrections to the slow-roll result could be large.

Since the potential $V(\phi, \psi) \approx V_0$ to a very good approximation, we can neglect the gravitational backreaction of the fields, and the equations of motion for linear perturbations in ψ and ϕ can be written as

$$\begin{aligned} \delta\psi + 3H\delta\psi + \left(\frac{k^2}{a^2} - \beta H^2 + \gamma\phi^2\right)\delta\psi \\ = -2\gamma\psi\phi\delta\phi - 3\lambda\psi^2\delta\psi, \end{aligned} \quad (24)$$

$$\begin{aligned} \delta\phi + 3H\delta\phi + \left(\frac{k^2}{a^2} + \alpha H^2\right)\delta\phi \\ = -\gamma\psi^2\delta\phi - 2\gamma\psi\phi\delta\psi. \end{aligned} \quad (25)$$

⁴Note that for $\beta > 1$ this also ensures that de Sitter remains a good approximation until very near the end of inflation.

Note that in region I, when $\psi=0$, the terms on the right-hand sides are zero and the evolution of $\delta\psi$ and $\delta\phi$ decouple. We

can write these equations, in terms of the canonically quantized fields $u \equiv a \delta\psi$ and $v \equiv a \delta\phi$, as

$$u_k'' + \left(k^2 - \frac{2 + \beta(1 - \eta^{2r})}{\eta^2} \right) u_k = 0, \quad (26)$$

$$v_k'' + \left(k^2 - \frac{2 - \alpha}{\eta^2} \right) v_k = 0, \quad (27)$$

where primes denote derivatives with respect to conformal time, $\eta = -1/aH$, and we have chosen $\eta = -1$ when $\phi = \phi_c$.

Since the mass of the ϕ field is constant, we can write an exact expression for the quantum fluctuations,

$$v_k(\eta) = \frac{\sqrt{\pi}}{2\sqrt{k}} e^{i(1-r)\pi/2} (-k\eta)^{1/2} H_{3/2-r}^{(1)}(-k\eta), \quad (28)$$

where $r = r_+$ is defined in Eq. (8). This has the correct flat-space limit as $-k\eta \rightarrow \infty$, $v_k \rightarrow e^{ik\eta}/\sqrt{2k}$, while as $\phi \rightarrow 0$, and $-k\eta \rightarrow 0$, we find

$$v_k(\eta) = \frac{C(r)}{\sqrt{2k}} e^{i(1-r)\pi/2} (-k\eta)^{r-1}, \quad (29)$$

where [28]

$$C(r) = 2^{-r} \frac{\Gamma(3/2-r)}{\Gamma(3/2)}. \quad (30)$$

This results in a scale-invariant spectrum of the growing-mode perturbations at horizon crossing with amplitude

$$\delta\phi_* = C(r) \frac{H}{2\pi}. \quad (31)$$

Note that the coefficient $C(r)$ gives a constant correction (independent of scale) to the usual amplitude of curvature perturbations [obtained in the slow-roll limit where $C(0) = 1$].

On the other hand, the effective mass of ψ changes with time and we cannot write down an exact solution. However we can understand the qualitative behavior by considering the effective Schrödinger equation for u_k , see Eq. (26), with the time-dependent potential $V(\eta) = -(2 + \beta)/\eta^2 + \beta/\eta^{2(1-r)}$. This has a maximum value V_{\max} at η_{\max} given by

$$V_{\max} = \frac{(\beta+2)r}{1-r} \left[\frac{\beta(1-r)}{\beta+2} \right]^{1/r}, \quad (32)$$

$$\eta_{\max} = \left[\frac{\beta+2}{\beta(1-r)} \right]^{1/2r}. \quad (33)$$

On small scales, $k^2 \gg V_{\max}$, the field u_k oscillates with almost constant amplitude until $-\eta \lesssim \sqrt{\beta+2}/k$, when it starts to diverge. As $k \rightarrow \infty$ we recover the constant-mass asymptotic ($\eta \rightarrow 0$) solution

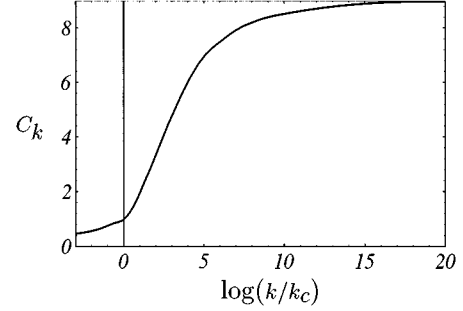


FIG. 1. The amplitude of quantum fluctuations of the symmetry-breaking field $\delta\psi$ in units of $H/2\pi$, as a function of scale k , evaluated when this scale left the horizon. Here k_c corresponds to the scale that left the horizon during the phase transition, when $\phi = \phi_c$. The figure corresponds to generic values of the parameters ($\alpha = 0.3, \beta = 8$). The asymptotic value as $k \rightarrow \infty$ is $C_\infty = 8.91$. The logarithm is to base 10.

$$u_k(\eta) = \frac{C(-s)}{\sqrt{2k}} e^{i(1+s)\pi/2} (-k\eta)^{-s-1}, \quad (34)$$

where $s = s_+$ is defined in Eq. (16). However, the amplitude of the quantum fluctuations decay exponentially for $k^2 < V(\eta)$. Thus modes with $k^2 < V_{\max}$ will be suppressed. The amplitude of the growing mode of the field perturbations at horizon crossing can then be written as

$$\delta\psi_* = C_k \frac{H}{2\pi}, \quad (35)$$

where the coefficient C_k is scale-dependent, as shown in Fig. 1 for the case $\beta = 8$.

We can understand qualitatively the behavior of this growing amplitude of quantum fluctuations. For modes $k \approx 1$ that leave the horizon near $\phi \approx \phi_c$, the ψ field is effectively massless and the amplitude of quantum fluctuations has the usual value $H/2\pi$, with coefficient $C_k \approx 1$. When $\phi \lesssim \phi_c(1 - 1/\beta)^{1/2}$, corresponding to scales $k > 1 + 1/2r\beta$, the magnitude of the (imaginary) mass of the ψ field becomes larger than H . Then the amplitude of quantum fluctuations even before horizon crossing, when $k/a \approx m_\psi$, is $\delta\psi \sim m_\psi/2\pi$, which is already greater than $H/2\pi$. As ϕ decreases and the corresponding scale increases, the effective mass of the ψ field approaches its bare value while the amplitude of quantum fluctuations grows. At very large k we recover the constant-mass scale-invariant value, $C_\infty = C(-s)$, see Eq. (30). For $\beta = 8$, this asymptotic value is $C_\infty = 8.91 \gg 1$, see Fig. 1. On the other hand, for $k^2 < V_{\max}$, the mass (squared) of the ψ field is large and positive at horizon crossing and the amplitude of these quantum fluctuations is suppressed.

IV. INFLATING TOPOLOGICAL DEFECTS

During inflation we must also consider the effect of short wavelength fluctuations that cross outside the horizon and perturb the coarse-grained background field on superhorizon scales. One can get a pretty good idea of the amplitude of perturbations on a scale corresponding to the time of the

phase transition by investigating the inflating topological defects produced at that time. For this purpose one should study the evolution of the fluctuations $\delta\psi$ at the moment of the phase transition.

Before the phase transition the field ψ is very heavy and its quantum fluctuations can be neglected. As we have seen, its fluctuations are generated when the mass of the ψ field becomes smaller than H , i.e., close to the phase transition. The exact duration of this stage is strongly model dependent, but with the parameters used in Ref. [18] the time before the phase transition when the fluctuation can grow is only about H^{-1} . As a result, one may (approximately) visualize the scalar field ψ at the moment of the phase transition as a sinusoidal wave with wavelength H^{-1} and amplitude $\delta\psi \sim H/\sqrt{2}\pi$,

$$\delta\psi_c(x) = \frac{H}{\sqrt{2}\pi} \sin Hx. \quad (36)$$

(Note that the amplitude here is $H/\sqrt{2}\pi$, whereas the standard expression $H/2\pi$ holds for the averaged amplitude $\langle \delta\phi^2 \rangle^{1/2}$, i.e., for the dispersion of the field.)

This representation is not exact since many different waves give a contribution to $\delta\psi(x)$, but their wavelengths are comparable and, for the models we consider, the amplitude of $\delta\psi$ in Eq. (36) is indeed of the order of $H/\sqrt{2}\pi$. During inflation the wavelength of these perturbations grows exponentially, $\lambda \sim H^{-1}a(t)/a(t_c)$, where $a(t_c)$ is the scale factor at the moment of the phase transition, but the amplitude of the field $\delta\phi$ also grows exponentially. This is the main reason why spontaneous symmetry breaking occurs despite the fact that formally the field ψ , averaged over the whole universe, always remains equal to zero. During each interval of time Δt , when the effective potential can be represented as $V_0 - m^2(t)\psi^2/2$ with $m^2(t) \ll H^2$, the amplitude of the field $\delta\psi$ in Eq. (36) grows as

$$\delta\psi_c \sim \frac{H}{\sqrt{2}\pi} \exp\left(\frac{m^2\Delta t}{3H}\right) \sin\left(\frac{a(t)Hx}{a(t+\Delta t)}\right), \quad (37)$$

whereas for $m^2(t) \gg H^2$ one has

$$\delta\psi_c \sim \frac{H}{\sqrt{2}\pi} \exp(m\Delta t) \sin\left(\frac{a(t)Hx}{a(t+\Delta t)}\right). \quad (38)$$

In both cases spontaneous symmetry breaking during inflation preserves the simple sinusoidal shape of the perturbation, until the field rolls down to its minimum and a more complicated nonlinear regime begins. But this occurs already at the very end of inflation.

Note that inflation near the cores of the topological defects continues for a while even after the field ψ reaches its minimum at $\psi = f \sim M_p$. Indeed, at that time the gradient energy density of the field distribution (38) is $\sim H^2 M_p^2 \exp(-2N_c)$, where the factor $\exp(-2N_c)$ appears because of the stretching of the wave during inflation. For large N_c it is much smaller than the potential energy density of the field ψ , given by $V(0) = 3H^2 M_p^2/8\pi$. As for the kinetic energy density of the field ψ it is always small near $\psi = 0$.

Therefore inflation in the vicinity of topological defects continues for a long time, until eventually the gradient energy becomes greater than $V(0)$.

As a result, after the field (38) approaches the minimum of the effective potential, on the exponentially large scale $\lambda \sim H^{-1}e^{N_c}$ we will have an extremely inhomogeneous matter distribution. Roughly speaking, in half of the volume of the Universe on that scale, the field will be near the minimum of its effective potential, whereas in the other half the scalar field will be close to the top of the effective potential with $\psi = 0$, and inflation will be still going on. This shows that we will have density perturbations $\delta\rho/\rho \sim 1$ on exponentially large scales corresponding to the time of the phase transition.

This effect is very general. It is related to the inflating topological defects discovered in Ref. [20]. The field may roll in any direction from $\psi = 0$, but stable regions with $\psi = 0$ are constantly being created, corresponding to inflating domain walls, strings, or monopoles. However, in Ref. [20] the curvature of the effective potential was much smaller than H^2 . In our case the second stage of inflation occurs even if $m_\psi^2 > H^2$, because the initial value of the field $\delta\psi_c(x) = H/\sqrt{2}\pi$ is much smaller than the amplitude of spontaneous symmetry breaking, so it takes a lot of time for the field ψ to roll down. This specific feature implies that there will be no eternal self-reproduction and no fractal structure of topological defects in our model.

Note that a similar effect may occur even in the models where topological defects are unstable if the decay rate of unstable defects is sufficiently small [29]. A possible example may be provided by the metastable electroweak cosmic strings or other ‘‘embedded defects’’ [30]. Such defects are unstable and usually do not cause any cosmological problems. However, they also have $\psi = 0$ in their cores. If they inflate [29] and decay only after the end of inflation, they lead to large density perturbations in the same way as the topologically stable inflating defects.

One may expect that large density inhomogeneities should lead to a copious black hole formation. The problem of black hole formation in a hot universe is not trivial because pressure gradients do not allow black holes to be formed unless density perturbations on the horizon scale are of the order of 1 or even greater [31,32,17,18,33,34]. But this is precisely the case for inflating topological defects. The centers of these defects remain forever at large density near the top of the effective potential, whereas far away from the cores the energy density gradually drops down to zero in an expanding universe. This corresponds to the delay of end of inflation $\delta N \gg 1$, and to density perturbations $\delta\rho/\rho > 1$. If these topological defects were isolated objects immersed in empty Minkowski space, one can show that their thickness is much *smaller* than their Schwarzschild radius [20]. Therefore one would come to the conclusion that there is no suppression for the probability of black hole formation in this scenario. However, it turns out that this is not what happens.

To investigate this issue note that the sinusoidal wave (36) is a particular example of a perturbation $\delta\psi$ which is symmetric with respect to $\psi \leftrightarrow -\psi$. The specific shape of these perturbations indicates that one cannot study them by considering Gaussian distributions centered away from $\psi = 0$. However, one could apply the theory of density perturbations to those parts of the defects away from $\psi = 0$. The

main ingredient is the calculation of the time delay for rolling of the field ψ down to the minimum of its effective potential. For the centers of topological defects this time delay is infinite, which corresponds to density perturbations $\delta\rho/\rho \gg 1$. Thus, one may expect important black hole formation near the centers of inflating topological defects. However, as we will see, the mere fact that topological defects correspond to large perturbations does not automatically imply that they form black holes, because the distribution of density near the cores of these defects is extremely non-Gaussian. Note that topological defects correspond to places where the distribution of the field ψ near their centers grows linearly, $\psi(x) \sim H^2 x$, see Eq. (36). Therefore the density distribution near the topological defects looks like a narrow peak. Meanwhile, the distribution of the scalar field produced by the usual density perturbations at $\psi \neq 0$ has relatively flat regions where the spatial derivative of the field ψ vanishes. These places have greater size, for the same amplitude of density contrast, and thus can be more easily converted into black holes.

To estimate the density contrast $\delta\rho/\rho$ in the vicinity of the topological defects we will use the fact that during the last stages of inflation the field ϕ in our model is already close to the minimum of the effective potential. Thus, let us consider the field (38) at the moment when it reaches the minimum of its effective potential at $\psi=f$. This happens approximately at a time t_e given by equation $(H/\sqrt{2\pi})e^{m_\psi t_e} = f$. At this moment the distribution of the field ψ , as a function of the distance $r = xe^{-Ht}$ from the center of the topological defect, can be written in a very crude approximation as

$$\delta\psi_c \sim \frac{H}{\sqrt{2\pi}} \exp(m_\psi t_e) \sin(Hr). \quad (39)$$

Let us rewrite its evolution in terms of f , counting time from t_e . For small r ,

$$\delta\psi_c(r) \sim f H r \exp(m_\psi t), \quad (40)$$

which remains a good approximation until $\delta\psi_c(r) \sim f$. The time delay until the field at a distance r reaches f is

$$\delta N(r) = H \delta t(r) = -\frac{H}{m_\psi} \ln H r. \quad (41)$$

This means that the density of the topological defect will exceed the average density by $\delta\rho/\rho \sim \delta N(r) \sim 1$ only in the core of the topological defect, at a distance

$$r \sim H^{-1} \exp\left(-\frac{m_\psi}{H}\right). \quad (42)$$

For the usual inflating topological defects considered in Ref. [20] one has $m_\psi/H \ll 1$, and therefore the deviation of density from the average density becomes large for the whole region from $r \sim H^{-1}$ to $r=0$. That is why one may expect such topological defects to look like huge black holes from the outside. Meanwhile in the model of Ref. [18] one has $m_\psi/H > 1$. As a result, the part of the volume inside the horizon where the energy density deviates considerably from

that on the horizon will be extremely small, being suppressed by $\exp(-3m_\psi/H)$. Therefore the total mass excess on the horizon scale is too small to cause the black hole formation. The same behavior of the density distribution is repeated at smaller scales as well. This suggests that inflating topological defects in the theories with $m_\psi > H$ do not necessarily lead to dangerous black hole formation.

The validity of this argument depends on many assumptions. For example, if the Universe is matter dominated then black holes are formed even if the density perturbations are not very large. Therefore topological defects may serve as seeds for small black holes which are formed at soon after inflation, before the Universe reheats and becomes radiation dominated. This may lead to very interesting consequences, to be discussed in Sec. VI. Meanwhile, black holes which are formed in the models with a prolonged second stage of inflation are very large, and they are formed very late, when the Universe is supposed to be radiation dominated. Our investigation indicates that large black holes do not come from topological defects. However, they may be produced by the usual density perturbations, which will be studied in the next section.

We should emphasize that our conclusions concerning black hole formation from inflating topological defects should be considered only as very preliminary. Indeed, here we deal with perturbations of density which are extremely large, $\delta\rho/\rho \gg 1$, and our simple analysis using methods of perturbation theory may be inapplicable. It is sufficient to remember that the interior of topological defects in our model continues to inflate for a long time after inflation ceased to exist in the outer space. Thus we have here a complicated wormhole-type geometry which requires a very careful analysis, similar to the numerical investigation performed in Ref. [35]. It is the first time when we need to know a detailed structure of the Universe filled by a gas of inflationary topological defects on the scale smaller than the size of the observable part of the Universe. We need to return to investigation of this fascinating question in the future. In the meantime we can only say that it would be premature to conclude that the model of Ref. [18] contradicts observational data solely on the basis of our investigation of density perturbations produced by inflating topological defects.

For this reason in the next section we will concentrate on the theory of usual (non-topological) density perturbations in models with two scalar fields and apply it to our model. The results that we have obtained from the investigation of topological defects suggest that, in the model under consideration, usual density perturbations may also happen to be very large. In what follows we will show that this is indeed the case.

V. DENSITY PERTURBATIONS

The most general scalar metric perturbations about a spatially flat Friedmann-Robertson-Walker metric with scale factor $a(t)$, may be written as

$$ds^2 = -(1+2A)dt^2 + 2B_{,i} dx^i dt + a^2(t)[(1-2\mathcal{R})\delta_{ij} + 2E_{,ij}]dx^i dx^j, \quad (43)$$

where A , B , \mathcal{R} , and E are scalar functions which can be decomposed into Fourier modes with different comoving wave numbers k whose evolution decouples in the linear approximation. Not all the perturbations represent physical degrees of freedom, so to remove gauge artifacts we can define gauge-invariant quantities [36,37]

$$\Phi \equiv A + (B - a^2 \dot{E}), \quad (44)$$

$$\Psi \equiv \mathcal{R} - H(B - a^2 \dot{E}). \quad (45)$$

In fact we have $\Phi = \Psi$ for all linear perturbations due to scalar field fluctuations.

Quantum fluctuations of the scalar fields are responsible for curvature perturbations on comoving (or equal energy density) hypersurfaces, $\mathcal{R}_\Sigma = H \delta\rho/\dot{\rho}$, where the density perturbation $\delta\rho$ is evaluated on a spatially flat hypersurface. In terms of the gauge-invariant potential Φ this can be written as [38,39]

$$\mathcal{R}_\Sigma = \Phi - \frac{H^2}{\dot{H}} (\Phi + H^{-1} \dot{\Phi}) + \frac{\nabla^2 \Phi}{3\dot{H}}. \quad (46)$$

The curvature perturbation at the end of inflation can be equated with the change in the time (or number of e -folds) it takes to end inflation. In the case of adiabatic perturbations, e.g., in single-field inflation driven by the field ψ , the amplitude of the curvature perturbation remains fixed on superhorizon scales, so it can be calculated as $\mathcal{R}_\Sigma = \delta N = [H \delta\psi/\dot{\psi}]_*$ at horizon crossing where $\delta\psi$ can be estimated as $H/2\pi$ [40,37]. The origin of this curvature perturbation is that the jump of the field in the direction opposite to its motion leads to a time delay in the end of inflation which can be estimated by $\delta t = \delta\psi/\dot{\psi}$. This equation immediately suggests that in the hybrid inflation model where the second stage of inflation after the phase transition begins at $\psi=0$, density perturbations on the length scale corresponding to the moment of the phase transition should be extremely large, since at that time $\dot{\psi}=0$. This is the standard situation in all models where inflation occurs near the local maximum of the effective potential. It does not lead to any troubles if the corresponding scale is many orders of magnitude greater than the present size of the observable part of the Universe [22]. But in the model of Ref. [18] this scale was supposed to be rather small, $l_c < 10$ Mpc. One certainly does not want to have the observable part of the Universe densely populated by large black holes.

However, these considerations are too naive. First of all, in our case we have two fields moving, ϕ and ψ , so even if the field ψ does not move at all, the whole field configuration evolves in time. Therefore the delay of the end of inflation is no longer given by the simple one-field expression $\delta t = \delta\psi/\dot{\psi}$. Also, in the presence of more than one field, the perturbations may not be adiabatic and, as a consequence, their amplitude need not be constant [41,39]. The effect of the perturbation must be integrated along the perturbed trajectory to the end of inflation, or until the evolution becomes adiabatic [42,39].

Finally, because we no longer have a unique trajectory in field space, we must consider which trajectories are likely to

be realized in practice. Because of the quantum fluctuations, the field ψ does not stay exactly at the point $\psi=0$ at the moment of the phase transition and we must investigate the dispersion of the probability distribution to see what the likely amplitude of density perturbations will be.

A. Single-field perturbations

Firstly we consider the simplest regime, before the phase transition, where single-field results apply. At large values of $\phi \gg \phi_c$ the ψ field has a large positive mass and remains fixed at $\psi=0$. The amplitude of ψ fluctuations crossing outside the horizon is negligible. Thus we need only consider adiabatic fluctuations $\delta\phi_*$ along the trajectory, given by Eq. (31), which perturb the time it takes to end inflation:

$$\delta N = \left[\frac{H \delta\phi}{\dot{\phi}} \right]_* = \frac{C(r)H}{2\pi r \phi_*}. \quad (47)$$

The power spectrum of curvature perturbations on comoving hypersurfaces, $\mathcal{R}_\Sigma = \delta N$, is then given by

$$\mathcal{R}_\Sigma^2(N) = \frac{C(r)^2}{4\pi^2} \frac{\gamma}{\beta r^2} e^{2r(N-N_c)}, \quad (48)$$

where N_c is the number of e -folds from the phase transition ($\phi = \phi_c$) to the end of inflation (which we have seen could be of order 20–30). Assuming these perturbations are responsible for the observed temperature anisotropies in the microwave background, they give a constraint on the parameters of the model. The small (of order 1 TeV) Hubble constant during inflation implies that the contribution of gravitational waves to the microwave background anisotropies will be negligible [11,12]. The low multipoles of the angular power spectrum measured by COBE [7] give a value $\mathcal{R}_\Sigma^2 \approx 3 \times 10^{-9}$ on the scale of our current horizon, corresponding to

$$N_{\text{CMB}} \approx 46 + \frac{2}{3} \ln \left(\frac{M}{10^{11} \text{ GeV}} \right) + \frac{1}{3} \ln \left(\frac{T_{\text{rh}}}{10^7 \text{ GeV}} \right). \quad (49)$$

For parameters α and β of order one, this requires $\gamma \leq 10^{-8}$. Note that for these parameters we will have $\phi_c \geq 10^4 \text{ TeV} \gg H_0$.

This is one of the few cases in inflationary cosmology where we have an almost exact expression for the amplitude of curvature perturbations [43]. The only approximation we have made is to assume that the energy density remains constant, $\dot{\phi}^2/2 + m^2 \phi^2/2 \ll M^4$, so that we can neglect the back-reaction on the metric. Using Eq. (47), we see that this will be true as long as

$$\frac{8C(r)^2}{3r\mathcal{R}_\Sigma^2} \frac{M^4}{M_p^4} \ll 1. \quad (50)$$

It follows that the allowed range of r is

$$\frac{8}{3\mathcal{R}_\Sigma^2} \frac{M^4}{M_p^4} \ll r \ll \frac{3}{2} - \sqrt{\frac{8}{9\pi\mathcal{R}_\Sigma^2} \frac{M^2}{M_p^2}}, \quad (51)$$

which for $M \ll 10^{16}$ GeV essentially leaves α as a free parameter in the range $0 < \alpha < 9/4$. At the upper limit, $r \sim 3/2$, the correction coefficient $C(r)$ becomes large, giving a significant amplification of the curvature perturbations \mathcal{R}_Σ compared to the usual slow-roll approximation.

Because the comoving scale at horizon crossing is just proportional to the scale factor $\propto e^N$, the scale dependence of the power spectrum in Eq. (48) readily gives the tilt of the spectrum as

$$n-1 \equiv \frac{d \ln \mathcal{R}_\Sigma^2}{d \ln k} = 2r, \quad (52)$$

which becomes $2\alpha/3$ in the slow-roll limit. Note that in principle for this model one could have any value of the tilt in the range $1 < n < 4$. A precise measurement of n , which may be possible with the next generation of satellite experiments [44], would give a tight constraint on α . Present limits give $n = 1.2 \pm 0.3$ [7]. Since n could be greater than one, there are also limits coming from production of black holes at small scales, $n < 1.4$ [17]. Together they give $\alpha \leq 0.6$, which constrains the size of the correction coefficient in Eq. (48) to lie in the range $0.9 \leq C < 1$.

B. Two-field perturbations

In this subsection we describe the interesting regime in which the system goes through the phase transition and quantum fluctuations of both fields become important. Here the curvature perturbation on a comoving hypersurface at the end of inflation cannot be given simply in terms of that at horizon crossing since it does not remain constant on super-horizon scales. In order to compute the amplitude of the curvature perturbation at the end of inflation, one usually integrated the coupled differential equations for the two fields' fluctuations and evaluated their amplitude at the end of inflation from that at horizon crossing. Only recently, Sasaki and Stewart [42] developed a formal method for computing the metric perturbations at a given hypersurface from the change in the number of e -folds to that hypersurface as a *local* function in field space, in the slow-roll approximation. This method was shown in Ref. [39] to be equivalent to the usual method of integrating the quantum field fluctuations. Unless one finds solutions for all trajectories in field space, the problem remains analytically intractable. In Ref. [39] we found particular cases, with separable potentials for the interacting fields, in which the field trajectories were integrable and we could write explicit expressions for the amplitude of curvature perturbations at the end of inflation. In principle, for a general model all we need is a computer to evaluate the change in the number of e -folds to the end of inflation due to quantum fluctuations of the fields, for all points in field space. This gives us the possibility to investigate density perturbations even for very complicated theories where it is not possible to express the final result in a compact analytical form. In the case of hybrid inflation we are fortunate to have a complete analytical solution for the classical evolution in regions I and II which takes us from $\phi = \phi_c$ to the end of inflation. This will allow us to compute in a compact way the amplitude of curvature perturbations at the end of inflation (where the system becomes adiabatic and the amplitude re-

mains constant on super-horizon scales) in terms of the field fluctuations at horizon crossing.

The perturbation in the number of e -folds from any point (ϕ_*, ψ_*) in region I to the end of inflation can be computed by evaluating the number of e -folds from a given point up until a surface of constant $\psi = \psi_m$ in region II, since the time from ψ_m to the end of inflation is fixed, see Eq. (17). Due to the overlap between the regions, this surface will also lie in region I for $\psi_m \leq \phi_c = \sqrt{\beta/\gamma} H_0$. The number of e -folds to ψ_m is given by

$$N(\phi_*, \psi_m) = N_m - \frac{1}{r} \ln \left(\frac{\phi_*}{\phi_m} \right), \quad (53)$$

where ϕ_m is a function of the trajectory parametrized by c_1 ,

$$\phi_m \simeq (c_1 A)^{r/s} \psi_m^{-r/s}. \quad (54)$$

Using the solution for $\psi(\phi)$ in region I, given by Eq. (13), we can eliminate c_1 to give

$$N(\phi_*, \psi_*) \simeq \frac{1}{s} \ln \left[\frac{\psi_e}{\psi_*} \right] + \frac{1}{s} \ln \left[\frac{\phi_*^\nu Y_\nu(\kappa \phi_*)}{A} \right], \quad (55)$$

only in terms of the fields at horizon crossing. This is one of the few cases in which such an integration can be done completely up to the end of inflation, see also Ref. [39].

Note that in general a trajectory beginning at a perturbed point $(\phi + \delta\phi, \psi + \delta\psi)$ may end up at a completely different point in field space compared with the nonperturbed trajectory. This can make the comparison of the lengths of the trajectories very complicated and could lead to entropy as well as curvature perturbations at the end of inflation. However, in our case all trajectories merge at the end of inflation, and this complication does not arise. The fact that by the end of inflation we are left with a single field (and thus all perturbations have become adiabatic) allows us to equate the amplitude of curvature perturbations on a comoving hypersurface at the end of inflation with perturbations on comoving hypersurfaces at late times and, in particular, at the surface of last scattering, see Ref. [39].

We can now evaluate the perturbation in the number of e -folds as

$$\delta N \simeq \frac{\kappa}{s} \frac{Y_{\nu-1}(\kappa \phi_*)}{Y_\nu(\kappa \phi_*)} \delta \phi_* + \frac{\delta \psi_*}{s \psi_*}. \quad (56)$$

Note that $Y_\nu(z) \sim z^{-\nu}$ for small z , and thus the first term vanishes if the point (ϕ_*, ψ_*) lies in region II, which gives $\delta N = \delta \psi_* / s \psi_*$, as required by Eq. (17).

For $\phi_* \simeq \phi_c$ we have $\partial N / \partial \phi_* \simeq 1/\phi_c \ll \partial N / \partial \psi_*$, for $\psi_* \ll \phi_c$. Since the quantum fluctuations $\delta \phi_*$ and $\delta \psi_*$ at this time are both of order $H/2\pi$, the amplitude of curvature perturbations is given by

$$\mathcal{R}_\Sigma = \delta N \simeq \frac{C_k H}{2\pi s \psi_*}, \quad (57)$$

where $C_k \simeq 1$, see Eq. (35). On the other hand, for $\phi_* \ll \phi_c$, the coefficient $C_k \gg 1$, see Fig. 1, which is then responsible for large curvature perturbations.

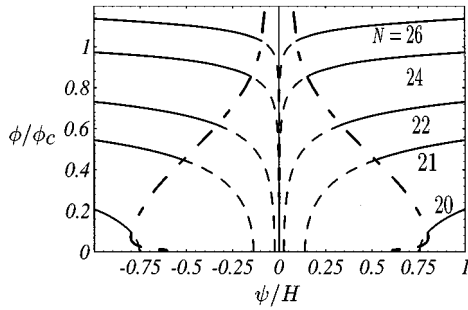


FIG. 2. The thick dash-dotted line corresponds to the $\delta N=1$ region within which it is not possible to define equal-time hypersurfaces, and density perturbations are of order 1. We also show a few equal-number-of- e -folds contours in field space $(\psi/H, \phi/\phi_c)$, for generic values of the parameters ($\alpha=0.3, \beta=8$). The dashed sections enter the $\delta N=1$ region where due to quantum fluctuations we cannot associate a definite number of e -folds to the end of inflation.

It is clear from Eq. (57) that for small values of ψ , the amplitude of curvature perturbations can become arbitrarily large. In Fig. 2 we show a few equal- N surfaces in field space $(\psi/H_0, \phi/\phi_c)$, around and below $\phi = \phi_c$, together with the line $\delta N=1$.

The amplitude of density perturbations on a comoving hypersurface when the curvature perturbations (57) reenter the horizon is given by [11]

$$\frac{\delta\rho}{\rho} = \frac{2+2w}{5+3w} \mathcal{R}_\Sigma, \quad (58)$$

where $p=w\rho$ is the equation of state of the Universe at reentry. We therefore expect large density perturbations on scales associated with the phase transition.

C. Quantum diffusion

So far we have discussed the classical evolution of the homogeneous field and the effect of perturbations about the classical trajectories on a given scale for values of ϕ above ϕ_c and below ϕ_c . However at some stage the role of quantum diffusion of the coarse-grained field ψ on superhorizon scales dominates over its classical motion. Purely classical trajectories in region I beginning with $\psi=0$ above ϕ_c are focused along $\psi=0$, due to the large effective mass of ψ at large ϕ , and continue to evolve close to $\psi=0$ long after the point ϕ_c when it becomes an unstable ridge. In practice we require quantum diffusion of the ψ field to move the field off the ridge and begin its roll down to the global minimum.

If diffusion washes out any trace of the classical motion as we cross $\phi = \phi_c$, it does not make sense to calculate the curvature perturbations in terms of the classical trajectories. This would destroy our notion of associating points in field space with a given number of e -folds from the end of inflation. Quantum diffusion close to ϕ_c could distort equal time hypersurfaces so much that we lose information about the origin of trajectories. This problem is analogous to that of trying to trace the path of photons beyond the (cosmological) last scattering surface. Beyond this surface, photons scatter many times and an observer at late times can no longer as-

sociate their pathlengths with a single smooth surface. Thermal diffusion is responsible for this loss of information in the trajectories of photons beyond last scattering. In our case it is quantum diffusion of the scalar field that determines the loss of information. Note that in the above-mentioned sense the region near the critical point $\phi = \phi_c$ becomes opaque to wavelengths equal to the wavelengths of perturbations formed at $\phi = \phi_c$, but it remains transparent to perturbations with much greater and much smaller wavelengths.

As argued above, our calculation of density perturbations relies only on being able to associate a given scale at late times, determined by the number of e -folds N , with a unique smooth surface in field space. Our results for the amplitude of perturbations near $\phi = \phi_c$ in Eqs. (56) and (57) show that there is indeed a region near $\phi = \phi_c$, $\psi=0$ for which δN becomes very large. In Fig. 2 the region under the $\delta N=1$ curve is the dangerous region. The question now is whether the parameters of the model are such that this affects a significant number of trajectories in field space. To determine this we should evaluate the probability distribution for the scalar fields and calculate how much of the initial wave packet suffers large perturbations in the number of e -folds, i.e., $\delta N \geq 1$. A heuristic constraint could be that the size of the scattering region should be much smaller than the size of the wave packet (very much like scattering of light of wavelength λ by a target with a diameter $a < \lambda$). These $\delta N \sim 1$ perturbations correspond to large curvature perturbations on asymptotic comoving hypersurfaces, which later become black holes. If a significant part of the packet enters the region where $\delta N > 1$, then at late times we cannot reconstruct the amplitude of the initial perturbation, corresponding to large scales ($\phi > \phi_c$). What happens is that, due to quantum diffusion, different scales will mix and their amplitudes will be undetermined for an asymptotic observer at late times.

According to [45], in single-field slow-roll inflation the regime of $\delta N \geq 1$ can be identified with quantum diffusion dominating over classical motion, $\delta\phi \geq \dot{\phi}/H$, i.e., with the well-known self-reproduction regime [46,45]. However, in two-field inflation this may no longer be the case. For example, fluctuations in one of the fields may not affect the time taken to end inflation. Even in our region II, where only ψ determines N , it is the asymptotic time delay that determines δN , not the instantaneous perturbation $[\delta\psi/\dot{\psi}]_*$ at horizon crossing.

Let us now calculate the probability distribution for the field ψ in region I, as the field $\phi = \phi_c e^{-r(N-N_c)}$ slowly rolls down its potential. This can be done using the stochastic approach to inflation [22,18]. Assuming an initial δ distribution for ψ at $\phi \gg \phi_c$, and an average quantum diffusion per Hubble volume per Hubble time $\approx H/2\pi$,⁵ the time-dependent probability distribution has the form

$$P(\psi, t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\psi^2/2\sigma^2(t)}, \quad (59)$$

⁵Note that our earlier analysis shows that this is an overestimate for $\phi \geq \phi_c$. However it should give a safe upper bound on the dispersion of the wave packet.

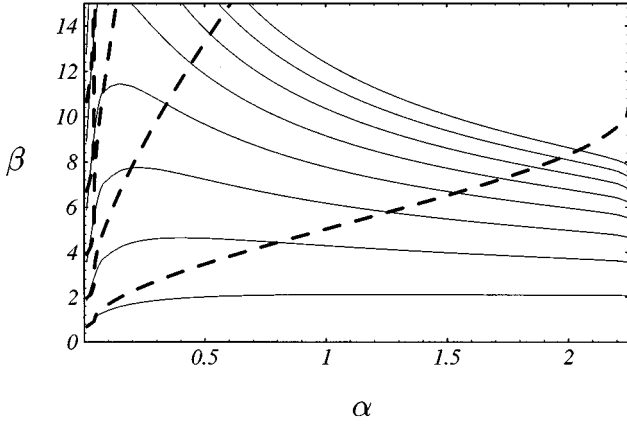


FIG. 3. Contour lines for the dispersion of the ψ distribution at the phase transition (dashed line) and one e -fold after the phase transition (continuous line), compared with the size of the $\delta N=1$ region, $\langle\psi^2\rangle^{1/2}=nH/2\pi s$ (with $n=1, \dots, 8$ from the bottom up) in parameter space (α, β) . In the region below the curves, the probability distribution for the ψ field cannot avoid the dangerous $\delta N=1$ region and large-amplitude perturbations will be expected at scales associated with the phase transition.

where the dispersion $\sigma^2(t)$ satisfies the evolution equation

$$\frac{d\sigma^2(t)}{dt} = \frac{H^3}{4\pi^2} + \frac{2\beta H}{3} \left(1 - \frac{\phi^2}{\phi_c^2}\right) \sigma^2(t). \quad (60)$$

Under a change of variables, $x \equiv \exp[-2r(N-N_c)]$ and $S(x) \equiv \sigma^2(t)/H^2$, this equation becomes

$$\frac{dS}{dx} = -\frac{1}{8\pi^2 r x} - \frac{\beta(1-x)}{3rx} S(x), \quad (61)$$

which has an exact solution

$$S(x) = \frac{1}{8\pi^2 r} \left(\frac{e^x}{ax}\right)^a \Gamma(a, ax), \quad (62)$$

where $a \equiv \beta/3r \approx \beta/\alpha$ is a constant and $\Gamma(a, z)$ is the incomplete γ function. The solution $S(N)$ characterizes the dispersion of the classical trajectories due to quantum fluctuations. Since the region where $\delta N \approx 1$ has a width $\psi \approx H/2\pi s$, see Eq. (57), at $\phi = \phi_c$, most classical trajectories will pass through this region. It is still possible that just one e -fold after the phase transition the distribution will have spread so much that only a small fraction of all the trajectories still goes through this region. We thus consider the dispersion of the ψ -distribution one e -fold after the phase transition, when $x = \exp(-2r)$. Note that the $\delta N \geq 1$ region is broader there, since $C_{k=e} > 1$ from Fig. 1, but the distribution (62) is also wider. If the spread of the probability distribution is still within the $\delta N \geq 1$ region at this stage, i.e.,

$$\langle\psi^2\rangle = \frac{H^2}{8\pi^2 r} \left[\frac{\exp(e^{-2r})}{ae^{-2r}}\right]^a \Gamma(a, ae^{-2r}) < \frac{H^2}{4\pi^2 s^2}, \quad (63)$$

it will be difficult to avoid large perturbations on this scale. Note that this condition is totally independent of the coupling γ . We have plotted this condition in Fig. 3, where we show

the contour plots of the dispersion of the distribution in units of the size of the $\delta N=1$ region, $\langle\psi^2\rangle^{1/2}=nH/2\pi s$, for $n=1, \dots, 7$ in parameter space (α, β) . The figure shows that in order for the distribution to have spread (one e -fold after the phase transition) several times the size of the $\delta N=1$ region one needs $\beta \gg 1$. The large separation of the lines indicates how difficult it is for the distribution to spread, unless $\beta \gg 1$. If they were closely packed it would mean that, for not very large β , the distribution would be much wider than this dangerous region and it would be possible for most trajectories to avoid this region. As it stands, for the values of the parameters in Ref. [18], most of the trajectories will go through this region.

In summary, we have shown that for $\alpha < 0.6$ (the range of parameters allowed by observations of the spectral tilt on large and small scales) we find large perturbations, $\delta N \sim 1$, along most trajectories at the phase transition, unless $\beta \gg 1$. This corresponds to large curvature perturbations on these scales and thus to large density perturbations after inflation which, as we shall see, leads inevitably to the formation of black holes.

VI. BLACK HOLE PRODUCTION

A. Probability of black hole formation

We have seen that quantum fluctuations of the fields can be responsible for large curvature perturbations on a comoving hypersurface at the end of inflation. These perturbations, on scales that left the horizon 20–30 e -folds before the end of inflation, as in the model we study, reenter the horizon during the radiation era and could in principle collapse to form primordial black holes. The theory of production of primordial black holes from initial inhomogeneities was first discussed in Ref. [31], see also [32]. There is an expression for the probability that a region of mass m , with initial density contrast $\delta(m) \equiv \delta\rho/\rho|_m$, becomes a primordial black hole,

$$P(m) \sim \delta(m) e^{-\bar{\beta}^4/2\delta^2}, \quad (64)$$

where $\bar{\beta}^2 \sim w$ [31]. In the derivation of this equation it was assumed that the Universe was a barotropic fluid ($p=w\rho$) during gravitational collapse, and that the initial density contrast $\delta(m)$ is much smaller than one. There seems to be disagreement over the value of the parameter $\bar{\beta}$ and the way to calculate it in a radiation dominated universe, see Ref. [18]. Novikov *et al.* [33,34] give a simpler prescription based on numerical calculations. They claim that even for density perturbations less than one at horizon crossing, a black hole will form as long as the perturbation in the metric $\delta g_{ab} = 2\Phi \delta_{ab}$ is of order 0.75–0.90, where Φ is the gauge-invariant Newtonian potential [33,34]. The analysis of a probability distribution for density perturbations with a peaked spectrum is beyond the scope of this paper, but we expect a probability distribution like that of Eq. (64) with $\bar{\beta}^2 \lesssim 1$, somewhat larger than that of Ref. [31]. Furthermore, Carr points out in Ref. [31] that, for a scale invariant spectrum with density contrast $\delta \sim 1$ at horizon crossing, the probability of black hole formation is $P \sim 1/2$ on all scales, and half of the mass of the Universe is always in the form of

primordial black holes. From the above discussion it is clear that for the large density contrasts produced during the phase transition, $\delta=(4/9)\delta N\sim 0.5$ at horizon crossing, see Eq. (57), there is no suppression of the probability of black hole formation on scales associated with the phase transition.

Note that in order to calculate the precise probability of black hole formation (64), we have to solve the ambiguity in the value of the parameter β . This would require a much more detailed investigation, see, e.g., [25]. However, for our purposes it is enough to realize that in a Gaussian distribution where the typical fluctuations of the density contrast are about 0.5, the fluctuations $\delta\sim 1$ are just one standard deviation away from $\delta\sim 0.5$. Therefore for every 10 horizon-sized regions with density contrast $\delta=0.5$ we typically find one region with $\delta\sim 1$, which can be expected to collapse and form a black hole.

We should note that the previous discussion of the probability of the black hole formation is based on investigation of conventional inflationary density perturbations. In our case, in addition to such perturbations, we have a dense gas of inflating topological defects. One might expect each of them to become a black hole, which would make the number of black holes much greater even than the one suggested by the estimates based on Eq. (64). (Here we are talking about the monopolelike inflating topological defects, since domain walls and strings with symmetry breaking $f\sim M_P$ lead to a cosmological disaster even if they do not inflate and form black holes.) However, we believe that this issue requires a more detailed analysis, see Sec. IV, and therefore in this paper we impose on the model of Ref. [18] only those constraints which follow from our investigation of the usual density perturbations.

Let us now evaluate the typical size and mass of the black holes produced by these perturbations. Suppose that after the phase transition the universe inflated e^{N_c} times. Then at the end of inflation the physical scale that left the horizon during the phase transition is $H_0^{-1}e^{N_c}$, where H_0 , as before, is the Hubble constant during inflation. Suppose that soon after inflation the equation of state became $p=\rho/3$, as for the ultrarelativistic gas. Then the scale factor of the Universe after inflation grows as $\sqrt{tH_0}$. The scale $H_0^{-1}e^{N_c}\sqrt{tH_0}$ becomes comparable to the particle horizon $\sim t$ at

$$t_h = H_0^{-1}e^{2N_c} \quad (65)$$

when the energy density becomes smaller than the inflationary energy density $\sim H_0^2 M_P^2$ by a factor e^{-4N_c} . At that time perturbations with density contrast $\delta\sim 1$ form black holes of size $H_0^{-1}e^{2N_c}$ and mass

$$M_{\text{BH}} \approx \frac{M_P^2}{H_0} e^{2N_c}. \quad (66)$$

For $N_c\sim 30$ one would have black holes with mass $\sim 10^{37}$ g, comparable to the masses of the black holes in the centers of galaxies. This is a very interesting mass scale, but copious production of such black holes would lead to catastrophic cosmological consequences.

By changing the parameters of our model one can make the duration of the second stage of inflation rather short. For small black holes with $N_c\sim 1$ in Eq. (65) should be modified

because they are formed at the stage when the energy may still be dominated by the oscillations of the inflaton field with the equation of state $p=0$. This changes a little our estimate for the time of formation of the black hole,

$$t_h = H_0^{-1}e^{3N_c} \quad (67)$$

and for the black hole mass,

$$M_{\text{BH}} \approx \frac{M_P^2}{H_0} e^{3N_c}. \quad (68)$$

The smallest black holes, corresponding to $N_c\sim 1$ (and $H_0\sim 10^3$ GeV, as in [18]), would have a mass of about 10^{11} g. Perturbations $\delta\sim 1$ giving rise to black holes in the mass interval $10^{11}-10^{37}$ g are clearly ruled out by the bounds of Ref. [17]. Thus, we should avoid at all costs the dangerous region $\delta N\approx 1$, since otherwise we will have too many large black holes.

A very interesting possibility arises when one considers such a peak in the spectrum, for not very massive black holes. From the bounds of Ref. [17] we see that density contrasts of order $\delta\sim 3\bar{\beta}^2/20$ are just enough to give $\Omega_0=1$ in the mass range $10^{15}-10^{30}$ g. Taking $\bar{\beta}^2\approx 1$ from [33,34] and using Eq. (69), we find that a parameter $s=3$ could indeed give the desired density contrast. This corresponds to a bare mass for the triggering field, $m_\psi\approx 4$ TeV, which is very natural. Furthermore, the associated mass scale can be computed from $N_c=32/3\sim 11$ as $M_{\text{BH}}=2\times 10^{20}$ g $=10^{-13}M_\odot$. Using the average density of our galaxy, $\rho_g\sim 10^{-25}$ g/cm³, we find that these small black holes may populate the halo of our galaxy and be separated from each other an average of 10^{15} cm or about six times the size of the solar system. They could very well constitute the missing mass in our galaxy, and still pass undetected by the microlensing surveys [47].

Note that changing slightly the parameters of the model one changes simultaneously the scale and the height of the peak in the black hole spectrum. This means that numerical values of the black hole masses and the distances between them can be made substantially different by modification of the hybrid inflation model. These numbers are very sensitive to the details of the theory of black hole formation, which still requires a more complete analysis. It is important, however, that in the context of the hybrid inflation scenario the possibility that black holes may contribute to the dark matter of the Universe becomes quite realistic.

The idea that dark matter may consist of black holes produced after inflation was explored earlier by Ivanov, Naselsky, and Novikov [25]. They performed a detailed investigation of the probability of formation of large black holes in such models, and in this respect their work can be extremely useful. Their model required the existence of a plateau in the effective potential, which would lead to a high peak in the spectrum of density perturbations. However, it is very difficult to obtain a realistic model of a single scalar field where one has almost exactly flat spectrum $\delta\rho/\rho\sim 5\times 10^{-5}$ on all scales from 10^{28} to 10^{20} cm, and a sharp peak with $\delta\rho/\rho\sim 1$ on a slightly smaller scale. Meanwhile, as we have shown, in the hybrid inflation scenario this possibility emerges in a very natural way.

Finally we note that if the density contrast at horizon crossing is sufficient to form black holes one would also expect gravitational radiation to be produced during non-spherical collapse⁶ whose energy density would be comparable with that of the black holes at formation. The typical frequency of these waves, $f_* \sim H_*$ at formation, would be redshifted to $f_0 \sim 10^{-7} (M_\odot / M_{\text{BH}})^{1/2}$ Hz today. However the energy density of the gravitational waves also redshifts away relative to that of the black holes so that for massive black holes (that do not evaporate) the present day $\Omega_{\text{GW}} \sim 10^{-12} (M_{\text{BH}} / M_\odot)^{1/2} \Omega_{\text{BH}}$. On the other hand, there could be many more regions where there is significant generation of gravitational waves even though the density contrast is not quite large enough to produce black holes. This might enhance the fraction of energy density in gravitational waves. For primordial black holes of order $10^{-6} M_\odot$ the gravitational wave spectrum has a maximum at a frequency ($f_0 \sim 10^{-4}$ Hz) which might be detected by a laser interferometer in space like LISA [48] if the fraction of energy density in gravitational waves were $\Omega_{\text{GW}} \geq 10^{-10}$. We will leave a detailed study of this interesting issue for future work. Furthermore, gravitational waves associated with black holes with masses $M_{\text{BH}} \sim 10^{15}$ g have a much higher frequency at the maximum, $f_0 \sim 100$ Hz, which make them reasonably good candidates for detection at the Laser Interferometric Gravitational Wave Observatory (LIGO) and VIRGO [49], for a similar energy density.

B. Reheating from black hole evaporation

A very interesting application of the above results comes when we consider a two-stage inflation with a sufficiently short period of inflation after the phase transition. With the parameters of the model [18] even the smallest black holes are very heavy and evaporate too late. However, by choosing a model of hybrid inflation with a sufficiently large Hubble constant and short second stage of inflation one can have a very interesting regime when the black holes will dominate the energy density of the Universe soon after the end of inflation and later evaporate before nucleosynthesis, reheating the Universe.

Let us first assume that small black holes were formed in a radiation dominated universe, soon after the usual stage of reheating after inflation. To evaluate the probability of black hole formation we need to know the density contrast at the horizon crossing during the radiation dominated stage, see Eqs. (57) and (58),

$$\delta = \frac{4}{9} \delta N \approx \frac{2 C_k H_0}{9 \pi s \psi} \approx \frac{4}{9 s}. \quad (69)$$

The number of e -folds in the second stage of inflation is given by $N_c = (1/s) \ln(\psi_e / \psi)$. With initial condition $\psi \sim H_0 / 2\pi$ we have

$$e^{N_c} \sim \left(\frac{2 M_P}{s H_0} \right)^{1/s}. \quad (70)$$

The time it takes a black hole to evaporate is given by

$$\tau \approx \frac{1}{g^* M_P} \left(\frac{M_{\text{BH}}}{M_P} \right)^3. \quad (71)$$

Here $g^* \sim 10^2$ is the effective number of particle species at the time of the black hole evaporation. For black holes with small N_c formed at the matter dominated stage we have

$$\tau \sim \frac{1}{g^* M_P} \left(\frac{M_P}{H_0} e^{3N_c} \right)^3. \quad (72)$$

Suppose that the fraction of matter in the black holes initially was only very small, and the Universe was radiation dominated from the time of black hole formation to the time they evaporate. Then the fraction of mass in black holes grows during this time as $a(t) \sim \sqrt{t}$ due of the more rapid decrease of the energy density of relativistic particles outside black holes. At the instant before the black holes finally evaporate, the fraction of energy in black holes has grown by a factor of

$$\sqrt{\frac{\tau}{t_h}} \sim \frac{M_P}{\sqrt{g^* H_0}} e^{3N_c}. \quad (73)$$

Therefore even if only a small fraction of energy was in the black holes initially, because the probability of their formation was suppressed by the exponential factor in Eq. (64), we only require $P(\delta) > (\tau/t_h)^{-1/2}$ for the black holes to come to dominate the energy density of the Universe before they evaporate.

To give a particular example, let us consider hybrid inflation with $H_0 \sim 10^{14}$ GeV (which is much greater than in the model of Ref. [18]). Let us take, e.g., $s \sim 3$, i.e., larger than the usual parameters of the models of Ref. [18] but much smaller than those of Ref. [10]. Then we have the total number of e -folds at the second stage of inflation $e^{N_c} \approx 40$. The density contrast in this case is $\delta \sim 1/7$. The fraction of matter in the black holes, according to Eq. (64), will be about 10^{-5} . In fact, this is a rather conservative estimate, since our investigation of topological defects suggests that this number may be much greater. The black holes will be produced at the moment $t_h \sim 6 \times 10^{-34}$ s. They will have mass $M_{\text{BH}} \sim 6 \times 10^4$ g, and will evaporate at $\tau \sim 2 \times 10^{-16}$ s, much earlier than the epoch of nucleosynthesis. Because of the large growth of the scale factor and redshift of energy of relativistic particles, at the time of the black hole evaporation practically all matter in the Universe will be in black holes. This means that practically all particles which exist in the universe at $t > 10^{-16}$ s are created at the moment of the black hole evaporation.

Note that in the above example, although the probability of black hole formation is very small, they still give the dominant contribution to the energy density at late times because the energy of relativistic particles decreases much faster than that of black holes. However, this black hole dominance may begin much earlier if they are formed before conventional reheating is complete and the equation of state is $p \approx 0$. This condition can be easily satisfied in the case of very small black holes. Then there will be no exponential suppression of the probability of the black hole production

⁶We thank John Barrow for drawing our attention to this interesting issue.

(64), and the fraction of energy in the black holes could be large from the very beginning.

The process of black hole evaporation could be responsible for the baryon asymmetry in the Universe, even though it is not very easy to get large baryon asymmetry by this mechanism [24]. Typically it is assumed that reheating and thermalization of the Universe occurs due to the inflaton field decay and the subsequent particle interactions, or through bubble collisions like in first-order inflation. The natural assumption was that the gravitational interaction at the stage of reheating could be neglected. Here we have another, very unusual mechanism of reheating. Even in the absence of bubble wall collisions or a large coupling of the inflaton to matter, a considerable fraction of matter after inflation could be in the form of small black holes. Unlike in the extended inflation scenario [50], in our case all such black holes are formed in the same mass range given by Eqs. (66) and (68). If the probability of black hole formation is not strongly suppressed, then very soon they dominate the energy density of the Universe [24]. Eventually the evaporation of these black holes could reheat the Universe. This opens up an interesting possibility of connecting the origin of matter in the Universe with black hole physics.

Let us estimate the reheating temperature of the Universe in this scenario. Black hole masses in the process of their evaporation decrease as $M_{\text{BH}}[1-t/\tau]^{1/3}$. (Here we have taken into account that the age of the Universe t_h at the moment of the black hole formation is much smaller than their evaporation time τ .) The main part of the energy release by the evaporating black holes occurs at the end of the time interval τ . Therefore one may simply use the standard temperature-time relation for the hot universe to get the following estimate of the reheating temperature T_r after the black hole evaporation:

$$T_r^2 \sim \frac{M_P}{4\pi\tau} \sqrt{\frac{45}{g^*\pi}} = \frac{\sqrt{45}g^*H^3}{4\pi\sqrt{\pi}M_P} e^{-9N_c}. \quad (74)$$

For a particular example which we studied ($H \sim 10^{14}$ GeV, $s \sim 3$, $g^* \sim 10^2$) we get $T_r \sim 2 \times 10^9$ GeV. This estimate is extremely sensitive to the choice of the parameters. One can easily get reheating temperature as high as 10^{10} GeV or even greater, or as small as 1 eV. The only real constraint on this temperature is that one should be able to produce the baryon asymmetry of the Universe during or after black hole evaporation, and before nucleosynthesis. This picture differs considerably from the standard theory of reheating due to the decay of the inflaton field, see, e.g., [51,52].

Perhaps one can appreciate a potential importance of this regime if one remembers that the standard reheating due to the inflaton decay often is very inefficient because of the small coupling of the inflaton to matter [51,5]. In such cases the Universe for a long time remains matter dominated ($p=0$). In some other cases reheating is extremely efficient in the very beginning, but later becomes inefficient, so that the Universe eventually may become matter dominated again [52]. But then formation of black holes is no longer suppressed by radiation pressure. In this case matter easily collapses into small black holes, which later evaporate and reheat the Universe. Thus, the absence of the usual reheating triggers black hole formation, which eventually leads to a

very efficient reheating of the universe. This is a win-win situation, where black holes can reheat the Universe even if the standard reheating mechanism is inoperative.

The energy density released in gravitational waves during the collapse of these small black holes is redshifted away together with any other type of radiation, as the black holes eventually dominate the expansion of the Universe. However, when the black holes evaporate, they emit a spectrum of gravitational waves that redshifts unperturbed since they do not thermalize with the other particles. Some estimates [53] of black hole evaporation suggest that 2–10 % of the Hawking radiation is in the form of gravitational waves. If all the radiation we observe today comes from primordial black hole evaporation, then the present fraction of energy density in gravitational waves is of order $\Omega_{\text{GW}} \sim 10^{-7}$ while the frequency at the maximum is

$$f_0 \sim 4 \times 10^{15} \text{ Hz} \left(\frac{M_{\text{BH}}}{10^5 g} \right)^{1/2}. \quad (75)$$

This frequency is far beyond detectability by laser interferometers like LIGO, VIRGO, LISA, etc., see Refs. [48,49]. However, there are recent proposals for gravitational wave detectors with certain crystals that are sensitive to frequencies of order 10^{15} Hz [54]. The possible detection of such a spectrum of gravitational waves is a subject for further investigation and would be a novel signature of this reheating mechanism.

VII. “NATURAL” HYBRID INFLATION

As we have seen, model (1), as well as the version proposed in [18], lead to a copious formation of huge black holes if one requires that (unlike in the original version of hybrid inflation) there is an additional inflationary stage after the phase transition. This problem occurs because typical classical trajectories in this model go very close to $\psi=0$. One can avoid this problem by a modification of the shape of the effective potential [13]. Also, as we have shown above, black hole production can be even useful if the second inflationary stage is very short and the black holes are very small. But there exists another problem, which we will consider now together with the first one.

The main reason why many authors are trying to implement hybrid inflation in supersymmetric theories is to protect the flatness of the effective potential in the ϕ direction. One may try to relate the small mass of the field ϕ to the gravitino mass $m_{3/2} \sim 1$ TeV, which appears because of supersymmetry breaking. If one argues that the parameter M^2 in Eq. (2) is of the order of the intermediate scale of supersymmetry breaking $m_{3/2}M_P$, then there appears to be no unexplained small parameters in the model. Still the appearance of the term $M^4 \sim (m_{3/2}M_P)^2 \cos^2(\psi/\sqrt{2}f)$ in Eq. (2) remains somewhat unclear to us. If one expands $M^4 \cos^2(\psi/\sqrt{2}f)$ in powers of ψ for $M \sim 10^{11}$ GeV and $f \sim 10^{18}$ GeV as in Ref. [18], one would find an extremely small coupling constant $M^4/f^4 \sim 10^{-30}$ in front of the term ψ^4 . It was pointed out in [18] that such couplings may appear in a natural way if one introduces certain superpotentials which lead to nonrenormalizable interactions. However, to study nonrenormalizable terms in an internally consistent way it would be necessary

to consider models based on supergravity, which was outside the scope of our investigation, as well as of the investigation performed in [18].

Fortunately, both smallness of the parameter M^4 and the shape of the potential can be explained if one interprets ψ as a pseudo Goldstone field similar to the axion field. One may consider a model of a complex scalar field $\Psi(x) \equiv [f(x)/\sqrt{2}] \exp i\theta(x)$, which after spontaneous symmetry breaking can be represented as $(f/\sqrt{2}) \exp[i\psi(x)/f]$. If the original effective potential was a function of $\Psi^* \Psi$, the field ψ will be massless. However, nonperturbative (instanton or wormhole) effects may give this field a small mass (see Ref. [55] for a recent discussion of this issue). This effect can be described by adding operators breaking initial U(1) symmetry of the effective potential. Consider the family of operators $g_n (\Psi \pm \Psi^*)^n f^{4-n}$. Since these operators appear because of nonperturbative effects, the coupling constants g_n may be exponentially small. One may take, for example, the simplest operator $g_1 (\Psi \pm \Psi^*) f^3$, and add to it a constant term $\sqrt{2} g_1 f^4$ normalizing the vacuum energy to zero. This gives the effective potential of the field ψ , which is completely analogous to the standard axion potential,⁷

$$V(\psi) = 2\sqrt{2} g_1 f^4 \cos^2\left(\frac{\psi}{2f}\right). \quad (76)$$

Note that in this potential ψ/f is an angular variable from 0 to 2π . This potential coincides with the effective potential of the field ψ in Eq. (2) up to an obvious change $2\sqrt{2} g_1 f^4 \rightarrow M^4$, $f \rightarrow f\sqrt{2}$. (Our definition of f corresponds to a canonical normalization of the field ϕ kinetic terms.) In this context both the shape of the potential for the field ψ and the smallness of the term $M^4 \cos^2(\psi/\sqrt{2}f)$ are explained in a natural way. Potentials of this type have been used in ‘‘natural inflation’’ models [26,27]. The problem with ‘‘natural inflation’’ is that for the ‘‘natural’’ value of symmetry breaking $f \lesssim M_p$ inflation is too short and the spectrum index n is significantly less than 1. There is no such problem in our model; the main purpose of the introduction of the field ψ is to support inflation *before* the phase transition rather than after it.

Thus it makes a lot of sense to explore cosine potentials such as Eq. (76) in the context of hybrid inflation. But with the interpretation of the field ψ as a pseudo Goldstone particle, one cannot couple it to the field ϕ in the way proposed in Eq. (2). Now ψ is the angular part of the field Ψ , and one cannot write any superpotentials for the fields Ψ and ϕ which would result in the simple interaction terms $\sim \psi^2 \phi^2$. However, since we already reinterpreted the cosine term in Eq. (2) as appearing from the anomalous term $g_1 (\Psi + \Psi^*) f^3$, we can go further and introduce an anomalous interaction term $g_2 (\Psi e^{-i\theta} - \Psi^* e^{i\theta})^2 f^2 \phi^2$. Note that we have introduced here for generality the phase shift θ between the two anomalous terms. Indeed, the origin of the

first and of the second anomalous terms may be different, and *a priori* one should not expect U(1) symmetry to be broken by these two terms in the same way. In what follows we will assume that θ is small. The resulting effective potential including the mass term of the field ϕ can be represented in the following form:

$$V(\phi, \psi) = 2\lambda_1^2 f^4 \cos^2 \frac{\psi}{2f} + \frac{\lambda_2^2}{2} f^2 \phi^2 \sin^2 \left(\frac{\psi}{f} - \theta \right) + \frac{m^2}{2} \phi^2. \quad (77)$$

Let us now analyze the shape of this potential and its relation to the more usual hybrid inflation potential (1). Consider first the case $\theta=0$. At large ϕ the dominant term involving ψ in Eq. (77) is the second one, which implies that at large ϕ the field ψ will settle in one of the minima at $\psi/f = n\pi$, where n is some integer. For odd values of n the energy density due to the self-interaction cosine-squared term also vanishes and we are left with conventional chaotic inflation with $V = m^2 \phi^2/2$. However for even values of n , the self-interaction term is nonzero and ψ is trapped in a false vacuum, like the model in Eq. (1). Near $\psi=0$ the potential in Eq. (77) is given by

$$V(\phi, \psi) = 2\lambda_1^2 f^4 - \frac{\lambda_1^2 f^2}{2} \psi^2 + \frac{\lambda_2^2}{2} \phi^2 \psi^2 + \frac{m^2}{2} \phi^2. \quad (78)$$

One concludes that about $\psi=0$ the bare mass squared of the field ψ is $m_\psi^2 = -\lambda_1^2 f^2$, but that the effective mass squared becomes positive for $\phi > \phi_c = \lambda_1 f / \lambda_2$. Near ϕ_c at $\psi=0$ the energy density is given by $2\lambda_1^2 f^2 (f^2 + m^2/4\lambda_2^2)$. The first term dominates, as in the usual hybrid inflation, for $m \ll \lambda_2 f$, and we have exactly the same as the model we have analyzed in the preceding sections where

$$H = \sqrt{\frac{\pi}{3}} \frac{4\lambda_1 f^2}{M_p}, \quad (79)$$

and the dimensionless parameters introduced in Sec. II are given by $\alpha = (3/16\pi) m^2 M_p^2 / \lambda_1^2 f^4$, $\beta = (3/16\pi) M_p^2 / f^2$, and $\gamma = \lambda_2^2$. The curvature perturbations produced at $\phi > \phi_c$ are then given by Eq. (48). For small α we have [10]

$$\frac{\lambda_2 \lambda_1^2 f^5}{M_p^3 m^2} \simeq 2 \times 10^{-6} \quad (80)$$

in order to agree with the COBE normalization [7].

There are two regimes one may consider in this theory. First of all, one may assume that the coupling constants are not extraordinarily small. Then, as was shown in Ref. [10], the conditions $m \ll H$ and $\delta\rho/\rho \sim 10^{-5}$ imply that there was no second stage of inflation in this model, i.e., everything is going on as in the first version of the hybrid inflation scenario [10]. One may consider, for example, the following parameters: $f \sim 10^{16}$ GeV (GUT scale), $m = 10^{10}$ GeV (intermediate SUSY-breaking scale), and $\lambda_1 \sim \lambda_2 \sim 10^{-3}$. In this case all conditions mentioned above will be satisfied. The parameters $\alpha \sim 10^{-2}$ and $\beta \sim 10^5$ so there will be no second stage of inflation and no anomalous black hole production. For $\theta=0$ there will be domain wall production. However, the domain walls formed in this scenario are unstable be-

⁷Note that in [12] a model of hybrid inflation was studied based on a supersymmetric Peccei-Quinn model. In that case it was the self-interaction energy of the radial degree of freedom $f(x)$ that gave the constant potential energy in the false vacuum.

cause they are always bounded by strings with $\Psi = 0$. Moreover, it is sufficient to consider models with a very small nonzero value of θ , so that all the evolution will go in one direction, and there will be no domain walls or other topological defects. One can easily understand this if one takes into account that at large ϕ the minimum of the effective potential with respect to the field ψ is at $\psi = \theta f \neq 0$. Thus it is enough to have $\theta > 10^{-15}$ to avoid black hole formation in our model.

As one might expect, the same model for a different choice of parameters can reproduce all the results of the model proposed in [18], including the second stage of inflation after the phase transition. This requires to take $\lambda_1^2 \sim 10^{-30}$, which is an extremely small number. However, in our case the existence of this small parameter is not surprising, because it could appear due to nonperturbative effects. Typically the value of this parameter is suppressed by factors such as $\exp(-8\pi^2/g^2)$ where g is the gauge coupling constant. In some models this suppression may not be very significant, but in general this suppression can easily give numbers much smaller than 10^{-30} [55]. In particular, in the usual axion theory with $f \sim 10^{12}$ GeV the corresponding constant is of the order of 10^{-130} . From this perspective it is more surprising that in this model the coupling constant λ_2^2 is not required to be equally small. This disparity can be easily alleviated if one does not insist that the masses of both fields as well as the Hubble constant at the end of inflation should be of the order of $m_{3/2}$.

For $\theta = 0$ in this model, just like in the model of Ref. [18], one obtains inflating topological defects, very large density perturbations on the scale corresponding to the moment of the phase transition, and catastrophic black hole production. However, it is no longer a generic property of the model. Remember that the dangerous area of the phase space is located very close to $\phi = \phi_c$ and $\psi = 0$. For example, large density perturbations are generated only at $\psi < H \sim 10^{-15}f$, for $H \sim 10^3$ GeV and $f \sim 10^{18}$ GeV. It is enough to have $\theta > 10^{-15}$ to avoid black hole formation in our model. Thus for generic values of θ inflationary trajectories never come close to $\psi = 0$, and the problem disappears. For noninteger values of θ/π hybrid inflation can occur along $\psi = \theta + n\pi$ for any integer value of n , with the false vacuum energy density equal to $2\lambda_1^2 f^4 \cos^2(\theta/2)$ for even n or $2\lambda_1^2 f^4 \sin^2(\theta/2)$ for odd values.

We do not want to pretend that the ‘‘natural’’ hybrid inflation model is necessarily very natural. In order to study this question one would have to investigate the appearance of the anomalous terms in a more detailed way, and to analyze the possible effects of adding a more standard term like $\Psi^* \Psi \phi^2$. Our main purpose was to show that the cosine terms with small coefficients can be incorporated into the hybrid inflation models, and that it is possible to avoid the problem of large density perturbations and black hole production in these models. However, as we have argued in the previous section, primordial black holes produced after inflation under certain conditions may lead to very interesting cosmological consequences. On the other hand, there is a simpler way to get rid of the black holes; one may simply return to the original hybrid inflation scenario without the second inflationary stage.

VIII. CONCLUSIONS

Inflationary theory was first proposed about 15 years ago, and its main principles are by now well understood. It is therefore surprising to see that slight modifications in simple and natural models may lead to important and sometimes absolutely unexpected consequences. For almost 15 years we knew that inflation exponentially dilutes the density of topological defects, but only two years ago did we learn that topological defects may inflate themselves [20]. It was also thought that old inflation did not work, and chaotic inflation always predicted that the Universe is flat. A year ago, however, it was found that the simplest hybrid model where one takes an old inflation potential for one of the fields and chaotic inflation potential for another one (even if these two fields do not interact with each other) leads to the Universe consisting of infinitely many separate universes with all possible values of $\Omega \leq 1$ [56]. Now we encountered one more surprising fact. For many years it seemed clear that inflation erased all preexisting inhomogeneities and did not leave much room for the production of primordial black holes, which had been the subject of active investigation in the end of the 70s. Now we see that in a very simple inflationary model one can easily obtain a large amount of black holes. They are formed only in a specific mass range, determined by the duration of inflation after the phase transition. Typically they are huge, but depending on the parameters of the model they can be very small as well.

Note that black holes are not necessarily a curse but could also be a blessing. In the simplest model studied here the probability of black hole formation is not suppressed at all, and their number appears to be large. We propose some modifications of this model where black hole formation is strongly suppressed. It is possible for certain values of the parameters of the model to have the right amount of relatively large black holes, $M \sim 10^{15} - 10^{30}$ g, that have not yet evaporated and may be responsible for the dark matter in the halos of galaxies. In a particular model considered in Sec. VI we have shown that the halo of our galaxy may consist of black holes of mass $\sim 10^{21}$ g. However, numerical values of the masses and abundances of the black holes are strongly model dependent. Depending on the parameters of the model there could be just enough black holes to give $\Omega_0 = 1$ in the Universe. If they are supermassive, one could speculate about their relation to the black holes in the centers of galaxies.

Suppressing the number of large black holes down to a desirable level requires a certain degree of fine tuning. But it is relatively easy to make the black holes very small and harmless by making the second stage of inflation short and by ending inflation at large H . For $H \sim 10^3$ GeV the smallest black hole masses $\sim M_p^2/H$ are still very large, about 10^{11} g, and they evaporate very late, at $t \sim 10^3$ s. But if, e.g., one takes the models with $H \sim 10^{14}$ GeV, one can obtain black holes with a mass $\sim 6 \times 10^4$ g, evaporating at $t \sim 10^{-16}$ s. Such black holes would dominate the Universe after their formation until evaporation. Evaporation of black holes may lead to baryon asymmetry production. During the last 15 years this mechanism of baryogenesis was largely ignored since it seemed impossible to produce many small black holes after inflation (see however Ref. [24]). We may now

return to the investigation of this interesting possibility. Independently of the issue of baryogenesis, one should emphasize that the possibility of the black hole dominance at the intermediate post inflationary stage may change completely the mechanism of reheating after inflation, which would proceed via black hole evaporation. Furthermore, there is the possibility of gravitational wave production during black hole evaporation. For a range of black hole masses it is possible that the associated stochastic spectrum of gravitational waves will be observed in future detectors.

In the course of our work we have further developed a method of investigation of density perturbations which can be applied even for complicated systems of several coupled scalar fields [42,39]. This method is rather simple and powerful. It gives analytical results in those cases in which the motion in field space is integrable, like in hybrid inflation and in theories with coupled inflaton and dilaton fields, but it can be used in a more general context as well. The method consists of three main parts. First of all, for any point in the (ϕ, ψ) space one finds (either analytically or numerically) an inflationary trajectory going from this point, and calculates the number of e -folds $N(\phi, \psi)$ for this trajectory. This problem is easy to solve numerically even for very complicated potentials. Then one perturbs the position of the initial point (ϕ, ψ) by adding to it inflationary jumps, which typically are of the order $H/2\pi$, but may be greater or smaller, see Sec. III. This gives us the perturbation of the number of e -folds δN , which is directly related to the density perturbations: $\delta\rho/\rho = (4/9)\delta N(\phi, \psi)$ at reentry during the radiation dominated era. Note that the resulting density perturbations for a given N (i.e., for a given wavelength) will depend on the place (ϕ, ψ) our trajectory came from. Thus the remaining step is to evaluate the probability that for a given number of e -folds N from the end of inflation the field was at any particular point (ϕ, ψ) . This problem can be solved by using the stochastic approach to inflation [22]. This approach tells us what is the probability to find density perturbations of a given amplitude with a given wavelength.⁸ Usually at the last stages of inflation we have a single inflationary trajectory, independent of initial conditions. In our case, however, in-

flationary trajectory is unstable at $\phi < \phi_c$, $\psi = 0$; all trajectories bifurcate due to quantum fluctuations. Therefore the calculation of the probability distribution was necessary to show that a *typical* amplitude of density perturbations produced near the point of the phase transition is very large.

As an important by-product of our investigation we have found a new type of inflating topological defect. They appear in the models where the curvature of the effective potential is somewhat greater than H^2 , but nevertheless the time necessary for symmetry breaking to occur is much greater than H^{-1} . These defects do not inflate eternally and do not form a fractal structure found in [20]. Still inflation in the cores of these defects continues for a while even after it ends outside of them. As a result, they lead to large density perturbations of a specific type. Until now inflating topological defects could be considered as an interesting but somewhat esoteric feature of certain inflationary models. Typically the distance from us to these defects was many orders of magnitude greater than the size of the observable part of the Universe. They were important for understanding of the global structure of the Universe, but not of our local neighborhood. However, in hybrid inflation models with two stages of inflation these defects are abundantly produced at the moment of the phase transition, and populate the part of the Universe which is accessible to our observations. We believe that the new type of inflating topological defects deserves separate investigation. It would be very interesting to understand whether they lead to black hole formation and to explore other possible observational consequences of these exotic objects. It is amazing that very simple models of two scalar fields can exhibit such a rich and interesting behavior.

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⁸Note that a similar problem of evaluation of probability appears also in the theory of a single inflaton field if one takes into account the possibility of nonperturbative effects due to large jumps along the inflationary trajectory [57]. However, in the theory of several scalar fields this issue becomes of more immediate importance.

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