Hara's theorem in the constituent quark model

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We show that the parity-violating electromagnetic matrix elements describing the hyperon decay in the nonrelativistic constituent quark model obey Hara's theorem in first-order perturbation theory, i.e., to $O(G_F)$, where $G_F \simeq 10^{-5} M_N^{-2}$ is the Fermi weak interaction coupling constant and M_N is the nucleon mass. Particular emphasis is put on questions of gauge invariance and of threshold behavior. We discuss our results and compare them with the literature. [S0556-2821(96)03821-0]

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I. INTRODUCTION

In a recent review of parity-violating (PV) radiative decays of hyperons [1], attention was focused on the purported breakdown of Hara's theorem in a nonrelativistic constituent quark model calculation [2]. Hara's theorem [3] is the statement that, in the limit of exact SU(3) symmetry, all parity-violating radiative hyperon decay matrix elements vanish. Although it is not directly applicable to the real world [with SU(3) breaking], it provides the necessary starting point on which to build more realistic approximations and an important benchmark to be used for testing of all model calculations. The claim of Hara's theorem violation in the constituent quark model was a serious surprise which has left the field with a deeply rooted mistrust of that model [1], at least in this application. That being no ordinary model, but rather the basic conceptual foundation underlying QCD, makes this discrepancy all the more important.

The purpose of the present paper is to show that Hara's theorem is valid in the nonrelativistic constituent quark model. The fault for the breakdown of Hara's theorem in Ref. [2] does not lie with the model, but rather with the gauge-variant calculation: Even though the starting point is a gauge-invariant relativistic quark-quark bremsstrahlung amplitude, its subsequent nonrelativistic reduction, as performed in Ref. [2], is incorrect; consequently, the calculation ends up violating gauge invariance and the correct threshold (on-shell photon point) behavior.

This is not the first time there has been trouble with maintaining the gauge invariance and the threshold behavior of parity-violating electromagnetic (EM) current nucleon matrix element of this kind: Exactly, the same type of Lorentz structure describes the *elastic* parity-violating nucleon matrix element of the EM current (the so-called anapole moment of the nucleon [4]) and exactly the same kinds of problems appear when one tries and calculates it in the nonrelativistic constituent quark model. In the latter case these problems were resolved in Ref. [5], so we will apply the methods developed there in order to try and resolve the Hara's theorem conundrum in the nonrelativistic constituent quark model. We will not try to extend the discussion here to the realistic case of hyperon radiative decays with SU(3) breaking.

II. MODEL-INDEPENDENT RESULTS

There are two independent relativistic parity-violating EM couplings (currents) describing the radiative transition of one baryon, say a hyperon, into another, degenerate, but otherwise distinguishable, baryon, e.g., a nucleon:

$$\langle N_2(p')|J_{\mu5}^{(\mathrm{I})}(0)|N_1(p)\rangle = F_1(q^2)\overline{u_2}(p') \times\left(\gamma_\mu - \frac{\not{q}}{q^2}q_\mu\right)\gamma_5 u_1(p) \quad (1)$$

and

$$\langle N_2(p')|J_{\mu5}^{(\mathrm{II})}(0)|N_1(p)\rangle = F_3(q^2)\overline{u_2}(p')(i\sigma_{\mu\nu}q^\nu)\gamma_5 u_1(p). \quad (2)$$

We use the Bjorken and Drell [6] conventions, in particular the metric has the signature (+ - - -). Each of these currents is separately gauge invariant. Absence of massless hadrons, according to Ref. [1], or simply gauge invariance according to Refs. [5,7], forces the first form factor $F_1(q^2)$ to vanish as $q^2 = q_0^2 - \mathbf{q}^2$ at the threshold, i.e., as $q^2 \to 0$. It is the failure of the first type of coupling, Eq. (1), to achieve the expected threshold behavior that invalidated Hara's theorem in earlier applications of the quark model to this reaction. The second coupling (2) is not constrained in this way, and hence may contribute for on-shell photons. Yet, in the exact SU(3) limit its contribution vanishes due to its symmetry properties (see Sec. 3.1 in [1]). The second coupling corresponds to type II operators, in the language of Ref. [2], which do not endanger Hara's theorem.

We will henceforth focus exclusively on type I operators. This type of coupling is not new, however: in the elastic scattering limit it corresponds to the parity-violating EM elastic matrix element, also known as the "anapole" moment. It never attracted the interest of the main-stream of the theoretical community, yet a few facts pertaining to it have been learned over the years. In particular, the anapole has been calculated in the constituent quark model to first order in perturbation theory in Ref. [5], where the troubles with achieving, rather than enforcing, the correct threshold behavior have also been met and resolved. This paper is to serve as

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an application of the lessons learned on the anapole to the weak radiative hyperon decay. We will work in the nonrelativistic limit, since that is the approximation in which the violation of Hara's theorem was reported. The nonrelativistic limit also corresponds to the long wavelength limit in this case. In the nonrelativistic limit, or in the Breit (brick-wall) frame, the matrix element equation (1) turns into

$$\mathbf{J}_5^{(\mathrm{I})} = F_1(q^2)[\boldsymbol{\sigma} - (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\hat{\mathbf{q}}], \tag{3}$$

where $F_1(-\mathbf{q}^2)$ is a function of $-\mathbf{q}^2 = q^2 = -Q^2$ with the expected long-wavelength limit:

$$F_1(-\mathbf{q}^2) = e\frac{G_\mathrm{F}}{\sqrt{2}}\mathbf{q}^2 H(\mathbf{q}^2), \tag{4}$$

where $H(\mathbf{q}^2)$ is a new parity-violating EM form factor which is regular (finite) at the threshold, i.e., $H(0) < \infty$, $G_F \simeq 10^{-5} M_N^{-2}$ is the Fermi weak interaction coupling constant, and M_N is the nucleon mass. We need to prove Eq. (4) in the constituent quark model.

While one could certainly start from a specific quark model and the definition of the parity-violating EM current matrix element, and then proceed to evaluate the form factor $F_1(q^2)$, it turns out that there is a general proof of the expected result, which is also a substantial simplification over any explicit calculation. This argument, first published by Serot [7], relies only on the multipole decomposition of the EM current matrix element and on the EM current conservation. The transverse nature of type I EM current operators is clearly visible in Eq. (3). Any transverse EM current matrix element can be decomposed

$$\int d\mathbf{R}\ \mathbf{J}_{fi}\cdot\hat{\varepsilon}_\lambda \exp(i\mathbf{q}\cdot\mathbf{R}) = -\sum_{J=1}^{\infty}\sqrt{2\pi(2J+1)}\, i^J[\lambda\langle\Psi_f|\hat{T}_{J\lambda}^{\mathrm{mag}}|\Psi_i\rangle + \langle\Psi_f|\hat{T}_{J\lambda}^{\mathrm{el}}|\Psi_i\rangle] \tag{5}$$

into transverse electric and magnetic current multipoles $\langle\Psi_f|\hat{T}_{J\lambda}^{\mathrm{el}}|\Psi_i\rangle, \langle\Psi_f|\hat{T}_{J\lambda}^{\mathrm{mag}}|\Psi_i\rangle$ with well-defined rotation-, parity-, and time-reversal properties [8]. The latter properties are used to eliminate, in this case, all but one multipole: the transverse electric dipole

$$\int d\mathbf{R}\ \mathbf{J}_{fi}\cdot\hat{\varepsilon}_\lambda \exp(i\mathbf{q}\cdot\mathbf{R}) = -i\sqrt{6\pi}\langle\Psi_f|\hat{T}_{1\lambda}^{\mathrm{el}}|\Psi_i\rangle, \tag{6}$$

where

$$\langle\Psi_f|\hat{T}_{1\lambda}^{\mathrm{el}}|\Psi_i\rangle = \frac{1}{i|\mathbf{q}|\sqrt{2}}\int d\mathbf{R}\ \{-\mathbf{q}^2(\mathbf{J}_{fi}\cdot\mathbf{R}) + (\nabla\cdot\mathbf{J}_{fi}) \times[1+(\mathbf{R}\cdot\boldsymbol{\nabla})]\} j_1(|\mathbf{q}|R) Y_{1M}(\hat{\mathbf{R}}), \tag{7}$$

and $\mathbf{J}_{fi} = \langle\Psi_f|\mathbf{J}|\Psi_i\rangle$ is the *exact* conserved, elastic parity-violating EM current matrix element, j_1 is a spherical Bessel function, and Y_{1M} is a spherical harmonic.[1] In order to ascertain the threshold properties of this multipole, we note that $j_1(|\mathbf{q}|R) \sim \frac{1}{3}|\mathbf{q}|R$ in the long-wavelength limit. Hence, the first and second terms on the right-hand side of Eq. (7) are in agreement and in conflict, respectively, with Eq. (4). In order to show that the offending term vanishes just use the EM current conservation

$$\boldsymbol{\nabla}\cdot\mathbf{J}_{fi} = -i\langle\Psi_f|[H,\rho]|\Psi_i\rangle = i(\mathrm{E}_i - \mathrm{E}_f)\rho_{fi}, \tag{8}$$

where $\rho_{fi} = \langle\Psi_f|\rho|\Psi_i\rangle$ is the associated charge density and $\mathrm{E}_{i,f}$ are the *exact* energy eigenvalues. Since we are working in the Breit frame, and despite our initial and final states not being identical (this is hyperon *decay* we are talking about), they are degenerate $E_i = E_f$, as a consequence of the assumed *exact* SU(3) symmetry. This means that the offending term is exactly zero, as announced earlier.

Thus, we have proven that the odd-parity, time-reversal-conserving, transverse electric dipole vanishes as $\mathbf{q}^2 = -q^2$ in the long-wavelength limit and thence that Hara's theorem holds. The above-shown proof is exact and model independent; hence, it ought to hold in the constituent quark model, too. The three crucial assumptions are (i) EM current conservation, and (ii) the use of *exact* nucleon wave functions, and (iii) the SU(3) symmetry is *exact*. If this theorem fails in some explicit calculation, it has to be because one or more of these three assumptions is violated. In the rest of this paper we will explore one scenario in which that possibility may actually occur: the perturbative treatment of the weak interaction. That is, as we shall show below, exactly as the situation in Ref. [2].

III. THE CONSTITUENT QUARK MODEL

In Ref. [2] the gauge-invariant set of four Feynman diagrams (Fig. 1) describing the so-called quark-quark bremsstrahlung in the Fermi interaction limit of the Salam-Weinberg model are used as the underlying theoretical model. These diagrams describing a relativistic transition (decay) amplitude were then reduced to a two-body (quark)[2] nonrelativistic PV EM interaction Hamiltonian (and a corresponding PV EM current), which apparently violates Hara's theorem. This last conclusion is unwarranted due to an incomplete analysis: Having made the nonrelativistic reduction of one set of operators, we must be careful, for the sake of consistency, to include all nonrelativistic contributions to the given order in p/m expansion in the model calculation. In particular, the aforementioned, two-body PV EM current corresponds to the so-called Z graphs, with negative energy

[1] Equation (7) may seem nonrelativistic due to the presence of three-momentum transfer and a three-dimensional scalar product, but, in the Breit frame, where the four-momentum transfer retains only its three-vector part, it is Lorentz covariant.

[2] In our nomenclature an operator describing say bremsstrahlung from a pair of quarks, which involves two (quark) creation and two destruction operators, is called a two-body, or two-quark operator. The same operator, in Kamal and Riazuddin's language [2] is called a four-quark operator. Analogous relation holds between the one- and two-quark operators in the two conventions.

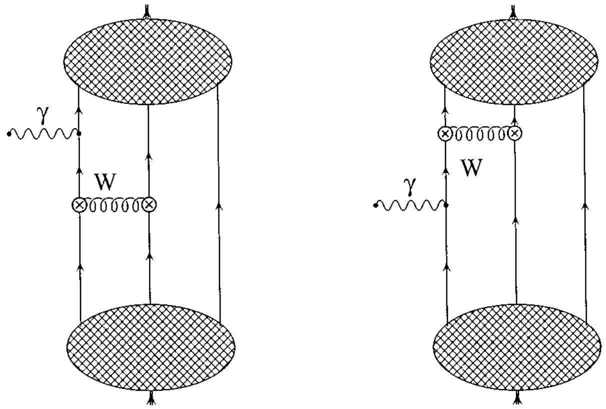

FIG. 1. Two of four Feynman diagrams describing the parity-breaking contribution to the EM axial-vector currents. The solid line denotes the quarks, the wavy line is the photon, and wiggly line is the charged intermediate vector boson $W^{\pm}$. The shaded blob together with the three solid lines and one double solid line leading to it denotes the nucleon wave function. The remaining two graphs have their photon lines attached to the ''other'' quark line.

intermediate states, in the time-ordered perturbation theory. There is another set of time-ordered graphs, however, to the same order in perturbation theory, which corresponds to terms with positive energy intermediate states. These latter terms are due to the ordinary one-body EM current and the PV admixtures in the hyperon and nucleon wave function induced by the action of the PV quark-quark potential. They were completely neglected in Ref. [2]. We will show that in a completely consistent analysis the one-body current contributions exactly cancel the offending two-body current terms, due to gauge invariance. Hence, we review the salient features of EM current conservation in the constituent quark model [9,10].

A. Conservation of the nuclear electromagnetic current

In configuration space the conservation of the electromagnetic current is reflected in the validity of the continuity equation

$$\nabla_{\mathbf{R}} \cdot \mathbf{J}(\mathbf{R}) \equiv \nabla \cdot \mathbf{J} = -\frac{\partial \rho(\mathbf{R})}{\partial t}. \tag{9}$$

In quantum mechanics, this can be written as an equation relating the divergence of the three current and the commutator of the Hamiltonian and the charge density:

$$\nabla \cdot \mathbf{J} = -i[H, \rho(\mathbf{R})], \tag{10}$$

where the total Hamiltonian

$$H = H_0^{\mathrm{PC}} + V^{\mathrm{PV}}$$

is the sum of the parity-conserving (PC) Hamiltonian H^{PC} [which equals the kinetic $T=\Sigma_{i=1}^{3}\mathbf{p}_i^2/2m_i$ plus the PC potential $V_{2-b}^{\mathrm{PC}}=\Sigma_{i<j}^{3}V_{2-b}^{\mathrm{PC}}(ij)$ energy] and the parity-violating potential $V_{2-b}^{\mathrm{PV}}=\Sigma_{i<j}^{3}V_{2-b}^{\mathrm{PV}}(ij)$ of the interacting system, and $\rho(\mathbf{R})$ is the charge density. In the following we will drop the ''label'' $\mathbf{R}$ in all EM currents and densities, except when necessary to avoid confusion. The one-body electromagnetic current is the usual sum of nonrelativistic convection and magnetization currents, which in configuration space reads

$$\begin{aligned}\mathbf{J}_{1-b}(i) &= \mathbf{J}_{1-b}^{\mathrm{c}}(i) + \mathbf{J}_{1-b}^{\mathrm{m}}(i) \\ &= \frac{e(i)}{2m_q}[\{\mathbf{p}_i, \delta(\mathbf{R}-\mathbf{r}(i))\} - \nabla_{\mathbf{R}} \\ &\quad \times \boldsymbol{\sigma}(i)\,\delta(\mathbf{R}-\mathbf{r}(i))]\end{aligned} \tag{11}$$

and, in momentum space,

$$\mathbf{J}_{1-b}(i) = \frac{e(i)}{2m_q}[(\mathbf{p}_i + \mathbf{p}_i') + i\boldsymbol{\sigma}(i) \times \mathbf{q}], \tag{12}$$

where $\mathbf{q}=\mathbf{p}_i'-\mathbf{p}_i$. The charge density $\rho(\mathbf{R})$ is equivalent to the one-body charge density

$$\rho_{1-b}(\mathbf{R}) = \sum_{i=1}^{3} e(i)\,\delta(\mathbf{R}-\mathbf{r}(i)),$$

where $e(i)=\frac{1}{2}[\frac{1}{3}+\tau_3(i)]$. Then, the divergence of the one-body electromagnetic current equals $-i$ times the commutator of the kinetic energy and the one-body charge density (we set $\hbar=1$):

$$\nabla \cdot \mathbf{J}_{1-b} = -i[T, \rho_{1-b}(\mathbf{R})]. \tag{13}$$

Thus, the simplest test of electromagnetic current density conservation is whether or not the two-body potential commutes with the one-body charge density. If not, then one needs a two-body electromagnetic current density[3] $\mathbf{J}_{2-b}$ to compensate for the induced temporal change of charge density:

$$\nabla \cdot \mathbf{J}_{2-b} = -i[V_{2-b}, \rho(\mathbf{R})]. \tag{14}$$

The complete potential is, of course, the sum of the parity-conserving and the parity-violating ones. Similarly, the two-body electromagnetic current density is a sum of a polar vector (parity-conserving) and an axial (parity-violating) vector terms $\mathbf{J}_{2-b}=\mathbf{J}_{2-b}^{\mathrm{PC}}+\mathbf{J}_{2-b}^{\mathrm{PV}}=\Sigma_{i<j}^{3}\mathbf{J}_{2-b}(ij)$. Each one of these must satisfy its own EM current conservation equation; one does not normally assume PC forces that exchange the electric charge[4] between two quarks, so we may set all PC two-body currents equal to zero. The PV strangeness-changing weak potential V_{2-b}^{PV} between two quarks is also (electric) charge changing, so we know that there will *have* to be some PV two-body currents $\mathbf{J}_{2-b}^{\mathrm{PV}}$ to account for that

$$\nabla \cdot \mathbf{J}_{2-b}^{\mathrm{PV}} = -i[V_{2-b}^{\mathrm{PV}}, \rho(\mathbf{R})]. \tag{15}$$

It is manifest that one can only recover Eq. (10) by adding Eqs. (13) and (15), i.e., *if the latter two are valid.* It is an immediate corollary that no calculation which employs only

[3]There is a vast literature on such two-body currents in nuclear physics where they are called meson-exchange currents [11].

[4]This is manifestly a model-dependent assumption, yet even if it were not true our proof would hold, only slightly more complicated.

the one-body, or only the two-body current for that matter, can be gauge invariant, i.e., neither satisfies Eq. (10).

Since Ref. [2] did not include the one-body EM current contribution, the above argument explains their breaking of EM current conservation and, consequently, of Hara's theorem. A number of authors did include the one-body EM current contributions, within the quark model version of the (modern) ''pole model'' [12–14],[5] but some of them omitted the PV two-body EM current. Yet, even calculations with the complete EM current *operator* may still have trouble maintaining the conservation of the EM current *matrix element*, which is the necessary condition of Hara's theorem.

The resolution of this puzzle lies in the fact that the baryon wave functions have entered our considerations by way of the second line in Eq. (8). Even if the EM current *operator* satisfies Eq. (10), it need not be enough to ensure the conservation of the EM current *matrix element*, and with it the validity of Hara's theorem, since the second line in Eq. (8) is only satisfied when the initial and final wave functions are the *exact* eigenstates of the total Hamiltonian H. The task of exactly solving the full three-body Schrödinger equation seems excessive for the purpose at hand, since the PV interactions are weak and hence tractable in first order perturbation theory: that is the approach adopted in Ref. [12]. In the following we will show that a necessary condition for the conservation of an EM current matrix element in perturbation theory is the retention of *all* excited (''intermediate'') states in the admixtures to the initial and final wave functions. Since most practical perturbative calculations keep only the lowest-lying ones, they are usually *not* gauge invariant, one exception being Ref. [12].

B. First-order perturbation theory

The PV weak interaction between quarks implies the existence of a small, $O(G_F)$, abnormal-parity, strangeness-violating admixture in the baryon (hyperon or nucleon) wave function. To determine this admixture, we shall use the first-order, time-independent (Rayleigh-Schrödinger) perturbation approximation to the quark dynamics described by the Hamiltonian $H=H_0^{\mathrm{PC}}+V^{\mathrm{PV}}$. The ground state of the nucleon or hyperon $|\Psi_0\rangle$ is given by

$$|\Psi_0\rangle=|\Phi_0\rangle+\sum_{n\neq 0}|\Phi_n\rangle\frac{\langle\Phi_n|V^{\mathrm{PV}}|\Phi_0\rangle}{E_0-E_n}+O(G_F^2) \quad (16a)$$

$$=|\Phi_0\rangle+\sum_{n\neq 0}\varepsilon_{n0}|\Phi_n\rangle+O(G_F^2), \quad (16b)$$

where $|\Phi_n\rangle$ are the *exact* eigenstates of the PC Hamiltonian H_0^{PC}: $H_0^{\mathrm{PC}}|\Phi_n\rangle=E_n|\Phi_n\rangle$, and

$$\varepsilon_{na}=\frac{\langle\Phi_n|V^{\mathrm{PV}}|\Phi_a\rangle}{E_a-E_n}=O(G_F) \quad (17)$$

are the admixture coefficients to the baryon a. The abnormal-parity admixtures in the initial and final wave functions generate a parity-violating electromagnetic current hyperon-nucleon matrix element equation (1). Since the PV potential is of $O(G_F)$, Eq. (15) demands that the PV two-body electromagnetic current also be of $O(G_F)$. Hence, to $O(G_F)$, the EM current matrix element reads

$$\mathbf{J}_{fi}=\langle\Phi_f|\mathbf{J}_{2-b}^{\mathrm{PV}}|\Phi_i\rangle+\sum_{n\neq i,f}\{\langle\Phi_f|\mathbf{J}^{\mathrm{PC}}|\Phi_n\rangle\varepsilon_{ni}+\varepsilon_{nf}^{*}\langle\Phi_n|\mathbf{J}^{\mathrm{PC}}|\Phi_i\rangle\}. \quad (18)$$

C. Conservation of the electromagnetic current in first-order perturbation theory

Equation (8) ought to hold order by order in perturbation theory, or equivalently, in the expansion in the weak coupling constant G_F. We shall now repeat the crucial argument which has already been presented in Ref. [5], in order to explicitly demonstrate the significance of keeping all intermediate states in the calculation. To evaluate the divergence start from Eq. (18):

$$\begin{aligned}\nabla\cdot\mathbf{J}_{fi}&=\nabla\cdot\langle\Phi_f|\mathbf{J}_{2-b}^{\mathrm{PV}}|\Phi_i\rangle+\sum_{n\neq i,f}\{\nabla\cdot\langle\Phi_f|\mathbf{J}^{\mathrm{PC}}|\Phi_n\rangle\varepsilon_{ni}\\&\quad+\varepsilon_{nf}^{*}\nabla\cdot\langle\Phi_n|\mathbf{J}^{\mathrm{PC}}|\Phi_i\rangle\}\\&=-i\langle\Phi_f|[V^{\mathrm{PV}},\rho]|\Phi_f\rangle+\sum_{n\neq i,f}\{\langle\Phi_f|[H_0^{\mathrm{PC}},\rho]|\Phi_n\rangle\varepsilon_{ni}\\&\quad+\varepsilon_{nf}^{*}\langle\Phi_n|[H_0^{\mathrm{PC}},\rho]|\Phi_i\rangle\}.\end{aligned}$$

Now, use the definition of the admixture coefficients ε_n, Eq. (17), with $a=i,f$; the fact that the unperturbed states are eigenfunctions of H_0^{PC}, implies that

$$\langle\Phi_f|[H_0^{\mathrm{PC}},\rho]|\Phi_n\rangle=(E_f-E_n)\langle\Phi_f|\rho|\Phi_n\rangle,$$

$$\langle\Phi_n|[H_0^{\mathrm{PC}},\rho]|\Phi_i\rangle=(E_n-E_i)\langle\Phi_n|\rho|\Phi_i\rangle$$

which, in turn, leads to

$$\begin{aligned}\nabla\cdot\mathbf{J}_{fi}&=\nabla\cdot\langle\Phi_f|\mathbf{J}_{2-b}^{\mathrm{PV}}|\Phi_i\rangle-i\sum_{n\neq f}(E_i-E_n)\\&\quad\times\langle\Phi_f|\rho|\Phi_n\rangle\frac{\langle\Phi_n|V^{\mathrm{PV}}|\Phi_i\rangle}{E_f-E_n}-i\sum_{n\neq i}\frac{\langle\Phi_f|V^{\mathrm{PV}}|\Phi_n\rangle}{E_i-E_n}\\&\quad\times(E_n-E_f)\langle\Phi_n|\rho|\Phi_i\rangle\\&=-i\langle\Phi_f|[V^{\mathrm{PV}},\rho]|\Phi_i\rangle-i\sum_{n\neq f}\langle\Phi_f|\rho|\Phi_n\rangle\langle\Phi_n|V^{\mathrm{PV}}|\Phi_i\rangle\\&\quad+i\sum_{n\neq i}\langle\Phi_f|V^{\mathrm{PV}}|\Phi_n\rangle\langle\Phi_n|\rho|\Phi_i\rangle+i(E_i-E_f)\\&\quad\times\langle\Phi_f|\rho|\Phi_i\rangle+i(E_i-E_f)\sum_{n\neq fi}\{\langle\Phi_f|\rho|\Phi_n\rangle\varepsilon_{ni}\\&\quad+\varepsilon_{nf}^{*}\langle\Phi_n|\rho|\Phi_i\rangle\}.\end{aligned} \quad (19)$$

The terms explicitly excluded from the sums in the first line of Eq. (19) are identically zero, due to the good parity of

[5]For example, Gavela *et al.* [12] explicitly included the contributions of the low-lying negative parity baryon multiplet (**70**,1^-). That turns out to be sufficient for maintaining Hara's theorem due to the simplicity of harmonic oscillator wave functions employed there.

unperturbed states $|\Phi_n\rangle$. Hence, the sums can be extended over *all* intermediate states, which form a complete set $\Sigma_n|\Phi_n\rangle\langle\Phi_n|=1$. The second line of Eq. (19) can be further simplified using Eq. (16b) for the initial and final states, leading to

$$\begin{aligned}\nabla\cdot\mathbf{J}_{fi}&=-i\langle\Phi_f|[V^{\rm PV},\rho]|\Phi_i\rangle-i\langle\Phi_f|\rho V^{\rm PV}-V^{\rm PV}\rho|\Phi_i\rangle\\&\quad+i(E_i-E_f)\langle\Psi_f|\rho|\Psi_i\rangle+O(G_F^2)\\&=i(E_i-E_f)\langle\Psi_f|\rho|\Psi_i\rangle+O(G_F^2)\end{aligned}\tag{20}$$

which, together with the degeneracy of nucleons and hyperons, $E_i=E_f$, in the SU(3) limit, leads to the final result

$$\nabla\cdot\mathbf{J}_{fi}=0.\tag{21}$$

Note that Eq. (8) differs from Eq. (20) only in that it involves the *exact* eigenenergies $\mathrm{E}_{i,f}$ which are also degenerate in the exact SU(3) limit,[6] rather than the perturbative ones $E_{i,f}$, i.e., to $O(G_F)$, the two equations coincide, as they should. This completes the proof that type I operators do *not* contribute to the parity-violating radiative hyperon decay to $O(G_F)$ in the exact SU(3) limit, in accord with Hara's theorem, provided that the sum over *all* intermediate states is complete.

IV. SUMMARY AND CONCLUSIONS

To summarize, we have shown how the correct threshold behavior, in the exact SU(3) limit of the constituent quark model, of the radiative PV hyperon decay amplitude is in agreement with Hara's theorem as long as the EM current matrix element is exactly conserved. This means not only that the EM current operator satisfies the continuity equation, but also that the exact wave functions are used. We provided several examples of tacit and explicit approximations that can lead to the violation of EM current conservation and hence to the breakdown of Hara's theorem. For example, Hara's theorem is violated in explicit perturbative calculations when the EM current operator is not conserved, and/or if the parity-violating admixtures in the wave functions are not properly included. So, the basic moral of this work is the understanding that one can neither omit the PV two-body current, nor abbreviate the expansion in excited states with impunity when dealing with parity-violating electromagnetic current matrix elements within perturbation theory.

Now, that the fundamental mathematical difficulty has been resolved, we can turn to the task of relaxing the assumption of good SU(3) symmetry and evaluating the finite contributions to the radiative hyperon decay matrix elements in this simple quark model. Should those results be found insufficiently close to the experiment, one may consider other physical mechanisms, such as the meson ''cloud,'' which might also contribute. It is essential not to lose sight of the fact that we have not tested any such model-dependent mechanism for Hara's theorem. Proving Hara's theorem in the exact SU(3) limit ought to be the first order of business when contemplating such additional mechanisms, so as to avoid false paradoxes.

[6]Strictly speaking, this is a self-contradictory statement, since the weak interactions break the flavor SU(3) by construction. Hence, Hara's assertion is a self-consistent theorem only to lowest order in perturbation theory, i.e., as proved above.

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