Spin dependence of the masses of heavy baryons

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It is argued from the systematics of spin-dependent forces between quarks that two proposed baryon states, named $\Sigma_c(2380)$ and $\Sigma_b(5760)$, do not exist. [S0556-2821(96)01421-X]

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Recently, Falk [1] has proposed that there exist two undiscovered heavy baryons: a $\Sigma_c(2380)$ that decays radiatively to the Λ_c, and a $\Sigma_b(5760)$ that decays radiatively to the Λ_b. In this Brief Report I use the systematics of the spin-dependent forces between quarks to argue against the existence of these two new states. The same ideas suggest that two of the proposed ''equal spacing rules'' [1] among heavy baryon mass differences should be replaced by inequalities.

Tensor and spin-orbit forces do not contribute perturbatively to the masses of ground-state baryons and so I confine myself to baryons without radial or orbital excitations. Then only the spin-spin interaction (sometimes called the color-magnetic or color-hyperfine interaction) survives in the perturbative approximation. More detailed discussions of these points have been given previously [2,3].

In addition to treating baryon mass splittings, I also consider the spin-dependent splittings of meson masses, as under certain assumptions there are inequalities relating meson and baryon mass differences. I neglect mass splittings among isospin multiplets, which means neglecting electromagnetic effects and the mass difference between the d and u quark. Also, as is often done, I let the symbol for a hadron denote its mass, averaged over isospin states if more than one exists.

In order to motivate certain inequalities among matrix elements of the spin-spin interaction, it is convenient to assume [2,3] that the form of the spin-spin interaction I_S between quarks in a baryon or between a quark and an antiquark in a meson has the form

$$I_S = -3\sum_{i<j} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j f(r_{ij})/(8m_i m_j), \tag{1}$$

where i and j denote quarks (or antiquarks), the λ_i and λ_j are color Gell-Mann matrices, the σ_i and σ_j are Pauli spin matrices, and $f(r_{ij})$ is a positive definite function of the distance between the two particles. The factor 3/8 is chosen for convenience. In the Fermi-Breit approximation to one-gluon exchange in QCD, the spin-spin interaction is a special case of this form: namely,

$$f(r_{ij}) = 4\pi\alpha_s \delta(r_{ij})/9. \tag{2}$$

It should be stressed that it is not necessary to assume the validity of Eq. (1). The inequalities among hadron masses may alternatively be obained from the systematics of the observed spin splittings of hadrons containing only light quarks. The essential point of this paper is that it is very reasonable that analogous inequalities should also hold for hadrons containing heavy quarks.

The expectation values for the color and spin operators in I_S can be taken explicitly [3]. The expectation value of the spatial operator can be given in terms of quantities R_{ij} for mesons and R_{ijk}, R_{ikj}, and R_{kji} for baryons. For mesons,

$$R_{ij} = 2\langle ij|f(r_{ij})|ij\rangle/(m_i m_j), \tag{3}$$

where $|ij\rangle$ is the unperturbed meson spatial wave function. For baryons,

$$R_{ijk} = \langle ijk|f(r_{ij})|ijk\rangle/(m_i m_j),$$

$$R_{ikj} = \langle ijk|f(r_{ik})|ijk\rangle/(m_i m_k),$$

$$R_{jki} = \langle ijk|f(r_{jk})|ijk\rangle/(m_j m_k), \tag{4}$$

where $|ijk\rangle$ is the unperturbed baryon spatial wave function. The ordering of the quarks in $|ijk\rangle$ is important here [4]. If all three quarks are different, the two lightest are the first two; if two are identical, these are the first two. Although the operator $f(r_{ij})$ for baryons is a two-quark operator, the expectation values R_{ijk}, etc. depend also on the third or ''spectator'' quark through the three-quark wave function. The R_{ijk} are symmetric under the interchange of their first two indices,

$$R_{ijk} = R_{jik}, \tag{5}$$

but, in general, (all quarks different),

$$R_{ijk} \neq R_{ikj} \neq R_{jki}. \tag{6}$$

However, because of the neglect of the mass difference between the u and d quarks, if i is a u quark and j is a d quark, then $R_{ukd} = R_{dku}$. From here on, I denote both u and d quarks by the symbol q.

Let M_{12}^* and M_{12} denote ground-state vector and pseudoscalar mesons containing quark 1 and antiquark 2. For baryons, if all three quarks have different flavors, let B_{123}^* denote the baryon of spin 3/2 containing quarks 1, 2, and 3, and B_{123}, and B_{123}' denote two different spin 1/2 baryons. The baryons B and B' are distinguished by the spin of the two lightest quarks 1 and 2; in B these quarks have spin 0 and in B' they have spin 1. If two quarks in the baryon have the same flavor, they are 1 and 2, and the state B is absent. If all three quarks have the same flavor, both B and B' are absent.

The R_{ij} and R_{ijk} contribute to the masses of these ground-state mesons and baryons as follows [3]:

$$M_{12}^{*}=E_{12}+R_{12}, \quad M_{12}=E_{12}-3R_{12}, \tag{7}$$

$$B_{123}^{*}=E_{123}+R_{123}+R_{132}+R_{231},$$

$$B_{123}'=E_{123}+R_{123}-2R_{132}-2R_{231},$$

$$B_{123}=E_{123}-3R_{123}, \tag{8}$$

where E_{12} and E_{123} are the meson and baryon masses in the absence of the spin-dependent force.

The forms of Eqs. (3) and (4) suggest the following inequalities [2] among the R_{ij} and R_{ijk},

$$R_{ij}>R_{il}, \quad R_{ijk}>R_{iln} \text{ if } m_j<m_l, \tag{9}$$

because the quark masses appear in the denominator in Eqs. (3) and (4). Furthermore, because the expression for R_{ij} in Eq. (3) contains a factor 2 compared to the expression for R_{ijk} in Eq. (4), it is plausible that

$$R_{ij}>2R_{ijk}. \tag{10}$$

Likewise, it is plausible that the inequality

$$R_{ijk}<R_{ijl} \text{ if } m_k<m_l, \tag{11}$$

holds [2]. The inequalities (10) and (11) should be valid for any function $f(r_{ij})$ of sufficiently short range, as the following argument shows: Because the short-range part of the quark-antiquark potential (arising from one-gluon exchange) is twice as large for mesons as for baryons, the meson wave function is not as spread out in space as is the baryon wave function. Because both wave functions are normalized to unity, the meson wave function must be larger than the baryon wave function at small spatial separations where $f(r_{ij})$ is large, so that

$$\langle ij|f(r_{ij})|ij\rangle>\langle ijk|f(r_{ij})|ijk\rangle. \tag{12}$$

Using this result in Eqs. (3) and (4), I obtain the inequality (10). The inequality (11) follows from the fact that for potentials such as the quark-quark potential, the radial extent of a two-particle wave function decreases as the reduced mass increases. This is principally a kinematic effect. If the mass of the spectator quark k is increased, the quarks i and j are pulled closer to k and, consequently, to each other. This reduces the radial extent of the wave function and, therefore, increases $\langle ijk|f(r_{ij})|ijk\rangle$, from which the inequality (11) follows. However, in the limit of heavy quark effective theory, i.e., k and l are considered to be infinitely heavy, the inequality (11) becomes an equality [1] because the reduced mass of either quark i or j is just its actual mass.

The R's can be obtained from Eqs. (7) and (8) in terms of the observed hadron masses:

$$R_{12}=(M_{12}^{*}-M_{12})/4, \tag{13}$$

$$R_{123}=(2B_{123}^{*}+B_{123}'-3B_{123})/12, \tag{14}$$

$$(R_{132}+R_{231})/2=(B_{123}^{*}-B_{123}')/6. \tag{15}$$

If quarks 1 and 2 are either identical or u and d quarks, respectively, then $R_{132}=R_{231}$. Now, Eqs. (13)–(15) may be taken as the definitions of the R's, and the inequalities (9)–(11) may be postulated to hold independently of the validity of the interaction (1). The inequalities may then be tested with the data.

In Ref. [3] the observed meson and baryon masses were used to obtain values of some of the R_{ij} and R_{ijk}. It is useful to repeat this procedure using Eqs. (13)–(15) with the new data that are available [5–8]. It should be noted, however, that some of the new baryon data are preliminary.

For mesons, I use the data from the Particle Data Group [5]. The results for mesons are (in MeV)

$$R_{qq}=158, \quad R_{qs}=99.5, \quad R_{qc}=35.5, \quad R_{sc}=35.4,$$

$$R_{cc}=29.2, \quad R_{qb}=11.5, \tag{16}$$

where the experimental errors are less than 1 MeV. Missing from Eqs. (16) is R_{ss} because neither the η nor the η' is a pure $\overline{ss}$ state. Also missing are R_{cb} and R_{bb} because of the absence of data. These results have changed little from those given in Ref. [3]. These values of R_{ij} satisfy all the meson inequalities, except that, within the errors, $R_{qc}=R_{sc}$. This fact may indicate that the shrinking of the wave function when q is replaced by s compensates for the replacement of m_q by m_s in the denominator of R_{ij}.

For baryons, I use the same data that Falk [1] used in his Table II, with two exceptions. Note that his paper should be consulted for the experimental references. The first exception is that I assign the Ξ_c' the mass 2573±4 MeV [8] as the error is smaller than the error in Falk's reference. The second exception is that, in addition to using the DELPHI data given by Feindt [7], as Falk does, I also use the earlier DELPHI data quoted by Jarry [6].

First, I use the conventional mass assignments for Σ_c. The results for baryons are (in MeV)

$$R_{qqq}=48.8, \quad R_{qqs}=51.2, \quad R_{qsq}=32.0, \quad R_{qss}=35.8,$$

$$R_{qqc}=54.8\pm1.2, \quad R_{qcq}=12.8\pm1.2, \quad R_{qqb}=52.6\pm2,$$

$$R_{qbq}=9.3\pm1.4, \quad R_{qsc}=38.1\pm1,$$

$$(R_{qcs}+R_{scq})/2=11.8\pm1, \tag{17}$$

where errors less than 1 MeV are omitted. In some instances, I have added statistical and systematic errors in quadrature. Other R_{ijk} are missing either because of absence of data or because Eqs. (14) and (15) are not sufficient to compute them.

Although in a few cases, the central values of the R_{ij}, R_{ijk}, and R_{ikj} do not satisfy the inequalities (9)–(11), these quantities do satisfy the inequalities within the errors except that R_{qbq} is too large by about three standard deviations. On the other hand, if instead of using the DEPHI data [7] quoted by Falk, I use the earlier DELPHI data quoted by Jarry [6]. $R_{qbq}=4.1\pm0.4$ MeV, a value which satisfies the inequalities, but $R_{qqb}=51.9\pm2$, a value which is in marginally greater disagreement with the inequalities. The DELPHI data are still preliminary. In the limit of heavy quark effective theory, $R_{qqc}=R_{qqb}$. It can be seen from the values of these quantities given in Eq. (17) that, within experimental error, the heavy quark limit has been reached.

I expect that more precise experiments will confirm all the inequalities of this paper. If not, it would mean that the systematics of the spin splittings which hold for hadrons containing only light quarks do not carry over to hadrons containing heavy quarks. It is, of course, premature to speculate on the possible reason for such a hypothetical departure of baryon masses from the regularities noted here.

On the other hand, with Falk's new assignments, I obtain (in MeV)

$$R_{qqc}=35.9\pm 1, \quad R_{qcq}=12.2\pm 1,$$
$$R_{qqb}=40.2\pm 2, \quad R_{qbq}=6.0\pm 1.4. \tag{18}$$

A comparison with R_{qqq}=48.8 MeV shows that the values of R_{qqc} and R_{qqb} grossly violate the inequality (11), a fact that leads me to the conclusion that the postulated states $\Sigma_c(2380)$ and $\Sigma_b(5760)$ do not exist.

I now turn to the mass equalities given in Falk's paper [1]. Two of these equalities are

$$\Sigma_c^* - \Sigma_c = \Omega_c^* - \Omega_c, \tag{19}$$

$$(2\Sigma_c^* + \Sigma_c)/3 - \Lambda_c = (2\Xi_c^* + \Xi_c')/3 - \Xi_c. \tag{20}$$

It can be seen from the inequalities satisfied by the R_{ijk}, etc. that Eqs. (19) and (20) get replaced by

$$\Sigma_c^* - \Sigma_c > \Omega_c^* - \Omega_c, \tag{21}$$

$$(2\Sigma_c^* + \Sigma_c)/3 - \Lambda_c > (2\Xi_c^* + \Xi_c')/3 - \Xi_c. \tag{22}$$

The inequality (21) cannot be tested at present because the Ω_c^* has not been observed. The inequality (22) is satisfied with the conventional assignments for the Σ_c and Σ_c^* but violated for Falk's assignments.

One of the main motivations for Falk's new assignments is that the equation

$$(\Sigma_b^* - \Sigma_b)/(\Sigma_c^* - \Sigma_c) = (B^* - B)/(D^* - D), \tag{23}$$

which follows from heavy quark effective theory, is badly violated with the conventional assignments. The left side is 0.73±0.13 while the right side is 0.33. The problem is that the mass difference $\Sigma_b^* - \Sigma_b$ is 56±8 MeV experimentally [7], a big change from the earlier experimental value of 25±5 MeV [6], whereas from heavy quark effective theory it should be 25 MeV. The situation is not much better with respect to my inequalities. From inequality (10) together with Eqs. (15) and (16), I obtain

$$\Sigma_b^* - \Sigma_b < 35 \text{ MeV}, \tag{24}$$

a value consistent with that given by heavy quark effective theory, but inconsistent with the most recent data [7]. Because the Σ_b and Σ_b^* data are preliminary, not even being in the full listings of the Particle Data Group [5], I think that the discrepancy between theory and experiment will eventually go away. The data on Σ_c, however, are well confirmed [5] and are consistent with the inequalities of this paper. Because Falk chooses to take the data on Σ_b and Σ_b^* seriously, he arrives at a value for the mass of the Σ_c which seriously violates an inequality of this paper.

Although I have arrived at the inequalities by considering a spin-spin interaction of the form (1) with short-range $f(r_{ij})$, this is really not necessary. The observed pattern of spin splittings in mesons and in baryons containing only light quarks is such as to satisfy all the inequalities of this paper. All that is really needed is the assumption that the pattern persists in heavy baryons.

In conclusion, if the heavy baryons have their conventional spin assignments, then inequalities in spin-dependent mass splittings which are satisfied for hadrons containing only light quarks are also satisfied for observed baryons containing heavy quarks. (There is an exception involving b baryons at the three standard deviation level.) However, if the heavy baryons are given Falk's new assignments, some of the inequalities of this paper are seriously violated not only for b baryons, where the data are preliminary, but also for c baryons, which are better measured.

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