PHYSICAL REVIEW D VOLUME 54, NUMBER 9 1 NOVEMBER 1996

Higgs boson mass bounds separate models of electroweak symmetry breaking

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(Received 5 December 1995)

Vacuum stability and metastability imply lower limits on the mass of the Higgs boson in the standard model (SM). In contrast, we present an improved calculation of the lightest Higgs boson mass in supersymmetric (SUSY) models, by summing to all orders in perturbation theory the leading and next-to-leading logarithms with a renormalization group equation technique, and by including finite two-loop QCD corrections. We believe our result to be the most accurate available in the literature. The mass calculation leads to an upper bound on the Higgs boson mass when the SUSY-breaking scale is sensibly restricted to $\lesssim 1$ TeV. In particular, our improvements to the SUSY Higgs boson mass calculation lower the minimal SUSY standard model (MSSM) upper limit by about 10 GeV. We study the possibility that these SM and MSSM bounds do not overlap, in which case a single Higgs boson mass measurement will distinguish between the two models. We find the following: (i) A gap emerges between the SM Higgs boson and the lightest MSSM Higgs boson at ~ 120 GeV for $m_t \sim 175$ GeV and $\alpha_s(M_Z^2)=0.118$, and for $m_t \sim 180$ GeV and more generous values $\sim(0.130)$ of α_s, and between the SM and the minimal plus singlet SUSY model [$(M+1)$SSM] Higgs bosons if the independent scalar self-coupling of the latter is perturbatively small or if the $\tan\beta$ parameter is small; these mass gaps widen with increasing m_t; (ii) the mass gap emerges with m_t 10 GeV lighter if only vacuum stability and not metastability is imposed; (iii) restricting $\tan\beta$ to the values (~ 1–2) preferred in supersymmetric grand unified theories, the lightest MSSM Higgs boson mass upper bound is reduced by at least 10 GeV (which implies no overlap between the SM and the MSSM bounds at even smaller values of m_t); for $m_t \sim 175$ GeV, the bound is $m_h \lesssim 110$ GeV. Thus, a measurement of the first Higgs boson mass will serve to exclude either the MSSM or $(M+1)$SSM Higgs sectors or the SM Higgs sector. In addition, we discuss the upper bound on the lightest Higgs boson mass in SUSY models with an extended Higgs sector. Finally, we comment on the discovery potential for the lightest Higgs bosons in these models. [S0556-2821(96)00821-1]

PACS number(s): 14.80.Cp, 12.15.Lk, 12.60.Fr, 14.80.Bn

I. INTRODUCTION

The simplest and most popular possibilities for the electroweak (EW) symmetry-breaking sector are the single Higgs doublet of the minimal standard model (SM), and the two Higgs doublet sector of the minimal supersymmetric standard model (MSSM). Experimentally, very little is known about the Higgs sector of the electroweak model. However, theoretically, quite a lot of Higgs physics has been calculated. The electroweak symmetry-breaking scale is known: the vacuum expectation value (VEV) of the complex Higgs field Φ is $\langle 0|\Phi|0\rangle = v_{\rm SM}/\sqrt{2} = 175$ GeV. This value is remarkably close to the top quark mass of $176\pm 8\pm 10$ GeV (which itself is very consistent with the values inferred from precision electroweak data, assuming the SM: $m_t=164\pm 25$ GeV in [1], and more recently, $m_t=156\pm 15$ GeV in [2]) announced by the Collider Detector at Fermilab (CDF) Collaboration at Fermilab [3]. Higgs boson mass bounds have been calculated, including loop corrections. One aspect of the mass bounds [4] which we quantify in this paper is the following: inputing the CDF value for the top quark mass into quantum loop corrections for the symmetry-breaking Higgs sector leads to mutually exclusive, reliable bounds on the SM Higgs boson mass and on the lightest MSSM Higgs boson mass [5,6]. From this we infer that, *independent of any other measurement, the first Higgs boson mass measurement will rule out one of the two main contenders for the electroweak theory: the SM, with no new physics below* $\sim 10^{10}$ GeV, *or the MSSM with supersymmetry-breaking scale* $M_{\rm SUSY} \lesssim 1$ TeV. Here, we improve our previous calculation [5] of the renormalized MSSM Higgs boson mass by including two-loop QCD corrections and then summing to all orders in perturbation theory the leading and next-to-leading logarithms with a renormalization group equation (RGE) technique [7,8]. We also use the improved stability [6,9] and metastability [10] lower bounds on the SM Higgs boson mass (which we summarize in Sec. II).

In the limit where the masses of the pseudoscalar, and heavy and charged Higgs bosons (these are m_A, m_H, and $m_{H^\pm}$, defined in Sec. III) are large compared to M_Z (of the order of a TeV for example), the Feynman rules connecting the light Higgs bosons in the MSSM to ordinary matter are approximately equal to the SM Feynman rules [11]. Therefore, in this limit, the MSSM light Higgs boson looks very much like the SM Higgs boson in its production channels and decay modes; the only difference, a vestige of the underlying supersymmetry, is that the constrained Higgs boson self-coupling requires the MSSM Higgs boson to be light, whereas SM vacuum stability requires the SM Higgs boson to be heavy. Thus, *it may only be possible to distinguish between the SM Higgs boson and the lightest Higgs boson of MSSM (with* $M_{\rm SUSY} \lesssim 1$ TeV) *by their allowed mass values.*

We demonstrate these allowed mass values in our Figs. 1 and 2. Furthermore, the mass of the lightest MSSM Higgs boson rises toward its upper bound as the ‘‘other’’ Higgs boson masses are increased.[1] Thus, for masses in the region where the SM lower bound and the MSSM upper bound overlap, the SM Higgs boson and the lightest MSSM Higgs boson may not be distinguishable by branching ratio or width measurements [13]. Only if the two bounds are separated by a gap is this ambiguity avoided.

In the SM and even in supersymmetric models the main uncertainty in radiative corrections is the value of the top quark mass. With the announcement of the top quark mass, this main uncertainty is greatly reduced. *The radiatively corrected observable most sensitive to the value of the top quark mass is the mass of the lightest Higgs particle in SUSY models* [14]: for a large top quark mass, the top and scalar-top ($\tilde{t}$) loops dominate all other loop corrections, and *the light Higgs boson mass squared grows as* $m_t^4 \ln(m_{\tilde{t}}/m_t)$.[2] We quantify this large correction, including two-loop QCD corrections and summing to all orders in perturbation theory the leading and next-to-leading logarithms, in Sec. III.

In addition to contrasting the MSSM with the SM, we also consider in Sec. IV supersymmetric models with a nonstandard Higgs sector, in particular the minimal-plus-singlet SUSY standard model [$(M+1)$SSM] containing an additional SU(2) singlet, and the low energy effective theory of SUSY models with a strongly interacting electroweak sector. A discussion of supersymmetric grand unified theories (SUSY GUT’s) is put forth in Sec. V; SUSY GUT’s impose additional constraints on the low energy MSSM, leading to a lower upper bound on the lightest Higgs boson mass. The discovery potential for the Higgs boson is analyzed in Sec. VI, and conclusions are presented in Sec. VII.

II. STANDARD MODEL VACUUM STABILITY BOUND

It has been shown that when the newly reported value of the top quark mass is input into the renormalized effective potential for the SM Higgs field, the broken-symmetry potential minimum remains stable when the renormalization scale is taken all the way up to the Planck mass only if the SM Higgs boson mass satisfies the lower bound constraint [9]

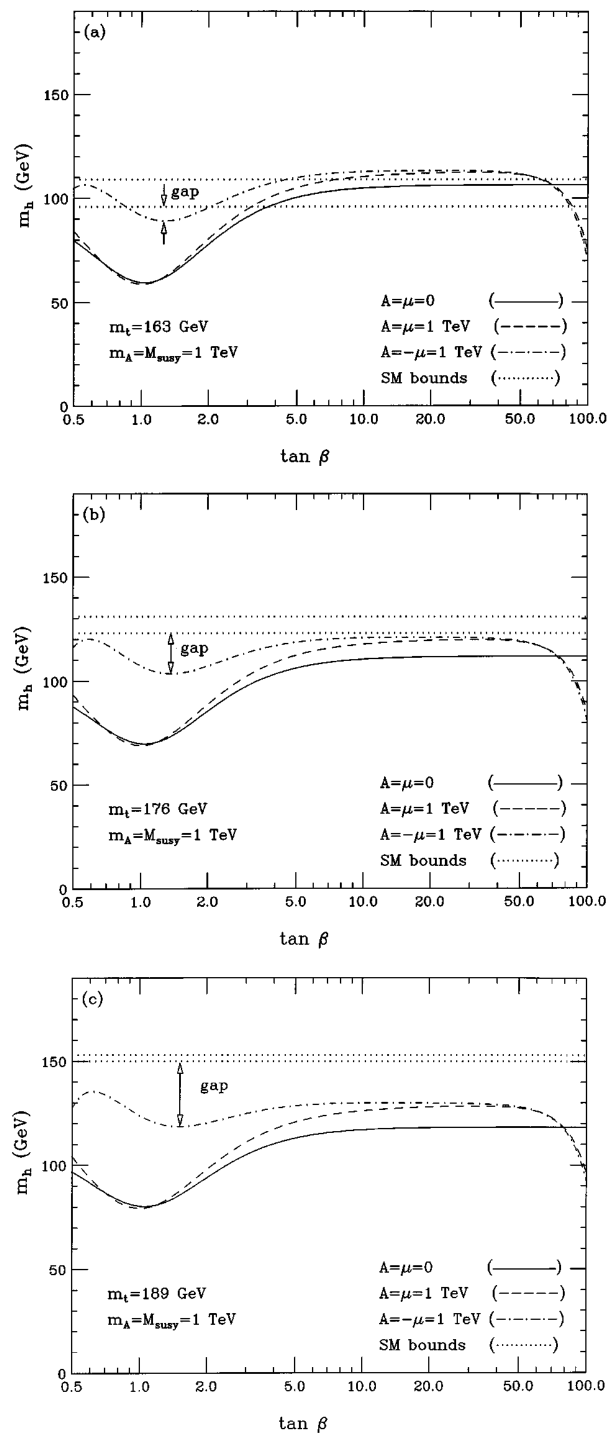

FIG. 1. The curves reveal the upper bound on the lightest MSSM Higgs particle vs $\tan\beta$, for top quark mass values of (a) 163 GeV, (b) 176 GeV, and (c) 189 GeV. Three extreme choices of SUSY parameters are invoked: the solid curve is for $\mu=A_t=A_b=0$, the dashed curve is for $\mu=A_t=A_b=1$ TeV, and the dot-dashed curve is for $\mu=-1$ TeV, $A_t=A_b=1$ TeV. In all cases, $m_A=m_{\tilde{q}}=1$ TeV and $m_b(M_Z)=4$ GeV are assumed. The horizontal dotted lines are the ($\tan\beta$-independent) SM lower bounds on the Higgs boson mass; the more restrictive stability bound derives from requiring that the EW VEV sits in an absolute minimum, while the less restrictive metastability bound derives from requiring that the VEV lifetime in the local EW minimum exceeds the age of the Universe.

[1]The saturation of the MSSM upper bound with increasing ‘‘other’’ Higgs boson masses is well known in the tree-level relations (the bound $m_h \leqslant M_Z|\cos(2\beta)|$ approaches an equality as Higgs boson masses increase) [12]. The MSSM upper bound still saturates with increasing ‘‘other’’ Higgs boson masses even when one-loop corrections are included.

[2]It is not difficult to understand this fourth power dependence; the contribution of the top loop to the SM Higgs self-energy also scales as m_t^4 However, in the SM the Higgs boson mass is a free parameter at the tree-level, and so any radiative correction to the SM Higgs boson mass is not measurable. In contrast, in the MSSM the lightest Higgs boson mass at the tree level is fixed by other observables, and so the finite renormalization is measurable.

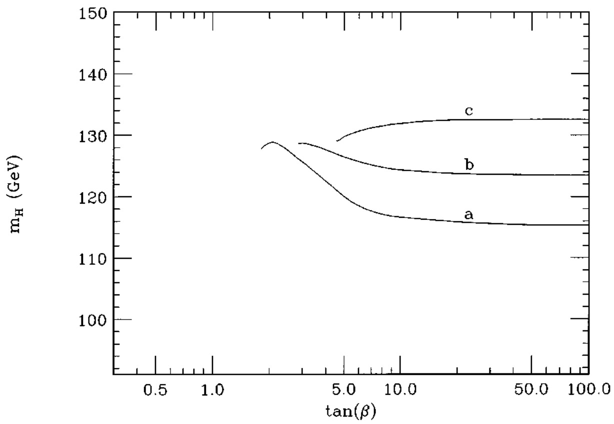

FIG. 2. Upper bound on the lightest $(M+1)$SSM Higgs boson vs $\tan\beta$, for the top quark mass values (a) 163 GeV, (b) 176 GeV, and (c) 189 GeV. All superparticles and Higgs bosons beyond the lightest are assumed to be heavy, of order of the chosen SUSY-breaking scale of 1 TeV. The GUT scale is taken as 10^{16} GeV.

$$m_H > 139 + 2.1(m_t - 176) - 4.5\left(\frac{\alpha_s - 0.118}{0.006}\right),$$
$$\Lambda = 10^{19}\ \text{GeV}. \tag{1}$$

In this equation, mass units are in GeV, and α_s is the strong coupling constant at the scale of the Z mass. The accuracy of the bound is estimated to be ~ 5–10 GeV. A similar but slightly lower bound is found in Ref. [6]:

$$m_H > 136 + 1.92(m_t - 176) - 4.25\left(\frac{\alpha_s - 0.118}{0.006}\right),$$
$$\Lambda = 10^{19}\ \text{GeV}, \tag{2}$$

valid in the range 150 GeV$<m_t<$200 GeV. These equations are the result of an analysis of the one-loop SM effective potential using two-loop β functions and the appropriate matching conditions. Here, the estimated accuracy is $\lesssim 3$ GeV from the theoretical calculation, and $\lesssim 1$ GeV from the linear fit resulting in Eq. (2).

The definition of the SM which we use requires no new physics (i.e., a desert) ''only'' up to the scale $\Lambda \sim 10^{10}$ GeV. We use the m_H vs m_t curves for various cutoff values in Ref. [6] to determine the coefficient of the m_t term at $\Lambda \sim 10^{10}$ GeV; and we run the SM renormalization group equations (RGE's) to determine the coefficient of the α_s term at $\Lambda \sim 10^{10}$ GeV. The resulting lower bound for $\Lambda \sim 10^{10}$ GeV is

$$m_H > 131 + 1.70(m_t - 176) - 3.47\left(\frac{\alpha_s - 0.118}{0.006}\right),$$
$$\Lambda = 10^{10}\ \text{GeV}. \tag{3}$$

The accuracy of this bound should approximate that of Eq. (2), $\lesssim 4$ GeV. Because the parameter space for a smaller SM desert is necessarily contained within the parameter space for a larger SM desert, a smaller desert implies weaker constraints on the model; accordingly, we see that the lower bound on the Higgs boson mass relaxes when the cutoff for new physics is reduced. In fact, it has been pointed out [9,6] that the discovery of a Higgs boson with low mass would place an upper limit on the scale of new physics.

This lower mass bound and the related ''triviality bound'' [15–17] are based on the physical requirement that the running Higgs self-coupling remains positive and finite up to the energy scale Λ. Below this energy scale Λ the SM is supposed to be valid. If the Higgs boson mass, given by $\sqrt{2\lambda}v_{\rm SM}$, is too small compared to the top quark mass, then the running Higgs self-coupling λ turns negative at a scale below the cutoff Λ [18]. On the other hand, if the Higgs boson mass is too large, then the running Higgs self-coupling λ diverges at a scale below the cutoff Λ. Thus, for a given cutoff scale Λ and top quark mass m_t, the Higgs boson mass is bounded from below by the vacuum stability bound, and bounded from above by the triviality bound. For large values of the cutoff, $\Lambda \gtrsim 10^{10}$ GeV, these bounds are only weakly dependent on the value of Λ [19,6]. By comparing Eq. (3) with Eqs. (1) and (2), we see that for a top quark mass $m_t = 176$ GeV and $\alpha_s = 0.118$, an increase in Λ from 10^{10} GeV to the Planck mass $\sim 10^{19}$ GeV raises the vacuum stability bound by only 5 to 8 GeV. To put it in simple terms: if the running Higgs self-coupling λ is going to diverge or become negative, it will do so at a relatively low energy scale.

It has been known for some time [20] that the SM lower bound rises rapidly as the value of the top quark mass increases through M_Z; below M_Z the bound is of order of the Linde-Weinberg value, ~ 7 GeV [21]. So what is new here is the inference from the large reported value for m_t that the SM Higgs boson lower mass bound dramatically exceeds 100 GeV. Adding the statistical and systematic errors of the CDF top quark mass measurement in quadrature gives a top quark mass[3] with a single estimated error of $m_t = 176 \pm 13$ GeV. The D0 Collaboration has also announced discovery of the top quark [23], with a top quark mass estimate of 199 ± 30 GeV, consistent with the (better-determined) CDF value. The main uncertainty in the SM vacuum stability bound remains the exact value of the top quark mass. The CDF 1σ uncertainty of 13 GeV in the top quark mass translates into a 22 GeV 1σ uncertainty in the bound of Eq. (3). The bound's dependence on the uncertainty in α_s, a better known parameter, is more mild.

It is possible that the observed vacuum state of our Universe is not absolutely stable, but only metastable with a small probability to decay via thermal fluctuations or quantum tunneling. If metastability rather than absolute stability is postulated, then a similar but weaker bound results [24]. In an accurate calculation of this metastability bound, next-to-leading logs are included in the effective potential and one-loop ring graph contributions to the Debye mass are summed [10].

SM metastability bounds are given in Ref. [10] in tabular form for $\alpha_s = 0.124$ and various values of Λ, and in analytic form for $\Lambda = 10^{19}$ GeV with various values of α_s. To derive

[3]A top quark mass limit independent of the top quark decay modes is provided by an analysis of the W boson width: $m_t > 62$ GeV at 95% confidence level [22].

the metastability bound for our cutoff value $\Lambda=10^{10}$ GeV and various α_s values, we do the following: We first obtain the bound for $\alpha_s=0.124$ and $\Lambda=10^{10}$ GeV by extrapolating the values given in Table of [10]. The α_s-dependent term at $\Lambda=10^{19}$ GeV is obtained from Eq. (30) in [10]. Based upon our experience with running the SM RGE's from $\Lambda=10^{19}$ GeV down to $\Lambda=10^{10}$ GeV for the SM stability bound, we note that the coefficient of the α_s-dependent term is renormalized down by 20% [compare Eq. (3) to Eqs. (1) and (2)]. So we reduce the coefficient of α_s by 20%. The change in the Higgs boson mass bound effected by this renormalization is small, ~ 1 or 2 GeV or less. The resulting metastability bound at $\Lambda=10^{10}$ GeV is

$$m_H > 123 + 2.05(m_t - 176) - 3.9\left(\frac{\alpha_s - 0.118}{0.006}\right)\left(\frac{m_t}{176}\right),$$
$$\Lambda = 10^{10}\ \text{GeV}. \tag{4}$$

According to Eq. (2), the linear fit is valid to better than 1 GeV for $m_h > 60$ GeV, and the overall theoretical error is negligible compared to the experimental errors in the α_s and m_t values.

In our figures, we will present both the stability and the metastability lower bounds. The metastability bound is necessarily lower than the stability bound. A comparison of Eqs. (3) and (4) shows that the ordering is maintained in the m_t region of interest, below 200 GeV; beyond $m_t=200$ GeV the fitted equations are no longer valid. The CDF top quark mass values including 1σ allowances are 163, 176, and 189 GeV. The vacuum stability bounds following from Eq. (3) for these top quark masses with $\alpha_s=0.118$ are 109, 131, and 153 GeV, respectively, whereas the metastability bounds are 96, 123, and 150 GeV, respectively.[4]

As is evident in Eqs. (1)–(4), the vacuum stability and metastability bounds on the SM Higgs boson mass are sensitive to the value of $\alpha_s(M_Z)$. We have taken $\alpha_s=0.118$ (the central value in the work of [9]) to produce the bounds displayed in Fig. 1. The 1994 world average derived by the Particle Data Group [26] is 0.117 ± 0.005. The value derived from fitting SM radiative corrections to precision electroweak data from the CERN e^+e^- collider LEP or SLAC Linear Collider (SLO) is $\alpha_s(M_Z)=0.124\pm0.005$ in [2], and $\alpha_s(M_Z)=0.122\pm0.005$ in [27]. Other LEP analyses, and deep inelastic leptoproduction (Euclidean) data extrapolated to the M_Z scale give lower values ~ 0.112; a comparison of low Q^2 deep-inelastic data to the Bjorken sum rule [28] yields [29] $\alpha_s(M_Z)=0.116^{+0.004}_{-0.006}$. The LEP working group [30] quotes a world average of $\alpha_s(M_Z)=0.120\pm0.006\pm0.002$, assuming the SM. If we use the generous value $\alpha_s=0.130$, the stability bound on the SM Higgs boson mass decreases by about 9 GeV for $m_t>160$ GeV, and the metastability bound decreases by about 8 GeV.

The vacuum stability bound on the SM Higgs boson mass rises roughly linearly with m_t, for $m_t \gtrsim 100$ GeV, whereas the upper limit on the lightest MSSM Higgs boson mass grows quadratically with m_t. Therefore, for very large values of the top quark mass m_t, the two bounds will inevitably overlap. In addition, for low values of m_t the two bounds may overlap. For example, for very large or very small values of $\tan\beta$ the MSSM upper bound is at least M_Z, but the SM lower bound is only 60 GeV for $m_t=130$ GeV [31]. However, for m_t heavy, but not too heavy, there may be no overlap. In what follows, we show that in fact for m_t around the value reported by the CDF Collaboration, there is little ($\alpha_s=0.130$) or no ($\alpha_s=0.118$) overlap between the SM Higgs boson mass lower bound and the MSSM upper bound. Thus, the first measurement of the lightest Higgs boson mass will probably suffice to exclude either the SM Higgs sector, or the MSSM Higgs sector.

[4] LEP experiments have established the *nonexistence* of the SM Higgs particle below a mass value of 64 GeV [25].

III. THE LIGHTEST HIGGS BOSON IN THE MSSM

The spectrum of the Higgs sector in the MSSM contains two CP-even neutral Higgs bosons, h and H, with $m_h<m_H$ by convention, one CP-odd neutral Higgs boson A and a charged Higgs boson pair $H^\pm$. A common convenience is to parametrize the Higgs sector by the mass of the CP-odd Higgs boson m_A and the VEV ratio $\tan\beta\equiv v_T/v_B$. These two parameters completely specify the masses of the Higgs particles at the tree level

$$m_{H,h}^2 = \tfrac{1}{2}(m_A^2+m_Z^2) \pm \tfrac{1}{2}\sqrt{(m_A^2-m_Z^2)^2\cos^2 2\beta + (m_A^2+m_Z^2)^2\sin^2 2\beta},$$
$$m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{5}$$

implying for example that $m_{H^\pm}>m_W$, that the upper bound on the lightest Higgs boson mass is given by

$$m_h \leq |\cos(2\beta)| M_Z, \tag{6}$$

that the lightest Higgs boson mass vanishes at the tree level if $\tan\beta=1$, and that the masses m_H, m_A, and $m_{H^\pm}$ all increase together as any one of them is increased. However, radiative corrections strongly modify the tree-level predictions in the neutral [14,32–34] and charged [35,33,36] Higgs sectors. Some consequences are that the charged Higgs boson can be lighter than the W gauge boson [36], that the $\tan\beta=1$ scenario, in which $m_h=0$ at the tree level, is viable due to the possibility of a large radiatively generated mass [34], and that the upper bound on the lightest Higgs boson mass is increased by terms proportional to $m_t^4\ln(m_{\tilde t}/m_t)$, as mentioned in our introduction[5] [14].

An important mechanism for the production of the neutral Higgs bosons in e^+e^- colliders is the bremsstrahlung of a

[5] Note that in the SUSY limit, $m_t=m_{\tilde t}$, and the fermion- and boson-loop contributions cancel each other. However, in the real world of broken SUSY, $m_t\neq m_{\tilde t}$, and the cancellation is incomplete. The top quark gets its mass from its Yukawa coupling to the electroweak VEV, whereas the scalar top quark mass arises from three sources, from D terms, from the top quark Yukawa coupling, but mainly from the insertion into the model of dimensionful soft SUSY-breaking parameters. The interplay of these diverse masses leads to the dramatic correction. Note that the correction grows logarithmically as $m_{\tilde t}$ gets heavy, rather than decoupling.

Higgs boson by a Z gauge boson. Relative to the coupling of the SM Higgs boson to two Z bosons, the ZZH coupling is $\cos(\beta-\alpha)$ and the ZZh coupling is $\sin(\beta-\alpha)$, where α is the mixing angle in the CP-even neutral Higgs boson mass matrix. The angle is restricted to $-\pi/2 \leqslant \alpha \leqslant 0$, and is given at the tree level by

$$\tan 2\alpha = \frac{(m_A^2 + m_Z^2)}{(m_A^2 - m_Z^2)} \tan 2\beta. \tag{7}$$

From Eq. (7) it is seen that the limit $m_A \to \infty$ is important for three reasons. First, it requires $\alpha \to \beta - \pi/2$, implying that $\cos(\beta-\alpha) \to 0$, i.e., the heavy Higgs boson decouples from the Z gauge boson. Second, it requires that $\sin(\beta-\alpha) \to 1$, i.e., the light Higgs boson behaves such as the SM Higgs boson. And third, $m_A \to \infty$ is the limit in which the tree level m_h saturates its maximal value given in Eq. (6) for any value of $\tan\beta$.

We use the diagrammatic technique with an on-shell renormalization scheme to calculate the renormalized lightest MSSM Higgs boson mass, m_h [7]. We include the full one-loop corrections from the top or bottom quarks and squarks, the leading-log corrections from the remaining fields (charginos, neutralinos, gauge bosons, and Higgs bosons),[6] the dominant two-loop corrections, and the full momentum dependence of the Higgs self-energies. We then perform a renormalization group equation (RGE) improvement [38] of these results in order to include the resummed leading and next-to-leading logarithms. The result is a highly accurate calculation of the lightest MSSM Higgs boson mass, perhaps the most accurate available in the literature.

We find the renormalized neutral Higgs boson masses by looking for the zeros of the determinant of the inverse propagator matrix, including the loop corrections [8]. The two solutions to

$$\Sigma_{11}^{\chi}(p^2)\Sigma_{22}^{\chi}(p^2) = [\Sigma_{12}^{\chi}(p^2)]^2, \tag{8}$$

are the pole Higgs boson masses $p^2 = m_h^2$ and $p^2 = m_H^2$. The propagators are calculated in a basis in which the CP-even Higgs fields χ_1 and χ_2 are unmixed at the tree level. We renormalize each matrix element of the inverse propagator matrix first, and later diagonalize it nonperturbatively. Furthermore, we keep the full momentum dependence of the self-energies in Eq. (8). This is equivalent to defining a momentum-dependent mixing angle $\alpha(p^2)$. With this procedure, we avoid the introduction of a mixing-angle counterterm, which allows us to calculate directly the renormalized mixing angle at the two physically relevant scales $\alpha(m_h^2)$ and $\alpha(m_H^2)$ [8].

Two-loop corrections are negative and decrease the upper bound of the Higgs boson mass by several GeV [39]. We include the dominant two-loop corrections of Ref. [39] which include the leading and next-to-leading logarithms. Finally, using an RGE technique, we extend the results of Ref. [39] by summing to all orders in perturbation theory these leading and next-to-leading logarithm terms. In order to do this, we solve the two-loop RGE [40] with a supersymmetric boundary condition at the scale M_{SUSY} to obtain the quartic Higgs self-coupling constant at the weak scale. In this way, the running Higgs boson mass squared is equal to λv^2, where $v^2 = v_T^2 + v_B^2$ (v_T and v_B are the VEV's of the two Higgs doublets.). This RGE improvement [34,39],

$$(\Delta m_h^2)_{\mathrm{RGE}} = \lambda v^2 - M_Z^2 \cos^2 2\beta - (\Delta m_h^2)_{lnll}, \tag{9}$$

depends of course on the value of the top quark mass. Here, $(\Delta m_h^2)_{lnll}$ contains the logarithmic part of the one- and two-loop corrections, the so-called leading and next-to-leading logarithms. For example, at $m_t \sim 176$ GeV, we find the RGE correction to be -2 to -3 GeV for large $\tan\beta$ and -5 to -7 GeV if $\tan\beta$ is small. We include this correction in all of our plots.

We choose m_A and all squark mass parameters to be large, equal to 1 TeV,[7] in order to find the maximum light Higgs boson mass. With respect to the squark mixing, we work in three extreme scenarios: (a) no mixing, i.e., $\mu = A_t = A_b = 0$, where μ is the supersymmetric Higgs boson mass parameter and A_i, $i = t,b$, are the trilinear soft supersymmetry-breaking terms; and maximal mixing; (b) with $\mu = A_t = A_b = 1$ TeV; (c) and $\mu = -1$ TeV, $A_t = A_b = 1$ TeV.

We mention again that our chosen definition for the MSSM is the conventional one, with M_{SUSY}, all of the soft supersymmetry-breaking terms, and μ, having a magnitude of at most 1 TeV. One of the motivations for this choice is that in supergravity models the electroweak symmetry can be broken radiatively without fine-tuning the initial parameters, if M_{SUSY} is not too large [44].

The resulting lightest Higgs boson mass as a function of $\tan\beta$ is shown in Fig. 1 for the CDF central value of the top quark mass and the $\pm 1\sigma$ mass values. Since the difference between the one- and two-loop bound calculated before the RGE resummation is ~ 10 GeV, the accuracy of our bound can be conservatively estimated to be $\lesssim 10$ GeV. For the case $\tan\beta \sim 1$, the SM lower bound and the MSSM upper bound are separated already at $m_t = 163$ GeV. Were it not for the SM *meta*stability lower bound, the gap would exist for all values of $\tan\beta$. However, with the SM metastability bound, it is not until $m_t \sim 175$ GeV that a gap exists for all values of $\tan\beta$. In particular, for the preferred CDF value of $m_t = 176$ GeV, the two bounds do not overlap, making it possible to distinguish the SM and the MSSM solely on the basis of a determination of the Higgs boson mass. Even for $m_t = 189$ GeV the gap is still increasing with increasing top quark mass, indicating that the eventual closing of the gap occurs at still higher values of m_t.

Should α_s turn out to be closer to 0.130 than to the value 0.118 assumed here, then the separation of the SM Higgs boson mass region from the MSSM Higgs boson mass region is not quite complete. We have seen that the stability and metastability lower bounds on the SM Higgs boson mass

[6]Calculations of full one-loop corrections from all particles [37] have shown that finite (i.e., nonlogarithmic) corrections due to loops with particles other than the top or bottom quarks and squarks are very small.

[7]We note that $\lesssim 1$ TeV emerges naturally for the heavier superparticle masses when the MSSM is embedded into a GUT [41–43].

decrease as α_s is increased. The MSSM mass upper bound also decreases with increasing α_s, but at a much smaller rate. We find that raising α_s from 0.118 to 0.130 shifts the MSSM Higgs boson mass bound by -0.5 GeV for $m_t=163$ and by -0.8 GeV for $m_t=189$ GeV. The result is that the gap apparent for all values of $\tan\beta$ in the $\alpha_s=0.118$, $m_t=176$ GeV case [displayed in our Fig. 1(b)], remains a gap in the $\alpha_s=0.130$ case only in the $\tan\beta\sim1$–2 region. However, the overlapping mass region for the remaining $\tan\beta$ values is small. The region of overlap is interesting only if the observed Higgs boson mass turns out to lie in this region. With a small overlap region, such an occurrence is *a priori* unlikely. A further (interesting) complication is that the best fit value for α_s, when MSSM radiative corrections are assumed and fitted to precision data, is [45] $\alpha_s(M_Z)=0.114\pm0.007$. This lower value suggests that it may be best to compare SM bounds with a given value assumed for α_s to MSSM bounds with a slightly lower value assumed for α_s.

In Fig. 1 we can see that scenario (c) gives us a significantly larger range of Higgs boson mass values close to $\tan\beta\sim1$. This can be understood in the $\tan\beta=1$ approximation: there are nonleading logarithmic contributions to the Higgs boson mass from loops involving the top quark and squarks that are proportional to powers of $(\mu-A_i)/m_t$ [34]. Also in Fig. 1 we see that scenarios (b) and (c) offer a larger value for the m_h maximum than does scenario (a), except for the region $\tan\beta\gg1$. The reason is that among the additional light Higgs boson mass terms in (b) is a negative term proportional to $-(\mu m_b/\cos\beta)^4$, which becomes large [38] when $\tan\beta\gg1$. More significant is the fact that the extreme values in (a), (b), and (c) yield a very similar absolute upper bound in the region of acceptable $\tan\beta$ values, thereby suggesting insensitivity of the MSSM upper bound to a considerable range of the squark mixing parameters.

In the literature there are three popular methods to calculate the renormalized Higgs boson mass. These are the RGE technique, the effective potential method, and the diagrammatic technique. It is informative to compare these techniques, and to point out the advantages of the approach we have undertaken. The RGE technique is used for example in Ref. [40], where the leading and next-to-leading logarithms are summed to all orders in perturbation theory to give the running Higgs boson mass. This technique is based on the fact that the Veltman functions [46] which appear in the diagrammatic method can be approximated by logarithms when there are two different scales in the problem. The RGE technique sums these logarithms to all orders, but drops all nonlogarithmic, finite terms. These terms are often very important [34,36]. Moreover, the reliability of the RGE treatment of the logarithmic terms decreases if the two scales are not very far apart (as is the case here, where the two scales are the EW and SUSY-breaking scales). Numerically, the Higgs boson mass calculated with the RGE method can differ by 10 GeV or more compared to the diagrammatic method, even if two-loop RGE's are used.

The renormalization group improvement [see our Eq. (9)] we use in our work replaces the logarithmic part of the corrections obtained with the diagrammatic method by the resummed logarithmic corrections as obtained with the renormalization group technique. Our results, therefore, incorporate both the important finite corrections at the two-loop level and the resummed leading and next-to-leading logarithmic corrections.

The second popular technique is the effective potential method. In Ref. [34] the effective potential method is compared with the diagrammatic technique. Working in an on-shell scheme in both methods, it is shown that the two techniques reproduce the same answer when the tree-level Higgs boson mass is zero and when all supersymmetric particles are included in the effective potential. However, in the more realistic case where the tree-level Higgs boson mass is nonzero, the effective potential answer has to be corrected using diagrammatic methods. With these diagrammatic corrections, the two methods become indistinguishable.

The effective potential method is used in Ref. [47]. There the $\overline{MS}$ renormalization scheme is also used and so the comparison with our on-shell diagrammatic method is not simple. A nontrivial ambiguity for the choice of the arbitrary scale is present in this method. A further limitation in this calculation is the inclusion in the effective potential of only SM particles. Important log terms arising from SUSY particle loops are, therefore, absent. When the SUSY particles are ignored, the only connection with supersymmetry is in the boundary condition for λ at the scale $M_{\rm SUSY}$. A partial compensation is made by including the threshold effects of SUSY particles in the form of step functions. What would be a full Veltman's function in the diagrammatic method is approximated in the effective potential method by a step-function shift [48] in the boundary condition: $\lambda=\frac{1}{4}(g^2+g'^2)\cos^2 2\beta+\Delta\lambda$.

In our diagrammatic method these approximations are not present since the effects of the nonlogarithmic terms are included in the full expressions of the Veltman's functions. For example, important nonlogarithmic effects are included, such as the decreasing of the Higgs boson mass when $\tan\beta\to\infty$, $\mu\sim1$ TeV, and $A=0$, as explained above and seen in Fig. 1. Also, the effect of large splitting in the masses of the top squarks is automatically taken into account in our diagrammatic method. These effects are not included in Ref. [47].

There are two further improvements that we have achieved. The first improvement is the use of different RGE's above and below the top quark mass. Below m_t the top quark mass decouples and the RGE for λ does not contain the top quark Yukawa coupling. This effect can be important. In principle, the RGE for the gauge couplings should also be modified. In practice, it is a negligible effect. (This modification is more complicated, since the electroweak gauge symmetry is broken. A careful analysis can be found in Ref. [38].) The second improvement is the consideration of the running of $\tan\beta$. In practice, this effect is numerically small [38].

We finish this section with some comments on the decay $b\to s\gamma$. It is known that the branching ratio $B(b\to s\gamma)$ has a strong dependence on the SUSY Higgs parameters [49–51]. However, when all squarks are heavy, as here, the contribution from the chargino or squark loops to $B(b\to s\gamma)$ is suppressed. In the case of heavy squarks, the charged-Higgs-boson–top-quark loop may seriously alter the rate, and strong constraints on the charged Higgs boson minimum mass result [52,51]. This constraint does not affect the present work, where we take m_A and therefore, $m_{H^\pm}$ and

m_H large in order to establish the light Higgs boson upper bound: in the large m_A, large squark mass limit, the ratio $B(b \to s\gamma)$ approaches the SM value, consistent with the CLEO bound [53].

IV. THE LIGHTEST HIGGS BOSON IN NONSTANDARD SUSY MODELS

The MSSM can be extended in a straightforward fashion by adding an SU(2) singlet S with vanishing hypercharge to the theory [54]. As a consequence, the particle spectrum contains an additional scalar, pseudoscalar, and neutralino. This extended model, the so-called $(M+1)$SSM, features four possible additional terms in the superpotential. Two of these terms, $\lambda S H_B \epsilon H_T$ and $\frac{1}{3}\kappa S^3$, enter into the calculation of the lightest Higgs boson mass; λ enters directly, while κ enters through the RG equations. ϵ is the usual antisymmetric 2 by 2 matrix.

At the tree level, a study of the eigenvalues of the scalar mass matrix gives an upper bound on the mass of the lightest Higgs boson:

$$m_h^2 \leqslant M_Z^2 \left\{ \cos^2 2\beta + 2\frac{\lambda^2}{g_1^2+g_2^2}\sin^2 2\beta \right\}. \tag{10}$$

The first term on the right-hand side is just the MSSM result of Eq. (6). The second term gives a positive contribution, and since the parameter λ is *a priori* free, weakens the upper bound considerably [55,56]. However, there are two scenarios in which the bound proves to be very restrictive. In the first scenario $\tan\beta$ is small, and therefore, $\cos^2 2\beta$ is necessarily $\gg \sin^2 2\beta$. In the second scenario the value of λ is limited by the assumption of perturbative unification. In this latter scenario, even if λ assumes a high value at the GUT scale, the renormalization group equations drive the evolving value of λ to a moderate value at the SUSY-breaking scale. The exact Higgs boson mass upper bound depends on the value of the top Yukawa coupling g_t at the GUT scale through the renormalization group equations. Above $M_{\rm SUSY}$ the running of the coupling constants is described by the $(M+1)$SSM renormalization group equations, whereas below this scale the SM renormalization group equations are valid. At $M_{\rm SUSY}$ the boundary conditions

$$\lambda^{\rm SM} = \frac{1}{8}(g_1^2+g_2^2)\left(\cos^2 2\beta + 2\frac{\lambda^2}{g_1^2+g_2^2}\sin^2 2\beta \right),$$

$$g_t^{\rm SM} = g_t \sin\beta, \tag{11}$$

incorporate the transition from the $(M+1)$SSM to the SM. Here, $\lambda^{\rm SM}$ and $g_t^{\rm SM}$ are the standard model Higgs self-coupling and top quark Yukawa coupling, respectively. The value of the Higgs boson mass is determined implicitly by the equation $2\lambda^{\rm SM}(m_h) v_{\rm SM}^2 = m_h^2$ This RGE procedure of running couplings from $M_{\rm SUSY}$ down to the weak scale takes into account logarithmic radiative corrections to the Higgs boson mass, including in particular, those caused by the heavy top quark.

In Fig. 2 we show the maximum value of the Higgs boson mass as a function of $\tan\beta$ for the chosen values of the top quark mass m_t. We have adopted a SUSY-breaking scale of $M_{\rm SUSY} = 1$ TeV; this value is consistent with the notion of stabilizing the weak-to-SUSY GUT hierarchy, and is the value favored by RGE analyses of the observables $\sin^2\theta_W$ and m_b/m_τ. The bounds in Fig. 2 are quite insensitive to the choice of $M_{\rm SUSY}$, increasing very slowly as $M_{\rm SUSY}$ increases [55]. We have assumed that all superpartners and all Higgs bosons except for the lightest one are heavy, i.e., $\sim M_{\rm SUSY}$. For low values of the top quark mass ($\sim M_Z$), the mass upper bound on the Higgs boson in the $(M+1)$SSM will be substantially higher than in the MSSM at $\tan\beta \lesssim$ a few. This is because $\lambda(m_h)$ is large for low m_t, and because $\sin^2 2\beta \gtrsim \cos^2 2\beta$ for $\tan\beta \lesssim$ a few. However, for a larger top quark mass, as in Fig. 2, the difference between the MSSM and $(M+1)$SSM upper bounds diminishes. This is because $\lambda(m_h)$ falls with increasing m_t, although there is an increasing minimum value for $\sin\beta = g_t^{\rm SM}/g_t$ [from the second of Eqs. (11)], and therefore, for $\tan\beta$, when $m_t \propto g_t^{\rm SM}$ is raised and g_t is held to be perturbatively small up to the GUT scale.[8] This increasing minimum value of $\tan\beta$ is evident in the curves of Fig. 2. A comparison of Figs. 1 and 2 reveals that the $(M+1)$SSM and MSSM bounds are very similar at $\tan\beta \gtrsim 6$. For m_t at or above the CDF value, only this $\tan\beta \gtrsim 6$ region is viable in the $(M+1)$SSM model.

In a fashion very similar to the $(M+1)$SSM, perturbative unificaton yields a bound on the mass of the lightest Higgs bosons in more complicated extensions of the MSSM. In general, the lowest eigenvalue of the scalar mass matrix is bounded by M_Z times a factor which depends on the dimensionless coupling constants in the Higgs sector. The renormalizaton group equations force these coupling constants to assume relatively low values at the SUSY-breaking scale, and as a consequence the mass bound on the lightest Higgs boson is of the order of M_Z.

Although a bound on the mass of the lightest Higgs boson exists in perturbative SUSY models, this is not the case in SUSY models with a strongly interacting symmetry-breaking sector. The low energy physics of this class of theories is described by a supersymmetric nonlinear σ model, which is obtained by imposing the constraint $H_T \epsilon H_B = \frac{1}{4} v_{\rm SM}^2 \sin^2 2\beta$ on the action of the MSSM [57]. This constraint is the only one possible in the MSSM Higgs sector that obeys supersymmetry, is invariant under SU(2)×U(1), and leaves the VEV in a global minimum.[9] As a result of this constraint one of the scalar Higgs bosons, the pseudoscalar, and one of the neutralinos are eliminated from the particle spectrum. The remaining Higgs boson has a mass $m_h^2 = M_Z^2 + (\hat{m}_T^2 + \hat{m}_B^2)\sin^2 2\beta$, and the charged Higgs bosons have

[8] Keeping g_t perturbatively small up to the GUT scale implies $m_t \leqslant$ its pseudo-fixed-point value of $\sim 200 \sin\beta$. Therefore, a measured top quark mass as large as that reported by CDF requires $\tan\beta > 1$ in the GUT scenario, and suggests saturation of the fixed point.

[9] This MSSM nonlinear σ model is not the formal heavy Higgs boson limit of the MSSM, but is a heavy Higgs boson limit of the $(M+1)$SSM; the MSSM does not contain an independent, dimensionless, quartic coupling constant λ in the Higgs sector which can be taken to infinity, whereas the $(M+1)$SSM (and the SM) does.

masses $m_{H^\pm}^2 = M_W^2 + (\hat{m}_T^2 + \hat{m}_B^2)$. Here, $\hat{m}_T^2$ and $\hat{m}_B^2$ are soft, dimensionful, SUSY-breaking terms; they may be positive or negative.

In order for the notion of a supersymmetric nonlinear model to be relevant, the SUSY-breaking scale is required to be much smaller than the chiral symmetry-breaking scale $4\pi v_{\rm SM}$. The natural magnitude for the parameters $\hat{m}_B^2$ and $\hat{m}_T^2$ is, therefore, of the order of M_Z^2 Consequently, both the neutral and the charged Higgs bosons have masses of at most a few multiples of M_Z in the nonlinear model. This formalism of the effective action allows a description of the low energy physics *independent* of the particular strongly interacting underlying theory from which it derives. Thus, we believe that the nonlinear MSSM model presented here is probably representative of a class of underlying strongly interacting SUSY models. The lesson learned then is that measuring a value for m_h at $\lesssim 300$ GeV cannot validate the SM, MSSM, $(M+1)$SSM, or any other electroweak model. However, the premise of this present article remains valid, that such a measurement should rule out one or more of these popular models.

V. SUPERSYMMETRIC GRAND UNIFIED THEORIES

Supersymmetric grand unified theories (SUSY GUT's) are the only simple models in which the three low energy gauge coupling constants are known to merge at the GUT scale, and hierarchy and parameter-naturalness issues are solved. Thus, it is well motivated to consider the grand unification of the low energy SUSY models. At low energies, SUSY GUT models reduce to the MSSM, but there are additional relations between the parameters [42]. The additional constraints must yield an effective low energy theory that is a special case of the general MSSM we have just considered. Therefore, the upper bound[10] on m_h in such SUSY GUT models is, in general, *lower* than that in the MSSM without any restrictions. The assumption of gauge coupling constant unification (with its implied desert between $M_{\rm SUSY}$ and $M_{\rm GUT}$) presents no significant constraints on the low energy MSSM parameters [42,58]. However, the further assumption that the top quark Yukawa coupling remains perturbatively small up to $M_{\rm GUT}$ leads to the low energy constraint $0.96 \leqslant \tan\beta$. This is because the RGE evolves a large but perturbative top quark Yukawa coupling at $M_{\rm GUT}$ down to its well-known infrared pseudo-fixed-point value at $M_{\rm SUSY}$ and below, resulting in the top quark mass value $\sim 200\sin\beta$ GeV. If the bottom quark Yukawa coupling is also required to remain perturbatively small up to $M_{\rm GUT}$, then $\tan\beta \leqslant 52$ [59] emerges as a second low energy constraint.

The pseudo-fixed point solution is not a true fixed-point, but rather is the low energy Yukawa value that runs to become a Landau pole (an extrapolated singularity, presumably tamed by new physics) near the GUT scale. The apparent CDF top quark mass value is within the estimated range of the pseudo-fixed-point value. Thus, it is attractive to assume the pseudo-fixed-point solution. With the additional assumptions that the electroweak symmetry is radiatively broken [60] (for which the magnitude of the top quark mass is crucial) and that the low energy MSSM spectrum is defined by a small number of parameters at the GUT scale (the SUSY Higgs boson mass parameter μ, which is also the Higgsino mass, and four universal soft SUSY-breaking mass parameters: the scalar mass, the bilinear and trilinear masses, and the gaugino mass), two compact, disparate ranges for $\tan\beta$ emerge: $1.0 \leqslant \tan\beta \leqslant 1.4$ [59], and a large $\tan\beta$ solution $\sim m_t/m_b$.[11] Reference to our Figs. 1 and 2 shows that the gap between the SM and MSSM is maximized in the small $\tan\beta$ region and minimized in the large $\tan\beta$ region, whereas just the opposite is true for the gap between the SM and $(M+1)$SSM models. Moreover, the $(M+1)$SSM model is an inconsistent theory in the small $\tan\beta$ region if $m_t \gtrsim 160$ GeV.

In fact, a highly constrained low $\tan\beta$ region ~ 1 and high $\tan\beta$ region $\gtrsim 40$–70 also emerge when bottom quark-τ Yukawa unification at the GUT scale is imposed on the radiatively broken model [61–64]. Bottom quark-τ Yukawa coupling unification is attractive in that it is natural in SUSY SU(5), SO(10), and E_6, and explains the low energy relation, $m_b \sim 3m_\tau$. With bottom-τ unification, the low to moderate $\tan\beta$ region requires the proximity of the top quark mass to its fixed-point value [65], while the high $\tan\beta$ region also requires the proximity of the bottom quark and τ Yukawa couplings to their fixed point; the emergence of the two $\tan\beta$ regions results from these two possible ways of assigning fixed points.

The net effect of the Yukawa-unification constraint in SUSY GUT's is necessarily to widen the mass gap between the light Higgs MSSM and the heavier Higgs SM, thus strengthening the potential for experiment to distinguish the models. The large $\tan\beta$ region is disfavored by proton stability [66]. Adoption of the favored low to moderate $\tan\beta$ region leads to a highly predictive framework for the Higgs boson and SUSY particle spectrum [63,64]. In particular, the fixed-point relation $\sin\beta \sim m_t/(200\ \text{GeV})$ fixes $\tan\beta$ as a function of m_t. For a heavy top quark mass as reported by CDF, one has $\tan\beta \sim (1, 2)$ for $m_t = (140, 180)$ GeV. Since $\tan\beta \sim 1$ is the value for which the m_h upper bound is minimized (the tree-level contribution to m_h vanishes), the top quark Yukawa fixed-point models offer a high likelihood for h^0 detection at LEP200. Reduced m_h upper bounds have been reported in [62,63]. The reduction in these bounds is due to the small $\tan\beta$ restriction, an inevitable consequence of assigning the top quark mass, but not the bottom quark mass, to the pseudo fixed point. These bounds are basically our bound in Fig. 1 for $\tan\beta \sim 1$, when allowance is made for small differences resulting from different methods and approximations.

Even more restrictive SUSY GUT's have been analyzed. These include the "no-scale" or minimal supergravity mod-

[10] In fact, the additional restrictions may be so constraining as to also yield a *lower* limit on the lightest Higgs boson mass, in addition to the upper limit. For example, $m_h > 85$ GeV for $\tan\beta > 5$ and $m_t = 170$ GeV is reported in Ref. [42], and a similar result is given in [43].

[11] It may be noteworthy that a fit of MSSM radiative corrections to the electroweak datum $R_b \equiv \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ reveals a preference for just these two $\tan\beta$ regions [45].

els [67], in which the soft mass parameters m_0 (universal scalar mass) and A are zero at the GUT scale; and its near relative, the superstring GUT, in which the dilaton VEV provides the dominant source of SUSY breaking and so m_0, A, and the gaugino mass parameter all scale together at the GUT scale [68]. Each additional constraint serves to further widen the SM or MSSM Higgs boson mass gap.

In radiatively broken SUSY GUT's with universal soft parameters, the superparticle spectrum emerges at $\lesssim 1$ TeV. If the spectrum, in fact, saturates the 1 TeV value, then as we have seen the Feynman rules connecting h^0 to SM particles are indistinguishable from the Feynman rules of the SM Higgs boson. Thus, it appears that if a SUSY GUT is the choice of nature, then the mass of the lightest Higgs boson, but not the Higgs boson production rate or dominant Higgs boson decay modes, may provide our first hint of grand unification.

VI. DISCOVERY POTENTIAL FOR THE HIGGS BOSON

The Higgs boson discovery potential of LEPII [69,70] depends on the energy at which the machine is run. An SM Higgs boson mass up to 105 GeV is detectable at LEPII with the $\sqrt{s}=200$ GeV option (LEP200), while a SM Higgs boson mass only up to 80 GeV is detectable with LEP178. As we have shown, with the large value of m_t reported by CDF, the upper limit on the MSSM h^0 mass is ~ 120 GeV. This limit is ~ 10 GeV lower than that reported in our previous work [5], as a result of the inclusion of RGE-resummed leading and next-to leading logarithms and two-loop finite QCD corrections. Near this upper limit the MSSM Higgs boson has the production and decay properties of the SM Higgs boson. Discovery of this lightest MSSM Higgs boson then argues strongly for the LEP200 option over LEP178. Furthermore, for any choices of the MSSM parameters, associated production of either h^0Z or h^0A is guaranteed at LEP200 as long as $m_{\tilde{t}} \lesssim 300$ GeV [69]. Even better would be LEP230, where detection of Zh^0 is guaranteed as long as $m_{\tilde{t}} \lesssim 1$ TeV [69]. At an NLC300 (the Next Linear Collider), detection of Zh^0 is guaranteed for MSSM or for $(M+1)$SSM [69]. Turning to hadron colliders [71,72], it is now believed that while the SM Higgs boson cannot be discovered at Fermilab's Tevatron with its present energy and luminosity, the mass range 80 GeV–130 GeV is detectable at any hadron collider with $\sqrt{s} \gtrsim 2$ TeV and an integrated luminosity $\int dt \mathcal{L} \gtrsim 10$ fb^{-1} [72]; the observable mass window widens significantly with increasing luminosity, but very little with increasing energy. For brevity, we will refer to this High Luminosity DiTevatron hadron machine as the ''HLDT.'' If the SM desert ends not too far above the electroweak scale, then the SM Higgs boson may be as heavy as $\sim$600–800 GeV[12] (but not heavier, according to the triviality argument), in which case only the LHC (and not even the NLC500) guarantees detection.

We present our conclusions on detectability for the CDF central m_t value, for the $m_t \pm 1\sigma$ values, and for a $m_t - 3\sigma$ value of 137 GeV: (i) If $m_t \sim 137$ GeV, the SM Higgs boson mass lower bound from absolute vacuum stability is equal to the experimental lower bound of $m_H = 64$ GeV, while the metastability bound allows a mass as low as 43 GeV[13]; a SM mass up to (80, 105, 130) GeV is detectable at (LEP178, LEP200, HLDT); and the MSSM h^0 is certainly detectable at LEP178 for $\tan\beta \sim 1$–2, and certainly detectable at LEP200 for all $\tan\beta$. (ii) If $m_t \sim 163$ GeV, then the abolute (metastability) SM lower bound rises to 109 (96) GeV, so the SM Higgs boson cannot be detected at LEP178 and probably not at LEP200, but is still detectable at the HLDT if its mass is below 130 GeV; the lightest MSSM Higgs boson is certainly detectable at LEP178 if $\tan\beta$ is very close to 1, and is certainly detectable at LEP200 if $\tan\beta$ is $\lesssim 3$. (iii) if $m_t \sim 176$ GeV, then the SM Higgs boson is above 131 (123) GeV, out of reach for LEPII and probably for the HLDT as well; the MSSM Higgs boson is certainly detectable at LEP200 if $\tan\beta \sim 1$–2. (iv) if $m_t \sim 189$ GeV, then the SM Higgs boson is above 153 (150) GeV in mass; at any $\tan\beta$ value, the MSSM Higgs boson is not guaranteed to be detectable at LEP200, but is certainly detectable at the HLDT if $\tan\beta \sim 1$–3.

For these mass bounds the value $\alpha_s = 0.118$ has been assumed. The MSSM mass upper bound is relatively insensitive to changes in α_s, whereas the SM mass lower bounds decrease about 3 GeV with each 0.005 increase in α_s. It is interesting that the h^0 mass range most accessible to experiment is $\tan\beta \sim 1$–3, just the parameter range favored by SUSY GUT's.

VII. DISCUSSION AND CONCLUSIONS

For a top quark mass ~ 176 GeV, the central value reported by CDF, and an α_s value of ~ 0.118, a measurement of the mass of the Higgs boson will distinguish the SM with a $\gtrsim 10^{10}$ GeV desert from the MSSM with a SUSY-breaking scale of about 1 TeV. For the $(M+1)$SSM with the assumption of perturbative unification, conclusions are similar to those of the MSSM. For α_s above 0.120 and $m_t \sim 176$ GeV, a small overlap of the SM and MSSM mass regions exists, but it is *a priori* unlikely that the Higgs boson mass will be found in this small range. Accordingly, the first Higgs boson mass measurement can be expected to eliminate one of these popular models.

Most of the range of the lightest MSSM Higgs boson mass is accessible to LEPII. The lightest MSSM Higgs boson is guaranteed detectable at LEP230 and at the LHC; and the lightest $(M+1)$SSM Higgs boson is guaranteed detectable at a NLC300 and at the LHC. Since there is no lower bound on the lightest MSSM Higgs boson mass other than the experimental bound, the MSSM h^0 is possibly detectable even at LEP178 for all $\tan\beta$, but there is no guarantee. In contrast, the SM Higgs boson is guaranteed detectable only at the LHC; if $m_t \sim 176$ GeV, then according to the vacuum stability (metastability) argument, the SM Higgs boson mass exceeds 131 (123) GeV, and so likely will not be produced until the LHC or NLC is available.

Thus, one simple conclusion is that LEPII has a tremen-

[12]Theorists would prefer an even lower value of $\lesssim 400$ GeV, so that perturbative calculations in the SM converge [73].

[13]Recall that for the SM vacuum stability and metastability bounds we assume a desert up to $\sim 10^{10}$ GeV.

dous potential to distinguish MSSM and $(M+1)$SSM symmetry breaking from SM symmetry breaking. If a Higgs boson is discovered at LEPII, the Higgs sector of the SM with a large desert is ruled out.

Note added. Carena *et al.*, Nucl. Phys. **B461**, 407 (1996), have extended their approximate analytical formulas for the MSSM Higgs boson masses, derived from an RGE-improved effective potential, to include the three model parameters which allow for squark nondegeneracy and additional left-right squark mixing. It can be seen in their figures that these further squark parameters do not affect the m_h upper bound.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy Grant No. DE-FG05-85ER40226, and the Texas National Research Laboratory Commission Grant No. RGFY93-303.

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