Supersymmetric solution to *CP* problems

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We analyze the minimal supersymmetric left-right model with nonrenormalizable interactions induced by higher scale physics and study its CP-violating properties. We show that it (i) solves the strong CP problem, and (ii) predicts the neutron electric dipole moment well within experimental limits (thus solving the usual SUSY CP problem). In addition, it automatically conserves R parity. The key points are that the parity symmetry forces the Yukawa couplings to be Hermitian, while supersymmetry ensures that the scalar potential has a minimum with real Higgs doublet vacuum expectation values. Gluino and $B-L$ gaugino masses are automatically real. The observed CP violation in the kaon system comes, as in the standard model, from the Kobayashi-Maskawa-type phases. These solutions are valid for any value of the right-handed breaking scale M_R, as long as the effective theory below M_R has only two Higgs doublets that couple fully to fermions (i.e., the theory below M_R is MSSM-like.) The potentially dangerous contributions from the $SU(2)_L$ gaugino one-loop diagram as well as from some higher dimensional terms to $\overline{\Theta}$ below M_R can be avoided if the left-right symmetry originates from a unified theory such as SO(10) and we discuss how this embedding is achieved for the SO(10) case. [S0556-2821(96)05421-5]

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I. INTRODUCTION

Quantum chromodynamics (QCD) is now widely accepted as the theory of strong interactions. The periodic vacuum structure of QCD has, however, the unpleasant implication that strong interactions violate CP. This CP-violating interaction, being flavor conserving, only manifests itself as an electric dipole moment of the neutron and leads to a value far above the present experimental upper limit unless the associated CP-violating coupling (usually labeled as $\overline{\Theta}$), which is left arbitrary by strong interaction dynamics, is somehow suppressed to the level of 10^{-9}. This problem of fine-tuning of the $\overline{\Theta}$ parameter in gauge theories is known as the strong CP problem [1]. There are many solutions to the strong CP problem [1]: The most well known of these is the Peccei-Quinn solution, which requires the complete gauge theory of electroweak and strong interactions to respect a global chiral U(1) symmetry. This symmetry must, however, be spontaneously broken in the process of giving mass to the W boson and fermions, leading to a pseudo Goldstone boson in the particle spectrum known in the field as the axion. There are two potential problems with this otherwise beautiful proposal: (i) The axion has not been experimentally discovered as yet, and the window is closing in on it, and (ii) if nonperturbative gravitational effects induced by black holes and wormholes are important in particle physics as is believed by some [2], then the axion solution would require fine-tuning of the gravitationally induced couplings by some 50 orders of magnitude. This will make the axion theory quite contrived.

A second class of solutions that does not lead to any near massless boson is to require the theory to be invariant under discrete symmetries [3,4]. In our opinion, the most physically motivated of such theories are the ones [3] based on the left-right symmetric theories of weak interactions [5]. These theories are based on the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ with quarks and leptons assigned in a left-right symmetric manner. Such models are also completely quark-lepton symmetric. To see how parity symmetry of the Lagrangian helps to solve the strong CP problem, let us note that the physical QCD-induced CP-violating phase can be written as

$$\overline{\Theta}=\Theta+\text{Arg}\ \det(M_uM_d), \quad (1)$$

where Θ is the parameter in $F\tilde{F}$ part of the QCD Lagrangian and M_u and M_d, are the up and down quark mass matrices, respectively. Invariance under parity sets $\Theta=0$ because $F\tilde{F}$ is odd under parity. Additionally, constraints of left-right symmetry imply that the Yukawa couplings of quarks responsible for the generation of quark masses are Hermitian. If furthermore the vacuum expectation values (VEV's) of the Higgs fields responsible are shown to be real, then this would automatically lead to $\overline{\Theta}=0$ at the tree level. If the one-loop contributions also preserve the Hermiticity of the quark mass matrices, then we have a solution to the strong CP problem. In the nonsupersymmetric left-right models with nontrivial CP violation, it is well known that in general VEV's of the Higgs field are not real. This in the past led to suggestions that either new discrete symmetries be invoked together with left-right symmetry or new vectorlike fermions be added to the theory [3]. Such theories also do not suffer from the Planck-scale-implied fine-tunings [6]. It always remained a challenge to solve the strong CP problem using only left-right symmetry since often new additional symmetries invoked are not motivated from any other consideration.

A second CP-related problem is connected with the minimal supersymmetric standard model (MSSM), which is currently a subject of intense discussion as the next level of physics beyond the standard model and is the so-called (usual) supersymmetric (SUSY) CP problem [7]. Namely, in the MSSM the complex phase in the gluino mass is arbitrary and the one-loop gluino contribution to the neutron electric dipole moment is larger by two or three orders of magnitude than the experimental upper bound.

There have been many proposals in the literature to solve one or both of these problems. For instance, one recent suggestion is to consider a supersymmetric extension of the Peccei-Quinn symmetry [8], which can solve the strong as well as the SUSY CP problems. Another proposal in the context of grand unified models assumes CP-conserving gaugino masses at the grand unified theory (GUT) scale thereby solving only the SUSY CP problem [9]. Other proposals employ spontaneous breaking of CP symmetry to achieve the same goal [10] or small gaugino masses with an approximate R symmetry [11]. None of the above approaches, however, address the important issue of R-parity conservation.

Our goal in this paper is to discuss a possible solution to both the strong CP and the SUSY CP problem in supersymmetry. The first point to note is that in supersymmetric theories the Θ receives an additional contribution from the phase of the gluino mass at the tree level [12,13]:

$$\overline{\Theta}=\Theta+\text{Arg det}(M_u M_d)-3\text{ Arg } m_{\tilde{g}}. \tag{2}$$

So any solution to the strong CP problem in supersymmetric theories must also require that the phase of the gluino mass must be naturally suppressed. Note that a solution to the SUSY CP problem also requires the suppression of the same phase though to a lesser degree. Clearly, therefore, a solution to to the strong CP problem automatically provides a solution to the weak CP problem.

In two recent Letters [14,15], it has been pointed out that if supersymmetry is combined with left-right symmetry, the strong CP problem is automatically solved without the need for any extra symmetry. Furthermore, in Ref. [14], it was pointed out that this model also provides a solution to the SUSY CP problem of MSSM; i.e., it does not lead to a large electric dipole moment of the neutron. As a bonus, these models automatically conserve R parity. In this paper we elaborate on the results of Ref. [14] and present some new ones which show the left-right scale independence of our result. We also discuss the question of possible embedding of left-right symmetry in grand unified theories.

This paper is organized as follows. In Sec. II, we discuss our supersymmetric solution to the strong CP problem; in Sec. III, we show how the solution remains regardless of whether the right-handed scale M_R is in the TeV range or much higher; in Sec. IV, we discuss our solution to the usual SUSY CP problem; in Sec. V, we show how the theory can be embedded into the SO(10) model; and in Sec. VI, we give our conclusions. We discuss the question of potential minimization in Appendix A, show the reality of Higgs VEV's in Appendix B, list the evolution equations for Yukawa couplings for a general four-doublet expansion of the MSSM in Appendix C, and discuss the doublet-doublet splitting in Appendix D.

II. SUPERSYMMETRIC SOLUTION TO THE STRONG *CP* PROBLEM

Let us recall the arguments of Ref. [14] and see how the supersymmetric left-right model solves the strong CP problem at the scale M_R.

TABLE I. Field content of the SUSY L-R model.

Fields	$SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}$ representation
Q	$(2,1,+\frac{1}{3})$
Q^c	$(1,2,-\frac{1}{3})$
L	$(2,1,-1)$
L^c	$(1,2,+1)$
$\Phi_{1,2}$	$(2,2,0)$
$\underline{\Delta}$	$(3,1,+2)$
Δ	$(3,1,-2)$
$\underline{\Delta^c}$	$(1,3,+2)$
Δ^c	$(1,3,-2)$

The gauge group of the theory is $SU(2)_L\times SU(2)_R \times U(1)_{B-L}$ with quarks and leptons transforming as doublets under $SU(2)_{L,R}$. In Table I, we denote the quark, lepton, and Higgs superfields in the theory along with their transformation properties under the gauge group. Note that we have chosen two bidoublet fields to obtain realistic quark masses and mixings (one bidoublet implies a Kobayashi-Maskawa matrix proportional to unity, because supersymmetry forbids $\hat{\Phi}$ in the superpotential).

The superpotential for this theory is given by (we have suppressed the generation index)

$$\begin{aligned} W=&\mathbf{Y}_q^{(i)}Q^T\tau_2\Phi_i\tau_2Q^c+\mathbf{Y}_l^{(i)}L^T\tau_2\Phi_i\tau_2L^c \\ &+i(\mathbf{f}L^T\tau_2\Delta L+\mathbf{f}_cL^{cT}\tau_2\Delta^cL^c)+\mu_\Delta\text{Tr}(\Delta\overline{\Delta}) \\ &+\mu_{\Delta^c}\text{Tr}(\Delta^c\overline{\Delta^c})+\mu_{ij}\text{Tr}(\tau_2\Phi_i^T\tau_2\Phi_j)+W_{\text{NR}}, \end{aligned} \tag{3}$$

where W_{NR} denotes nonrenormalizable terms arising from higher scale physics such as grand unified theories or Planck scale effects. At this stage all couplings $\mathbf{Y}_{q,l}^{(i)}$, μ_{ij}, μ_Δ, μ_{Δ^c}, $\mathbf{f}$, and $\mathbf{f}_c$ are complex with μ_{ij}, $\mathbf{f}$, and $\mathbf{f}_c$ being symmetric matrices.

The part of the supersymmetric action that arises from this is given by

$$\mathcal{S}_W=\int d^4x\int d^2\theta\, W+\int d^4x\int d^2\overline{\theta}\, W^\dagger. \tag{4}$$

The terms that break supersymmetry softly to make the theory realistic can be written as

$$\begin{aligned} \mathcal{L}_{\text{soft}}=&\int d^4\theta\sum_i m_i^2\phi_i^\dagger\phi_i+\int d^2\theta\,\theta^2\sum_i A_iW_i \\ &+\int d^2\overline{\theta}\,\overline{\theta}^2\sum_i A_i^*W_i^\dagger+\int d^2\theta\,\theta^2\sum_p m_{\lambda_p}\widetilde{W}_p\widetilde{W}_p \\ &+\int d^2\overline{\theta}\,\overline{\theta}^2\sum_p m_{\lambda_p}^*\widetilde{W}_p^*\widetilde{W}_p^*. \end{aligned} \tag{5}$$

In Eq. (5), $\widetilde{W}_p$ denotes the gauge-covariant chiral superfield that contains the $F_{\mu\nu}$-type terms with the subscript going over the gauge groups of the theory including $SU(3)_c$. W_i denotes the various terms in the superpotential, with all superfields replaced by their scalar components and

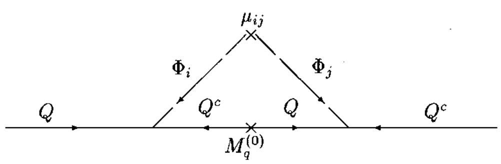

FIG. 1. Higgs contribution to one-loop calculation of $\overline{\Theta}$.

with coupling matrices which are not identical to those in W. Equation (5) gives the most general set of soft breaking terms for this model.

In Sec. I we saw that left-right symmetry implies that the first term in Eq. (1) is zero. Let us now see how supersymmetric left-right symmetry also requires the second term in this equation to vanish naturally. We choose the following definition of left-right transformations on the fields and the supersymmetric variable θ:

$$Q \leftrightarrow Q^{c*},$$
$$L \leftrightarrow L^{c*},$$
$$\Phi_i \leftrightarrow \Phi_i^\dagger,$$
$$\Delta \leftrightarrow \Delta^{c\dagger},$$
$$\overline{\Delta} \leftrightarrow \overline{\Delta}^{c\dagger},$$
$$\theta \leftrightarrow \overline{\theta},$$
$$\widetilde{W}_{\mathrm{SU}(2)_L} \leftrightarrow \widetilde{W}^*_{\mathrm{SU}(2)_R},$$
$$\widetilde{W}_{B-L,\mathrm{SU}(3)_c} \leftrightarrow \widetilde{W}^*_{B-L,\mathrm{SU}(3)_c}. \tag{6}$$

Note that this corresponds to the usual definition $Q_L \leftrightarrow Q_R$, etc. With this definition of L-R symmetry, it is easy to check that

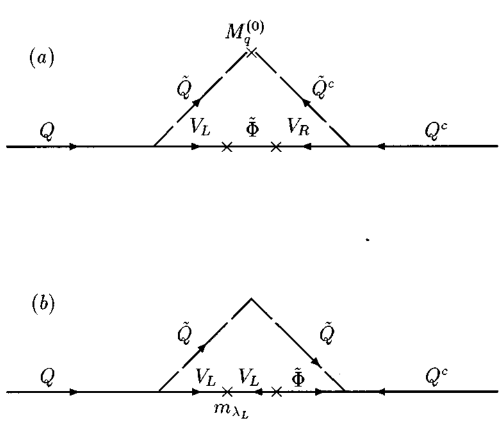

FIG. 2. Examples of gaugino contributions to one-loop calculation of Θ. $V_{L,r}$ are left and right gauginos, respectively. The gaugino mass m_{λ_L} is in general complex. There is an analogous graph to (b) that involves right-handed gauginos.

$$\mathbf{Y}_{q,l}^{(i)} = \mathbf{Y}_{q,l}^{(i)\dagger},$$
$$\mu_{ij} = \mu_{ij}^*,$$
$$\mu_\Delta = \mu_{\Delta^c}^*,$$
$$\mathbf{f} = \mathbf{f}_c^*,$$
$$m_{\lambda_{\mathrm{SU}(2)_L}} = m^*_{\lambda_{\mathrm{SU}(2)_R}},$$
$$m_{\lambda_{B-L\mathrm{SU}(3)_c}} = m^*_{\lambda_{B-L,\mathrm{SU}(3)_c}},$$
$$A_i = A_i^\dagger. \tag{7}$$

We will make extensive use of Eqs. (7) in this paper.[1] The first point to note is that the gluino mass is automatically real in this model: as a result, the last term in the equation for Θ above is naturally zero. We now therefore have to investigate only the quark mass matrices in order to guarantee that Θ vanishes at the tree level. For this purpose, we note that the Yukawa matrices are Hermitian[2] and the mass terms involving Higgs bidoublets in the superpotential are real. If we can show that the vacuum expectation values of the bidoublets are real, then the tree level value of Θ will be naturally zero.

As in [14], for W_{NR} we will use a single operator $(\lambda/M)[\mathrm{Tr}(\Delta^c \tau_m \Delta^c)]^2$, in order to be able to have vanishing sneutrino VEV's, as shown in Appendix A. The M could be equal to M_{Pl} or M_U. The other allowed nonrenormalizable operators do not effect our result and could be easily included in our discussion.

In this case we have made a detailed analysis of the Higgs potential and find that, at the minimum of the potential, the $\langle \Phi_i \rangle$ are always real. This result is not at all trivial because of large number of VEV's that enter, and one might naively think that spontaneous CP violation is possible. However, a recent analysis [16] has shown that a general supersymmetric model with two pairs of Higgs doublets (of which SUSY L-R is a special case) cannot break CP spontaneously. We give the application of this calculation to the SUSY L-R case in Appendix B. It is now clear that the quark mass matrices are Hermitian and therefore $\Theta = 0$ naturally at the tree level.

In Ref. [14] it was also shown that no strong CP-violating phase is generated at the one-loop level. Examples of one-loop diagrams are shown in Figs. 1 and 2; the Higgs diagram (Fig. 1) and some of the gaugino diagrams [Fig. 2(a)] generate only Hermitian contributions, while the other gaugino diagrams [Fig. 2(b)] are always real, if the gaugino masses are assumed to be real, as happens when our model is embedded into a grand unified theory (see later). Thus in the total contribution at the one-loop level the Yukawa matrices

[1]Note that the dagger in the last equation for A terms indicates that squark mass matrices $\mathbf{h}$ are Hermitian by L-R symmetry, although they of course do not have to be proportional to Yukawa mass matrices below some high scale.

[2]It is interesting that more general definitions of left-right transformations in the flavor sector are possible. For example, invariance under $Q \to U_1 Q^{c*}$ and $Q^c \to U_2 Q^*$, where U_1 and U_2 are some SU(3) matrices, gives *non*-Hermitian Yukawa matrices, but which still have a real determinant.

are still Hermitian. We concluded that the first nonzero contribution to $\bar{\Theta}$, if any, arises only at the two-loop level and is thus consistent with present limits.

III. SOLUTION TO THE STRONG *CP* PROBLEM HOLDS BELOW SCALE M_R

We have seen how the supersymmetric left-right model solves the strong CP problem at the scale of the $SU(2)_R$ breaking M_R. If this scale is of the order of the weak scale, then we are done, because the mass matrices in the expression for $\bar{\Theta}$ are defined at that scale and no further phase can be generated. Let us investigate what happens if M_R is some higher (intermediate) scale.[3] Two questions must be answered: Does the determinant of the Yukawa matrices stay real below M_R? The one-loop contribution of the $SU(2)_L$ gaugino is no longer canceled by the heavy $SU(2)_R$ gaugino. Can we avoid this contribution? As we will show, below the answer to both questions is yes.

Above the scale M_R the Hermiticity property of Yukawa couplings stays intact because L-R symmetry is not broken (see Appendix C). However, running of the Yukawa matrices below M_R will necessarily spoil the Hermiticity of Yukawa matrices because of breaking of parity (for example, the right-handed neutrino is excluded in running below M_R). Thus one might naively think that a nontrivial $\bar{\Theta}$ will be generated and that one must put constraints on M_R. However, we will now show that for the simplest case, when the field content below M_R is that of the MSSM, namely, two Higgs doublets, the determinants of the Yukawa matrices stay real.

Let us denote by $\mathbf{y}_i$ the Yukawa coupling of a Higgs doublet H_i ($i=1,2$). In the MSSM the one-loop running of the Yukawa couplings is of the form [17]

$$\frac{d}{dt}\,\mathbf{y}_i=\mathbf{y}_i\mathbf{T}, \tag{8}$$

where $\mathbf{T}$ is a matrix in flavor space which is a sum of terms of the form $\mathbf{y}_j^{\dagger}\mathbf{y}_j$, $\mathrm{Tr}(\mathbf{y}_j^{\dagger}\mathbf{y}_j)\mathbf{1}$, $g_a^2\mathbf{1}$ (see Appendix C). From Eq. (8) one can easily obtain the Jacobi identity for the determinant

$$\frac{d}{dt}\,\det\,\mathbf{y}_i=\det\,\mathbf{y}_i\ \mathrm{Tr}\ \mathbf{T}. \tag{9}$$

However, Tr $\mathbf{T}$ is always real, and since the determinant of $\mathbf{y}_i$ is real at the scale M_R, it will be real at any scale below M_R. We conclude that *although the Yukawa matrices will in general* not *be Hermitian anymore at the lower scale*, their determinants will nevertheless stay real.

The VEV's of the Higgs doublets in the MSSM can always be rotated so that both are real. Thus we conclude that $\Theta_{\rm tree}=0$.

Let us consider the one-loop contributions to $\bar{\Theta}$ below M_R. Typical diagrams that contribute at scale M_R are shown in Figs. 1 and 2. Since the running Yukawa matrices have a real determinant, the diagrams that have at vertices only Yukawa matrices or bidoublet masses (which are real) will not contribute. However, since the right-handed gaugino will decouple below M_R, the phase in the diagram involving the mass of the left gaugino will not cancel. The easiest way to circumvent this problem is to assume that the gaugino masses are real [15]. It is then easy to see that in the one-loop running the left gaugino mass stays real. Indeed, in Sec. V we show that the reality of the $SU(2)_{L,R}$ gaugino masses comes out naturally in an SO(10) model with a generalized left-right symmetry.

Let us next address the effect of the trilinear supersymmetry-breaking term involving squarks and the Higgs boson (i.e., $\mathbf{h}_u m_0\tilde{Q}H_u\tilde{u}^c$ and the corresponding term with u replaced by d) on $\bar{\Theta}$. Above the M_R scale, the matrices $\mathbf{h}_{u,d}$ are Hermitian due to the constraint of left-right symmetry (like the $\mathbf{Y}_{u,d}$). Therefore their contribution to $\bar{\Theta}$ involving the gluino at the one-loop level automatically vanishes above the scale M_R. (Here we used the fact that left-right symmetry requires that the gluino masses be real.) As we extrapolate it down to the M_Z scale using the renormalization group equations [17], we have to see if the det $h_{u,d}$ develop any imaginary part. A look at the one-loop renormalization group equation makes it clear that such an imaginary part (denoted by δ_A) could develop; let us therefore estimate its effect on the gluino mass as well as the quark mass matrices. A rough order of magnitude of the CP-violating phase in the gluino mass can be estimated as follows. Since the $\mathbf{h}_{u,d}$ are Hermitian and proportional to the Yukawa couplings $\mathbf{Y}_{u,d}$ at some scale above the M_R scale, let us go to a basis where $\mathbf{Y}_d$ and $\mathbf{h}_d$ are diagonalized. Then we find that, at the scale of proportionality, if any one of the off-diagonal elements of $\mathbf{Y}_u$ and (hence $\mathbf{h}_u$) are set to zero, the theory becomes completely CP conserving and cannot generate a CP-violating phase at any scale below M_R. It is then clear that the one-loop graph that generates a phase in the gluino mass can lead to the gluino phase $\delta_{\tilde{g}}$, which is at most

$$\delta_{\tilde{g}}\simeq\frac{V_{ub}V_{bc}V_{cd}V_{du}\alpha_s}{64\pi^3}\ln\frac{M_R}{M_Z}, \tag{10}$$

leading to $\delta_{\tilde{g}}\leqslant 10^{-8}$, which is close to the upper limits on the $\bar{\Theta}$. Similar arguments can be given for the one-loop contribution to the $\tilde{Q}\tilde{Q}^c$ mass matrix to show that their contribution to $\bar{\Theta}$ is around 10^{-8}.

Let us say a word about the finite contributions to $\bar{\Theta}$. These were shown [12] to be small in a supergravity theory with universality condition at a high scale, as we assume here. Of course, if the supersymmetry breaking is gauge mediated at scales slightly above the weak scale, the squarks will be highly degenerate and the solution to the strong CP problem is automatic [18].

It is worth pointing out at this stage that in the above discussion we have assumed that the theory below M_R is the MSSM (except, of course, the fact that the ''obnoxious'' R-parity-violating terms are naturally absent). In Appendix D, we discuss one way of obtaining the MSSM in the framework of our model.

[3]Such scales can be desired in grand unification schemes with L-R models as intermediate steps, because of the seesaw scenarios of neutrino masses.

In the end, let us consider what happens if we consider an effective four-Higgs-doublet model below M_R. The runnings of Yukawa couplings at one loop are listed in Appendix C. We note that the running of Yukawa matrices does not have a form of Eq. (8). There are additional terms on the right-hand side of the form $\mathbf{y}_j \mathrm{Tr}(\mathbf{y}_j^\dagger \mathbf{y}_i)$ $(i \neq j)$, thus invalidating the Jacobi identity for determinants. Indeed, such a term will in general produce phases of order $V_{cb}^2/(16\pi^2) \approx 10^{-5}$–$10^{-6}$. In this case, we also expect additional suppression coming from the fact at some very high scale the theory becomes *CP* conserving if any off-diagonal element of Y_u is set to zero. Barring enough suppression from this, it may be necessary to impose some additional symmetry to suppress the Yukawa couplings of the second pair of Higgs doublets[4] [16].

In conclusion, if the effective theory below M_R has the MSSM-like field content, and if the left gaugino mass is real, no observable Θ will be generated for all values of M_R from some intermediate scale ($\approx 10^{12}$ GeV) all the way down to 1 TeV.

IV. SOLUTION TO THE SUSY *CP* PROBLEM

Let us now turn to the discussion of the SUSY *CP* problem. The main issue here is the potentially large contribution to the electric dipole moment of the neutron at the one-loop level. An analysis of the various aspects of the problem has been reviewed in Ref. [19]. In the standard parametrization of the MSSM interactions at the electroweak scale, the large contributions to d_e^n comes from two sources: the phases of the $(Am_{\tilde{g}})$ and $(\mu v_u m_{\tilde{g}}/v_d)$ terms. Another way to state this is to note that the first term originate from the same trilinear scalar SUSY-breaking terms $h_{u,d}$ discussed in the previous section, whereas the second term arises from the F-term contribution extrapolated down to the electroweak scale. We work in a basis where the diagonal block matrices in the squark $(\tilde{q}-\tilde{q}^c)$ mass matrices are diagonalized. We will then be interested in the 11 entry of the gluino one-loop contribution to electric dipole moment operator for both the up and down sectors.

The first point to note is that in our model, above the M_R scale, the Hermiticity of $\mathbf{h}_{u,d}$ and $\mathbf{Y}_{u,d}$ together with the reality of the gluino mass implies that there is no one-loop contribution to d_ϵ^n. Garisto [19] has argued that if the above parameters are real at any high energy scale, their contribution to d_ϵ^n remains small at the electroweak scale. For example, in our case as shown above, the phase of the gluino mass at the scale M_Z is of order 10^{-8}. As far as the $\mathbf{h}_{u,d}$ and $\mathbf{Y}_{u,d}$ terms are concerned, we have not succeeded in showing that once extrapolated down to the M_Z scale, the 11 term of the gluino-induced dipole moment matrix remains real. However, using the already stated argument above, the Hermiticity of the Yukawa matrices above the scale M_R implies that that any departure from reality is at most of order $\delta \equiv V_{ub}V_{bc}V_{cd}/(16\pi^2)\ln(M_R/M_Z) \simeq 10^{-6}$. We then expect that the maximum contribution to d_e^n from them to

$$(d_e^n)^{\max} \leqslant \frac{8e\alpha_s m_d}{27\pi M_{\tilde{q}}^2}\,\delta \leqslant 10^{-28}\ e\ \mathrm{cm}, \qquad (11)$$

where we have assumed that $m_d \simeq 10$ MeV and $M_{\tilde{q}} \simeq 100$ GeV. This is safely within the present experimental upper limit. Thus our model provides simultaneously a solution to the SUSY *CP* problem without the need for any new symmetries.

Let us note that in this paper we do not address the usual SUSY flavor problems with KK mixings, etc., that require a certain degree of fine tuning in the structure of squark and quark matrices, since this is beyond the scope of our paper. Left-right symmetry implies that the Yukawa matrices are Hermitian, but the flavor properties such as hierarchy and alignment must come from the underlying flavor theory, and we refer the reader to the existing solutions [20] which may be employed here as well. We note, however, that the strongest requirements come actually from the *CP* problems that are solved here.

V. SO(10) EMBEDDING AND REALITY OF WEAK GAUGINO MASSES

In this section, we address the question of embedding the left-right model into an SO(10) theory so that we not only have a grand unified version of our theory, but also we guarantee the reality of the gaugino masses (i.e., $m_{\lambda_{L,R}} = m^*_{\lambda_{L,R}}$). The reality of the gaugino masses follows from the combination of two things: The requirement of left-right symmetry implies, as shown earlier, that $m_{\lambda_L} = m^*_{\lambda_R}$; on the other hand, SO(10) unification implies that the two gauginos, being part of the same **45** dimensional representation, have equal mass. The main task for us in this section is to show that there exists a definition of left-right symmetry which preserves the hermiticity of the Yukawa couplings.

To prove the Hermiticity of the Yukawa couplings, we will exhibit only the simplest model and not attempt to address the issues such as doublet triplet splitting, etc. Let us consider the Higgs fields belonging to **10** (denoted by **H**), **45** (denoted **A**), and **126** (denoted by Δ) (plus $\bar{\mathbf{\Delta}}$) representations. The superpotential involving all these fields can be written as

$$\begin{aligned} W_{\mathrm{GUT}} = {} & Y^s_{ab}\psi_a^T B\Gamma_i \psi H_i + Y^A_{ab}\psi_a^T B\Gamma_i\Gamma_j\Gamma_k\psi_b (H_i A_{jk})_{\mathrm{anti}}/M \\ & + Y'_{ab}\psi_a^T B\Gamma_i\psi_b H_j A_{ij}/M \\ & + \int_{ab}\psi_a^T B\Gamma_i\Gamma_j\Gamma_k\Gamma_l\Gamma_m\psi_b\Delta_{ijklm} \\ & + \text{terms involving } \Delta \text{ in order } \frac{1}{M}. \end{aligned} \qquad (12)$$

It is well known that Y^s, Y', and f are symmetric matrices, whereas Y^A is an antisymmetric combination since we have

[4] Note, however, that in the second paper of Ref. [16] a general four-Higgs-doublet model with arbitrary Yukawa couplings was considered, and the additional symmetry was needed to suppress too large *CP* violation in KK mixing; strong *CP* violation was too large in that model. In our case Yukawa couplings have the constraint that they come from Hermitian matrices at the M_R scale and the additional global symmetry is enough to solve the strong *CP* problem.

projected out the **120** dim. representation from the **10** and **45** product in the h^A term. Let us now define that under parity transformation

$$\psi \to D_\psi B^{-1}\psi^*, \quad H_i \to -D_H H_i^* D_H^{-1},$$

$$A_{ij} \to -D_A A_{ij}^* D_A^{-1}, \quad \Delta \to -D_\Delta \Delta^* D_\Delta^{-1}. \tag{13}$$

Here B is the charge conjugation matrix for SO(10): D is the operator that implements the left-right transformation inside the SO(10) multiplets ψ. H, etc.[5] For instance, operating on the **10** dimensional representation (H_i), it changes $H_7 \to -H_7$ and leaves all other components unchanged: Similarly, in **45**, it changes the sign of all elements that carry the index 7, etc. [Note that the choice of the seventh component is basis dependent and we are working in a basis where $I_{3L}=\frac{1}{4}(\Sigma_{90}-\Sigma_{78})$ and $I_{3R}=\frac{1}{4}(\Sigma_{90}+\Sigma_{78})$, where Σ_{ij} are the generators of SO(10).]

Now, using the fact that $B^T=-B$ and $B^{-1}\Gamma_i B=-\Gamma_i^T$, it is then easy to show that h^s_{ab}, h'_{ab}, and f_{ab} are real, whereas h^A_{ab} is imaginary. Together with the symmetricity properties, this implies that all these matrices are Hermitian.

Now, if the SO(10) symmetry is broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ by the **45** VEV as $\langle A \rangle = i\tau_2 \mathrm{diag}(v,v,v,0,0)$, then the effective low energy theory has two bidoublets and also general Hermitian Yukawa coupling of quarks. This leads to the embedding of our solution to the strong CP problem in an SO(10) model.

Another implication of the SO(10) embedding of our model is that it imposes restrictions on the terms in the superpotential involving the Δ and Δ^c fields. For instance, for the renormalizable term in Eq. (3) involving these fields, we get μ_Δ to be real. It will also imply that nonrenormalizable terms of the form $\Delta^c \Delta^c \phi_i \phi_j$ will have real couplings. This will have important bearing on their contribution to Θ, as shown in Appendix A.

We do not discuss here the unification of gauge couplings in theories with $SU(2)_L \times SU(2)_R$ as intermiediate symmetries, but note that examples of successful scenarios exist which could implement our mechanism. It has, for instance, been argued in Ref. [21] that a stringy embedding of our model in SO(10) can be achieved if $M_R > 10^8$ GeV or so. Our discussions will then apply to these models.

VI. CONCLUSION

We have shown that if the minimal supersymmetric extension of the standard model (MSSM) is embedded in the supersymmetric left-right model at higher energies, both the strong and weak CP problems of the MSSM are automatically cured. Adding this to the already known result that R-parity conservation is restored as an exact symmetry in the SUSYLR model, thereby providing a naturally stable neutralino that can act as the cold dark matter of the universe, makes this embedding quite attractive. The left-right symmetry is then incorporated into an SO(10) grand unified theory where the scale of right-handed symmetry breaking may be quite high. We show how the conclusion about the vanishing of the strong CP parameter remains unchanged in this case.

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APPENDIX A: AVOIDING SNEUTRINO VEV's

In this appendix we will show that if in the minimal SUSY L-R model one includes nonrenormalizable Planck-scale-induced terms, the ground state of the theory can be $Q^{\rm em}$ conserving even for $\langle \tilde{\nu}^c \rangle = 0$. For this purpose, let us briefly recall the argument of Ref. [22]. The part of the potential containing L^c, Δ^c, and $\bar{\Delta}^c$ fields only has the form (see Appendix B or [22])

$$V = V_0 + V_D. \tag{A1}$$

where

$$V_0 = \mathrm{Tr}|i\mathbf{f}^\dagger L^c L^{cT}\tau_2 + \mu_\Delta^* \bar{\Delta}^c|^2 + \mu_1^2 \mathrm{Tr}(\Delta^c \Delta^{c\dagger}) + \mu_2^{22}\mathrm{Tr}(\bar{\Delta}^c \bar{\Delta}^{c\dagger}) + \mu_3^2 \mathrm{Tr}(\Delta^c \bar{\Delta}^c) + \mu_4 \tilde{L}^{cT}\tau_2 \Delta^c L^c \tag{A2}$$

and

$$V_D = \frac{g^2}{8}\sum_m |\tilde{L}^{c\dagger}\tau_m \tilde{L}^c + \mathrm{Tr}(2\Delta^{c\dagger}\tau_m \Delta^c + 2\bar{\Delta}^{c\dagger}\tau_m \bar{\Delta}^c)|^2 + \frac{g'^2}{8}|\tilde{L}^{c\dagger}\tilde{L}^c - 2\mathrm{Tr}(\Delta^{c\dagger}\Delta^c - \bar{\Delta}^{c\dagger}\bar{\Delta}^c)|^2. \tag{A3}$$

Note that if $\langle \tilde{\nu}^c \rangle = 0$, then the vacuum state for which $\Delta^c = (1/\sqrt{2})v\tau_1$ and $\bar{\Delta}^c = (1/\sqrt{2})v'\tau_1$ is lower than the vacuum state

$$\Delta^c = v\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

and

$$\bar{\Delta}^c = v'\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

However, the former is electric charge violating. The only way to have the global minimum conserve electric charge is to have $\langle \tilde{\nu}^c \rangle \neq 0$. On the other hand, if we have nonrenormalizable terms included in the theory, the situation changes: For instance, let us include nonrenormalizable gauge-invariant terms of the form (inclusion of other nonrenormal-

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[5]This is similar to the L-R definition (6). This comes because, for example, strictly speaking Q^c is not a doublet under $SU(2)_R$, but rather $Q'^c \equiv \tau_2 Q^c$. So the L-R definition (6) in terms of the gauge multiplets would be $Q - \tau_2 Q'^{c*}$. The operator τ_2 plays a similar role as the operator D above in the SO(10) case.

izable gauge-invariant terms simply enlarges the parameter space where our conclusion holds)

$$W_{\rm NR}=\frac{\lambda}{M}[\mathrm{Tr}(\Delta^c\tau_m\overline{\Delta^c})]^2. \tag{A4}$$

This will change V to the form

$$V=V_0+V_{\rm NR}+V_D, \tag{A5}$$

where V_0 and V_D are given before and $V_{\rm NR}$ is given by

$$V_{\rm NR}=\frac{\lambda\mu}{M}[\mathrm{Tr}(\Delta^c\tau_m\overline{\Delta^c})]^2+\frac{4\lambda\mu_\Delta}{M}[\mathrm{Tr}(\Delta^c\tau_m\overline{\Delta^c})]\times[\mathrm{Tr}(\Delta^{c\dagger}\tau_m\Delta^c)]+\Delta^c\leftrightarrow\overline{\Delta^c}+\text{etc.} \tag{A6}$$

For the charge-violating minimum above, this term vanishes, but the charge-conserving minimum receives a nonzero contribution. Note that the sign of λ is arbitrary, and therefore, by appropriately choosing $\mathrm{sgn}(\lambda)$, we can make the electric-charge conserving vacuum lower than the $Q^{\rm em}$-violating one. In fact, one can argue that, since we expect $v^2-v'^2\approx f^2(M_{\rm SUSY})^2/16\pi^2$ in typical Polonyi-type models, the charge-conserving minimum occurs for $f<4\pi(4\lambda\mu_\Delta/M)^{1/4}v/M_{\rm SUSY}$. For $\lambda\approx1$, $\mu_\Delta\approx v\approx M_{\rm SUSY}\approx1$ TeV and $M=M_{\rm Pl}$, we get $f\leqslant10^{-3}$ if $v\approx M_{\rm SUSY}$. We have assumed that the right-handed scale is in the TeV range. The constraint on f, of course, becomes weaker for larger values of μ_Δ. We wish to note that a possible nonrenormalizable term of the form $\lambda_1\mathrm{Tr}(\Delta^c\tau_m\overline{\Delta^c})\mathrm{Tr}(\Phi_i\tau_m\Phi_j)(1/M_{\rm Pl})$ can induced a complex effective mass for the bidoublets, but its magnitude is given by $v_R^2/M_{\rm Pl}$ if λ_1 and μ_Δ are complex. Its presence, therefore, will affect the solution to the strong *CP* problem outlined in the paper for values of v_R above the intermediate scale $\approx10^6-10^7$ GeV, depending on the value of λ_1. It is interesting to note that if the model is embedded in the SO(10) theory, as already noted, both λ_1 and μ_Δ are real and there are no new contributions to Θ from these new terms. In this case, our solution is useful for v_R in the range of $10^{11}-10^{12}$ GeV.

Furthermore, it is also important to point out that since Planck scale effects are not expected to respect any global symmetries, the coupling parameters of the higher dimensional terms in Eq. (A3) involving Δ and Δ^c will be different. This difference will help in the realization of the parity-violating minimum as the global minimum of the theory.

APPENDIX B: REALITY OF BIDOUBLET VEV's

Here we show that the VEV's of the bidoublet Higgs fields in the supersymmetric left-right model are real. The scalar potential is given by

$$V=V_F+V_{\rm soft}+V_D+V_{\rm NR}(\Delta^c,\overline{\Delta^c}), \tag{B1}$$

where

$$V_F=\sum_p|\mathbf{Y}^{(i)}_{qpr}\tau_2\Phi_i\tau_2Q^c_r|^2+\sum_r|\mathbf{Y}^{(i)}_{qpr}Q_p\tau_2\Phi_i\tau_2|^2+\sum_i\mathrm{Tr}|\mathbf{Y}^{(i)T}_qQQ^{cT}+\mathbf{Y}^{(i)T}_lLL^{cT}+2\mu_{ij}\Phi_j|^2+\sum_p|\mathbf{Y}^{(i)}_{lpr}\tau_2\Phi_i\tau_2L^c_r+2if_{pr}\tau_2\Delta L_r|^2+\sum_r|\mathbf{Y}^{(i)}_{lpr}L_p\tau_2\Phi_i\tau_2+2if^*_{pr}L^c_p\tau_2\Delta^c|^2+\mathrm{Tr}|i\mathbf{f}^TLL^T\tau_2+\mu_\Delta\overline{\Delta}|^2+\mathrm{Tr}|i\mathbf{f}^\dagger L^cL^{cT}\tau_2+\mu^*_\Delta\overline{\Delta^c}|^2+|\mu_\Delta|^2\mathrm{Tr}(\Delta\Delta^\dagger+\Delta^c\Delta^{c\dagger}), \tag{B2}$$

$$V_{\rm soft}=m_q^2(\tilde{Q}^\dagger\tilde{Q}+\tilde{Q}^{c\dagger}\tilde{Q}^c)+m_l^2(\tilde{L}^\dagger\tilde{L}+\tilde{L}^{c\dagger}\tilde{L}^c)+m^2_{\Phi_i}\Phi_i^\dagger\Phi_i+m_\Delta^2\mathrm{Tr}(\Delta^\dagger\Delta+\Delta^{c\dagger}\Delta^c)+m_{\bar\Delta}^2\mathrm{Tr}(\overline{\Delta}^\dagger\overline{\Delta}+\overline{\Delta^c}^\dagger\overline{\Delta^c})+[A_{q,i}\mathbf{Y}^{(i)}_q\tilde{Q}^T\tau_2\Phi_i\tau_2\tilde{Q}^c+A_{l,i}\mathbf{Y}^{(i)}_l\tilde{L}^T\tau_2\Phi_i\tau_2\tilde{L}^c+A_Li(\mathbf{f}\tilde{L}^T\tau_2\Delta L+\mathbf{f}^*\tilde{L}^{cT}\tau_2\Delta^cL^c)+A_\Delta[\mu_\Delta\mathrm{Tr}(\Delta\overline{\Delta})+\mu^*_\Delta\mathrm{Tr}(\Delta^c\overline{\Delta^c})]+A_\Phi\mu_{ij}\mathrm{Tr}(\tau_2\Phi_i^T\tau_2\Phi_j)+\text{H.c.}], \tag{B3}$$

$$V_D=\frac{g^2}{8}\sum_m|\tilde{L}^\dagger\tau_m\tilde{L}+\mathrm{Tr}(2\Delta^\dagger\tau_m\Delta+2\overline{\Delta}^\dagger\tau_m\overline{\Delta}+\Phi^\dagger\tau_m\Phi)|^2+\frac{g^2}{8}\sum_m|\tilde{L}^{c\dagger}\tau_m\tilde{L}^c+\mathrm{Tr}(2\Delta^{c\dagger}\tau_m\Delta^c+2\overline{\Delta^c}^\dagger\tau_m\Delta^c+\Phi\tau_m^T\Phi^\dagger)]^2+\frac{g'^2}{8}|\tilde{L}^{c\dagger}\tilde{L}^c-\tilde{L}^\dagger\tilde{L}+2\mathrm{Tr}(\Delta^\dagger\Delta-\Delta^{c\dagger}\Delta^c-\overline{\Delta}^\dagger\overline{\Delta}+\overline{\Delta^c}^\dagger\overline{\Delta^c})|^2, \tag{B4}$$

and $V_{\rm Nr}$ is defined in Appendix A.

We assume the following fields get the VEV's:

$$\langle\Delta^c\rangle=\begin{pmatrix}0&0\\ \Delta^0e^{-i\beta_\Delta}&0\end{pmatrix},\quad\langle\overline{\Delta^c}\rangle=\begin{pmatrix}0&\delta^0\\0&0\end{pmatrix}, \tag{B5}$$

and

$$\langle\Phi_1\rangle=\begin{pmatrix} v_1 & 0 \\ 0 & v_2e^{i\delta_2}\end{pmatrix},\quad \langle\Phi_2\rangle=\begin{pmatrix} v_3e^{i\delta_3} & 0 \\ 0 & v_4e^{i\delta_4}\end{pmatrix}, \tag{B6}$$

where we have rotated away the nonphysical phases.

The VEV of the scalar potential is

$$\langle V\rangle=|2\mu_{11}v_1+2\hat{\imath}_{12}v_3e^{i\delta_3}|^2+|2\mu_{11}v_2e^{i\delta_2}+2\mu_{12}v_4e^{i\delta_4}|^2+|2\mu_{21}v_1+2\mu_{22}v_3e^{i\delta_3}|^2+|2\mu_{21}v_2e^{i\delta_2}+2\mu_{22}v_4e^{i\delta_4}|^2+|\mu_\Delta|^2$$
$$\times(\Delta^{02}+\delta^{02})+m_\Delta^2\Delta^{02}+m_{\bar{\Delta}}^2\delta^{02}+m_{\Phi_1}^2(v_1^2+v_2^2)+m_{\Phi_2}^2(v_3^2+v_4^2)+A_\Delta|\mu_\Delta|\Delta^0\delta^0\cos[\mathrm{B}_\Delta+\mathrm{Arg}(\mu_\Delta)]+A_\Phi\mu_{11}2v_1v_2\cos\delta_2$$
$$+A_\Phi\mu_{12}2[v_1v_4\cos\delta_4+v_2v_3\cos(\delta_2+\delta_3)]+A_\Phi\mu_{22}2v_3v_4\cos(\delta_3+\delta_4)+\langle V_D\rangle+V_{\mathrm{NR}}(\langle\Delta^c\rangle,\langle\overline{\Delta^c}\rangle), \tag{B7}$$

where $\langle V_D\rangle$ is the VEV of the D term:

$$\langle V_D\rangle=\frac{g^2}{8}[v_1^2-v_3^2-v_2^2-v_4^2]^2+\frac{g^2}{8}[2(\Delta^{02}+\delta^{02})+v_1^2+v_3^2$$
$$-v_2^2-v_4^2]^2+\frac{g'^2}{8}[2(\Delta^{02}+\delta^{02})]^2. \tag{B8}$$

Note that the phases of the bidoublets δ_i, $i=2,3,4$, come in the terms

$$v_1v_i\cos\delta_i,\quad i=2,3,4,$$
$$v_2v_3\cos(\delta_2+\delta_3),$$
$$v_2v_4\cos(\delta_2-\delta_4),$$
$$v_3v_4\cos(\delta_3+\delta_4). \tag{B9}$$

Also, powers of the bidoublet VEV's which are higher than 2 come only in the D term and there in one only combination $g(\mathbf{v})=v_1^2+v_3^2-v_2^2-v_4^2$. This is exactly the situation in general four-Higgs-doublet supersymmetric models with real mass parameters. In Ref. [16] it was shown, by using a simple geometrical interpretation for the minimum equations for the three phases, that the minimum in such a model is CP conserving. Thus we conclude that in the SUSY L-R model the VEV's of the doublets are real. This conclusion holds for general $A_{\Phi_{ij}}$, which can be different for different i,j.

The phase of the VEV of the triplet β_Δ is in general nonzero (e.g., induced by the phase of the coupling μ_Δ), but it does not couple to the VEV's of the doublets. Thus it is irrelevant since it does not enter the calculation of Θ at the tree level or one loop. However, as noted in Appendix A, when the theory is embedded into the SO(10) group, the triplet VEV's become real.

APPENDIX C: ONE-LOOP RUNNING OF YUKAWA COUPLINGS

1. Four-Higgs-doublet SUSY model

Here we list the one-loop running of Yukawa couplings for a general *four*-Higgs-doublet supersymmetric model. The Yukawa matrices of Higgs-doublets that couple to down quarks are denoted by $\mathbf{y}_1$ and $\mathbf{y}_3$ and, similarly $\mathbf{y}_2$ and $\mathbf{y}_4$ for the up-type quarks:

$$L_Y=D\mathbf{y}_1QH_1+D\mathbf{y}_3QH_3+U\mathbf{y}_2QH_2+U\mathbf{y}_2QH_2+U\mathbf{y}_4QH_4$$
$$+E\mathbf{y}_1^eLH_1+E\mathbf{y}_3^eLH_3 \tag{C1}$$

$$\frac{d}{dt}\mathbf{y}_1=\frac{1}{16\pi^2}\{\mathbf{y}_1[\mathrm{Tr}(3\mathbf{y}_1^\dagger\mathbf{y}_1+\mathbf{y}_1^{e\dagger}\mathbf{y}_1^e)+3\mathbf{y}_1^\dagger\mathbf{y}_1+y_2^\dagger\mathbf{y}_2+\mathbf{y}_3^\dagger\mathbf{y}_3$$
$$+\mathbf{y}_4^\dagger\mathbf{y}_4]+\mathbf{y}_3[\mathrm{Tr}(3\mathbf{y}_3^\dagger\mathbf{y}_1+\mathbf{y}_3^{e\dagger}\mathbf{y}_1^e)+2\mathbf{y}_3^\dagger\mathbf{y}_1]$$
$$-\mathbf{y}_1(\tfrac{7}{15}g_1^2+3g_2^2+\tfrac{16}{3}g_3^2)\},$$

$$\frac{d}{dt}\mathbf{y}_2=\frac{1}{16\pi^2}\{\mathbf{y}_2[\mathrm{Tr}(3\mathbf{y}_2^\dagger\mathbf{y}_2)+\mathbf{y}_1^\dagger\mathbf{y}_1+3\mathbf{y}_2^\dagger\mathbf{y}_2+\mathbf{y}_3^\dagger\mathbf{y}_3+\mathbf{y}_4^\dagger\mathbf{y}_4]$$
$$+\mathbf{y}_4[\mathrm{Tr}(3\mathbf{y}_4^\dagger\mathbf{y}_2)+2\mathbf{y}_4^+\mathrm{y}_2]-\mathbf{y}_2(\tfrac{13}{15}g_1^2+3g_2^2+\tfrac{16}{3}g_3^2)\},$$

$$\frac{d}{dt}\mathbf{y}_3=\frac{1}{16\pi^2}\{\mathbf{y}_3[\mathrm{Tr}(3\mathbf{y}_3^\dagger\mathbf{y}_3+\mathbf{y}_3^{e+}\mathbf{y}_3^e)+\mathbf{y}_1^\dagger\mathbf{y}_1+\mathbf{y}_2^\dagger\mathbf{y}_2+3\mathbf{y}_3^\dagger\mathbf{y}_3$$
$$+\mathbf{y}_4^\dagger\mathbf{y}_4]+\mathbf{y}_1[\mathrm{Tr}(3\mathbf{y}_1^+\mathbf{y}_3+\mathbf{y}_1^{e\dagger}\mathbf{y}_3^e)+2\mathbf{y}_1^+\mathbf{y}_3]-\mathbf{y}_3(\tfrac{7}{15}g_1^2$$
$$+3g_2^2+\tfrac{16}{3}g_3^2)\},$$

$$\frac{d}{dt}\mathbf{y}_4=\frac{1}{16\pi^2}\{\mathbf{y}_4[\mathrm{Tr}(3\mathbf{y}_4^\dagger\mathbf{y}_4)+\mathbf{y}_1^\dagger\mathbf{y}_1+\mathbf{y}_2^\dagger\mathbf{y}_2+\mathbf{y}_3^\dagger\mathbf{y}_3+3\mathbf{y}_4^\dagger\mathbf{y}_4]$$
$$+\mathbf{y}_2[\mathrm{Tr}(3\mathbf{y}_2^\dagger\mathbf{y}_4)+2\mathbf{y}_2^\dagger\mathbf{y}_4]-\mathbf{y}_1(\tfrac{13}{15}g_1^2+3g_2^2+\tfrac{16}{3}g_3^2)\},$$

$$\frac{d}{dt}\mathbf{y}_1^e=\frac{1}{16\pi^2}\{\mathbf{y}_1^e[\mathrm{Tr}(3\mathbf{y}_1^\dagger\mathbf{y}_1+\mathbf{y}_1^{e\dagger}\mathbf{y}_1^e)+3\mathbf{y}_1^{e+}\mathbf{y}_1^e+\mathbf{y}_3^{e\dagger}\mathbf{y}_3^e]$$
$$+\mathbf{y}_3^e[\mathrm{Tr}(3\mathbf{y}_3^\dagger\mathbf{y}_1+\mathbf{y}_3^{e\dagger}\mathbf{y}_1^e)+2\mathbf{y}_3^{e\dagger}\mathbf{y}_1^e]-\mathbf{y}_1^e(\tfrac{9}{5}g_1^2+3g_2^2)\},$$

$$\frac{d}{dt}\mathbf{y}_3^e=\frac{1}{16\pi^2}\{\mathbf{y}_3^e[\mathrm{Tr}(3\mathbf{y}_3^\dagger\mathbf{y}_3+\mathbf{y}_3^{e\dagger}\mathbf{y}_3^e)+\mathbf{y}_1^{e\dagger}\mathbf{y}_1^e+3\mathbf{y}_3^{e\dagger}\mathbf{y}_3^e]$$
$$+\mathbf{y}_1^e[\mathrm{Tr}(3\mathbf{y}_1^\dagger\mathbf{y}_3+\mathbf{y}_1^{e\dagger}\mathbf{y}_3^e)+2\mathbf{y}_1^{e\dagger}\mathbf{y}_3^e]-\mathbf{y}_3^e(\tfrac{9}{5}g_1^2+3g_2^2)\}. \tag{C2}$$

The equations for a two-Higgs-doublet model (i.e., the MSSM) are easily obtained by setting, for example, Yukawa matrices $\mathbf{y}_3$, $\mathbf{y}_4$, and $\mathbf{y}_3^e$ to zero in the equations above. They indeed have the form of Eq. (8).

We see that Eq. (C2) part from the form of Eq. (8) because of Higgs doublet wave function renormalization terms (for example the term $\mathbf{y}_3\mathrm{Tr}(3\mathbf{y}_3^\dagger\mathbf{y}_1)$ in the equation for $\mathbf{y}_1$).

One can still write an equation in form Eq. (9) with new terms in $\mathbf{T}$ which are not real in general. For example, the phase will appear in $\mathbf{y}_1^{-1}\mathbf{y}_3\mathrm{Tr}(3\mathbf{y}_3^{\dagger}\mathbf{y}_1)$, and it will depend on the structure of the Yukawa matrices how large the phase is.

2. SUSY *L-R* model

It is easy to generalize the above one-loop runnings for the case of Yukawa couplings in the SUSY *L-R* model with two bidoublet:[6]

$$L_Y = Q^c\mathbf{Y}_1 Q\Phi_1 + Q^c\mathbf{Y}_2 Q\Phi_2 + L^c\mathbf{Y}_1^e L\Phi_1 + L^c\mathbf{Y}_2^e L\Phi_2 . \tag{C3}$$

We simply take $\mathbf{y}_1=\mathbf{y}_2=\mathbf{Y}_1$ (and similarly for other Yukawa couplins), add the right-handed neutrino, and compute the contribution from gauge couplings. Alternatively, we use general formulas [17]. In any case we obtain

$$\begin{aligned}
\frac{d}{dt}\mathbf{Y}_1 = \frac{1}{16\pi^2}\{&\mathbf{Y}_1[\mathrm{Tr}(3\mathbf{Y}_1^{\dagger}\mathbf{Y}_1+\mathbf{Y}_1^{e\dagger}\mathbf{Y}_1^{e})+4\mathbf{Y}_1^{\dagger}\mathbf{Y}_1 2\mathbf{Y}_2]\\
&+\mathbf{Y}_2[\mathrm{Tr}(3\mathbf{Y}_2^{\dagger}\mathbf{Y}_1+\mathbf{Y}_2^{e\dagger}\mathbf{Y}_1^{e})+2\mathbf{Y}_2^{\dagger}\mathbf{Y}_1]\\
&-\mathbf{Y}_1(\tfrac{1}{6}g_{B-L}^2+3g_L^3+3g_R^2+\tfrac{16}{3}g_3^2)\},
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\mathbf{Y}_2 = \frac{1}{16\pi^2}\{&\mathbf{Y}_2[\mathrm{Tr}(3\mathbf{Y}_2^{\dagger}\mathbf{Y}_2+\mathbf{Y}_2^{e\dagger}\mathbf{Y}_2^{e})+2\mathbf{Y}_1^{\dagger}\mathbf{Y}_1+4\mathbf{Y}_2^{\dagger}\mathbf{Y}_2]\\
&+\mathbf{Y}_1[\mathrm{Tr}(3\mathbf{Y}_1^{\dagger}\mathbf{Y}_2+\mathbf{Y}_1^{e\dagger}\mathbf{Y}_2^{e})+2\mathbf{Y}_1^{\dagger}\mathbf{Y}_2]\\
&-\mathbf{Y}_2(\tfrac{1}{6}g_{B-L}^2+3g_L^2+3g_R^2+\tfrac{16}{3}g_3^2)\},
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\mathbf{Y}_1^e = \frac{1}{16\pi^2}\{&\mathbf{Y}_1^e[\mathrm{Tr}(3\mathbf{Y}_1^{\dagger}\mathbf{Y}_1+\mathbf{Y}_1^{e}\mathbf{Y}_1^{e})+4\mathbf{Y}_1^{e\dagger}\mathbf{Y}_1^e+2\mathbf{Y}_2^{e\dagger}\mathbf{Y}_2^e]\\
&+\mathbf{Y}_2^e[\mathrm{Tr}(3\mathbf{Y}_2^{+}\mathbf{Y}_1+\mathbf{Y}_2^{e\dagger}\mathbf{Y}_1^{e})+2\mathbf{Y}_2^{e\dagger}\mathbf{Y}_1^e]\\
&-\mathbf{Y}_1^e(\tfrac{3}{2}g_{B-L}^2+3g_L^2+3g_R^2)\},
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dt}\mathbf{Y}_2^e = \frac{1}{16\pi^2}\{&\mathbf{Y}_2^e[\mathrm{Tr}(3\mathbf{Y}_2^{+}\mathbf{Y}_2+\mathbf{Y}_2^{e\dagger}\mathbf{Y}_2^{e})+2\mathbf{Y}_1^{e\dagger}\mathbf{Y}_1^e\\
&+4\mathbf{Y}_2^{e+}\mathbf{Y}_2^e]+\mathbf{Y}_1^e[\mathrm{Tr}(3\mathbf{Y}_1^{+}\mathbf{Y}_2+\mathbf{Y}_1^{e\dagger}\mathbf{Y}_2^{e})+2\mathbf{Y}_1^{e\dagger\mathbf{Y}_2^e}]\\
&-\mathbf{Y}_2^e(\tfrac{3}{2}g_{B-L}^2+3g_L^2+3g_R^2)\}.
\end{aligned} \tag{C4}$$

It is easy to see that the Hermiticity of Yukawa couplings is preserved throughout the running in the $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ phase (i.e., about m_R), as expected.[7] This is in contrast to case given in Appendix C 1 above where below M_R running of matrices necessarily spoils Hermiticity (both in the MSSM and the four-Higgs-doublet model), because then the *L-R* symmetry is broken.

[6]We skipped triplet couplings for simplicity.

APPENDIX D: DOUBLET-DOUBLET SPLITTING

In this appendix, we show how a left-right symmetric theory with two bidoublets above the scale M_R reduces to the MSSM with only one pair of H_u, H_d). We will call this phenomenon doublet-doublet splitting. The simplest way to achieve this is by a fine-tuning of the parameters of the superpotential (3) involving the ϕ_1 and ϕ_2 fields, i.e., μ_{ij}. To make this explicit, consider the part of the superpotential

$$W_\phi = \sum_{ij} \tfrac{1}{2}\mu_{ij}\mathrm{Tr}\phi_a^T\tau_2\phi_b\tau_2 , \tag{D1}$$

where the symbols a,b go over 1,2. This leads to the following superpotential in terms of the standard model doublets:

$$W_\phi = \mu_{11}H_{u1}H_{d1}+\mu_{22}H_{u2}H_{d2}+\mu_{12}(H_{u1}H_{d2}+H_{u2}H_{d1}). \tag{D2}$$

Now it is clear that, if the parameters μ_{ab} are so chosen that we have $\mu_{11}\mu_{22}-\mu_{12}^2=0$ and that each μ_{ij} are of order v_R, then below the scale v_R the model has only two standard model doublets as in the MSSM. The surviving doublets are then linear combinations of the original four doublets in the theory. If, however, one wanted ''pure'' doublets surviving below the v_R scale (such as, say, H_{u1} and H_{d2}), then one can use a superpotential of the type:

$$W'_\phi = \frac{\lambda_{12}}{M_{\mathrm{Pl}}}\mathrm{Tr}\phi_1^T\tau_2\tau_i\phi_2\mathrm{Tr}\Delta^c\tau_i\overline{\Delta^c}+\mu_{12}\mathrm{Tr}\phi_1^T\tau_2\phi_2\tau_2 . \tag{D3}$$

In this case fine-tuning of the parameters $\lambda_{12}v_R^2/M_{\mathrm{Pl}}+\mu_{12}=0$ leaves the pure low energy doublets H_{u2} and H_{d1}.

[7]For example, note that in the equation for $\mathbf{Y}_1$, we have a sum of terms $\mathbf{Y}_1\mathbf{Y}_2^{\dagger}\mathbf{Y}_2+\mathbf{Y}_2\mathbf{Y}_2^{\dagger}\mathbf{Y}_1$, which is Hermitian if Y_1 amd Y^2 are Hermitian.

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