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Phase transitions and vacuum tunneling into charge- and color-breaking minima in the MSSM

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The scalar potential of the MSSM may have local and global minima characterized by nonzero expectation values of charged and colored bosons. Even if the true vacuum is not color and charge conserving, the early Universe is likely to occupy the minimum of the potential in which only the neutral Higgs fields have nonzero VEV's. The stability of this false vacuum with respect to quantum tunneling imposes important constraints on the values of the MSSM parameters. We analyze these constraints using some novel methods for calculating the false vacuum decay rate. Some regions of the MSSM parameter space are ruled out because the lifetime of the corresponding physically acceptable false vacuum is small in comparison to the present age of the Universe. However, there is a significant fraction of the parameter space that is consistent with the hypothesis that the Universe rests in the false vacuum that is stable on a cosmological time scale. [S0556-2821(96)00421-3]

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I. INTRODUCTION

In the standard model color and electric charge are automatically conserved because the only fundamental scalar field is the Higgs boson, a colorless electroweak doublet. The Higgs potential has a continuum of degenerate minima, but these are all physically equivalent, and without loss of generality one can always define the unbroken U(1) generator to be the electric charge. This is not the case in the minimal supersymmetric standard model (MSSM), which employs a pair of Higgs doublets as well as a number of other scalar fields, the supersymmetric partners of quarks and leptons. Although the relative alignment of the two Higgs doublets' vacuum expectation values (VEV's) in group space is physical, the minimum of the Higgs potential (at least at the tree level) preserves electric charge[1] as long as the squark and slepton fields have vanishing classical values (see, e.g., Ref. [1] and references therein). However, the full scalar potential of the MSSM may have additional charge- and/or color-breaking (CCB) minima due to the vacuum expectation values of charged and/or colored scalars.

The existence of the CCB minima in the MSSM in addition to the acceptable standard-model-like (SML) minimum may have important physical consequences. One might expect that the regions of parameter space for which there is a global CCB minimum could be automatically excluded, thereby further restricting theoretical predictions for the MSSM spectrum. However, one must be careful in drawing such conclusions. Just as the cup being the lowest point on the golf course by no means guarantees that the ball will end up there after being struck, the Universe at present may not be in its lowest possible energy state. Instead, it may rest in a false vacuum whose lifetime is large on a cosmological time scale. The fundamental reason that makes this possible is that quantum tunneling, a nonperturbative effect responsible for the first-order phase transitions in field theory, naturally introduces a time scale that is exponentially larger than the typical scale that characterizes the effective potential. Consequently, the relaxation to the lowest energy state from some excited state may take a very long time. In particular, parameters for which the local SML "false vacuum" has a lifetime large in comparison to the age of the Universe may be acceptable, provided of course that the SML minimum was populated first in the evolution of the Universe.

The existence of local CCB minima which were populated temporarily during the early stages in the evolution of the Universe would also have dramatic implications for cosmology and astrophysics. In particular, since baryon and lepton numbers are spontaneously violated in the CCB vacua, their existence might have important consequences for baryogenesis.

Previous attempts [2–8] to elucidate the structure of the CCB minima in the MSSM met with serious difficulties. Some analyses [2,4,5,8] attempted to find analytic constraints on CCB minima. However, such conditions are generally neither necessary nor sufficient [4] except for overly simplified toy models which resemble the MSSM in some features, but cannot be used to draw firm conclusions about the MSSM. For this reason, recent studies [6–8] have employed extensive numerical analyses. Second, the determination of whether or not a global minimum is "dangerous," must rely on a trustworthy calculation of the tunneling rates at present and in the early Universe. There is no reason why the Universe cannot be resting in a false vacuum which has a very long (on the cosmological scale) lifetime. We therefore disagree with the restrictions imposed by a number of au-

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[1]This need not be the case in a general model with two Higgs doublets.

thors [5,7,8] on the allowed MSSM parameter space, which did not consider the corresponding tunneling rates. The calculation of the transition probability is more or less straightforward in the case of a single scalar but becomes extremely difficult for a potential that depends on several fields. Below, we address these difficulties and employ a new technique [9] to determine the lifetime of the false vacuum in the case of the MSSM.

We will see, in fact, that the SML minimum is effectively stable with respect to the transitions to the corresponding CCB minima for a substantial part of the allowed parameter space in the MSSM. We will also argue that, due to the specific nature of the CCB minima, they would not have been populated during the early stages of the evolution of the Universe, except for some small regions of parameters. On the other hand, the stability of the color- and charge-conserving vacuum on the time scales of order of the age of the Universe imposes important constraints on theoretical models and can provide guidance for future experimental searches.

The paper is organized as follows. In Sec. II, we discuss general features of the SML vacuum decay in the MSSM. In Sec. III, we consider the electroweak phase transition in the early Universe, in particular the issue of the $SU(3)\times SU(2)\times U(1)$-symmetric vacuum stability before the transition. In Secs. IV and V, we study the zero-temperature tunneling rates numerically. The method used to compute the transition probability is described in Appendix A, while Appendix B contains an approximate description of tunneling in the limit of a very deep true minimum.

II. ESSENTIAL ASPECTS OF THE MSSM VACUUM STABILITY

We begin by considering a simplified version of the MSSM potential. As was emphasized in [3,6], the third generation requires the most attention in connection with the issue of color and charge breaking, because the the CCB minima associated with the the large Yukawa coupling are the most dangerous.[2] We will see shortly that the tunneling rate into a CCB minimum is roughly proportional to $\exp(-c/y^2)$, where y is the corresponding Yukawa coupling and c is a constant.

We begin by considering a model defined by the superpotential:

$$W = y t_L t_R H_2 + \mu H_1 H_2, \tag{1}$$

where t_L and t_R denote the top quark superfields, and the H_1 and H_2 are the MSSM Higgs bosons. At this point we ignore the leptons, lighter quarks, and the electrically charged Higgs components. The resulting scalar potential, including the soft supersymmetry (SUSY-) breaking terms, is, at the tree level,

$$V = V_2 + V_3 + V_4, \tag{2}$$

where

$$V_2 = m_1^2 H_1^{0^2} + m_2^2 H_2^{0^2} + 2 m_3^2 H_1^0 H_2^0 + m_{\tilde{t}_L}^2 \tilde{t}_L^2 + m_{\tilde{t}_R}^2 \tilde{t}_R^2, \tag{3}$$

$$V_3 = -2 A H_2^0 \tilde{t}_L \tilde{t}_R - 2 \mu H_1^0 \tilde{t}_L \tilde{t}_R, \tag{4}$$

$$V_4 = \tilde{t}_L^2 \tilde{t}_R^2 + \tilde{t}_L^2 H_2^{0^2} + \tilde{t}_R^2 H_2^{0^2} + V_D. \tag{5}$$

For the $SU(3)\times SU(2)\times U(1)$ gauge group, the D terms are

$$V_D = \frac{1}{8y^2}\left[g_1^2\left(H_1^{0^2} - H_2^{0^2} - \frac{1}{3}\tilde{t}_L^2 + \frac{4}{3}\tilde{t}_R^2 \right)^2 + g_2^2 (H_1^{0^2} - H_2^{0^2} + \tilde{t}_L^2)^2 + \frac{4}{3} g_3^2 (\tilde{t}_L^2 - \tilde{t}_R^2)^2 \right]. \tag{6}$$

Here the color indices are suppressed. We have absorbed the Yukawa coupling in Eqs. (2)–(6) by the redefinition of the fields $\phi \to \phi/y$ and of the scalar potential $V \to y^2 V$. Also, all the fields are made real by a rephasing, and the complex phases are absorbed into the definitions of A and μ parameters. (There are strong experimental limits on such phases, which force A and μ to be nearly real; see Ref. [11] for reviews of these constraints.)

In the $\tilde{t}_L = \tilde{t}_R = 0$ hyperplane, Eq. (2) describes the usual MSSM Higgs potential. The constraint

$$m_1^2 m_2^2 < (m_3^2)^2 < \left(\frac{m_1^2 + m_2^2}{2} \right)^2 \tag{7}$$

ensures the existence (first inequality) and stability (second inequality) of the minimum with the correct pattern of electroweak symmetry breaking. The latter inequality results from requiring that the quadratic term is positive definite along the flat directions of the quartic term V_4.

Much of our discussion will concentrate on the effects of the trilinear terms V_3. If A and/or μ are large enough, the potential acquires an additional local, or global, minimum at some point outside the $\tilde{t}_L = \tilde{t}_R = 0$ hyperplane. In this case, the electromagnetic U(1), color SU(3), as well as some other symmetries [e.g., the global $U(1)_{\text{baryon}}$] will be spontaneously broken by the nonzero VEV of $\tilde{t}_L$ and $\tilde{t}_R$. For example, the potential (2) has, for appropriate values of A and μ, four degenerate CCB minima mapped onto each other by the following reflections:

[2]It was argued in [8] that a global CCB minimum need not be associated with a large Yukawa coupling because in the limit of small Yukawa coupling both the cubic and the quartic terms are small and only their relative values affect the depth of the CCB minimum. This is true, as long as one ignores the quartic D terms which, in fact, become more important in the small Yukawa coupling limit. However, significant tunneling rates from the metastable SML vacuum are only possible if the CCB minima are associated with a large Yukawa coupling: as was shown in Ref. [10], the height of the barrier separating the SML minimum from the CCB minimum is roughly proportional to $1/y_{\min}^2$ where $y_{\min}$ is the smallest Yukawa coupling associated with the fields that acquire nonzero VEV in the CCB minimum. The corresponding tunneling rates are greatly suppressed for small y.

$$S_1=\left\{\begin{array}{c} H_1\to -H_1 \\ H_2\to -H_2 \\ \tilde{t}_L\to -\tilde{t}_L \end{array}\right\},$$

$$S_2=\left\{\begin{array}{c} H_1\to -H_1 \\ H_2\to -H_2 \\ \tilde{t}_R\to -\tilde{t}_R \end{array}\right\},$$

$$S_3=S_1S_2=\left\{\begin{array}{c} \tilde{t}_L\to -\tilde{t}_L \\ \tilde{t}_R\to -\tilde{t}_R \end{array}\right\} \tag{8}$$

The gauge $SU(3)\times SU(2)\times U(1)$ symmetry is broken by the nonzero $\tilde{t}_L$, $\tilde{t}_R$, and H_2 vacuum expectation values down to an SU(2) subgroup of the color SU(3).

Evidently, for some otherwise reasonable values of the parameters, the potential may have a global CCB minimum. If this is the case, we would like to estimate the tunneling probability from the SML to the CCB minimum.

The semiclassical calculation of the false vacuum decay width was done in Refs. [12–14] for the case of a single scalar field, $\phi(x)$. For a recent review of tunneling we refer the reader to Ref. [15]. The corresponding path integral is dominated by the field configuration $\phi(x)$ called the ''bounce'' and can be evaluated using the saddle point method [13,14]. The bounce, being the stationary point of the Euclidean action, is the nontrivial solution of the corresponding Euler-Lagrange equation obeying certain boundary conditions.

The transition probability per unit volume in the semiclassical limit [14] is

$$\Gamma/V=Ae^{-S[\bar{\phi}]/\hbar}, \tag{9}$$

where $S[\bar{\phi}]$ is the Euclidean action of the bounce, a classical solution to the variational equation $\delta S=0$.

For the Universe to have decayed to the global minimum, the transition has to take place within a four-volume of size, roughly, t_0^4, where $t_0\sim 10^{10}$ yr is the age of the Universe. Taking the preexponential factor in Eq. (9) to be of order $(100\ \text{GeV})^4$, one obtains $(\Gamma/V)t_0^4\sim 1$ for $S_E[\bar{\phi}]/\hbar\sim 400$. Therefore, a false vacuum whose decay rate is characterized by $S_E[\bar{\phi}]/\hbar>400$ can safely be considered stable.

The presence of many scalar fields in the potential introduces a number of complications which will be addressed below.[3] However, our immediate goal is to obtain a crude estimate of the false vacuum decay rate. We therefore make a further simplification and reduce the tunneling problem to that of a one-component case.

Suppose the energy difference between the two minima $\Delta V=\epsilon^4$ is small in comparison to the height of the barrier. Then the thin-wall approximation is appropriate and the Euclidean action of the bounce of size R is given by

$$y^2S_E=-2\pi^2\frac{R^4}{4}\Delta V+2\pi^2R^3\int_{\phi_{\rm SML}}^{\phi_{\rm CCB}}\sqrt{2V(H)}\,d\phi, \tag{10}$$

where the appearance of the Yukawa coupling on the left-hand side of the equation is due to the fact that the fields have been scaled by a factor y in the beginning.

The bounce $\bar{\phi}(x)$ corresponds to the extremum of the Euclidean action with respect to R, which is reached for the critical size of the ''bubble'' $R_c=3S_1/\epsilon^4$, where $S_1=\int\sqrt{2V(H)}\,d\phi$. The corresponding action is

$$S_E[\bar{\phi}]\approx\frac{27\pi^2}{2y^2}\frac{S_1^4}{\epsilon^{12}}. \tag{11}$$

Some comments are in order. First, we observe that, as was asserted above, the tunneling rate is very sensitive to the value of the Yukawa coupling. Therefore, the minima associated with the third generation of squarks are the most interesting. Second, it is well known that the thin-wall approximation works well only for very small values of ϵ. On the other hand, $S_E[\bar{\phi}]<400$ in Eq. (11) corresponds to $\epsilon/S_1^{1/3}>[(27\pi^2/2y^2)/400]^{1/12}\approx 0.9$, which is not the thin-wall regime. This means that whenever the thin-wall limit is a good approximation, the transition we are interested in will not take place on the relevant time scale. The ''dangerous'' CCB minima lie, as a rule, outside the domain of validity of the thin-wall approximation.

In Appendix B, we find an approximate representation of the bounce in the opposite limit, which we call a ''thick-wall approximation.'' Unfortunately, the phenomenologically acceptable values of the trilinear term can be approximated by neither thin-wall nor thick-wall limiting expressions. For this reason, one must resort to a numerical analysis to determine the fate of the false SML vacuum in the presence of the lower lying CCB vacua.

We will focus on the CCB minima associated with the scale of order the electroweak and the SUSY breaking scales, in contrast to other studies [18,19] that discussed the possibility of a potential CCB minimum characterized by a Planckian, or a GUT-scale scale VEV.

It was argued in Ref. [18] that, if the MSSM is to be incorporated in some grand unified theory (GUT) characterized by the scale M_G, some new constraints should be imposed on the MSSM parameters to eliminate the possibility of a global CCB minimum developing at some scale Q, $1\ \text{TeV}\ll Q<M_G$. We note that symmetry restoration at a large energy scale is not required for the self-consistency of a spontaneously broken gauge theory. The latter is characterized by a symmetric action and an asymmetric vacuum. Cosmological data may, at least in principle, provide a test of whether the symmetric ground state existed in the early Universe. However, the finite-temperature field theory describing an expanding universe has a different effective potential from that of the $T=0$ case. Most of the flat directions of the tree-level potential are lifted by terms of order SUSY breaking scale.[4] Therefore, the CCB minima of the kind dealt with in Ref. [18] may not be present in the early Universe. Fur-

[3]See also Refs. [9,16,17].

[4]In addition to finite-temperature and tree-level breaking of super-

thermore, as was pointed out in Ref. [19], the existence of a global CCB minimum at such a large value of the VEV is irrelevant for the low-energy physics because the color- and charge-conserving vacuum is effectively stable with respect to tunneling into the CCB minimum of that kind.

This is an example of a phenomenon which is similar to the usual (perturbative) decoupling. Tunneling into a ''very deep'' vacuum occurs at a rate which is independent of the depth of such a minimum and is determined only by the magnitude of the field and the steepness of the potential at some well-defined point, the ''escape point,'' ϕ_e (cf. the discussion in our Appendix B). The shape of the effective potential at the scale $Q \gg \phi_e$ has no effect on the transition probability, and therefore the low-energy physics is independent of the physics at $Q \gg \phi_e$. This is analogous to decoupling in perturbation theory, even though the perturbative decoupling theorems may not apply to nonperturbative effects such as tunneling.

In practice, this decoupling allows us to treat the ultradeep minima on the same footing as the CCB minima of the tree-level potential. Also, since all the relevant dimensionful quantities, including ϕ_e, are of order 100 GeV to 1 TeV, the radiative corrections to the effective potential have a very small effect on the tunneling rates. Therefore, it is well justified to use a tree-level potential for evaluating the stability of the SML vacuum and determining the allowed regions of parameters in the MSSM. In a future study, we plan to further refine our present results by taking into account the radiative corrections to the effective potential.

Closely related is the issue of the directions in the scalar sector of the MSSM along which the tree-level potential appears to be unbounded from below (UFB). These directions are chosen to zero the quartic terms in the tree-level potential. Usually, the one-loop radiative corrections rectify the situation by introducing positive definite quartic terms of the type $STrM^4 \ln(M^2/Q^2)$. If this is the case, the full effective potential turns out to have only a very deep CCB minimum and is not unbounded from below. The latter may be separated from any other vacuum by a high enough barrier to make the presence of such a CCB minimum irrelevant. To determine whether a certain region of the parameter space must be excluded, one must again examine the corresponding tunneling rates. However, since in any case the tunneling rate is determined by the shape of the potential at the VEV's of order a few TeV, one can treat the UFB directions as if they were leading to a very deep minimum. We stress that this is true, regardless of whether the given direction is a UFB direction of the exact effective potential, or only of its finite-order (tree-level, one-loop, etc.) approximation. Thus, the tunneling rates for the UFB direction may be calculated by the same technique as for the ordinary CCB minima considered in this work. However, different fields may be involved, and a detailed investigation of the allowed parameter regions is beyond the scope of this paper.

III. CCB MINIMA IN THE EARLY UNIVERSE

If the effective potential has more than one minimum, then the determination of the physical vacuum requires consideration of the evolution of the Universe. For the MSSM, the analysis is complicated by the presence of flat directions (see, e.g., Ref. [21]), lifted only by terms of order the SUSY breaking parameters, along which the scalar field may acquire a large VEV and fall out of thermal equilibrium in the early Universe. We leave this question to future study and assume that the electroweak phase transition proceeds in the usual manner, from an $SU(3)\times SU(2)\times U(1)$-symmetric to a broken phase.

This assumption is well justified for the types of CCB minima we consider. It is well known that in inflationary models the large fluctuations of the scalar fields may populate some color- and charge-breaking minima. This is true of both local and global minima. If the vacuum expectation value of the scalar field in that minimum is large in comparison to the reheating temperature, the Universe may ''freeze'' in that minimum. However, the CCB minima we consider, unlike those of Refs. [18,19], have VEV's of the order of the electroweak scale and usually disappear at temperatures of order 1 TeV, except for the models with nontrivial (and atypical) symmetry restoration pattern like that of Ref. [6]. Since most inflationary models predict much higher reheating temperatures, one can assume that at $T \sim 1$ TeV the $SU(3)\times SU(2)\times U(1)$ is unbroken.

The main effect of the temperature-dependent corrections to the effective potential is contributions of order T^2 to the quadratic terms. The trilinear terms also receive some corrections, linear in T.

The depth of the CCB minima depend on the relative values of the squark mass terms (m_0) on one hand, and A or μ on the other. At finite temperature, positive mass-squared terms proportional to T^2 appear in the effective potential and lead, at some critical temperature, $T_c \sim 100$ GeV, to the disappearance of the SML minimum. Since A, μ, and m_0 are allowed to be large in comparison to T_c, it is possible that as the Universe cooled the negative energy CCB minima formed at some $T_{\rm CCB} > T_c$, before the Universe was cold enough to undergo the electroweak phase transition to the SML minimum. This would allow for the possibility of a transition from an $SU(3)\times SU(2)\times U(1)$-symmetric to the color- and charge-breaking phase. One then must consider whether the transition to the CCB minimum actually occurred, and, if so, what happened subsequently.

Then, *a priori* there are three possibilities: (i) the Universe may stay in the symmetric minimum until the temperature reaches T_c and the usual electroweak transition takes place; (ii) the Universe may go to the CCB minimum and freeze there; and (iii) the transition from the symmetric to the SML minimum may occur in two stages: first the transition from the unbroken to the CCB phase takes place and only then the Universe may tunnel to the SML vacuum. Clearly, the second possibility is excluded empirically.

Possibility (iii) is intriguing. The idea of a multistage phase transition, in the course of which the gauge group might change a number of times finally arriving at the standard model group, is not new. An evolution of this kind could have important implications for magnetic monopoles [22], charge asymmetry of the Universe, and magnetic field generation [23], baryogenesis [24], etc. The sufficient (though not necessary) conditions for the first part of this scenario, the transition from the symmetric to the CCB

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symmetry, there are SUSY breaking terms associated with the metastability of the false vacuum [20].

phase, will be derived below. However, we have not found a model in which a second-order phase transition from the CCB minimum to the SML minimum would subsequently take place. It appears to be a generic feature that even when the CCB minima can be populated via a second-order phase transition, they are separated from the SML minimum by a thick barrier. If the Universe is stuck in the false CCB vacuum (which, in this scenario, must have a higher vacuum energy than the SML phase), this would trigger a new stage of inflation which could only be ended by a first-order phase transition characterized by a relatively low tunneling rate. It was shown in Ref. [25] that this sort of transition proceeds via bubble nucleation at a rate which is too slow to catch up with the expanding volume. The bubbles of the true vacuum would neither collide, nor percolate, preventing the Universe from reheating. Therefore, the two-stage electroweak phase transition could not have taken place in the early Universe.

Let us consider the possibility that a CCB minimum exists at some temperature $T>T_c$. At that temperature, the mass matrix of the third generation squarks (ignoring the rest of the squarks and sleptons) in the $SU(2)\times U(1)$-symmetric phase ($H_1=H_2=0$) is of the form

$$\begin{pmatrix} m^2_{\tilde{t}_L}+c_L T^2 & 0 \\ 0 & m^2_{\tilde{t}_R}+c_R T^2 \end{pmatrix}. \tag{12}$$

Suppose that it has at least one negative eigenvalue, which makes the $SU(2)\times U(1)$-symmetric vacuum unstable. Since $T>T_c$, the SML minimum does not yet exist. The decay of the unstable vacuum will result in the creation of the CCB condensate by a second-order phase transition. At the same time, we assume that all the necessary conditions have been applied to constrain the masses squared of squarks to be positive (and large enough) in the SML minimum at zero temperature, where the mass matrix is

$$\begin{pmatrix} m^2_{\tilde{t}_L}+m_t^2 & AH_2+\mu H_1 \\ AH_2+\mu H_1 & m^2_{\tilde{t}_R}+m_t^2 \end{pmatrix}, \tag{13}$$

where the m_t is the top quark mass.

We would like to see whether there is a region of parameter space in which the matrix (13) has only positive eigenvalues, while the matrix (12) has a negative eigenvalue. It is easy to see that this is only possible if either $c_L T_c^2<m_t^2$ or $c_R T_c^2<m_t^2$. The shaded region shown in Fig. 1 corresponds to the additional domain of parameters which can be ruled out by requiring stability of the $SU(2)\times U(1)$ symmetric minimum above the electroweak transition temperature. This domain comprises two regions in which $m^2_{\tilde{t}_L}$ and $m^2_{\tilde{t}_R}$ differ in sign (Fig. 1). The hyperbola $(m^2_{\tilde{t}_L}+m_t^2)(m^2_{\tilde{t}_R}+m_t^2)=(AH_2+\mu H_1)^2$, where the values of H_1 and H_2 are computed at the SML minimum, outlines the domain of positive determinant of the matrix in Eq. (13). If one requires that the squark masses be greater than 45 GeV, in accordance with the current experimental limits, it would further reduce the area of the corresponding (shaded) regions in Fig. 1.

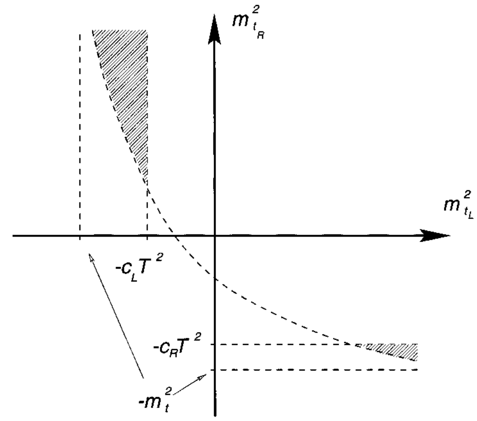

FIG. 1. Region of parameters which can be ruled out by requiring stability of the $SU(2)\times U(1)$-symmetric minimum above the electroweak transition temperature.

The coefficients c_L and c_R are of order 1; their exact values depend on the spectrum of the MSSM and have been computed, e.g., in Ref. [26]. For new constraints to arise from the requirement of symmetric vacuum stability at $T\geqslant T_c$, the inequality $T_c\sim 100\ \mathrm{GeV}<m_t/\sqrt{c_{L,R}}$ must be satisfied.

We note that the high-temperature expansion [27] is not expected to be accurate for $T\sim T_c$. Therefore, the accuracy to which one can determine the boundaries of the shaded regions in Fig. 1 is limited by one's inability to determine the effective potential accurately for $T\sim T_c$ due to the limitations of the theoretical framework [27]. It is clear, however, that the size of the shaded regions in Fig. 1 is rather small, so their exclusion cannot cause an appreciable reduction in the MSSM parameter space. For the generic values of parameters outside the shaded regions, the second-order phase transition into the CCB minimum does not take place.

We have demonstrated that the symmetric vacuum of the MSSM is generally stable with respect to second-order phase transitions to a CCB minimum at $T>T_c$. However, the question of a first-order transition at finite temperature remains open. The probability of tunneling in the high-temperature limit is suppressed by the factor [28]

$$\Gamma/V\propto e^{-S_3[\tilde{\phi}]/\hbar T}, \tag{14}$$

where $S_3[\tilde{\phi}]$ is the three-dimensional action of the $d=3$ bounce. The time allowed for the transition is roughly $t_T=m_{\mathrm{Pl}}/T^2$, the age of the Universe when the temperature equals T, which means that $S_3[\tilde{\phi}]/\hbar T$ must be less than about 45 for the transition to take place.

Thus, it appears most likely that if T_c is not too small (of order 100 GeV, as is generally believed), then the second-order transition to the CCB minimum of the type discussed above is not likely, and the Universe is driven towards the color and charge conserving SML minimum of the scalar potential. While, admittedly, this is not a rigorous theorem, the color- and charge-conserving minimum appears to be favored by the thermal evolution of the Universe. We leave the detailed investigation of this issue for future work.

Our next question is what happens after the Universe cools down in the SML minimum of the potential: will the false vacuum be effectively stable at $T \approx 0$ even in the presence of some deeper CCB minima, or not.

IV. NUMERICAL ANALYSIS OF TUNNELING RATES AT ZERO TEMPERATURE

To make a definitive determination of whether the SML vacuum is stable with respect to decay into a lower CCB minimum, one has to compute the tunneling probability numerically. Analytic computation is usually feasible only in the thin-wall limit.

In the semiclassical limit, the zero-temperature tunneling probability [12–14,16,17] per unit volume is

$$\Gamma/V = C_G \left[\int |\overline{\phi(x)}|^2 d^4x \right]^{N/2} \left(\frac{S[\overline{\phi}]}{2\pi\hbar} \right)^2 e^{-S[\overline{\phi}]/\hbar} \times \left| \frac{\det'[-\partial_\mu^2 + U''(\overline{\phi})]}{\det[-\partial_\mu^2 + U''(0)]} \right|^{-1/2} [1 + O(\hbar)], \quad (15)$$

where $S[\overline{\phi}]$ is the Euclidean action of the bounce, a solution to the variational equation $\delta S = 0$, det$'$ stands for the determinant with all the zero eigenvalues omitted, N is the number of Goldstone zero modes and the C_G is the group-theoretical coefficient [16,17].

Suppose the scalar potential $U(\phi_1, \ldots, \phi_n)$ has a local minimum at $\phi_i = \phi_i^f$, $i = 1,2, \ldots, n$, as well as at least one additional (local or global) minimum at $\phi_i = \phi_i^t$, $i = 1,2, \ldots, n$; $U(\phi^t) < U(\phi^f)$. Then the bounce $\overline{\phi}(x) = (\overline{\phi}_1(x), \ldots, \overline{\phi}_n(x))$ is a nontrivial O(4)-symmetric [29] solution $\overline{\phi}(r), r = \sqrt{x^2}$, of the system of Euler-Lagrange equations:

$$\Delta \overline{\phi_i}(r) = \frac{\partial}{\partial \overline{\phi_i}} U(\overline{\phi_1}, \ldots, \overline{\phi_n}) \quad (16)$$

with the boundary conditions

$$(d/dr)\overline{\phi_i}(r)|_{r=0} = 0, \qquad \overline{\phi_i}(\infty) = \phi^f. \quad (17)$$

In the case of a potential that depends on a single scalar field, one can solve Eq. (16) with the boundary condition (17) numerically. The straightforward technique is to assume some value for the unknown quantity $\overline{\phi}(0) = \phi^e$, the so-called "escape point." Then one can integrate Eq. (16) numerically and vary the escape point until the proper limit $\overline{\phi_i}(\infty) = \phi^f$ is reached. Here one uses the fact that if for some value $\phi^e = \phi_-^e$ the corresponding $\overline{\phi_i}(\infty) > \phi^f$, and for some other value $\phi^e = \phi_+^e$ $\overline{\phi_i}(\infty) < \phi^f$, then the true escape point must lie somewhere between ϕ_-^e and ϕ_+^e. This strategy fails when one has to deal with a potential that depends on several scalar fields. The peculiarity of one-dimensional topology no longer allows one to find the true escape point as the compromise between the "undershoot" and the "overshoot."[5] It is a generic property of the bounce that it is a saddle point of the Euclidean action [13], and not a minimum, and thus small changes in the initial conditions result in large changes in the form of the solution. Therefore, it is impossible to find the bounce numerically in the case of a multicomponent field using the procedure just mentioned.

This difficulty was realized by the authors of [3], who proposed an iterative procedure to look for the bounce as a special point of the discretized action on a lattice. Since the solution in question is not a minimum, but a saddle point, they tried to minimize the action with respect to random variations, while maximizing the same action with respect to scaling $r \to \lambda r$. We find that the iterative procedure of this kind is ill defined and cannot have a meaningful limit. First, it is impossible to separate the variations corresponding to scaling from those orthogonal to scaling in a practical numerical simulation. Second, although it is true that the bounce maximizes the action with respect to scaling, it is easy to see that the generator of such a variation cannot be the eigenvector of the second variation operator corresponding to the negative eigenvalue.

Instead, we use a new method proposed in [9] to find the solution of Eqs. (16) and (17). The idea is to turn the saddle point of a Euclidean action into a true minimum by adding to it some auxiliary terms which vanish for $\phi = \overline{\phi}(x)$. The resulting "improved action" will have a minimum at the point corresponding to the desired solution, which can now be found by minimizing the discretized version of improved action on the lattice. Details of the application of this method are given in Appendix A.

V. THE MSSM VACUUM STABILITY AT ZERO TEMPERATURE

For the reasons explained above, we consider the MSSM potential with the third generation only. We also neglect all of the trilinear couplings except those proportional to the largest Yukawa couplings, y_t and y_b. This justifies dropping the squarks of the first two generations. Also, although we allow the values of tanβ from ~ 1 to ~ 60, the most stringent constraints come from the small tanβ region, where the top Yukawa coupling is larger. As was pointed out in Ref. [8], the noteworthy CCB minima can only develop in the directions along which one and only one of the trilinear terms is nonzero. Thus, it is the size of the largest Yukawa coupling that affects the lifetime of the false vacuum the most. We therefore consider the following subset of the MSSM scalars: $\Phi = \{H_1^0, H_2^0, H_1^-, H_2^+, \widetilde{Q}_L, \widetilde{t}_R, \widetilde{b}_R\}$, where $\widetilde{Q}_L = \{\widetilde{t}_L, \widetilde{b}_L\}$.

The superpotential

$$W = y_t Q_L H_2 t_R + y_b Q_L H_1 b_R - \mu H_1 H_2 \quad (18)$$

[5] Even in the case of a single scalar field, the shooting method may turn out to be ineffective for a potential with sufficiently degenerate minima. This is because one may be required to specify the trial value for the escape point with an exceedingly high precision to compute the action of the bounce to a given accuracy.

corresponds to the following scalar potential (with the soft SUSY breaking terms included) at the tree level:

$$V = V_{2,H} + V_2 + V_3 + V_4, \tag{19}$$

where $V_{2,H}$ comprises the terms quadratic in the Higgs fields,

$$V_2 = +m^2_{\tilde{t}_L}\tilde{t}^2_L + m^2_{\tilde{t}_R}\tilde{t}^2_R + m^2_{\tilde{b}_L}\tilde{b}^2_L + m^2_{\tilde{b}_R}\tilde{b}^2_R, \tag{20}$$

$$\begin{aligned} V_3 = & -y_t A_t (H_2^0 \tilde{t}_L - H_2^+ \tilde{b}_L)\tilde{t}_R - y_b A_b (H_1^- \tilde{t}_L - H_1^0 \tilde{b}_L)\tilde{b}_R \\ & - y_t \mu (H_1^0)^* \tilde{t}_L \tilde{t}_R - y_b \mu (H_2^+)^* \tilde{t}_L \tilde{b}_R \\ & - y_t \mu (H_1^-)^* \tilde{b}_L \tilde{t}_R - y_b \mu (H_2^0)^* \tilde{b}_L \tilde{b}_R + \text{H.c.}, \end{aligned} \tag{21}$$

and V_4 is comprised of the D terms, as well as the terms of the form $y^2_{t,b}\phi_1^2\phi_2^2$, where $\phi_1 \in \Phi$.

We now come to the issue of radiative corrections. It has been argued (see, e.g., [7]) that if the CCB minimum occurs at a scale which is within 1 or 2 orders of magnitude from the electroweak scale, then the (logarithmic) radiative corrections are too small to cause a noticeable distortion in the shape of the effective potential, and therefore can be ignored. While this conclusion is correct, we would like to emphasize that, as was discussed in Sec. II, it is not so much the position of the CCB minimum, as the size of the bounce and the value of the escape point that determine the scale to which the bounce will ''probe'' the effective potential. The shape of the effective potential beyond that scale has no effect on the tunneling rate. In our numerical calculations, a typical size of the bounce for realistic values of of the MSSM parameters is of order $(1-0.1\ \text{TeV})^{-1}$ (except for deliberately fine-tuned cases of highly degenerate minima in which the bounce blows up to the size of its thin-wall limit, $R \sim 1/\epsilon$, and the tunneling becomes highly improbable). This justifies *post factum* the neglect of the one-loop contribution in the effective potential. As long as one never encounters very small (in length units) bounces, one can be certain that only the shape of the potential around the electroweak scale is relevant for tunneling.

We search the MSSM parameter space by generating randomly the values of the parameters that enter in the scalar potential (19). Then the minima are found numerically and the tunneling probability is computed using the method of Appendix A.

The results of the numerical analyses are plotted in Fig. 2, where the action of the bounce S is shown as a function of the relative depth of the CCB minimum with respect to the SML minimum, while the other MSSM parameters are varied randomly. For each point plotted, the global minimum of the potential is not color and charge conserving. Nevertheless, one observes that S takes values on both sides of the critical value, $S=400$. Therefore, for some values of parameters, namely those for which $S>400$, the false SML vacuum is stable on a time scale large compared to the age of the Universe. Such color- and charge-breaking minima are ''safe,'' and the corresponding parameters are allowed. In contrast, those points that correspond to $S<400$ are ruled out by the mere existence of the world as we know it. Another lesson one learns from Fig. 2 is that the tunneling rate is not

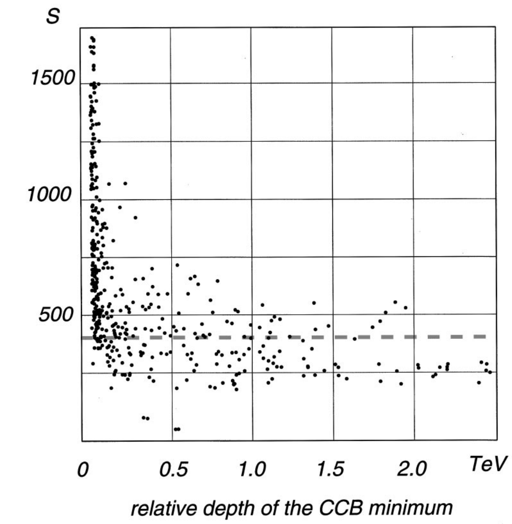

FIG. 2. The action of the bounce, S, does not depend sensitively on the depth of the CCB minimum, ΔV, except in the ''thin-wall'' limit (small ΔV).

very sensitive to the depth of the CCB minimum, except in the limit of nearly degenerate minima, the so-called ''thin-wall'' limit. This is to be expected, as was discussed in Sec. II.

The domain of stability of the false SML vacuum with respect to tunneling is delineated by stars in Fig. 3. The lighter top squark in the presence of the large trilinear couplings forces the barrier, which keeps the system in the metastable vacuum, to be thinner and lower. That results in a higher likelihood of tunneling. The points labeled by boxes fall into the domain that is excluded by the existence of our (color- and charge-conserving) Universe. On the other hand, if both left-handed and right-handed stops are heavy, and if the trilinear terms are small, then the false vacuum is stable and the presence of a global CCB minimum is irrelevant. We

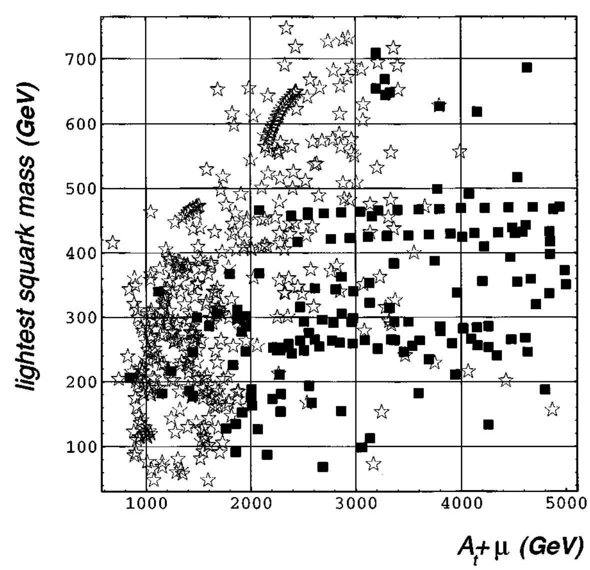

FIG. 3. The domains of stability (stars) and instability (boxes) of the false SML vacuum with respect to tunneling into the global CCB minimum. Light top squark and large trilinear couplings generally correspond to a lower and thinner barrier and, thus, higher probability of tunneling.

note that a number of models favor the left lower corner of the plot in Fig. 3, where only a few ''dangerous'' CCB minima occur.

It would be useful to derive some empirical algebraic constraints to distinguish between the allowed and excluded domains of parameter space based on the numerical results. For instance, it has been argued [8] that the inequality

$$A_t^2+3\mu^2<3(m_{\tilde{t}_L}^2+m_{\tilde{t}_R}^2) \tag{22}$$

is a reasonably good condition for excluding a global CCB minimum. This inequality would force one to be above the dotted line in Fig. 4. This is in good agreement with our numerical results. However, if one takes into account the tunneling rates, the constraint (22) is relaxed significantly. The empirical inequality which should replace Eq. (22),

$$A_t^2+3\mu^2<7.5(m_{\tilde{t}_L}^2+m_{\tilde{t}_R}^2), \tag{23}$$

is depicted by a thick dashed line in Fig. 4. The simple condition (23), should be applied with caution because, strictly speaking, it is neither necessary nor sufficient. It is an approximate empirical inequality that may be useful for a crude determination of whether the CCB minima are ''dangerous.''

For the phenomenologically attractive values of $|\mu|<2$ TeV, $|A|<4$ TeV, it is generally true that ''the larger the trilinear coupling, the more dangerous is the corresponding CCB minimum.'' However, it is instructive to examine what happens to the tunneling probability in the limit of very large μ and A_t (and large enough squark mass terms to ensure the existence of the SML minimum). In that limit, as the CCB minimum moves away from the SML minimum, the barrier separating the two becomes thicker, and the false vacuum should become more stable. This is, in fact, what happens. The set of points in Fig. 5 includes those points (located in the lower left corner) shown in Figs. 2–4. In addition, Fig. 5 displays the points corresponding to some very large values of A_t and μ. As expected, the tunneling probability diminishes for very large values of A_t and μ, and $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$.

To summarize, if the global CCB minimum is nearly degenerate with the local SML minimum (thin-wall limit), then the tunneling probability is extremely small. As the trilinear couplings increase, the false vacuum decay rate increases because the escape point of the bounce moves out of the flat vicinity of the global minimum into the region in which the gradient of the potential is significant. However, a further increase in the size of the trilinear couplings, as well as the consequent increase in the squark mass terms, makes the barrier thicker and pushes the escape point away from the SML minimum. This eventually causes a decrease in the tunneling rate. In accordance with one's intuition, the low-energy physics is unaffected by the physics at the very high-energy scales.

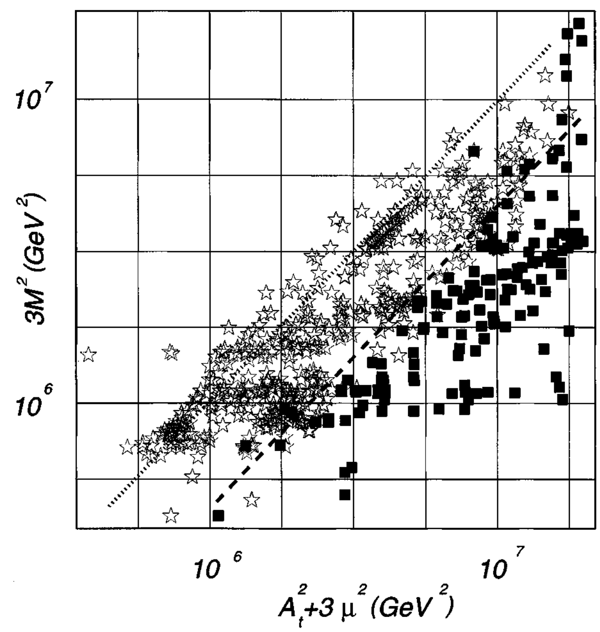

FIG. 4. Each point represents the set of the MSSM parameters for which the global minimum of the potential is color and charge breaking. The stars correspond to the SML false vacua whose lifetime is large compared to the age of the Universe. The boxes indicate those points in the parameter space for which the false SML vacuum should have decayed via quantum tunneling. The dotted line represents the empirical criterion for the absence of the global CCB minima: $A_t^2+3\mu^2<3M^2$, where $M^2=m_{\tilde{t}_L}^2+m_{\tilde{t}_R}^2$ Taking into account the tunneling rates relaxes this constraint to, roughly, $A_t^2+3\mu^2<7.5M^2$, shown as the dashed line. The scale is logarithmic.

VI. CONCLUSION

The color- and charge-conserving minimum may not be the global minimum of the MSSM potential. It is possible that the Universe rests in a false vacuum whose lifetime is large in comparison to the present age of the Universe. Under fairly general conditions, the SML vacuum may be favored by the thermal evolution of the Universe, even if it does not represent the global minimum.

The existence of the CCB minima of the scalar potential results in some important constraints on models with low-energy sypersymmetry. However, the commonly imposed (see, e.g., Refs. [30,31]) requirement that the SML minimum be global is too strong and may overconstrain the theory. In fact, for a large portion of the parameter space the presence of the global CCB minimum is irrelevant because the time required for the Universe to relax to its lowest energy state may exceed its present age. The basic reason for this is that the quantum tunneling is a nonperturbative phenomenon that is naturally associated with the energy scale that is exponentially smaller (suppressed by a factor $\exp\{-S\}$) than the typical scale in the theory.

We have computed numerically the SML false vacuum decay rates for a variety of values of the MSSM parameters. Our results indicate clearly that the MSSM vacuum stability with respect to tunneling into a CCB minimum imposes important constraints on models with low-energy supersymmetry. Similar considerations apply to UFB directions, although

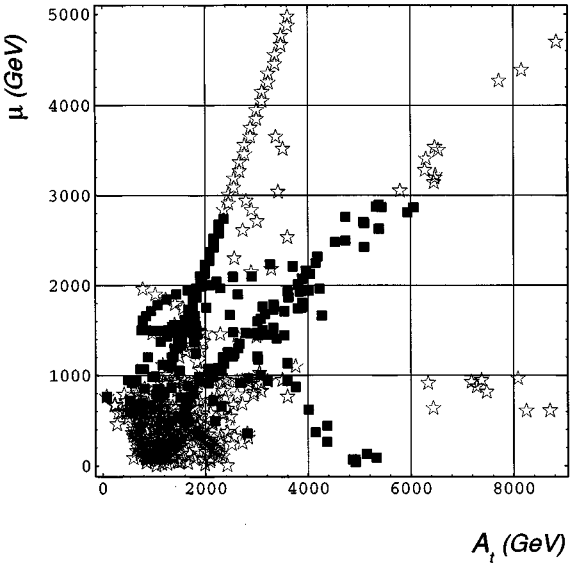

FIG. 5. Tunneling probability for unphysically large values of A_t and μ. As the CCB minimum moves farther away, it becomes "less dangerous." As before, the stars mark the points with $S>400$, while the boxes depict those with $S<400$.

we have not carried out a detailed study of the allowed parameter space.

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APPENDIX A

The numerical algorithm for computing the tunneling probability comprises several steps. First, the positions of the minima, ϕ^t and ϕ^f of the potential $U(\phi)$ are determined numerically for a given set of values of the MSSM parameters. If there is no minimum below the SML vacuum, a new set of parameters is chosen. We define the Euclidean action on the L-point lattice:

$$S[\phi]=T[\phi]+V[\phi], \tag{A1}$$

$$T[\phi]=2\pi^2\Delta^4\sum_{m=1}^{L-1}(d_{m+1}-d_m)d_m^3\left(\sum_{i=1}^{n}\frac{(\phi_i^{(m+1)}-\phi_i^{(m)})^2}{2(d_{m+1}-d_m)^2\Delta^2}\right), \tag{A2}$$

$$V[\phi]=2\pi^2\Delta^4\sum_{m=1}^{L-1}(d_{m+1}-d_m)d_m^3U(\phi_1^{(m)},\ldots,\phi_n^{(m)}), \tag{A3}$$

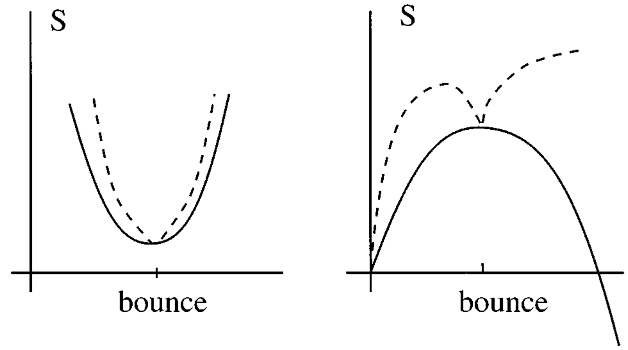

FIG. 6. The improved action $\widetilde{S}[\phi]$ (dashed curve) has a minimum that coincides with the saddle point of the actual Euclidean action $S[\phi]$ (solid curve), as shown in two projections. The origin on the right corresponds to the trivial solution $\phi(x)\equiv\phi^f$ for which $\widetilde{S}[\phi]=S[\phi]=0$. The size of the "gap" between the two curves is a function of λ.

where d_m is the position of the m's site of the lattice in dimensionless units, and Δ is the length parameter that determines the overall scale and is chosen so as to optimize the computation.

The next step is to define the improved action [9], for which the bounce is the minimum. We do that by adding to the action some auxiliary terms which (i) vanish as $\phi(x)$ approaches the bounce, and (ii) make the bounce $\phi(r)$ a minimum of the improved action [9]:

$$\widetilde{S}[\phi]\equiv S[\phi]+\lambda|T[\phi]+2V[\phi]|^{1/2}, \tag{A4}$$

where λ is some arbitrary (dimensionless) Lagrange multiplier.

The effect of adding the auxiliary terms is shown qualitatively in Fig. 6, where two projections of the saddle point of $S[\phi]$ are depicted symbolically.

The bounce $\phi(x)$ is always a minimum of the improved action $\widetilde{S}[\phi]$. However, unless λ is chosen to be large enough, the difference $\widetilde{S}[\phi]-S[\phi]$ may appear to be too small in the vicinity of the bounce. In this case, it is possible that the small perturbations will take one over the barrier (Fig. 6) towards the trivial solution of zero action $\phi(x)\equiv\phi^f$. Thus, it is crucial to take λ large enough for the numerical simulation to succeed.

Initially, one may choose

$$\phi_i^{(m)}=\begin{cases}\phi_i^0, & m<L/2,\\ \phi_i^f, & m\geqslant L/2,\end{cases} \tag{A5}$$

where ϕ^0 is a zero of $V(\phi)$. Alternatively, one may start from a different profile for the bounce, e.g., the thick-wall ansatz described below in the Appendix B.

Then we find the optimized value for the overall scale Δ by maximizing the action with respect to Δ. After that, we allow random variations of the values of the scalar field at each lattice site to minimize the improved action (A4). The iterations stop when further variations do not lead to a reduction in the improved action. As an independent criterion of the quality of the fit, we require that $T[\phi]/(-2V[\phi]) = 1 \pm 0.01$ (see, e.g., Ref. [9] and references therein for a discussion of this identity).

As was explained in Secs. II and V, tunneling into a ''UFB direction'' is equivalent to a transition into a very deep CCB minimum. The details of the effective potential at scales much larger than ϕ^e do not affect the decay probability. In particular, it is irrelevant whether the alleged ''UFB direction'' is heading towards a very deep minimum, or minus infinity. This allows one to introduce an effective cutoff to stop the runaway fields from going to infinity. In practice, we did not allow the value of ϕ to run beyond 10 TeV. Therefore, the ''UFB directions'' were treated as if they lead to a CCB minimum with a 10 TeV VEV. In each particular case this procedure is justified *post factum* by ensuring that the value of the escape point is small compared to a 10 TeV cutoff.

APPENDIX B

In the limit of nearly degenerate minima separated by a high barrier, the so-called thin-wall limit, the bounce can be approximated by a smoothed out step function [13]. This approximation proved very useful in estimating the tunneling rates and was used in our analysis of the toy model in Sec. II. In practice, however, one rarely encounters a situation in which the thin-wall approximation is in good agreement with the numerical results (c.f. Ref. [3]). The tunneling rates are usually very small in the thin-wall limit, and therefore in many physically interesting models the first-order phase transition takes place when the energy difference between the true and the false vacuum is not small in comparison to the height of the barrier. Then it is necessary to go beyond the thin-wall approximation, which is usually done by means of a numerical calculation.

If, however, the energy difference ΔV between the two vacua is larger than the height of the barrier, one can find a simple approximation to the bounce in what we call a ''thick-wall'' limit. As ΔV increases, the so-called ''escape point,'' the value of the field in the center of the bounce $\phi_e \equiv \bar{\phi}(0)$, moves away from the minimum of the potential. Then Eq. (16) can be linearized in the vicinity of the escape point because $\partial U(\phi)/\partial\phi$ is a constant independent of ϕ for $\phi \approx \bar{\phi}(0)$. Therefore, in the vicinity of the center of the O(4)-symmetric bounce $\bar{\phi}(r)$,

$$\overline{\phi''}(r) + \frac{3}{r}\overline{\phi'}(r) = -a, \qquad \overline{\phi'}(0) = 0, \tag{B1}$$

where the constant $a = |(\partial U/\partial\phi)(\phi_e)|$. This is a linear equation whose only solution satisfying the boundary condition is of the form

$$\overline{\phi}(r) = -\frac{a}{8}r^2 + \phi_e . \tag{B2}$$

Outside the small neighborhood of the origin, the approximation (B1) is not valid and $\bar{\phi}(r)$ falls off exponentially, just as in the thin-wall limit [13]. The equation for the bounce $\bar{\phi}(r)$ for large r (and, therefore, small ϕ) becomes

$$\overline{\phi''}(r) = \partial U(\phi)/\partial\phi \approx m^2 \overline{\phi^2}, \qquad \overline{\phi(\infty)} = 0, \tag{B3}$$

where m is the mass term for the potential $U(\phi) = (m^2/2)\phi^2 + \cdots$. The solution of Eq. (B3) is

$$\overline{\phi(r)} = Ce^{-mr}, \tag{B4}$$

where C is an arbitrary constant. We now sew the approximate solution for the bounce from the two asymptotics, (30) and (32), at some point $r=R$. The values of C and R are determined by requiring continuity and differentiability of the solution at $r=R$. The resulting ansatz is

$$\overline{\phi(r)} = \begin{cases} \phi_e - (a/8)r^2, & r<R, \\ \dfrac{aR}{4m}e^{-m(r-R)}, & r \geqslant R, \end{cases} \tag{B5}$$

where

$$R = \frac{1}{m}[\sqrt{1+8m^2\phi_e/a} - 1], \tag{B6}$$

and the value of ϕ_e is an unknown parameter which can be found either from requiring that

$$0 = \frac{d}{d\phi_e}S = \frac{d}{d\phi_e}(T+V), \tag{B7}$$

or, equivalently, from solving the equation

$$T = -2V, \tag{B8}$$

where

$$T = 2\pi^2\int_0^\infty r^3 dr \frac{1}{2}\left(\frac{d\overline{\phi(r)}}{dr}\right)^2$$

and

$$V = 2\pi^2\int_0^\infty r^3 dr U(\overline{\phi(r)}) \tag{B9}$$

are functions of ϕ_e .

The representation of the bounce in the thick-wall limit described above is approximate and is not very useful in application to the MSSM. However, it exhibits the essential features of tunneling in this limiting case. We also found it convenient to use this approximate solution as an initial profile for the bounce in the numerical procedure described in Appendix A.

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