Heavy quark solitons in the Nambu–Jona-Lasinio model

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The Nambu–Jona-Lasinio (NJL) model is extended to incorporate heavy quark spin symmetry. In this model baryons containing one heavy quark are analyzed as bound states of light baryons, represented as chiral solitons, and mesons containing one heavy quark. From related studies in Skyrme-type models, the ground-state heavy baryon is known to arise for the heavy meson in a P-wave configuration. In the limit of an infinitely large quark mass the heavy meson wave function is sharply peaked at the center of the chiral soliton. Therefore, the bound state equation reduces to an eigenvalue problem for the coefficients of the operators contained in the most general P-wave ansatz for the heavy meson. Within the NJL model a novel feature arises from the coupling of the heavy meson to the various light quark states. In this respect conceptual differences to Skyrme-model calculations are discovered: The strongest bound state is given by a heavy meson configuration, which is completely decoupled from the grand spin zero channel of the light quarks. [S0556-2821(96)03521-7]

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I. INTRODUCTION AND MOTIVATION

Recently, there has been considerable attention given to properties of hadrons containing a single heavy quark with mass M_Q much larger than the typical scale of strong interactions, $\Lambda_{\rm QCD}$. Henceforth, these particles will be referred to as ''heavy hadrons.'' These ''heavy hadrons'' are described frequently within the heavy quark effective theory (HQET) [1,2], which represents a $1/M_Q$ expansion of the heavy quark content of QCD. The heavy quark effective Lagrangian is constructed from the heavy quark transformation

$$Q_v^{(l,s)}(x)=\frac{1\pm \not{v}}{2}\exp(iM_Q v\cdot x)Q(x), \tag{1}$$

which disentangles the large (l) and small (s) components of the heavy quark spinor, $Q(x)$. In Eq. (1), v_μ refers to the velocity of the heavy quark spinor which, e.g., in its rest frame takes the form, $v_\mu=(1,\mathbf{0})$. The Dirac operator for the large component is expanded as $v\cdot D+O(1/M_Q)$. Since the leading order term commutes with the quark spin operator, heavy mesons differing only by their spins, as e.g., pseudoscalar and vector mesons, are degenerate in the heavy quark limit, $M_Q\to\infty$. The empirical spectrum of heavy mesons [3], $m_D=1.87$ GeV$\approx m_{D*}=2.01$ GeV and $m_B=5.28$ GeV $\approx m_{B*}=5.32$ GeV represents the most convincing evidence of the heavy quark symmetry.[1] This degeneracy is in contrast to the spectrum of mesons containing only light quarks. Here the pseudoscalars are much lighter than the vector mesons. This is understood by interpreting the light pseudoscalar mesons as the would-be Goldstone bosons of spontaneous chiral symmetry breaking.

The incorporation of the heavy quark symmetry into effective meson theories is accomplished by introducing the heavy meson field $\mathcal{H}^a_{lh}$ in the heavy quark rest frame [4]. Here, l and h denote the light and heavy quark spins, respectively, while $a=u,d$ refers to the isospin of the light quark. This field contains pseudoscalar (P^a) and vector (V^a_μ) mesons on an equal footing [4]:

$$\mathcal{H}^a_{lh}=iP^a\delta_{lh}+\sigma^i_{lh}V^a_i. \tag{2}$$

According to the heavy quark symmetry, the heavy spin index (h) decouples completely in effective theories.[2] The interactions of $\mathcal{H}^a_{lh}$ with the light mesons proceeds via the light quark indices (l,a) and is governed by chiral symmetry. Such effective models have been employed to study not only the properties of heavy mesons, (spectrum, form factors, weak decays) [1,2], but also the spectrum of heavy baryons [5–10]. In this context the soliton picture for baryons is commonly adopted. Within this approach the baryon number is carried by a solitonic configuration of the light meson fields. Heavy baryons emerge when a heavy meson is bound in the background of this soliton. This picture relies on the bound state approach to strange baryons initiated by Callan and Klebanov [11,12]. The main difference here is that for the heavy meson both the pseudoscalar and vector meson degrees of freedom have to be taken into account as a consequence of the above-mentioned degeneracy. A continuous transition from the hyperons to the heavy baryons containing c or b quarks instead of an s-quark suggests a P-wave configuration for the bound meson.

In the heavy quark limit the heavy meson wave function $\mathcal{H}^a_{lh}$ is confined to the center ($r=0$) of the soliton. For a quantitative determination of this effect, it is necessary to take into account the interaction amongst the heavy quark degrees of freedom before performing the transformation (1) [9]. However, to the leading order in the $1/M_Q$ expansion, it is consistent to investigate the interaction of a strongly peaked heavy meson wave function and the light quark de-

[1]See [2] for further manifestations of the heavy quark symmetry.

[2]Note that in the rest frame of the heavy quark the time component of the vector field is suppressed in the heavy quark limit by virtue of the Proca-type equation of motion.

grees of freedom while, henceforth, discarding that part of the action, which describes only heavy quark degrees of freedom. Up to $O(1/M_Q)$ corrections the binding energy of the heavy baryon is, thus, determined by the properties of the soliton at $r\approx 0$ [5–7]. In that case the bound state equation reduces to an eigenvalue problem for the coefficients of the operators in the general P-wave ansatz for $\mathcal{H}^a_{lh}$.

Here we study such bound systems within the Nambu–Jona-Lasinio (NJL) model [13] extended to incorporate heavy quark symmetry [14].[3] In contrast to Skyrme-type models the soliton in the NJL model contains components of nonvanishing grand spin. The grand spin is the vector sum of total angular momentum and isospin, $\boldsymbol{G}=\boldsymbol{J}+\boldsymbol{\tau}/2$. The appearance of additional modes is due to the quark substructure of the soliton and leads to a novel coupling scheme between the heavy meson and the soliton as will become clear in the course of this paper. In particular, the largest binding energy is obtained for a heavy meson configuration, which decouples completely from the grand spin zero channel of the light quarks.

In Sec. II we review the extension of the NJL model to include heavy quarks. In Sec. III the bound state wave function of the heavy meson in the background of the soliton in the NJL model is constructed. The numerical results are presented in Sec. IV and concluding remarks are contained in Sec. V. Some technical details are relegated to an appendix.

II. HEAVY QUARKS WITHIN THE NJL MODEL

In NJL-type quark models the interaction Lagrangian is given by a four quark contact term. The heavy quark symmetry is incorporated by only taking into account the large component $Q_v\equiv Q_v^{(l)}$ which is defined in Eq. (1). Furthermore, that part of the Lagrangian, which only contains heavy quarks, Q, is discarded. We refer to Ref. [14] for the explicit form of the four quark interaction between the heavy and light quarks. Following the path integral bosonization procedure [16], a composite field, H, is introduced such that the Lagrangian is bilinear in the quark fields,

$$\mathcal{L}^{hl}=\overline{Q_v}(iv\cdot\partial)Q_v-\overline{Q_v}H\tilde{q}-\bar{\tilde{q}}\overline{H}Q_v+\frac{1}{2G_3}\mathrm{tr}(\overline{H}H),\quad (3)$$

where $\tilde{q}$ denotes the Dirac spinor of the light quarks in the chirally rotated representation $\tilde{q}=\Omega q$ [17]. Denoting by ξ the root of the chiral field, $U=\xi\xi$ in unitary gauge, we have $\Omega=\xi+\xi^\dagger+(\xi-\xi^\dagger)\gamma_5$. The second and third terms in Eq. (3) represent the interaction between the heavy and light quarks, which is mediated by the heavy meson field, $H=(H^u,H^d)$ [4]

$$H^a=\frac{1}{2}(1+\not{v})(i\gamma_5P^a+\not{V}^a),\quad a=u,d,$$

$$\overline{H^a}=\gamma_0H^{\dagger a}\gamma_0\qquad \begin{pmatrix}0 & 0\\ \mathcal{H}^a & 0\end{pmatrix},\quad \text{for } v_\mu\to(1,\mathbf{0}).\quad (4)$$

[3]Similar studies have been carried out in Ref. [15].

Finally, the last term in Eq. (3) is purely mesonic. The coupling constant G_3 stems from the four quark interactions between the heavy and light quark fields. In Eq. (3) only the leading terms of the $1/M_Q$ expansion have been maintained. Therefore, the kernel $(iv\cdot\partial)$ for Q_v is that of a massless nonrelativistic particle. Integrating out the fermion fields yields

$$A=A_{\mathcal{F}}+A_m^l+A_m^h,$$

$$A_{\mathcal{F}}=\mathrm{Tr}_\Lambda\ln\mathbf{D},$$

$$A_m^h=\frac{1}{2\mathrm{G}_3}\int d^4x\,\mathrm{tr}(\overline{H}H).\quad (5)$$

The functional trace stems from the path integral over all quark fields and involves the inverse propagator,

$$\mathbf{D}=\begin{pmatrix}i\not{D}_l' & -\overline{H}\\ -H & iv\cdot\partial\end{pmatrix},$$

$$i\not{D}_l'=\Omega i\not{D}_l\Omega^\dagger=\Omega[i\not{\partial}-m(U)^{\gamma_5}]\Omega^\dagger,\quad (6)$$

which acts on the light-heavy spinor $\binom{\tilde{q}}{Q_b}$. Here m denotes the light quark constituent mass. As a consequence of the chiral rotation and the omission of the light (axial) vector mesons, the light meson fields appear only in A_m^l. Its explicit form may, e.g., be found in Ref. [17]. Finally, $\not{D}_l$ refers to the Dirac operator for the light quarks, u,d, and describes the interaction between these quarks and soliton constructed from the light mesons which in turn are quark composites. The explicit form of $\not{D}_l$ will be presented in Sec. III.

In Eq. (5) we have indicated already the need for regularization. For the ongoing discussion we will employ Schwinger's proper-time regularization [18]. In the NJL model this implies a continuation to Euclidean space ($\tau=x_4=ix_0$). This defines the Euclidean Dirac operator $\mathbf{D}_E$ and the Euclidean action $A_{\mathcal{F}}^{(E)}=\mathrm{Tr}_\Lambda\ln\mathbf{D}_E$. Subsequently, its real part is replaced by a parameter integral

$$A_{\mathcal{F}}^R=\tfrac{1}{2}\mathrm{Tr}_\Lambda\ln\mathbf{D}_E\mathbf{D}_E^\dagger=-\tfrac{1}{2}\int_{1/\Lambda^2}^\infty\frac{ds}{s}\mathrm{Tr}\exp(-s\mathbf{D}_E\mathbf{D}_E^\dagger)\quad (7)$$

with the high momentum contribution chopped off. The imaginary part

$$A_{\mathcal{F}}^I=\tfrac{1}{2}\mathrm{Tr}\ln\mathbf{D}_E(\mathbf{D}_E^\dagger)^{-1}\quad (8)$$

is finite and remains unchanged. After computing the functional trace, the action is continued back to Minkowski space. To adjust the model parameters, we demand the physical pion mass and decay constant, $m_\pi=135$ MeV and $f_\pi=93$ MeV, respectively. Then all but one parameter of the light quark sector are determined [16,19]. For transparency this is commonly chosen to be the constituent quark mass m. Subsequently, the heavy quark coupling constant G_3 is fitted to the B-meson decay constant $f_B\approx 180$ MeV as estimated from lattice QCD [20] and QCD sum rules [21]. Since the present treatment follows closely the computation of Ref. [14], we dispense with further details although one remark is in order. As mentioned above, the light (axial)

TABLE I. The proper-time cutoff, Λ, the heavy quark coupling constant G_3, the root of the Bethe-Salpeter equation $\triangle M$, and the heavy meson binding energy E_M as functions of the constituent quark mass m.

m (MeV)	350	400	450	500	600
Λ (MeV)	641	631	633	642	672
G_3 (10^{-5} MeV^{-2})	1.53	1.68	1.82	1.94	2.17
$\triangle M$ (MeV)	327	353	369	401	449
E_M (MeV)	23	47	81	99	151

vector mesons are omitted in the present study. As a consequence of the missing $\pi-a_1$ mixing the relation between f_π and Λ is modified, whence reducing the numerical value of Λ. This also effects the prediction for G_3. We display the resulting values in Table I. For the system without the light (axial) vector mesons, we have re-evaluated the binding energy, $E_M=m-\triangle M$, of the heavy meson by determining the root, $\triangle M$ of the Bethe-Salpeter equation for the heavy meson field H. For simplicity we expand this Bethe-Salpeter equation in powers of $v\cdot p$, with $p_\mu=P_\mu-M_Q v_\mu$ being the residual four-momentum of H. The coefficients in this expansion depend on the proper-time cutoff, Λ. It turns out that the expansion in $v\cdot p$ converges less rapidly than in the model with light (axial) vector mesons. An expansion up to at least cubic order proved necessary to gain reliable results. In Table I we present the predicted root $\triangle M$ and the binding energy E_M. These results were obtained from an expansion up to fifth order in $v\cdot p$. The analytic form of this expansion is provided in the Appendix cf. Eqs. (A2) and (A3). Apparently, the binding energy increases with the constituent quark mass. It should be stressed that these results correspond to the leading order of the $1/M_Q$ expansion.

Equation (6) shows that only the residual momentum, p_μ, is affected by the regularization as a consequence of the heavy quark transformation (1). This is physically meaningful and crucial for a consistent interpretation of the model. Indeed, the residual momentum of the heavy quark in a hadron containing a single heavy quark arises entirely from its interaction with the light degrees of freedom (in this case light quarks) and is thus cut off at the same scale Λ. Note that the cutoff Λ is significantly smaller than the c- or b-quark masses (cf. Table I).

III. BARYONS WITH A HEAVY QUARK

Before describing heavy baryons as bound systems, it is appropriate to explain briefly the emergence of the soliton. When integrating out the light quarks, an energy functional for the light meson fields can be extracted [22,19] by first defining the one-particle Dirac Hamiltonian h from $i\beta \not{D}_l = i\partial_t - h$, cf. Eq. (6). Adopting the hedgehog ansatz for the chiral field $U=\exp[i\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\Theta(r)]$, this Hamiltonian reads $h=\boldsymbol{\alpha}\cdot\boldsymbol{p}+m\beta(\cos\Theta+i\gamma_5\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\sin\Theta)$. The total energy functional is expressed in terms of the eigenvalues, ϵ_μ, of h. Formally it is composed, respectively, of valence, vacuum, and meson contributions:

$$E_{\rm tot}[\Theta]=N_C\eta_{\rm val}|\epsilon_{\rm val}|+\frac{N_C}{2}\sum_\mu \int_{1/\Lambda^2}^{\infty}\frac{ds}{\sqrt{4\pi s^3}}{\rm e}^{-s\epsilon_\mu^2} +m_\pi^2 f_\pi^2\int d^3r[1-\cos\Theta(r)]. \tag{9}$$

Here $N_C=3$ is the number of color degrees of freedom. Furthermore, $\eta_{\rm val}$ denotes the occupation number of the valence quark level. This level is defined to be the one with the smallest module $|\epsilon_\mu|$. Its occupation number is adjusted to describe a configuration possessing unit baryon number, i.e., $\eta_{\rm val}=1+(1/2)\Sigma_\mu{\rm sgn}(\epsilon_\mu)$. Finally the chiral angle $\Theta(r)$ is self-consistently determined by minimizing $E_{\rm tot}$ [23,19].

In order to compute the binding energy of the heavy meson in the soliton background, the bosonized action (5) is first expanded up to the quadratic order in the meson field, $H(\boldsymbol{r},t)=\int(d\omega/2\pi)\tilde{H}(\boldsymbol{r},\omega)\exp(-i\omega t)$,

$$A_{hl}^{(2)}=\int\frac{d\omega}{2\pi}\Bigg[\frac{2\pi}{N_C G_3}{\rm tr}(\bar{\tilde{H}}\tilde{H})-\eta_{\rm val}\frac{\langle{\rm val}|\Omega^\dagger\beta\bar{\tilde{H}}\tilde{H}\Omega|{\rm val}\rangle}{\omega+\epsilon_{\rm val}} -\sum_\mu\langle\mu|\Omega^\dagger\beta\bar{\tilde{H}}\tilde{H}\Omega|\mu\rangle R_\Lambda(\omega,\epsilon_\mu)\Bigg], \tag{10}$$

where the arguments $(\boldsymbol{r},\omega)$ of the heavy fields have been omitted. In analogy to the static energy functional (9), this part of the action is composed of a purely mesonic piece as well as valence and vacuum contributions. The regularization function, R_Λ is obtained as an expansion in the frequency ω:

$$R_\Lambda(\omega,\epsilon)=\frac{1}{2\epsilon}\left[1-{\rm sgn}(\epsilon){\rm erfc}\left(\left|\frac{\epsilon}{\Lambda}\right|\right)\right]-\frac{\omega}{2\epsilon^2}[1-{\rm sgn}(\epsilon)] +\frac{\omega^2}{2\epsilon^3}\left\{\left(1-{\rm sgn}(\epsilon){\rm erfc}\left(\left|\frac{\epsilon}{\Lambda}\right|\right)\right) -\frac{2\Lambda}{\sqrt{\pi}\epsilon}(1-{\rm e}^{-\epsilon^2/\Lambda^2})\right\}+\cdots. \tag{11}$$

This expansion is equivalent to the one in terms of $v\cdot p$ for the meson sector. For completeness, we list R_Λ up to fifth in the appendix, cf. Eq. (A6). It is not surprising that this result for $A^{(2)}$ can be obtained equivalently from the expression found for the fluctuations of kaon fields in the background of the NJL model soliton [24] once the eigenenergies for the strange quark levels are set to zero. We note that in the unregularized formulation, only the negative energy states contribute to the vacuum part of $A^{(2)}$ because

$$\lim_{\Lambda\to\infty}R_\Lambda(\omega,\epsilon)=\frac{1}{2}\,\frac{1-{\rm sgn}(\epsilon)}{\omega+\epsilon}. \tag{12}$$

In the many-body interpretation of the functional integral $A_{\mathcal{F}}$, one would phrase this result such that in the limit $\Lambda\to\infty$, only the occupied quark orbits couple to the heavy meson. The unregularized expression (12) may be derived independently when considering

$$A_{hl}^{(2)}(\mathrm{unreg}) = \mathrm{Tr}[(i\not{D}_l)^{-1}\overline{H}G_v H] \tag{13}$$

in Minkowski space. Here

$$G_v(t,t') = \langle t|(i\partial_t)^{-1}|t'\rangle = \lim_{\eta\to 0}\int \frac{dk}{2\pi}\frac{\mathrm{e}^{-ik(t-t')}}{k+i\eta} \tag{14}$$

denotes the nonrelativistic propagator in the heavy quark rest frame. However, the regularization is unavoidable since $\Sigma_\mu R_\infty(\omega,\epsilon_\mu)$ diverges logarithmically. Nevertheless, the limit (12) serves as an independent check on our computations. From expression (12) it also becomes apparent that the total action (10) is continuous as the energy eigenvalue associated with the valence quark orbit changes its sign.

Next we study the coupling of a grand spin $\frac{1}{2}$, P-wave heavy meson to the soliton. Such a configuration can be expressed in terms of three radial functions [9]:

$$\widetilde{\mathcal{H}}_{lh}^{a}(\boldsymbol{r},\omega) = \hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}^{ab}[u_A(r,\omega)\delta_{lh}\delta^{bc} + u_B(r,\omega)\boldsymbol{\tau}^{bc}\cdot\boldsymbol{\sigma}_{lh} + u_C(r,\omega)\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}^{bc}\hat{\boldsymbol{r}}\cdot\boldsymbol{\sigma}_{lh}]\chi^c. \tag{15}$$

Here χ^a denotes a (constant) isospinor. One might have expected the appearance of three different isospinors. However, such an ansatz is waived because it is not an eigenstate of the grand spin projection operator. Skyrme-model studies [8] also indicate that the light grand spin, which is defined as the grand spin of the operator multiplying χ, has to vanish for the meson configuration with the largest binding energy. In the heavy quark limit the binding energy of this heavy meson is determined by the potential it experiences at the center of the soliton, $r=0$. In order to extract this potential it is convenient (and consistent) to adopt radial functions in Eq. (15), which are nonvanishing radial functions at $r=0$. Consequently, the heavy meson field couples only to quark levels, which have nonvanishing radial functions at $r=0$. As can be seen from the decomposition (4) and the coupling scheme exhibited in the Lagrangian (3), these nonvanishing radial functions must occur in the lower components of the light quark fields. At first sight this appears to be surprising because the lower component of the valence quark eigenfunction happens to vanish at $r=0$. However, one has to be aware of the fact that the heavy meson couples to the light quark states via the chiral rotation, which is off diagonal at the origin in the presence of the soliton since $\Theta(r=0)=-\pi$,

$$\Omega(\boldsymbol{r}=0) = \begin{pmatrix} 0 & i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}} \\ i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}} & 0 \end{pmatrix}, \tag{16}$$

allowing the valence quark to couple to the heavy meson. In addition to the channel with grand spin zero, which includes the valence quark orbit, only the channel carrying grand spin one contains states, which do not vanish at $r=0$.

Carrying out the angular integrals as well as the flavor and Dirac traces finally yields the action as a function of the values, which the meson wave functions in Eq. (15) assume at $r=0$. Denoting these by $A(\omega)$, $B(\omega)$, and $C(\omega)$, respectively, we find

$$A_{lh}^{(2)} = -\frac{1}{2}\int\frac{d\omega}{2\pi}(\chi^\dagger\chi)[A^*(\omega),B^*(\omega),C^*(\omega)]\mathcal{G}(\omega) \times\begin{pmatrix} A(\omega) \\ B(\omega) \\ C(\omega)\end{pmatrix}. \tag{17}$$

The inverse propagator is a 3×3 matrix

$$\mathcal{G}(\omega) = \frac{8\pi}{N_C G_3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 1\end{pmatrix} + f_0(\omega)\begin{pmatrix} 1 & -3 & -1 \\ -3 & 9 & 3 \\ -1 & 3 & 1\end{pmatrix} + f_1(\omega)\begin{pmatrix} 3 & 3 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 3\end{pmatrix}. \tag{18}$$

Denoting by $g_\mu(r)$ the radial parts of the upper components of the light quark wave functions, which are eigenstates of h, the spectral functions are expressed as[4]

$$f_0(\omega) = \eta_{\mathrm{val}}\frac{|g_{\mathrm{val}}(0)|^2}{\omega+\epsilon_{\mathrm{val}}} + \sum_{\mu,G=0}|g_\mu(0)|^2 R_\Lambda(\omega,\epsilon_\mu),$$

$$f_1(\omega) = \sum_{\mu,G=1}|g_\mu(0)|^2 R_\Lambda(\omega,\epsilon_\mu). \tag{19}$$

The sums refer to distinct quark grand spin (G) channels.

The equation of motion for the heavy meson reduces to a homogeneous matrix equation for A, B, and C. Its solution requires adjusting ω to ω_0 such that $\mathcal{G}(\omega_0)$ possesses vanishing eigenvalues. It is easy to show that there exist two distinct solutions to this problem:

$$(1):\quad \frac{2\pi}{N_C G_3} + f_0(\omega_0) = 0, \quad \text{with } (A,B,C)\propto(1,-1,0), \tag{20}$$

$$(2):\quad \frac{2\pi}{N_C G_3} + f_1(\omega_0) = 0, \quad \text{with } (A,B,C)\propto(3+\zeta,1,\zeta). \tag{21}$$

The vectors of set (2) obviously span a two-dimensional vector space causing the associated eigenstates to be degenerate. The set (1) decouples from the $G=1$ states while the set (2) is annihilated by the $G=0$ states. In particular, the set (1) corresponds to the bound state wave function found in the heavy quark limit of properly extended Skyrme-type models [8]

$$\mathcal{H}_{lh}^{a}(\mathrm{Skyrme}) = u(r)(\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau})_{al}(\tau^2_{hh'}\chi_{h'}) = \frac{u(r)}{2}[\hat{\boldsymbol{r}}\cdot\boldsymbol{\tau}(\delta_{lh} - \boldsymbol{\tau}\cdot\boldsymbol{\sigma}_{lh})\chi]_a. \tag{22}$$

[4]The single quark wave functions, g_μ, carry the dimension (energy)$^{3/2}$.

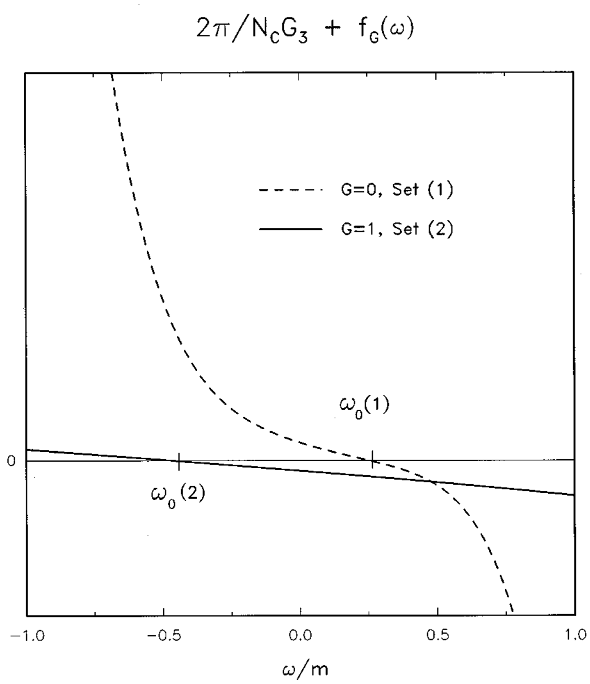

FIG. 1. The spectral functions and their roots for the eigenvectors defined in Eqs. (20) and (21). The constituent quark mass has been taken to be $m=400$ MeV.

The interpretation of this situation is obvious: If the background soliton field contains only $G=0$ states, as it is the case in Skyrme-type models, the only solution will be given by set (1). However, if also $G=1$ states are present, an alternative solution will exist. The soliton dynamics, from which the spectral functions f_0 and f_1 are computed, then determines which of these two modes, (1) or (2), leads to a larger binding and, therefore, has to be interpreted as the lightest baryon containing a heavy quark.

IV. NUMERICAL RESULTS FOR THE HEAVY BARYON

In analogy with the meson sector, we have expanded the regularization function, $R_\Lambda(\omega,\epsilon)$, (11) up to fifth order in the frequency ω. In Fig. 1 the resultant frequency dependence of the spectral functions (20) and (21) is shown for the special case $m=400$ MeV. As a result of the heavy quark transformation (1) the threshold for the (unphysical) decay of the heavy meson into a quark-antiquark pair is given by the energy of the lowest accessible light quark state. Therefore, a singularity appears at $\omega\approx-\epsilon_{\rm val}$ for the mode coupling to the $G=0$ channel, while the radius of convergence is much larger in the case of the $G=1$ channel. The latter mode apparently leads to a smaller eigenfrequency, ω_0.

The total energy of the system consisting of the soliton and the bound meson is given by $M_Q+E_{\rm tot}+\omega_0$. As the total energy of the heavy meson in the trivial background is $M_Q+\triangle M$, the binding energy of the heavy baryon becomes $E_B=\triangle M-\omega_0$. This binding energy is measured with regard to a decay into a soliton and a heavy meson. When increasing the heavy quark coupling constant G_3, the curves shown in Fig. 1 get shifted downwards by a constant amount. Hence, the roots decrease and the binding energies of the baryon states containing a heavy quark become larger when

TABLE II. The roots, ω_0, of the spectral functions (20) and (21) for the heavy meson modes, which couple to the grand spin zero and one quark channels, respectively. $E_B=\triangle M-\omega_0$ represents the associated binding energy of the heavy baryon. All data are in MeV.

m		350	400	450	500	600
Set (1)	ω_0	94	102	113	114	86
	E_B	233	251	257	287	363
Set (2)	ω_0	−128	−182	−223	−261	−341
	E_B	455	535	592	662	790

the coupling gains strength. Obviously, a heavy baryon constructed from an eigenmode of type (2) is more strongly bound than a type (1) heavy meson. In Table II the predicted roots, ω_0, as well as the associated binding energies, E_B, of the heavy baryon are presented. For the modes of type (1) the eigenfrequency acquires a maximum for $m\approx500$ MeV. Since $\triangle M$ increases with the constituent quark mass m the binding energy for the associated heavy baryon exhibits almost no variation for $m\leqslant500$ MeV. When m is further increased, the eigenfrequency decreases leading to a larger binding energy. This behavior of ω_0 is due to the decrease of $\epsilon_{\rm val}$ when m grows. On the contrary, the eigenfrequency corresponding to the eigenvectors of type (2) appears to be a quickly, monotonously decreasing function of m. This causes the binding energy of the corresponding heavy baryon to increase significantly with m. Since studies in the light quark sector of the NJL model favor 400 MeV$\leqslant m\leqslant$450 MeV [19], the results displayed in Table II suggest the interpretation that the NJL model predicts the baryon with one heavy quark to have a binding energy of about 560 MeV in leading order of the $1/M_Q$ expansion. A less strongly bound state is obtained possessing about half of that binding energy. As discussed above the latter state corresponds to the one which is identified as the most strongly bound baryon in Skyrme-type models. In this context it has to be remarked that a state associated with the mode (2) does not appear in Skyrme-type models because in those models the background soliton is a pure $G=0$ configuration. If the NJL model soliton had contained no $G=1$ components, the most strongly bound baryon would have been of the same structure as in purely mesonic soliton models.

As in the bound state approach [11], heavy baryon states with good spin and isospin are generated by quantizing canonically the collective isospin rotation, $A(t)$, of the meson fields. This adds a term of the form $C_i{\rm tr}[\chi\chi^\dagger A^\dagger(t)\dot A(t)]$ to the action. The index $i=1,2$ refers to the distinct sets in Eqs. (20), (21). The time derivative of the collective rotation defines the angular velocity $\boldsymbol{\omega}$ via $A^\dagger(t)\dot A(t)=(i/2)\boldsymbol{\omega}\cdot\boldsymbol{\tau}$. Empolying (iso)rotational invariance these coefficients can, hence, be computed from the matrix elements[5] $\langle\mu|\tau_3|\nu\rangle\langle\nu|\Omega^\dagger H^\dagger\beta H\Omega|\mu\rangle$. As argued above, H only couples to quark states with $G=0,1$. The bound state of set

[5]For the case of the bound state approach to hyperons these matrix elements have been discussed thoroughly by Weigel *et al.* in Ref. [12].

(1) has nonvanishing matrix elements only for quark levels with $G=0$. In that case the matrix element of τ_3 vanishes, hence $C_1=0$, in agreement with the Skyrme-model results [8]. For the set (2) there is indeed a coupling when both quark levels ($|\mu\rangle$ and $|\nu\rangle$) are from the $G^\pi=1^+$ channel. We find $C_2=\mathcal{C}_2[(\zeta+2)(\zeta+6)]/[(\zeta+2)^2+2]$, where the positive definite denominator is due to the normalization associated to the metric induced by the coefficient matrix of f_1 in Eq. (18). Although the collective rotation removes the degeneracy in ζ, we will argue that $\mathcal{C}_2$ is negligibly small without going into the details of the calculation. For quark states ($|\mu\rangle$ and $|\nu\rangle$) from the $G^\pi=1^+$ channel, the above-mentioned matrix elements are similar to those entering the evaluation of the moment of inertia α^2 [22]. As the collective quantization gives rise to a factor $1/\alpha^2$, $\mathcal{C}_2$ can be estimated by the contribution of the $G^\pi=1^+$ channel to the total moment of inertia. This contribution is very small because α^2 is dominated by the valence quark level, which resides in the $G^\pi=0^+$ channel, e.g., for $m=400$ MeV we find a contribution of only 2%. Hence, we conclude that there is (almost) no hyperfine splitting in the heavy quark limit. For a finite heavy quark mass, this conclusion should change because H will couple to channels with $G\geqslant 2$ as well.

V. CONCLUSIONS

We have analyzed fluctuations of mesons with a heavy quark in the background of the NJL model chiral soliton in the leading order of the heavy quark mass expansion. We have succeeded in showing that at this order the associated Bethe-Salpeter equation possesses the bound state solution known from Skyrme-model studies. We have, furthermore, seen that this Bethe-Salpeter equation contains an additional solution, which arises from the coupling of the heavy meson field to vacuum quark states. Also, the novel bound state appears in degenerate pairs. The actual computation reveals that in the NJL model, the novel states are more strongly bound than those configurations known from the Skyrme model. Although this certainly represents an interesting feature in its own, future studies are required to find out whether or not this novel state is an artifact of applying the well-established $1/M_Q$ expansion to the NJL model. In comparison, a few aspects of the Skyrme-model calculation for finite M_Q [9,10] are worthwhile to be mentioned. First, the restriction to the leading order of the $1/M_Q$ expansion has been found to be a somewhat crude approximation to the realistic cases causing states, which are predicted to be degenerate at leading order, to acquire distinct binding energies when corrections of subleading order are taken into account. We assume that a similar mechanism will remove the degeneracy of the novel states in the NJL model. Second, Table (1) of Ref. [9] shows that the transition from infinite to realistic values of M_Q is indeed continuous. In particular, the order of bound states remains unchanged. Hence, one is inclined to conjecture that the novel states persist for finite M_Q. Third, not all bound states, which are predicted within the large M_Q limit are permitted for finite M_Q because the limits $M_Q\to\infty$ and $r\to 0$ do not commute necessarily. This represents the only mechanism, which would prohibit the existence of the novel states in the case of finite M_Q. These issues certainly raise interest for generalizing the NJL model studies to finite M_Q. Such studies are considerably more involved because it is crucial to first perform the heavy quark transformation and to regularize subsequently the functional trace to have a consistent interpretation of the model.

In practice the binding energies of heavy baryons are measured with respect to the decay into a nucleon and the lightest meson containing the corresponding heavy quark. The experimentally observed binding energies are $\sim 613\pm 50$ MeV and ~ 625 MeV for the $\Lambda_B(5641\pm 50)$ and $\Lambda_C(2285)$, respectively [3]. It should be noted that these baryons are degenerate in the heavy quark limit. In reasonable agreement with these data, our numerical analysis yields a binding energy of about 560 MeV for the most strongly bound heavy baryon when the only free parameter of the model is adjusted to reproduce the properties of the light baryons. The less strongly bound heavy baryon with a predicted binding energy around 250 MeV may be associated eventually with $\Lambda_C(2625)$. This baryon is bound by about (~ 285 MeV) against the decay into a nucleon and a D meson. We would like to emphasize that the main purpose of the present paper was to discuss the novel coupling scheme between the heavy meson and the soliton rather than to achieve a precise agreement with the experimentally observed binding energies.

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APPENDIX

In this appendix we will provide briefly the expansions we have been using for the regularization functions, which enter the Bethe-Salpeter equations for the heavy quark meson.

In case no soliton is present, these functions are obtained from the quark loop in the self-energy, $\Pi(v\cdot p)$ [cf. Eq. (47) of Ref. [14]]:

$$\mathrm{tr}[\overline{H}\Pi(v\cdot p)H]=-iN_C\int\frac{d^4k}{(2\pi)^4}\times\frac{\mathrm{tr}[(\not{k}-\not{p}+m)\overline{H}H]}{[(k-p)^2-m^2](v\cdot p+i\epsilon)}. \quad \text{(A1)}$$

Again, v denotes the velocity of the heavy quark inside the meson while p labels the momentum of the meson. We refer to the literature [15,14] on the treatment of this quantity in Euclidean space and regularization of the quark loop. When $v\cdot p$ is smaller than the quark-antiquark threshold, which in the heavy quark limit is identical to the light quark constituent mass, the self-energy has the Taylor expansion

$$\Pi(v\cdot p)=\sum_{n=0}^{\infty}\Pi^{(n)}(0)(v\cdot p)^n, \quad \text{(A2)}$$

where the superscript refers to the derivative with respect to $v\cdot p$. In the proper-time regularization scheme of Ref. [14], which actually defines the model, the first five coefficients of the expansion (A2) are

$$\Pi^{(0)}(0)=\frac{N_C m^2}{16\pi^2}\left[\Gamma\left(-1,\frac{m^2}{\Lambda^2}\right)+2\frac{\Lambda\sqrt{\pi}}{m}\exp\left(-\frac{m^2}{\Lambda^2}\right)-2\pi\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right],$$

$$\Pi^{(1)}(0)=\frac{N_C m}{8\pi^2}\left[\Gamma\left(0,\frac{m^2}{\Lambda^2}\right)+\frac{\Lambda\sqrt{\pi}}{m}\exp\left(-\frac{m^2}{\Lambda^2}\right)-\pi\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right],$$

$$\Pi^{(2)}(0)=\frac{N_C}{16\pi^2}\left[2\Gamma\left(0,\frac{m^2}{\Lambda^2}\right)+\pi\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right],$$

$$\Pi^{(3)}(0)=\frac{N_C}{16m\pi^2}\left[\frac{4}{3}\exp\left(-\frac{m^2}{\Lambda^2}\right)+\pi\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right],$$

$$\Pi^{(4)}(0)=\frac{N_C}{16m^2\pi^2}\left[\left(\frac{4}{3}+\frac{\sqrt{\pi}m}{2\Lambda}\right)\exp\left(-\frac{m^2}{\Lambda^2}\right)+\frac{\pi}{4}\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right],$$

$$\Pi^{(5)}(0)=\frac{N_C}{16m^3\pi^2}\left\{\left(\frac{8}{15}\left[1+\frac{m^2}{\Lambda^2}\right]+\frac{\sqrt{\pi}m}{2\Lambda}\right)\exp\left(-\frac{m^2}{\Lambda^2}\right)+\frac{\pi}{4}\mathrm{erfc}\left(\frac{m}{\Lambda}\right)\right\}. \tag{A3}$$

We then obtain the binding energy, $\triangle M$, of the heavy meson by determining the root from the trunctated expansion and including the contribution from the purely mesonic part of the action, A_m^h, (5):

$$-\frac{1}{2G_3}+\sum_{n=0}^{5}\Pi^{(n)}(0)(\triangle M)^n=0. \tag{A4}$$

The decay constant, f_H, of the meson with the heavy quark is obtained by coupling external electroweak sources [14]. This then relates f_H to the field normalization $\sqrt{\Pi^{(1)}(0)}$:

$$f_H=\frac{1}{G_3\sqrt{\Pi^{(1)}(0)M_H}}, \tag{A5}$$

where M_H refers to the mass of the heavy meson. Using the B-meson parameters $f_B=180$ MeV and $M_B=5.3$ GeV yields the numerical results for $\triangle M$ and G_3 listed in Table I.

As the procedure leading to the regularization function $R_\Lambda(\omega,\epsilon)$ has been explained already in Sec. III, it sufficices to just list the expansion up to order ω^5:

$$\begin{aligned}R_\Lambda(\omega,\epsilon)=&\frac{1}{2\epsilon}\left[1-\mathrm{sgn}(\epsilon)\mathrm{erfc}\left(\left|\frac{\epsilon}{\Lambda}\right|\right)\right]-\frac{\omega}{2\epsilon^2}[1-\mathrm{sgn}(\epsilon)]\\&+\frac{\omega^2}{2\epsilon^3}\left\{\left[1-\mathrm{sgn}(\epsilon)\mathrm{erfc}\left(\left|\frac{\epsilon}{\Lambda}\right|\right)\right]-\frac{2\Lambda}{\sqrt{\pi}\epsilon}(1-e^{-\epsilon^2/\Lambda^2})\right\}-\frac{\omega^3}{2\epsilon^4}[1-\mathrm{sgn}(\epsilon)]\\&+\frac{\omega^4}{2\epsilon^5}\left\{\left[1-\mathrm{sgn}(\epsilon)\mathrm{erfc}\left(\left|\frac{\epsilon}{\Lambda}\right|\right)\right]+\frac{4\Lambda^3}{\sqrt{\pi}\epsilon^3}(1-e^{-\epsilon^2/\Lambda^2})-\frac{4\Lambda}{\sqrt{\pi}\epsilon}\right\}\\&-\frac{\omega^5}{2\epsilon^6}[1-\mathrm{sgn}(\epsilon)].\end{aligned} \tag{A6}$$

[1] E. Eichten and F. Feinberg, Phys. Rev. D **23**, 2724 (1981); M. B. Voloshin and M. A. Shifman, Yad. Fiz. **45**, 463 (1987) [Sov. J. Nucl. Phys. **45**, 292 (1987)]; N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); H. Georgi, *ibid.* **230**, 447 (1990).

[2] M. Neubert, Phys. Rep. **245**, 259 (1994).

[3] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1171 (1994).

[4] J. D. Bjorken, in *Results and Perspectives in Particle Physics*, Proceedings of the 4th Rencontres de Physique de la Vallee D' Aoste, La Thuile, Italy, 1990, edited by M. Greco (World Scientific, Singapore, 1990), p. 583; M. Wise, Phys. Rev. D **45**, R2188 (1992); F. Hussain, J. Körner, and G. Thompson, Ann. Phys. (N.Y.) **206**, 334 (1991).

[5] E. Jenkins and A. V. Manohar, Phys. Lett. B **294**, 173 (1992); Z. Guralnik, M. Luke, and A. V. Manohar, Nucl. Phys. **B390**, 474 (1993); E. Jenkins, A. V. Manohar, and M. Wise, *ibid.* **396**, 27 (1993).

[6] M. Rho, Heavy Quark Baryons as Skyrmions, in *Baryons as Skyrme Solitons*, edited by G. Holzwarth (World Scientific, Singapore, 1994) p. 193; D. P. Min, Y. Oh, B. Y. Park, and M. Rho, ''Soliton structure of heavy baryons,'' Seoul Report No. SNUTP 92-78, hep-ph/9209275 (unpublished); H. K. Lee, M. A. Novak, M. Rho, and I. Zahed, Ann. Phys. (N.Y.) **227**, 175 (1993); M. A. Novak, M. Rho, and I. Zahed, Phys. Lett. B **303**, 130 (1993).

[7] K. S. Gupta, M. A. Momen, J. Schechter, and A. Subbaraman, Phys. Rev. D **47**, 4835 (1993); M. A. Momen, J. Schechter, and A. Subbaraman, *ibid.* **49**, 5970 (1994).

[8] J. Schechter and A. Subbaraman, Phys. Rev. D **51**, 2311 (1995).

[9] J. Schechter, A. Subbaraman, S. Vaidya, and H. Weigel, Nucl. Phys. **A590**, 655 (1995); Y. Oh and B. Y. Park, Phys. Rev. D **51**, 5016 (1995).

[10] D. P. Min, Y. Oh, B. Park, and M. Rho, Int. J. Mod. Phys. E **4**, 47 (1995).

[11] C. Callan and I. Klebanov, Nucl. Phys. **B262**, 365 (1985); C. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B **202**, 269

(1988); I. Klebanov, Strangeness in the Standard Skyrme Model, in *Hadrons and Hadronic Matter*, edited by D. Vautherin, J. Negele, and F. Lenz (Plenum Press, New York, 1989), p. 223.

[12] J. Blaizot, M. Rho, and N. Scoccola, Phys. Lett. B **209**, 27 (1988); N. Scoccola, H. Nadeau, M. A. Novak, and M. Rho, *ibid.* **201**, 425 (1988); D. Kaplan and I. Klebanov, Nucl. Phys. **B335**, 45 (1990); Y. Kondo, S. Saito, and T. Otofuji, Phys. Lett. B **256**, 316 (1991); M. Rho, D. O. Riska, and N. Scoccola, Z. Phys. A **341**, 341 (1992); H. Weigel, R. Alkofer, and H. Reinhardt, Nucl. Phys. **A576**, 477 (1994).

[13] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[14] D. Ebert, T. Feldmann, R. Friedrich, and H. Reinhardt, Nucl. Phys. **B434**, 619 (1995).

[15] M. A. Novak, M. Rho, and I. Zahed, Phys. Rev. D **48**, 4370 (1993); W. A. Bardeen and C. T. Hill, *ibid.* **49**, 409 (1994).

[16] D. Ebert and H. Reinhardt, Nucl. Phys. **B271**, 188 (1986).

[17] H. Reinhardt and B. V. Dang, Nucl. Phys. **A500**, 563 (1989); R. Alkofer, H. Reinhardt, J. Schlienz, and H. Weigel, Z. Phys. A **354**, 181 (1996).

[18] J. Schwinger, Phys. Rev. **82**, 664 (1951).

[19] R. Alkofer, H. Reinhardt, and H. Weigel, Phys. Rep. **265**, 139 (1996).

[20] C. R. Allton *et al.*, Nucl. Phys. **B340**, 589 (1991); C. Alexandrou *et al.*, Phys. Lett. B **256**, 60 (1991); R. Sommer, ''Beauty Physics in Lattice Gauge Theories,'' DESY Report No. hep-lat/9401037 (unpublished).

[21] P. Colangelo, G. Nardulli, A. A. Ovchinnikov, and N. Paver, Phys. Lett. B **269**, 201 (1991); S. Narison and K. Zalewski, *ibid.* **320**, 369 (1994); S. Narison, *ibid.* **322**, 247 (1994); **325**, 197 (1994).

[22] H. Reinhardt, Nucl. Phys. **A503**, 825 (1989).

[23] H. Reinhardt and R. Wünsch, Phys. Lett. B **215**, 577 (1988); **230**, 93 (1989); T. Meissner, F. Grümmer, and K. Goeke, *ibid.* **227**, 296 (1989); R. Alkofer, *ibid.* **236**, 310 (1990).

[24] H. Weigel, H. Reinhardt, and R. Alkofer, Phys. Lett. B **313**, 377 (1993).