Decuplet reexamined in chiral perturbation theory

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This paper deals with two issues. First, we explore the quantitative importance of higher multiplets for properties of the Δ decuplet in chiral perturbation theory. In particular, it is found that the lowest order one-loop contributions from the Roper octet and the Δ(1600) to the decuplet masses and magnetic moments are substantial. The relevance of these results to the chiral expansion in general is discussed. The exact values of the magnetic moments depend upon delicate cancellations involving ill-determined coupling constants. Second, we present new relations between the magnetic moments of the Δ decuplet that are independent of all couplings. They are exact at the order of the chiral expansion used in this paper. [S0556-2821(96)04321-4]

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I. INTRODUCTION

The success of chiral perturbation theory (χPT) [1] for understanding properties of the pseudoscalar mesons is now well established [2]. The approach is based on the existence of a systematic expansion in derivatives of the pion's field and the pion's mass, whereby m_π divided by some large scale, generated by the theory itself and typically $\sim 4\pi f_\pi$, becomes the perturbative expansion parameter. For the purely mesonic sector this expansion is, in fact, quadratic in the pion's mass, so that even for the $\mathrm{SU}_f(3)$ generalization, $(m_K/4\pi f_\pi)^2$ is still a reasonably small parameter.

The application to the baryon sector has, however, from the outset been confronted with a variety of difficulties [3]. For example, how to handle the nucleon's mass was a problem solved only relatively recently [4,5]. One unavoidable complication when including baryons is that the chiral expansion is itself more complicated [3,4] than that in the purely mesonic case, and the expansion parameter is now only linear in m_π (m_K). A second pertinent complication involves the issue of resonances.[1]

Originally, it was conjectured [4] that all such resonances (and most notably the Δ) need not be included as an explicit degree of freedom, i.e., that they could be ''integrated out.'' One notably active group [6] has maintained this viewpoint.[2] Others [7–12] have included the decuplet on the same footing as the octet, assigning [13] chiral power 1 to the mass difference $M_{10}-M_8$. The Δ degree of freedom was first introduced into χPT by Jenkins and Manohar in Ref. [7]. Recently, the importance of the $\Lambda^*(1405)$ for understanding threshold kaon–nucleon scattering lengths has also been realized [14–16].

[1]Because of the large mass difference between the ρ and the pion, as well as the quadratic nature of the chiral expansion, this issue does not arise when considering the sector of purely pseudoscalar, Goldstone bosons. The effects of the ρ and other such states are incorporated as part of the needed input coupling constants of the theory.

[2]Arguing that for a very limited region near threshold in the $\mathrm{SU}_f(2)$ defined theory, the Δ-nucleon mass difference can be considered large.

In this paper we discuss the role of higher multiplets for the properties of the Δ decuplet at the one-loop level in χPT. We consider the $O(p^3)$ correction to the decuplet masses and the $O(p^2)$, one-loop correction to the magnetic moments of the decuplet (the electromagnetic vertex has chiral power -1, excluding whatever power may be assigned to electric charge). Our criterion for which multiplets to consider is that the average mass splitting δ_h between the multiplet and the Δ decuplet be less than the mass of the kaon:[3]

$$\delta_h=|M_H-M_{10}|<m_K. \tag{1}$$

This criterion is based on the fact that when it is met, an expansion in m_K/δ_h is not justified so that loop effects involving these higher-multiplet members as intermediate states *cannot* be absorbed into higher-order terms in χPT.[4] Such loop effects place a limitation on any formulation of χPT that omits the higher multiplet as an explicit degree of freedom, *even if the chiral expansion was then executed to all orders.*

Following the convention of Ref. [13], we set the chiral power of all δ to be 1. For the case of the nucleon octet Eq. (1) is clearly met,

$$\delta_N=M_{10}-M_8=226\ \text{MeV} \tag{2}$$

($M_{10}=1377$ MeV and $M_8=1151$ MeV), and is indeed the driving phenomenological reason for expecting that the Δ cannot be ignored for descriptions of the nucleon.

We focus here on the properties of the Δ decuplet as opposed to those of the nucleon octet because of the simple reason that the mass splitting δ_h satisfies the criterion specified by Eq. (1) for at least two resonances.

We will show that the Roper and Δ(1600) [the so-called ''Roper'' of the Δ(1232)] has a nontrivial effect on both decuplet mass splittings and magnetic moments. We should also note that these results lead to a more general statement.

[3]m_η is obtained at this order in χPT using the work of Gell-Mann, Oakes, and Renner (GMOR) [17].

[4]It may be possible to formally sum the contributions to all orders. Such an approach is, however, beyond the context of χPT.

When adding a loop one must also add resonances whose masses are within a kaon mass of the resonances already included.

The rest of this paper is organized as follows. In Sec. II we enumerate the various multiplets considered and their interactions (and experimentally obtained couplings) with the Δ decuplet utilizing the heavy baryon formalism of Jenkins and Manohar [5]. In Sec. III we present the one-loop, $O(p^3)$ contributions to the decuplet masses focusing on the violation to the decuplet equal spacing (DES) rule [18], the sole quantity for which χPT makes a prediction at this order in the chiral expansion [8,13]. In Sec. IV we consider the one-loop, $O(p^2)$ results for the magnetic moments of the decuplet, a subject first discussed in χPT in Ref. [10] where though adherence to the order of the chiral expansion was not strictly maintained. We demonstrate later that strict adherence is crucial for renormalizability. The focus in both Secs. III and IV is the quantitative importance of the higher multiplets. In addition, new relations for the magnetic moments at this order in χPT are derived that are independent of the intermediate multiplets considered. Their violation would be a clear measure of the importance of higher-chiral power terms in the expansion. In Sec. V we conclude with a discussion of the consequence of these results to the loop expansion, in general, in χPT.

II. HIGHER MULTIPLETS: DEFINITIONS AND COUPLINGS

A number of multiplets[5] satisfy the criteria equation (1). Fortunately, most of these can be eliminated due to symmetry constraints. For example, flavor singlets such as the $\Lambda^*(1405)$ do not couple to a decuplet via an $\mathrm{SU}_f(3)$ octet (the Goldstone bosons). Only slightly less straightforward, a $1/2^-$ octet [e.g., the $N(1535)$ multiplet] couples only through the lower components of the baryon spinors,

$$\mathcal{L}^i \sim \overline{T^\mu}\gamma_5 A_\mu B^*, \tag{3}$$

which vanishes to lowest order in the heavy baryon expansion [5]. (Such states would, in principle, need to be considered in higher-order calculations.) Coupling to the $5/2^-$ octet likewise vanishes at lowest order. With these eliminations, we conclude that only octets or decuplets of baryons with quantum numbers $1/2^+$, $3/2^+$, $3/2^-$, and $5/2^+$ need be considered.

The most important $1/2^+$ multiplet (beyond, of course, the nucleon's) is the octet containing the Roper, $N(1440)$. A slight difficulty arises in determining

$$\delta_R = M_8(1440) - M_{10} \tag{4}$$

because one member of the Roper multiplet, the Ξ^*, has not yet been identified. To get a reasonably approximate value for its mass, we use the corresponding Gell-Mann–Okubo (GMO) relation [24]

$$M_{\Xi^*} = \tfrac{3}{2} M_{\Lambda^*}(1600) + \tfrac{1}{2} M_{\Sigma^*}(1660) - M_{N^*}(1440), \tag{5}$$

by which one estimates that $M_{\Xi^*} = 1790$. The average value of the Roper multiplet is thereby

$$M_{8^*} = \tfrac{1}{8}\,(2M_{\Xi^*} + 3M_{\Sigma^*} + M_{\Lambda^*} + 2M_{N^*}) = 1630 \ \text{MeV}, \tag{6}$$

from which one gets that

$$\delta_R = 253 \ \text{MeV}. \tag{7}$$

For the $\Delta N^*\pi$ interaction one has

$$\mathcal{L}^i = \widetilde{C}(\overline{T^\mu} A_\mu B^* + \text{H.c.}) \tag{8}$$

in complete analogy to the leading $\Delta N\pi$ interaction in the heavy Fermion limit [7]. The coupling $\widetilde{C}^2$ can be obtained from the observed decay of the $N^*(1440) \to \Delta\pi$. For these purposes we use Hoehler's determination [19,20] of the Roper's pole position (1385 MeV) and absorptive part of the propagator ($-2 \times$ imaginary part) = 164 MeV and Manley and Saleski's determination [19,21] of the branching ratio $B_{N^* \to \Delta\pi} = 0.22$. We similarly use Hoehler's pole position of the Δ (1209 MeV) and take $m_\pi = m_\pi^0 = 135$ MeV (as most of the mass difference $m_{\pi^+} - m_{\pi^0}$ is of electromagnetic origin).[6] Comparing this with the decay of the Δ, $\Gamma_{\Delta \to N\pi}$ one obtains that

$$\widetilde{C}^2 \approx \frac{1}{2}\mathcal{C}^2. \tag{9}$$

In principle, one other $1/2^+$ multiplet meets our criteria, the octet containing the $N(1710)$, with a $\delta_h \approx 470$ MeV. However, best estimates [19] for the partial width for $N(1710) \to \Delta\pi$ is ≈ 30 MeV, which implies that the relevant coupling constant is significantly suppressed compared to that in Eq. (9). We, therefore, ignore this multiplet in our subsequent calculations as it amounts to only a small correction to those of the Roper octet.

The most important $3/2^+$ resonance for the properties of the Δ decuplet is the $\Delta(1600)$, the so-called ''Roper'' of the $\Delta(1232)$. Unfortunately, no other member of the decuplet has been identified,[7] so that all properties of the full decuplet must be inferred from this single state. For the mass splitting we take

[5]See, e.g., Table 30.4 in the Particle Data Group [19]. As we are at present only concerned with the average coupling of these multiplets with the Δ decuplet, we ignore potentially interesting questions as to the exact $\mathrm{SU}_f(3)$ composition of any particular excited state [23]. We also omit from consideration possible exotics.

[6]There is little difference between the extracted value of $\widetilde{C}^2$ thus obtained and the value obtained from the naive use of ''resonance'' masses and widths reported by the Particle Data Group (PDG) [19] for the N^*, i.e., $\Gamma_{N^*(1440)\to\Delta(1232)\pi(139)} \approx 350/4$ MeV. However, there is a significant difference in the results of the two methods when analyzing the $\Delta(1600)$.

[7]Indeed, the PDG only lists the $\Delta(1600)$ itself as a three star resonance.

$$\delta_{10}=341\text{ MeV}, \tag{10}$$

the difference of the pole positions of the $\Delta(1600)$ and $\Delta(1232)$ as determined by Hoehler [19,20]. The coupling between the two $3/2^+$ decuplets is identical in form to that within each decuplet:

$$\mathcal{L}^i=\widetilde{H}(\overline{\Delta^\mu}\gamma_\nu\gamma_5 A^\nu\Delta^*_\mu+\text{H.c.}). \tag{11}$$

We determine $\mathcal{H}^*$ from the decay $\Gamma_{\Delta*\to\Delta\pi}$. Unfortunately, no single analysis in the literature provides all the data we need. For the position of the pole we use 1550 MeV [19,20,22]. For the decay width we use Manley and Saleski's result [19,21] that $\Gamma_{\Delta*\to\Delta\pi}\approx 300$ MeV. We thus obtain that

$$\widetilde{H}^2=2.66. \tag{12}$$

As in the values of other couplings, it must be emphasized that there is a significant uncertainty in this result.

For the lowest lying $3/2^-$ octet, we obtain directly from the experimentally measured masses [19] that[8]

$$\delta_{8*}=296\ \text{ MeV}. \tag{13}$$

The interaction with the Δ decuplet is, to leading order in the chiral Lagrangian,

$$\mathcal{L}^i=C^*(\overline{T^\mu}\gamma_\nu A^\nu T^*_\mu+\text{H.c.}). \tag{14}$$

The coupling C^* can be determined from the observed decay of the $N^*(1520)$, $\Gamma[N^*(1520)\to\Delta\pi]\approx 25$ MeV, when one finds that

$$\left(\frac{C^*}{\widetilde{C}}\right)^2\approx\frac{1}{25}. \tag{15}$$

This relative suppression results both from the smaller branching width as well as overall kinematic factors that otherwise tend to enhance $N^*(1520)\to\Delta\pi$ with respect to $N^*(1440)\to\Delta\pi$.

There are two other $3/2^-$ multiplets, one octet and the other decuplet, listed in the Particle Data Group that could potentially satisfy our criteria, Eq. (1). Each is very poorly determined, containing merely one member each, the $N(1700)$ and the $\Delta(1700)$, respectively. In the case of the $N(1700)$, its coupling to the $\Delta\pi$ is experimentally negligible and hence can be safely ignored. On the other hand, the coupling with the $\Delta(1700)$ is not so readily ignored, having a decay width $\Gamma[\Delta(1700)\to\Delta\pi]\approx 120$ MeV. We, therefore, explicitly keep this decuplet, assigning for its intermultiplet spacing with the Δ decuplet a value

$$\delta_{10*}=1700-1232=468\ \text{ MeV}. \tag{16}$$

From the aforementioned decay width, and an interaction of the form equation (14), we obtain for the $\Delta^{-*}\Delta\pi$ coupling

$$\mathcal{H}^{*2}\approx 0.15. \tag{17}$$

[8]As a self-consistent test of the assignment of hadrons to this multiplet, note that the violation to the corresponding GMO relation for this octet is only 15 MeV.

Here, we have used the convention of Ref. [8] for the $SU_f(3)$ algebra factors (whereby, for the $\Delta\Delta\pi$ coupling, $\mathcal{H}^2\approx 4$ is typical). As in the case of the $N(1520)$ overall kinematic factors, in addition to the available phase space, yield a rather suppressed value of the coupling. Indeed, as we will soon see, due to Eqs. (15) and (17) little would have been lost had we ignored the $3/2^-$ multiplets altogether. Nevertheless, they have been included for completeness.

We come finally to the $5/2^+$ states. The lowest such multiplet is the $N(1680)$ octet. It has an intermultiplet mass splitting with the Δ decuplet of $\delta_h=496$ MeV. We conclude that there is no $5/2^+$ multiplet that meets our criteria equation (1).

This then concludes our examination of the relevant multiplets. By far, the most important, as we will presently see, are the Roper octet and $\Delta(1600)$ decuplet.

III. DECUPLET EQUAL SPACING RULE

The one-loop, $O(p^3)$ results for the masses of the decuplet involving intermediate Δ decuplet and nucleon octet states have been published previously [13]. The contribution δM_{10} from the $3/2^-$ multiplet, for the case $m>\delta_{8*}$, is

$$\delta M_{10}=\frac{-3\beta\delta_{8*}}{16\pi^2 f_\pi^2}\Bigg[\left(\delta_{8*}^2-\frac{1}{2}m^2\right)\left(\frac{1}{\epsilon}-\gamma_E+\ln(4\pi)+2-\ln\frac{m^2}{\mu^2}\right)+2\,\delta_{8*}\sqrt{m^2-\delta_{8*}^2}\times\left(\frac{\pi}{2}-\arctan\frac{\delta_{8*}}{\sqrt{m^2-\delta_{8*}^2}}\right)\Bigg], \tag{18}$$

where β represents SU(3) algebra factors [8,13]. As discussed in Ref. [13], the counterterms necessary to renormalize these terms are either of the form $\delta_{8*}^2\mathcal{L}_0^{\pi N}$ (for the δ_{8*}^3 divergences) or $\delta_{8*}\mathcal{L}_1^{\pi N}$ (for the $\delta_{8*}m^2$ divergences). As the decuplet equal spacing (DES) rule is exact for all terms through m^2, all counterterms (divergences) cancel at this order of the chiral expansion in violation to the DES rule.

Including all multiplets, the one-loop, $O(p^3)$, violation to the DES rule is

$$\begin{aligned}(M_{\Sigma*}-M_\Delta)-(M_{\Xi*}-M_{\Sigma*})&=(M_{\Xi*}-M_{\Sigma*})-(M_{\Omega^-}-M_{\Xi*})\\&=\frac{1}{2}\{(M_{\Sigma*}-M_\Delta)-(M_{\Omega^-}-M_{\Xi*})\}\\&=\frac{2}{9}[\mathcal{C}^2V(-\delta_N)+\widetilde{C}^2V(\delta_R)+(C^*)^2V^*(\delta_{8*})]\\&\quad-\frac{20}{81}[\mathcal{H}^2V(0)+\widetilde{H}^2V(\delta_{10})+\mathcal{H}^{*2}V^*(\delta_{10*})].\end{aligned} \tag{19}$$

V and V^* are given[9] by

$$V(\delta)=W(m_K,\delta,\mu)-\frac{3}{4}W(m_\eta,\delta,\mu)-\frac{1}{4}W(m_\pi,\delta,\mu), V^*(\delta)=W^*(m_K,\delta,\mu)-\frac{3}{4}W^*(m_\eta,\delta,\mu)-\frac{1}{4}W^*(m_\pi,\delta,\mu), \tag{20}$$

wherein the function $W(m,\delta,\mu)$ is [25,26][10]

$$\delta=0,\quad W(m,\delta,\mu)=\frac{1}{16\pi f_\pi^2}m^3,$$

$$m>|\delta|,\quad W(m,\delta,\mu)=\frac{1}{8\pi^2 f_\pi^2}(m^2-\delta^2)^{3/2}\times\left(\frac{\pi}{2}-\arctan\frac{\delta}{\sqrt{m^2-\delta^2}}\right)-\frac{3\delta}{32\pi^2 f_\pi^2}\left(m^2-\frac{2}{3}\delta^2\right)\ln\frac{m^2}{\mu^2}, \tag{21}$$

$$|\delta|>m,\quad W(m,\delta,\mu)=\frac{-1}{16\pi^2 f_\pi^2}(\delta^2-m^2)^{3/2}\times\ln\frac{\delta-\sqrt{\delta^2-m^2}}{\delta+\sqrt{\delta^2-m^2}}-\frac{3\delta}{32\pi^2 f_\pi^2}\left(m^2-\frac{2}{3}\delta^2\right)\ln\frac{m^2}{\mu^2}, \tag{22}$$

and $W^*(m,\delta,\mu)$ is

$$m>|\delta|,\quad W^*(m,\delta,\mu)=\frac{3\delta^2}{8\pi^2 f_\pi^2}\sqrt{m^2-\delta^2}\times\left(\frac{\pi}{2}-\arctan\frac{\delta}{\sqrt{m^2-\delta^2}}\right)-\frac{3\delta}{16\pi^2 f_\pi^2}\left(\delta^2-\frac{1}{2}m^2\right)\ln\frac{m^2}{\mu^2},$$

$$|\delta|>m,\quad W^*(m,\delta,\mu)=\frac{3\delta^2}{16\pi^2 f_\pi^2}\sqrt{\delta^2-m^2}\times\ln\frac{\delta-\sqrt{\delta^2-m^2}}{\delta+\sqrt{\delta^2-m^2}}-\frac{3\delta}{16\pi^2 f_\pi^2}\left(\delta^2-\frac{1}{2}m^2\right)\ln\frac{m^2}{\mu^2}. \tag{23}$$

[9]Note that unlike our convention in [13], all δ_h are now strictly positive and hence the explicit sign in the function V above.

[10]Note the correction from [13] regarding the arctangent term in the case $m>|\delta|$.

As was already mentioned in Sec. II, the contribution from the $3/2^-$ multiplets is essentially negligible due to their suppressed coupling constants, Eqs. (15) and (17). Explicitly, we find that

$$\frac{2}{9}C^{*\,2}V^*(+\delta_{8*})\approx-0.06\ \text{MeV},\qquad -\frac{20}{81}\mathcal{H}^{*2}V^*(+\delta_{10*})\approx+0.3\ \text{MeV}, \tag{24}$$

which are indeed negligible. Hence, we omit further considerations of the $3/2^-$ multiplets.

This is not true of the Roper octet or $\Delta(1600)$ decuplet. There is a tendency for these two contributions to cancel. Taking $\mathcal{C}^2=2.6$ [5] (and $\mathcal{H}^2=4.6$ to obtain the average experimental result for the GMO of 6.8 MeV) and using results (9) and (12), one obtains that

$$\frac{2}{9}\widetilde{C}^2V(+\delta_R)=-7.4\ \text{MeV},\qquad -\frac{20}{81}\widetilde{H}^2V^*(+\delta_{10})=+15.2\ \text{MeV}. \tag{25}$$

Added together they are, in absolute magnitude, greater than the average, experimental value of 6.8 MeV. They clearly cannot be ignored.

We have described the algorithm we followed to evaluate the four coupling constants, C^{*2}, $\mathcal{H}^{*2}$, $\widetilde{C}^2$, and $\widetilde{H}^2$. While our method is reasonable we do not suggest that it cannot be improved. We are dealing with transitions of broad resonances. There are many sources of ambiguities and uncertainties. The experimental inputs may also undergo revision. These uncertainties are not so material for the contributions of the odd parity $3/2^-$ octet and decuplet as their contributions are small. But for the contributions of the Roper octet and decuplet the issue is more serious. One should anticipate possible revisions of the numbers appearing in Eq. (25). But the facts that the individual contributions are comparable to the experimental value of the violation of the DES rule and that the two contributions tend to cancel each other will continue to be valid.

IV. DECUPLET MAGNETIC MOMENTS

The topic of the magnetic moments of the decuplet in the context of chiral perturbation theory was first discussed in the work of Ref. [10]. Apart from the inclusion of other resonances as an intermediate state, our work differs from Ref. [10] in the treatment of $SU_f(3)$ symmetry. In our calculation, the symmetry of the decuplet states is broken through the meson masses appearing in the one-loop calculation. The meson masses are taken to be proportional to the current quark masses with the up- and down-quark masses being equal. The strangeness [8] and charge dependences [12] of the baryon masses are regarded as effects of chiral power 1 or more. The quantity f_K-f_π has chiral singularity $\sim m_\pi^2\ln m_\pi^2$ [2] and has chiral power 2. Our calculations of the decuplet mass splittings and magnetic moments are limited to chiral power $O(p^3)$ and $O(p^2)$, respectively. Hence, we set $f_K=f_\pi$ and do not include in one-loop calculation the sigma terms from $\mathcal{L}_1$ which produce strangeness dependence of baryon masses at the tree level. We ignore charge dependence of the masses altogether. The advantage of this strategy in the calculation of baryon masses is well known [8,13]. The counterterms which appear at one-loop level simply renormalize the sigma terms. We find a similar result in the magnetic moment calculation at one-loop level, namely, that the counterterms are strictly proportional to the baryon charge and hence renormalize the tree-level decuplet magnetic moment term, Eq. (26) below. These advantages are lost if f_K is not set equal to f_π [10].

The lowest order term in the chiral Lagrangian for the magnetic moment of the ith member of the Δ decuplet is given by [9]

$$\mathcal{L}_M=-\imath\frac{e}{M_N}\mu_c q_i \overline{T_i^\mu} T_i^\nu F_{\mu\nu}, \tag{26}$$

where q_i is the charge of the ith member. The one-loop, $O(p^2)$ corrections to the magnetic moments result from vertex corrections in which the external photon attaches to the meson propagator [10] and receives contributions from intermediate states with either a $3/2^+$ or $1/2^+$ baryon. Photon attachments to the intermediate baryon are m_π/M_N further suppressed as are also the contributions from $3/2^-$ baryons. These latter are, hence, ignored as they form part of the higher-order contribution in the chiral expansion. Note that the η meson, being electrically neutral, also does not contribute at the order being considered.

Following the notation of [9], the magnetic moment of the decuplet members μ_i^{10} at the $O(p^2)$, one-loop contribution in the chiral expansion is, in nuclear magneton units $(e/2M_N)$:

$$\mu_i^{10}=q_i\mu_c+\sum_{j=\pi,K}\frac{M_N}{32\pi^2 f_\pi^2}\Bigg(\alpha_j^i\left[\frac{4}{9}\mathcal{H}^2\mathcal{F}(0,m_j,\mu)-\frac{4}{9}\widetilde{H}^2\mathcal{F}(\delta_{10},m_j,\mu)\right]\beta_j^i[\mathcal{C}^2\mathcal{F}(-\delta_N,m_j,\mu)+\widetilde{C}^2\mathcal{F}(\delta_R,m_j,\mu)]\Bigg). \tag{27}$$

The function $\mathcal{F}(\delta,m,\mu)$ is ultraviolet divergent and given by

$$m>|\delta|,$$

$$\mathcal{F}(\delta,m,\mu)=\delta\left[-\frac{1}{\epsilon}+\gamma_E-\ln(4\pi)-\frac{4}{3}+\ln\left(\frac{m^2}{\mu^2}\right)\right]-2\sqrt{m^2-\delta^2}\left(\frac{\pi}{2}-\arctan\frac{\delta}{\sqrt{m^2-\delta^2}}\right),$$

$$|\delta|>m,$$

$$\mathcal{F}(\delta,m,\mu)=\delta\left[-\frac{1}{\epsilon}+\gamma_E-\ln(4\pi)+\ln\left(\frac{m^2}{\mu^2}\right)\right]+\sqrt{\delta^2-m^2}\ln\frac{\delta+\sqrt{\delta^2-m^2}}{\delta-\sqrt{\delta^2-m^2}}. \tag{28}$$

The expression for the nonanalytic terms in $\mathcal{F}(\delta,m,\mu)$ appeared in Ref. [10].

The coefficients α_j^i and β_j^i are simply related to the coefficients α_{ij} and β_{ij} of Ref. [10]. We multiply the coefficients α_{ij} by 3 so that they add up to the charge of decuplet i. Unlike Ref. [10], we use the same mass for all members of a baryon multiplet. Accordingly, we add the contributions of $\pi^\pm$ and of $K^\pm$. The sum over j in Eq. (27) runs over two terms, π and K. The resulting coefficients have a surprising simplicity. First, $\alpha_j^i=\beta_j^i$. Second, they may be expressed in terms of any two of the following three, charge q^i, isospin I_3^i, and hypercharge Y^i of decuplet i. All three are traceless in any SU(3) multiplet space. We choose to use charge and isospin:

$$\alpha_\pi^i=\beta_\pi^i=\frac{2}{3}I_3^i, \tag{29}$$

$$\alpha_K^i=\beta_K^i=-\frac{2}{3}I_3^i+q^i, \tag{30}$$

$$\alpha_\pi^i+\alpha_K^i=\beta_\pi^i+\beta_K^i=q^i. \tag{31}$$

Equation (31) is the key result for renormalizability. As a consequence of these relations, the counterterm for the ultraviolet divergences in $\mathcal{F}(\delta,m,\mu)$ (which are m independent) is simply proportional to $\delta\mathcal{L}_M$, Eq. (26), and, therefore, absorbed into a redefinition of μ_c. Note that this is precisely the same procedure as for the one-loop, δ-dependent contributions to the masses. We emphasize that this procedure, and hence renormalizability, is tightly wedded to the systematics of the chiral expansion [whereby δ and f_π have fixed values in Eq. (27)].

The simplicities of the coefficients α_j^i and β_j^i allow a great simplification of Eq. (27). We introduce the combination:

$$\mathcal{G}_j=\frac{M_N}{32\pi^2 f_\pi^2}\left[\frac{4}{9}\mathcal{H}^2\mathcal{F}(0,m_j,\mu)+\frac{4}{9}\widetilde{H}^2\mathcal{F}(\delta_{10},m_j,\mu)-\mathcal{C}^2\mathcal{F}(-\delta_N,m_j,\mu)-\widetilde{C}^2\mathcal{F}(\delta_R,m_j,\mu)\right], \tag{32}$$

and rewrite the decuplet magnetic moments in the form:

$$\mu_i^{10}=q_i(\mu_c+\mathcal{G}_K)+\tfrac{2}{3}I_3^i(\mathcal{G}_\pi-\mathcal{G}_K). \tag{33}$$

Note that the form of Eq. (33), in particular the modification of the coefficient of charge, reflects the choice of charge and isospin as the two traceless quantities. The form would be different if we chose charge and hypercharge or some other combination of isospin and hypercharge. The second term is present only because the SU(3) symmetry of the states is broken through the difference in π and K masses. If we had used the same masses the states would be pure decuplets and the Wigner-Eckart theorem for $SU_f(3)$ would ensure that the magnetic moments are simply proportional to the charge.

At the tree level the decuplet magnetic moments are proportional to the charges only. The main one-loop result is the appearance of the second term of Eq. (33). Now, we need two magnetic moments to fix the two coefficients in Eq. (33), viz., $\mu_c+\mathcal{G}_K$ and $\mathcal{G}_\pi-\mathcal{G}_K$. The only decuplet magnetic moment which has been measured is $\mu_{\Omega^-}=-1.94\pm0.17\pm0.14$ nbm [19]. It fixes the coefficient of charge in Eq. (33). For the other magnetic moment we choose μ_{Δ^0}.[11] While not measured yet, it is given entirely by the loop effect.

$$\mu_{\Omega^-}=-\mu_c-\mathcal{G}_K, \quad \mu_{\Delta^0}=\frac{1}{3}(\mathcal{G}_K-\mathcal{G}_\pi). \tag{34}$$

We can express the magnetic moments of all other decuplets in terms of these two magnetic moments. Specifically, we derive the new relation that

$$\mu_i^{10}=-q_i\mu_{\Omega^-}-2\,I_3^i\mu_{\Delta^0}. \tag{35}$$

Explicit predictions for the eight other decuplet magnetic moments at the one-loop, $O(p^2)$ level are listed below:

$$\mu_{\Delta^{++}}=-2\mu_{\Omega^-}-3\mu_{\Delta^0}, \quad \mu_{\Delta^+}=-\mu_{\Omega^-}-\mu_{\Delta^0},$$
$$\mu_{\Delta^-}=\mu_{\Omega^-}+3\mu_{\Delta^0}, \quad \mu_{\Sigma^{*+}}=-\mu_{\Omega^-}-2\mu_{\Delta^0},$$
$$\mu_{\Sigma^{*-}}=\mu_{\Omega^-}+2\mu_{\Delta^0}, \quad \mu_{\Xi^{*-}}=\mu_{\Omega^-}+\mu_{\Delta^0},$$
$$\mu_{\Xi^0}=-\mu_{\Delta^0}, \quad \mu_{\Sigma^{*0}}=0. \tag{36}$$

Independent of the explicit multiplets included as intermediate states, violations to these relations are *strictly* due to higher-order terms in the chiral expansion.

Note that the magnetic moment of the Σ^{*0} continues to be zero at this order in the expansion. Analogous relations follow for the quadrupole moments [27]. We note that these relations are not obeyed by quenched lattice QCD [28]. This last result is perhaps not surprising as the quenched calculations do not contain disconnected, quark loop diagrams [29].

The explicit expression for μ_{Δ^0} in terms of the functions $\mathcal{F}(\delta,m,\mu)$ is given below.

$$\mu_{\Delta^0}=\frac{M_N}{96\pi^2f_\pi^2}\Bigg\{\mathcal{H}^2\frac{4}{9}[\mathcal{F}(0,m_K,\mu)-\mathcal{F}(0,m_\pi,\mu)] + \widetilde{H}^2\frac{4}{9}[\mathcal{F}(\delta_{10},m_K,\mu)-\mathcal{F}(\delta_{10},m_\pi,\mu)] - \mathcal{C}^2(\mathcal{F}(-\delta_N,m_K,\mu)-\mathcal{F}(-\delta_N,m_\pi,\mu)) - \widetilde{C}^2(\mathcal{F}(\delta_R,m_K,\mu)-\mathcal{F}(\delta_R,m_\pi,\mu))\Bigg\}. \tag{37}$$

[11]The Ω^-, decaying only weakly, is sufficiently long lived to allow such measurements. Since all other members of the decuplet decay through the strong interaction, it is a challenge to extract their magnetic moments from experiment.

With the help of Eqs. (28) it is easy to verify that μ_{Δ^0} is renormalization scale independent.

Since the magnetic moment of the Δ^0 is given strictly by loop effects, it is an appropriate measure of the relative importance of the Roper and $\Delta(1600)$ at the one-loop level. Explicitly, one has from Eq. (37) that

$$\mu_{\Delta^0}=-0.057\mathcal{H}^2-0.035\widetilde{H}^2+0.229\mathcal{C}^2+0.086\widetilde{C}^2. \tag{38}$$

As in the case of the masses [8] there is a strong cancellation between the Δ decuplet and nucleon octet intermediate states. This implies that μ_{Δ^0} is a very delicate function of $\mathcal{H}^2$ and $\mathcal{C}^2$ and, therefore, potentially very sensitive to the Roper and $\Delta(1600)$ contributions. Ambiguity in this regard resides in the fact that $\mathcal{H}^2$ and $\mathcal{C}^2$ are not sufficiently well known that a reliable prediction of μ_{Δ^0} can be made. To illustrate this point, we quote the results using two ''representative'' values for the couplings, both with and without the Roper and $\Delta(1600)$ included. For the couplings used in the previous section, that fit the average DES violation, one obtains only a mild dependence on these resonances:

$$\mu_{\Delta^0}(\mathcal{H}^2=4.5,\mathcal{C}^2=2.6,\widetilde{H}^2=\widetilde{C}^2=0)=0.33,$$
$$\mu_{\Delta^0}(\mathcal{H}^2=4.5,\mathcal{C}^2=2.6,\widetilde{H}^2=2.66,\widetilde{C}^2=1.3)=0.36, \tag{39}$$

while for the couplings used by Ref. [10], we find

$$\mu_{\Delta^0}(\mathcal{H}^2=4.84,\mathcal{C}^2=1.44,\widetilde{H}^2=\widetilde{C}^2=0.0)=0.05,$$
$$\mu_{\Delta^0}(\mathcal{H}^2=4.84,\mathcal{C}^2=1.44,\widetilde{H}^2=2.66,\widetilde{C}^2=0.72)=0.02. \tag{40}$$

The difference in μ_{Δ^0} from these two parameter sets is clearly sizable, as is the relative importance of the Roper and $\Delta(1600)$.

If instead, we chose to use as input the recent, model-dependent extraction [30] of the magnetic moment of the Δ^{++}, $\mu_{\Delta^{++}}=4.5\pm0.5$ to infer μ_{Δ^0} using the relations (36), one then obtains that $\mu_{\Delta^0}=-0.2\pm0.2$. If this is indeed the data, then by Eq. (38) the contribution of the Roper and $\Delta(1600)$ is, in absolute magnitude, significant. As in the case of the mass splittings, a formulation of χPT without this resonance as an explicit degree of freedom is intrinsically incapable of predicting such ''data.''

V. CONCLUSIONS

From the results of the last two sections a few points are worth discussing. First, we should remind the reader that we have calculated the lowest-order, one-loop contributions to the DES rule violations and the Δ decuplet magnetic moments. These expressions have corrections at next order in the chiral expansion due to higher-order terms in the chiral Lagrangian, including $1/M$ corrections to the heavy Fermion limit. Beyond that, there are two-loop renormalization effects, including SU(3) breaking in decay constants. Formally, these corrections should be of the order $m/M,\delta/M$ and $(m/M)^2,(\delta/M)^2$ respectively. Unfortunately, as we have discussed elsewhere [13,31] the convergence of the expansion is by no means ensured.

Second, while the magnetic moment of the Δ^0 depends sensitively on the cancellation between terms depending on relatively ill-determined coupling constants, the relations between the magnetic moments of the Δ decuplet given in Eq. (36) are rigorous predictions of χPT at $O(p^2)$. We urge experimental activity to confront these predictions with data. A new measurement of μ_{Ω^-} with higher precision will be most useful. At least two other decuplet magnetic moments need to be measured, hopefully with a precision of $\sim$0.1 nm.

Third, that on the level of one-loop corrections in χPT, the contributions of the Roper octet and $\Delta(1600)$ decuplet to properties of the Δ decuplet is as important as contributions from the nucleon octet or delta decuplet itself. We have seen this result quantitatively in the case of the DES rule and the magnetic moment of the Δ^0. Both of these quantities are good measures of the one-loop effects as they are each zero at lower order in the chiral expansion. We expect that these results are illustrative and that they generalize to all one-loop calculations for the Δ decuplet. Since the mass splitting δ_R between the Roper octet and Δ decuplet [or δ_{10}, between the $\Delta(1600)$ and Δ decuplets] is less than that of the kaon's mass, a Taylor expansion in m_K/δ_R is not permissible. Hence, these loop effects cannot be absorbed within higher-order terms of the chiral expansion of a theory not containing the Roper as an explicit degree of freedom. In such a theory it is indeed difficult to justify going to one loop or higher without inclusion of the Roper. Phenomenologically successful results would have to be considered merely fortuitous unless shown to be a result of more general considerations [as in the relations of Eq. (36) for the magnetic moments].

The above argument can be repeated in kind for the one-loop corrections to the Roper resonance and $\Delta(1600)$. That is, even higher multiplets, separated by δ_h in mass from the Roper octet [$\Delta(1600)$ decuplet] by an amount less than the kaon mass, will be *a priori* as important quantitatively as the Δ decuplet for properties of the Roper [$\Delta(1600)$] at the one-loop level. Since such corrections are necessary for a two-loop calculation[12] of the baryon masses, we are led to conclude that the loop expansion in general in the baryon sector of χPT is inevitably wedded to the necessity of including more and more multiplets in the theory as fundamental fields. While such a result may not be valid in a particular limit of QCD (e.g., $m_{u,d,s}\rightarrow 0$ or $N_c\rightarrow\infty$), it is a consequence of the experimental fact that, on the average, $m_\pi \approx \delta_h$.

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mass for all members of a given baryon multiplet, would exhibit the same dependence on charge and isospin as the magnetic moments. In analogy with Eq. (35), the resulting quadrupole moments would be expressible as

$$Q_i^{10} = -q_i\, Q_{\Omega^-} - 2\, I_3^i\, Q_{\Delta^0}, \tag{41}$$

and a set of relations analogous to Eq. (36) may be written down.

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